Analog Communication

by

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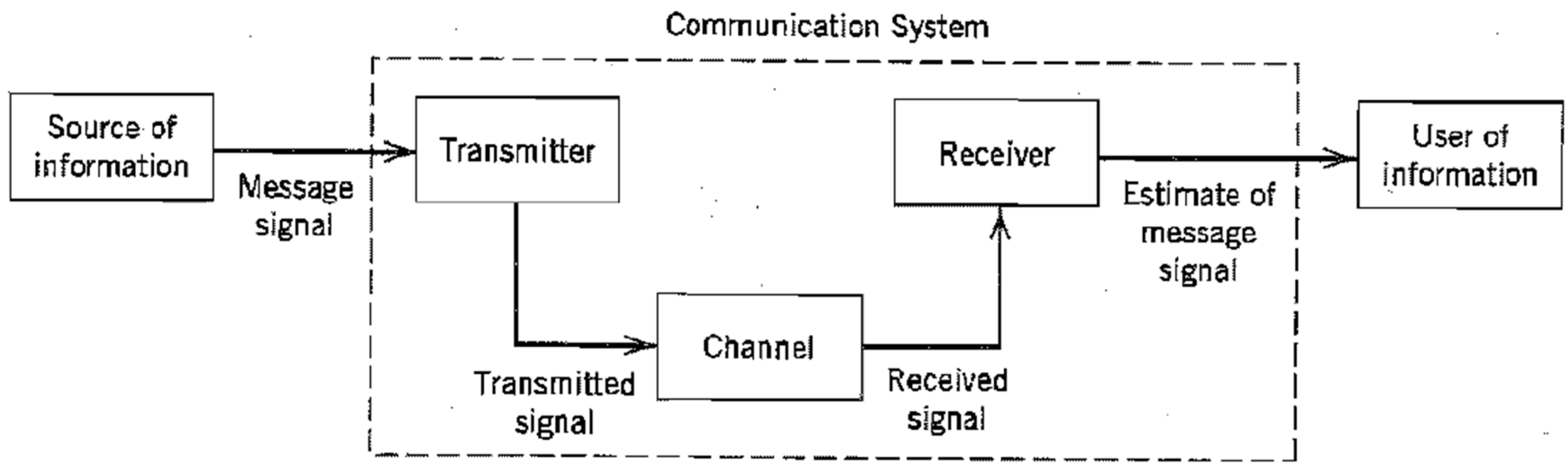
Analog Signal Transmission and Reception

- **The purpose of a communication system is to transmit information-bearing signals through a communication channel separating the transmitter from the receiver.**

- **A large number of such information sources are analog sources. Speech, image and video are examples of analog sources of information.**

- **Each analog source is characterized by its bandwidth, dynamic range, and the nature of the signal.**

Analog Signal Transmission and Reception



Elements of a Communication System.

There are two basic modes of communication:

- **Broadcasting:** Involves the use of a single powerful transmitter and numerous receivers, that are relatively inexpensive to build.

- **Point-to-point Communication:** It takes place over a link between a single transmitter and a receiver.

Radio Frequency Spectra

Frequency	Designation	Abbreviation
30-300 Hz	Extremely Low Frequency	ELF
300-3000 Hz	Voice Frequency	VF
3-30 kHz	Very Low Frequency	VLF
30-300 kHz	Low Frequency	LF
300 kHz-3 MHz	Medium Frequency	MF
3-30 MHz	High Frequency	HF
30-300 MHz	Very High Frequency	VHF
300 MHz-3 GHz	Ultra High Frequency	UHF
3-30 GHz	Super High frequency	SHF
30-300 GHz	Extra High Frequency	EHF

Note: **Communication Systems are often categorized by the frequency of the carrier.**

Analog Signal Transmission and Reception

Communication channels

- **The communication channel provides the connection between the transmitter and the receiver.**

- **The physical channel may be a pair of wires that carry the electrical signal, or an optical fiber that carries the information on a modulated light beam, or an underwater ocean channel in which the information is transmitted acoustically, or free space over which the information-bearing signal is radiated by use of an antenna.**

- **One common problem in signal transmission through any channel is additive noise.**

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Communication channels

- When such noise and interference occupy the same frequency band as the desired signal, its effect can be minimized by proper design of the transmitted signal and its demodulator at the receiver.

- Other types of signal degradations that may be encountered in transmission over the channel are signal attenuation, amplitude and phase distortion, and multipath distortion.

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Communication channels

- The effects of noise may be minimized by increasing the power in the transmitted signal.

- However, equipment and other practical constraints limit the power level in the transmitted signal.

- Another basic limitation is the available channel bandwidth.

- A bandwidth constraint is usually due to the physical limitations of the medium and the electronic components used to implement the transmitter and the receiver.

- These two limitations result in constraining the amount of data that can be transmitted reliably over any communications channel. Shannon's basic results relate the channel capacity to the available transmitted power and channel bandwidth.

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Communication channels

Based on guided propagation

- Telephone channels using twisted pairs.
- Coaxial cables.
- Optical fibers.

Based on free propagation

- Wireless broadcast channels.
- Mobile radio channels.
- Satellite channels.

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Communication Channels based on Guided Propagation

Telephone channels using twisted pairs

- **A telephone network uses circuit switching to establish an end-to-end communication link, on a temporary basis. In this mode of communication the message source is the sound produced by the speaker's voice and the ultimate destination is the listener's ear.**

- **The telephone channel only supports the transmission of electrical signals.**

- **A microphone is placed near the speaker's mouth to convert sound waves into an electrical signal. The electrical signal is converted back into acoustic form by a moving coil receiver placed near the listener's ear.**

- **The telephone channel is a bandwidth limited channel.**

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Communication Channels based on Guided Propagation

Telephone channels using twisted pairs

- A practical solution to the **telephone communication problem** must therefore minimize the channel bandwidth requirement, subject to a satisfactory transmission of human voice.

- A **speech signal** is essentially limited to a band **300 to 3100 Hz**.

- The **telephone channel** is built using twisted pairs for signal transmission. A twisted pair consists of **two solid copper conductors**, each of which is encased in a PVC sheath.

- Typically each pair has a **twist rate** of 2 to 12 twists per foot and a **characteristic impedance** of **90 to 110 ohms**. **Twisted pairs are naturally susceptible to electromagntic interference (EMI)**.

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Communication Channels based on Guided Propagation

Coaxial cables

- A coaxial cable consists of an inner conductor and an outer conductor, separated by a dielectric insulating material.

- The inner conductor is made of a copper wire encased inside the dielectric material.

- The outer conductor is made of copper, tinned copper or copper-coated steel.

- Typically a coaxial cable has a characteristic impedance of 50 or 75 ohms. Compared to a twisted pair cable a coaxial cable offers a greater degree of immunity to EMI.

- Because of their much higher bandwidth coaxial cables can support transmission of digital data at much higher bit rates (upto 20 Mb/s) than twisted pairs.

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Communication Channels based on Guided Propagation

Optical fibers

- **An optical fiber is a dielectric wave guide that transports light signals from one place to another.**

- **It consists of a central core within which the propagating electromagnetic field is confined and which is surrounded by a cladding layer, which is itself surrounded by a thin protective jacket.**

- **The core and cladding are both made of pure silica glass and the jacket is made of plastic.**

- **Optical fibers have enormous potential bandwidth, low transmission losses, immunity to electromagnetic interference, small size and weight, high ruggedness and flexibility and it is very popular for low cost line communication.**

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Communication Channels based on Free Propagation

Wireless broadcast channels

- **Wireless broadcast channels support the transmission of radio and television signals.**

- **The transmission originates from an antenna that acts as the transition or matching unit between the source of the modulated signal and electromagnetic waves in free space.**

- **The objective in designing the antenna is to excite the waves in required direction or directions, as efficiently as possible.**

- **By virtue of the phenomenon of diffraction, a fundamental property of wave motion, radio waves are bent around the earth's surface. At the receiving end, an antenna is used to pick up the radiated waves, establishing a communication link to the transmitter.**

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Communication Channels based on Free Propagation

Mobile radio channels

- A **mobile radio channel** introduces **mobility** in public telecommunication network by virtue of its **ability to broadcast**.

- In a **mobile radio environment**, there is a **multipath phenomenon** in which the various incoming radio waves reach their destination from different directions and with different time delays.

- There may be multiple **propagation paths** with different **electrical strengths** and their contribution to the received signal can combine in a **variety of ways**.

- Consequently, the **received signal strength** varies with location in a very complicated fashion, and so a mobile radio channel may be viewed as a **linear time-varying channel** that is statistical in nature.

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Communication Channels based on Free Propagation

Satellite channels

- **Satellite channels provide broad-area coverage both in continental and intercontinental sense.**

- **Access to remote areas not covered by conventional cable or fiber communications is also a distinct feature of satellites.**

- **In almost all satellite communications the satellites are placed in geostationary orbit.**

- **In a typical satellite communication system, a message signal is transmitted from an Earth station via an uplink to a satellite, gets amplified in a transponder on board the satellite and then is retransmitted from the satellite via a downlink to another Earth station.**

Analog Signal Transmission and Reception

Analog signal transmission

Baseband communication

It does not use *modulation*.

Carrier communication

It makes use of *modulation*.

Baseband refers to the band of frequencies representing the original signal as delivered by a source of information. The baseband in Telephony is the audio band with the range: 0 – 3.5 kHz and in Television is the video band with the range: 0 – 4.3 MHz.

Analog Signal Transmission and Reception

Features

- In **baseband communication**, signals are transmitted without any shift in the range of frequencies of the signal. However baseband signals produced by various information sources are not always suitable for direct transmission.

- These signals are further modified to facilitate transmission, using a process called **modulation**.

- **Modulation** causes a shift in the range of frequencies. This is called **carrier communication**.

- A **carrier** $c(t)$ is a sinusoid of high frequency, and one of its parameters i.e. amplitude, frequency, or phase, is varied in proportion to the baseband signal $m(t)$.

Analog Signal Transmission and Reception

Analog modulation techniques

Amplitude Modulation (AM)

Amplitude of $c(t)$ is varied with $m(t)$.

Angle Modulation

Instantaneous phase or frequency of $c(t)$ is varied with $m(t)$.

When carrier frequency is modulated, it is called Frequency Modulation (FM). When carrier phase is modulated, it is called Phase Modulation (PM).

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Purpose of Modulation …

To *translate* the low-pass signal in frequency to the passband of the channel so that the spectrum of the transmitted bandpass signal match the passband characteristics of the channel.

To *accommodate* for simultaneous transmission of signals from several message sources using frequency-division multiplexing (FDM).

To *expand* the bandwidth of the transmitted signal in order to increase its noise-immunity in transmission over a noisy channel and to optimize SNR.

To *overcome* hardware limitation because transmitting such lower frequencies require antennas with miles in wavelength.

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Purpose of Modulation …

Example: **For a voice signal, with *f*=3 kHz, we have:**

$$\lambda = c/f = 3\times10^8 / 3\times10^3 = 10^5 \text{ m}$$

If we modulate a carrier wave, f_c = 100 MHz, with this voice signal, then we have:

$$\lambda = c/f = 3\times10^8 / 100\times10^6 = 3 \text{ m}$$

What is Demodulation ??

At the receiving end of the system, the original bandpass signal is restored by performing a process called demodulation. Demodulation can be viewed as the reverse of the modulation process.

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Modulation Process

For all ***modulation processes*** analog signal $m(t)$ is considered as a low-pass signal of bandwidth W i.e. $M(f) \equiv 0$, for $|f| > W$. The signal is assumed as a **power signal with power** P_m:

$$P_m = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |m(t)|^2 dt$$

This ***message signal*** is carried through the communication channel by impressing it on a carrier signal $c(t)$:

$$c(t) = A_c \cos(2\pi f_c t + \phi_c)$$

A_c: carrier amplitude, f_c: carrier frequency, ϕ_c: carrier phase.

Review of Fourier Transform

- **The Fourier integral transform pair can be given as:**

$$G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt$$

$$g(t) = \int_{-\infty}^{\infty} G(f) e^{j2\pi ft} df$$

where *f* = *frequency measured in Hz*. This pair can be used to describe the time-frequency relationship for nonperiodic signals. The relationship between the time and frequency domains is indicated by the double arrow, given as:

$$g(t) \leftrightarrow G(f)$$

- ***G(f)* is specified by a magnitude characteristic and phase characteristic:**

$$G(f) = |G(f)| e^{j\theta(f)}$$

Review of Fourier Transform

Properties of the Fourier transform

Property	Mathematical Description
1. Linearity	$ag_1(t) + bg_2(t) \rightleftharpoons aG_1(f) + bG_2(f)$ where a and b are constants
2. Time scaling	$g(at) \rightleftharpoons \frac{1}{\lvert a \rvert} G\left(\frac{f}{a}\right)$ where a is a constant
3. Duality	If $g(t) \rightleftharpoons G(f)$, then $G(t) \rightleftharpoons g(-f)$
4. Time shifting	$g(t - t_0) \rightleftharpoons G(f) \exp(-j2\pi f t_0)$
5. Frequency shifting	$\exp(j2\pi f_c t) g(t) \rightleftharpoons G(f - f_c)$
6. Area under $g(t)$	$\int_{-\infty}^{\infty} g(t)\, dt = G(0)$

Review of Fourier Transform

Properties of the Fourier transform

Property	Mathematical Description
7. Area under $G(f)$	$g(0) = \int_{-\infty}^{\infty} G(f)\, df$
8. Differentiation in the time domain	$\frac{d}{dt} g(t) \rightleftharpoons j2\pi f G(f)$
9. Integration in the time domain	$\int_{-\infty}^{t} g(\tau)\, d\tau \rightleftharpoons \frac{1}{j2\pi f} G(f) + \frac{G(0)}{2} \delta(f)$
10. Conjugate functions	If $g(t) \rightleftharpoons G(f)$, then $g^*(t) \rightleftharpoons G^*(-f)$
11. Multiplication in the time domain	$g_1(t) g_2(t) \rightleftharpoons \int_{-\infty}^{\infty} G_1(\lambda) G_2(f - \lambda)\, d\lambda$
12. Convolution in the time domain	$\int_{-\infty}^{\infty} g_1(\tau) g_2(t - \tau)\, d\tau \rightleftharpoons G_1(f) G_2(f)$

Review of Fourier Transform

Fourier transform pairs

Time Function	Fourier Transform
$\text{rect}\left(\frac{t}{T}\right)$	$T\,\text{sinc}(fT)$
$\text{sinc}(2Wt)$	$\frac{1}{2W}\text{rect}\left(\frac{f}{2W}\right)$
$\exp(-at)u(t), \quad a > 0$	$\frac{1}{a + j2\pi f}$
$\exp(-a\|t\|), \quad a > 0$	$\frac{2a}{a^2 + (2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\begin{cases} 1 - \frac{\|t\|}{T}, & \|t\| < T \\ 0, & \|t\| \geq T \end{cases}$	$T\,\text{sinc}^2(fT)$
$\delta(t)$	1
1	$\delta(f)$

Review of Fourier Transform

Fourier transform pairs

Time Function	Fourier Transform
$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$
$\exp(j2\pi f_c t)$	$\delta(f - f_c)$
$\cos(2\pi f_c t)$	$\frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$
$\sin(2\pi f_c t)$	$\frac{1}{2j}[\delta(f - f_c) - \delta(f + f_c)]$
$\mathrm{sgn}(t)$	$\frac{1}{j\pi f}$
$\frac{1}{\pi t}$	$-j\,\mathrm{sgn}(f)$
$u(t)$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
$\sum_{i=-\infty}^{\infty} \delta(t - iT_0)$	$\frac{1}{T_0}\sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_0}\right)$

RANDOM PROCESSES

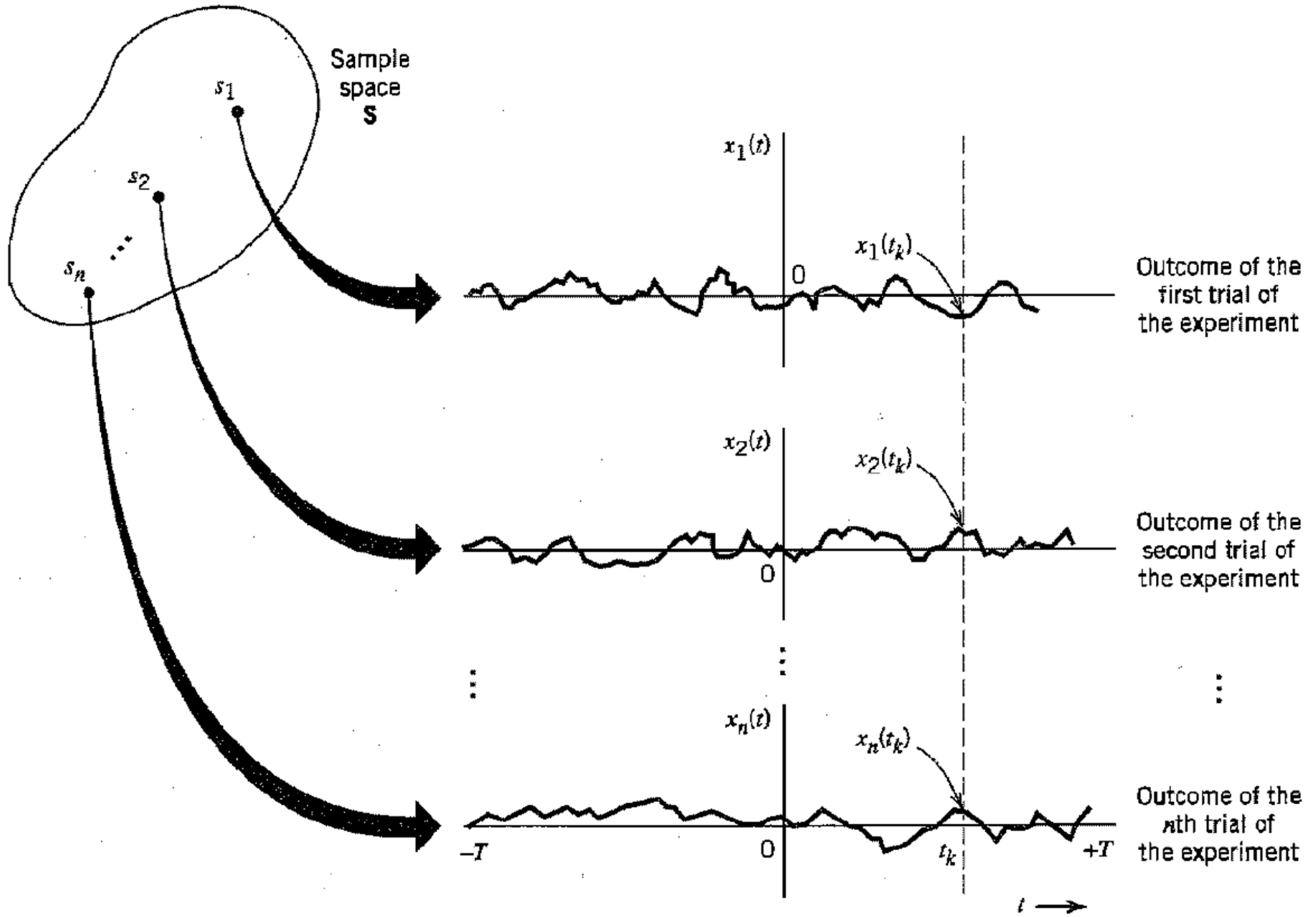
Random Processes

These are those processes in nature that are best characterized in statistical terms. For example, air temperature and air pressure fluctuating randomly as functions of time, thermal noise voltages generated in resistors of electronic devices etc.

Random signals are usually modeled as infinite-duration, infinite-energy signals. The set of all possible waveforms is called an *ensemble* of time functions or a *random process*.

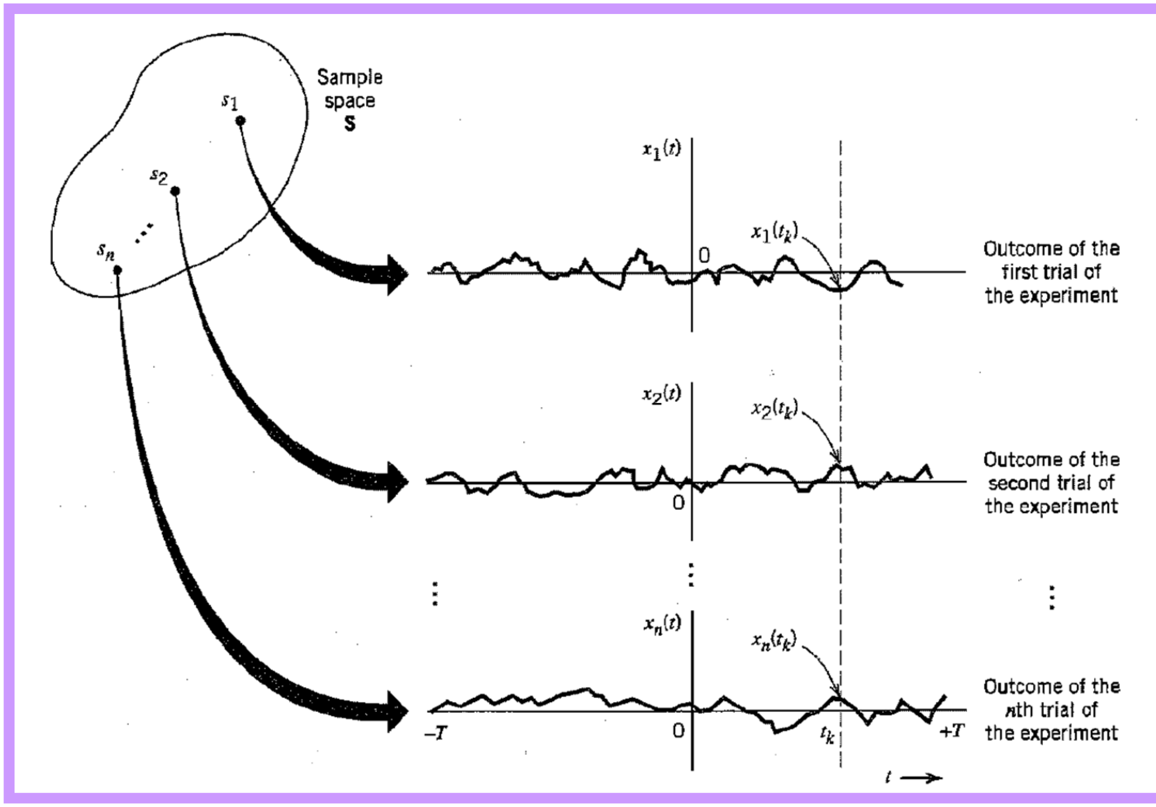
The set (ensemble) of *all possible waveforms of a random process* is denoted as $X(t,S)$, where t is the time index and S represents the *set* or *sample space* of all possible sample functions. A single waveform in this set is called $x(t,s)$. Usually s or S is dropped from the notation.

Random Processes



An ensemble of sample functions.

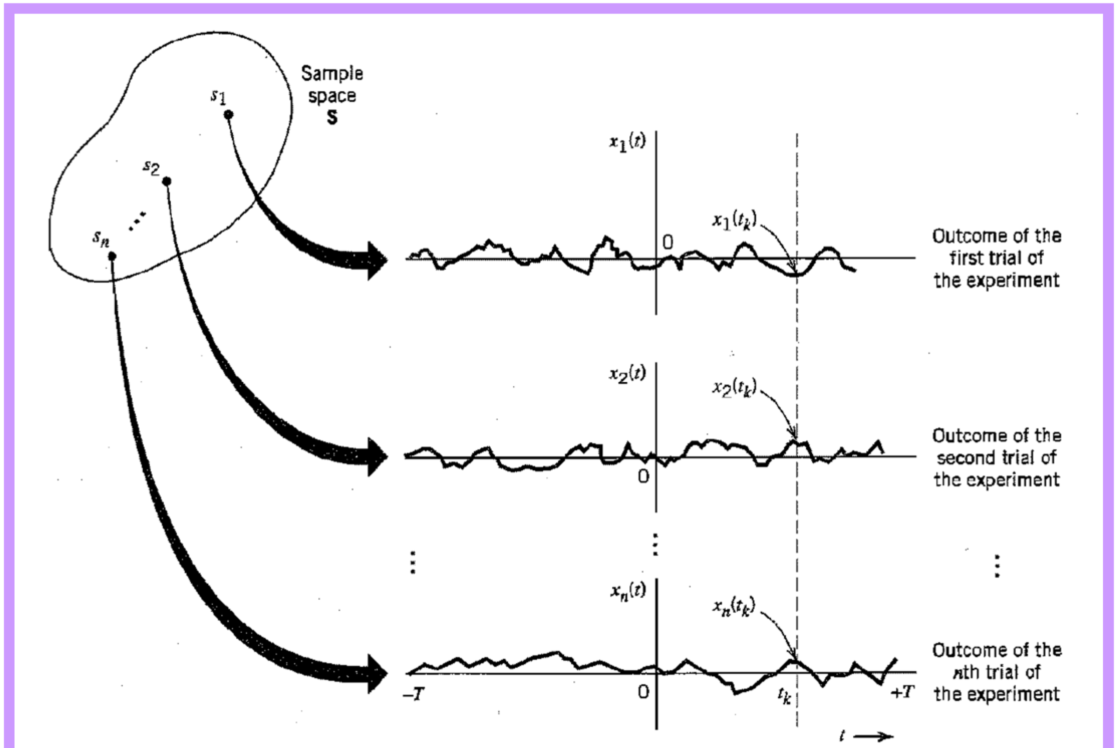
Random Processes



For a fixed sample point s_j, the graph of the function $X(t, s_j)$ versus time t is called a realization or sample function of the random process. These sample functions are denoted as:

$$x_j(t) = X(t, s_j), \quad j = 1, 2, \cdots, n$$

Random Processes



For a fixed time t_k inside the observation interval, the set of numbers

$$\left[x_1(t_k), x_2(t_k), \cdots, x_n(t_k)\right] = \left[X(t_k, s_1), X(t_k, s_2), \cdots, X(t_k, s_n)\right]$$

constitutes a *random variable*. This gives an indexed ensemble (family) of random variables $\{X(t,s)\}$, called a random process.

Random Processes

Let us consider a random process $X(t)$ at different time instants $t_1 > t_2 > \ldots > t_n$, n is a positive integer. In general, the samples $X_{t_i} \equiv x(t_i)$, $i = 1,2,\ldots,n$ are n random variables statistically characterized by their joint probability density function (PDF) denoted as $p(x_{t_1}, x_{t_2}, \ldots, x_{t_n})$ for any n.

What is the difference between a random variable and a random process ???

- **For a random variable, the outcome of a random experiment is mapped into a number.**
- **For a random process, the outcome of a random experiment is mapped into a waveform that is a function of time.**

Stationary Random Processes

- **Let us consider we have n samples of the random process $X(t)$ at $t = t_i$, $i = 1,2,\ldots,n$, and another set of n samples displaced in time from the first set by τ, as, $X_{t_i+\tau} \equiv X(t_i+\tau)$, $i = 1,2,\ldots,n$.**

- **If the joint PDFs of the two sets of random variables are identical i.e. $p(x_{t_1}, x_{t_2},\ldots, x_{t_n}) = p(x_{t_1+\tau}, x_{t_2+\tau},\ldots, x_{t_n+\tau})$ for all τ and n, the random process is called *stationary in the strict sense*.**

↓

- **The statistical properties of a stationary random process are invariant to a translation of the time axis.**

Autocorrelation Function

- Let a random process $X(t)$ be sampled at $t = t_i$. Then $X(t_i)$ is a random variable with PDF $p(x_{t_i})$. For a stationary process, $p(x_{t_i+\tau}) = p(x_{t_i})$ for all τ. Hence the PDF is independent of time.

- Let us consider two random variables $X_{t_i} = X(t_i)$, $i = 1,2$, corresponding to samples of $X(t)$ taken at $t = t_1$ and $t = t_2$. The *autocorrelation function* of the random process $X(t)$ is measured by the expectation of the product of the two random variables X_{t_1} and X_{t_2}. Mathematically speaking,

$$R_{XX}(t_1,t_2) = R_X(t_1,t_2) = E\left(X_{t_1} X_{t_2}\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_{t_1} x_{t_2}\, p\left(x_{t_1}, x_{t_2}\right) dx_{t_1}\, dx_{t_2}$$

Autocorrelation Function

- For a stationary process $X(t)$, the joint PDF of the pair (X_{t_1}, X_{t_2}) is identical to the joint PDF of the pair $(X_{t_1+\tau}, X_{t_2+\tau})$, for any arbitrary τ. Hence the ***autocorrelation function*** of $X(t)$ depends on the time difference, $t_1 - t_2 = \tau$.

- For a stationary, real-valued random process, the ***autocorrelation function*** is:

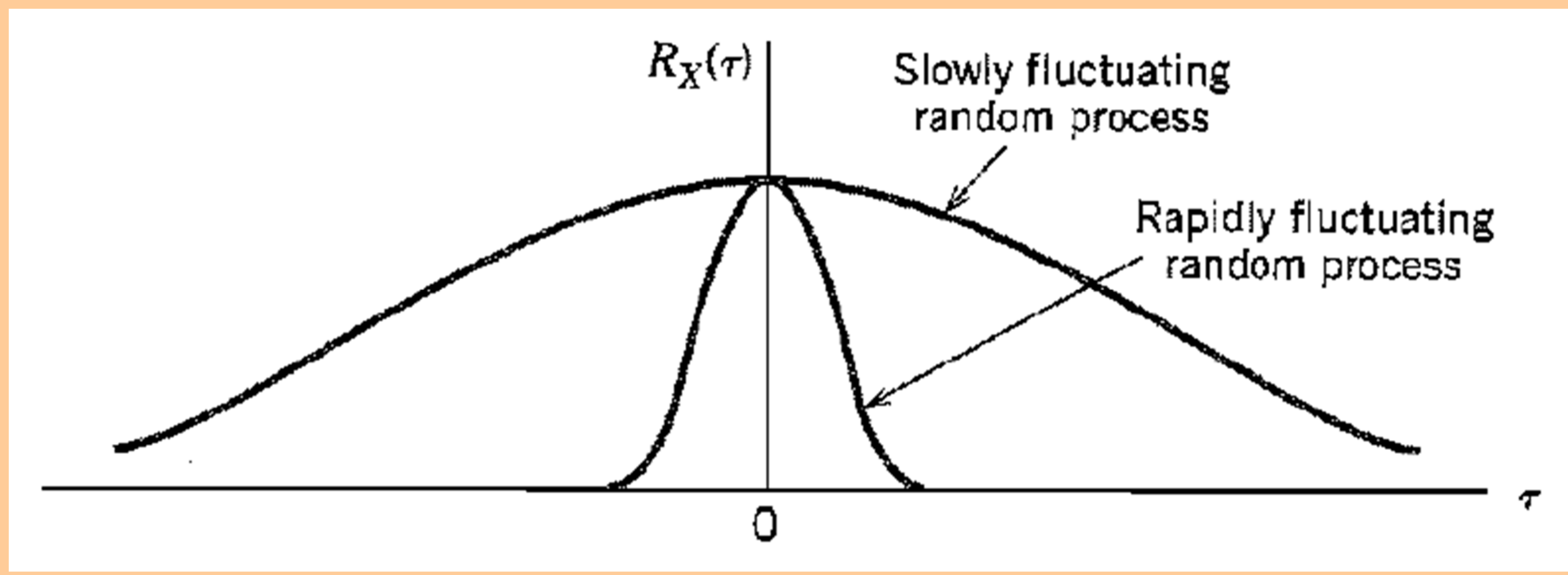
$$R_{XX}(t_1, t_2) = R_{XX}(t_1 - t_2) = R_{XX}(\tau) = E\left(X_{t_1+\tau} X_{t_1}\right)$$

- On the other hand:

$$R_{XX}(-\tau) = E\left(X_{t_1-\tau} X_{t_1}\right) = E\left(X_{t_1'} X_{t_1'+\tau}\right) = R_{XX}(\tau)$$

$$R_{XX}(0) = E\left(X_{t_1}^2\right) = \text{average power of the random process}$$

Autocorrelation Function



- **The physical significance of the autocorrelation function is that it provides a means of describing the interdependence of two random variables obtained by observing a random process $X(t)$ at times τ seconds apart.**

- **The more rapidly the random process $X(t)$ changes with time, the more rapidly will the autocorrelation function $R_X(\tau)$ decrease from its maximum $R_X(0)$ as τ increases.**

Crosscorrelation Function

- **The crosscorrelation function of $X(t)$ and $Y(t)$ is defined by the joint moment as:**

$$R_{XY}(t_1,t_2)=E\left(X_{t_1}Y_{t_2}\right)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}x_{t_1}y_{t_2}\,p\left(x_{t_1},y_{t_2}\right)dx_{t_1}dy_{t_2}$$

- **The random processes $X(t)$ and $Y(t)$ are said to be statistically independent iff:**

$$p\left(x_{t_1},x_{t_2},\cdots,x_{t_n},y_{t_1'},y_{t_2'},\cdots,y_{t_m'}\right)=p\left(x_{t_1},x_{t_2},\cdots,x_{t_n}\right)p\left(y_{t_1'},y_{t_2'},\cdots,y_{t_m'}\right)$$

for all choices of t_i, t_i' and for all positive integers n and m.

- **The random processes $X(t)$ and $Y(t)$ are said to be uncorrelated if:**

$$R_{XY}(t_1,t_2)=E\left(X_{t_1}\right)E\left(Y_{t_2}\right)$$

The Wiener-Khintchine Theorem

- Let x_n be a real signal. Then:

$$R_{XX}(l) \overset{F}{\leftrightarrow} S_{XX}(\omega)$$

- This means the energy spectral density of an energy signal is the Fourier transform of its autocorrelation sequence.

- This result is very important because it implies that the autocorrelation sequence of a signal and its energy spectral density contain the same information.

- The *power spectral density* or *power spectrum* of a stationary process $X(t)$ is defined as:

$$S_{XX}(f) = S_X(f) = \int_{-\infty}^{\infty} R_{XX}(\tau) e^{-j2\pi f\tau} d\tau$$

Power Spectral Density

- The **power spectral density** $S_{XX}(f)$ **and the autocorrelation function** $R_{XX}(\tau)$ **of a stationary process** $X(t)$ **form a Fourier transform pair:**

$$S_{XX}(f) = \int_{-\infty}^{\infty} R_{XX}(\tau) e^{-j2\pi f\tau} d\tau$$

$$R_{XX}(\tau) = \int_{-\infty}^{\infty} S_{XX}(f) e^{j2\pi f\tau} df$$

- **The *cross power density spectrum* or *cross spectral density* of two jointly stationary random processes** $X(t)$ **and** $Y(t)$ **is defined as:**

$$S_{XY}(f) = F(R_{XY}(\tau)) = \int_{-\infty}^{\infty} R_{XY}(\tau) e^{-j2\pi f\tau} d\tau$$

Power Spectral Density (PSD)

Properties …

- **The zero-frequency value of the PSD of a stationary process equals the total area under the graph of the autocorrelation function.**

$$S_{XX}(0) = \int_{-\infty}^{\infty} R_{XX}(\tau)\,d\tau$$

- **The average power of a stationary process equals the total area under the graph of the PSD.**

$$E\left(X_t^2\right) = R_{XX}(0) = \int_{-\infty}^{\infty} S_{XX}(f)\,df$$

- **The PSD of a stationary process is always nonnegative and the PSD of a real-valued random process is an even function of frequency.**

$$S_{XX}(-f) = S_{XX}(f)$$

Noise in Communication Systems

There are many potential sources of noise (external or internal) in a communication system.

The external sources of noise include atmospheric noise, galactic noise, man-made noise etc.

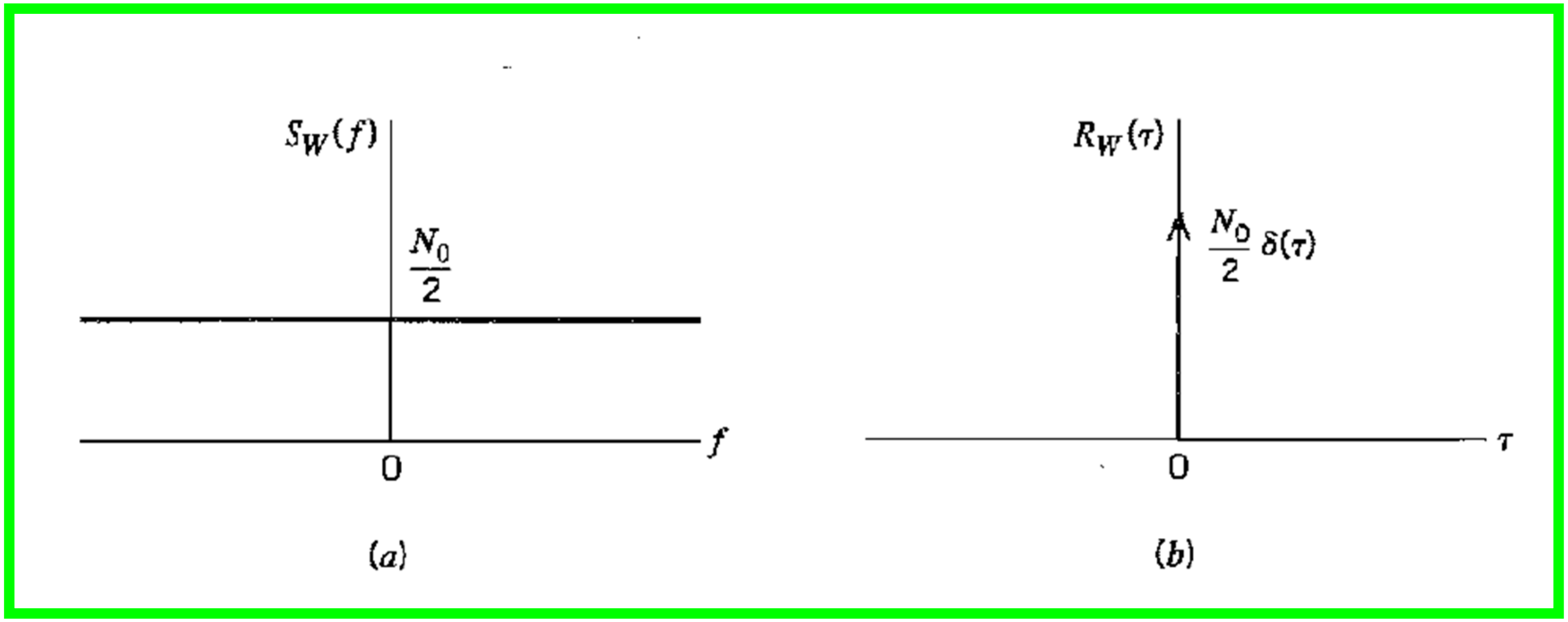
The internal sources of noise include spontaneous fluctuations of currents or voltage in electrical circuits. Two most common examples are *shot noise* and *thermal noise*.

- **Shot noise arises in electronic devices e.g. *diodes and transistors*, because of the discrete nature of current flow in these devices.**

- **Thermal noise arises from the random motion of electrons in a conductor.**

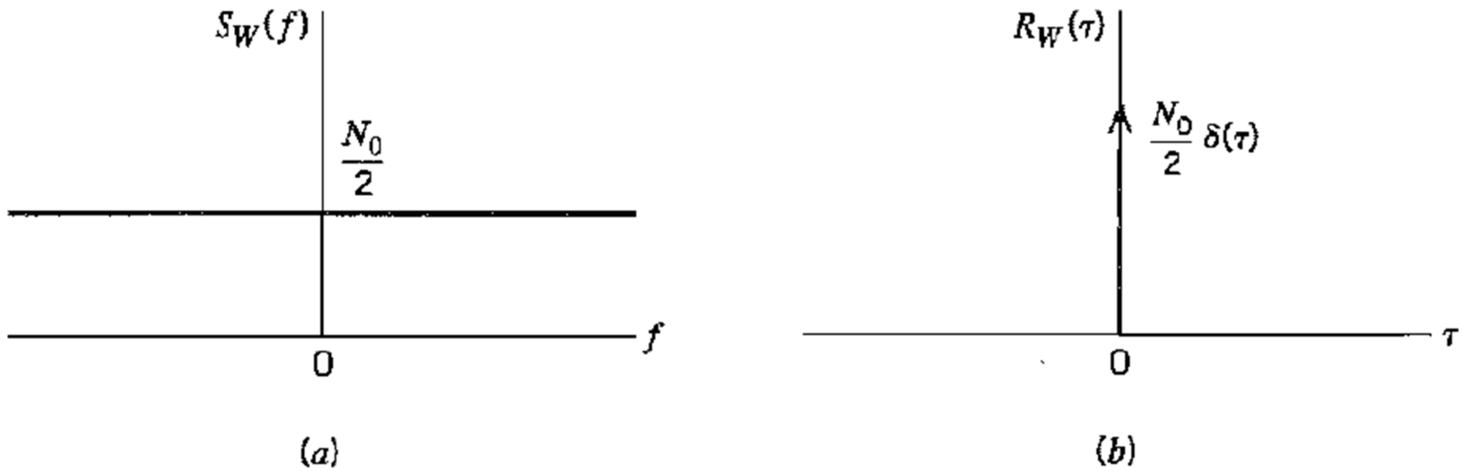
White Noise

The noise analysis of communication systems is customarily based on an idealized form of noise called *white noise*. The PSD of white noise is independent of operating frequency.



Characteristics of white noise. (a) Power spectral density and (b) Autocorrelation function.

White Noise



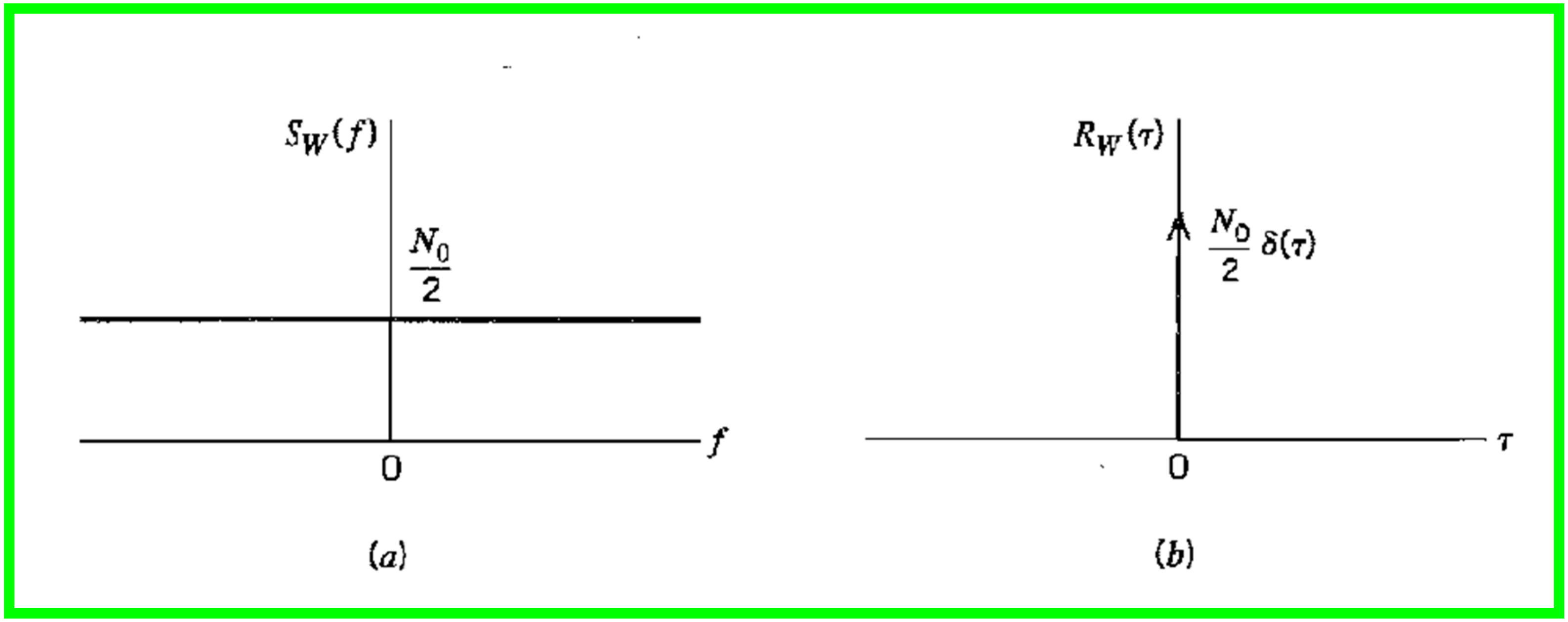
The PSD of white noise:

$$S_W(f) = \frac{N_0}{2}$$

N_0 is in watts per Hertz.

The parameter N_0 is usually referred to the input stage of the receiver of a communication system. It is given as $N_0 = kT_e$ where k = Boltzmann's constant and T_e = the equivalent noise temperature of the receiver.

White Noise



Since the autocorrelation function is the inverse Fourier transform of the PSD, for white noise, we have:

$$R_W(\tau) = \frac{N_0}{2}\,\delta(\tau)$$

White Noise

Strictly speaking, white noise has infinite average power and, as such, it is not physically realizable.

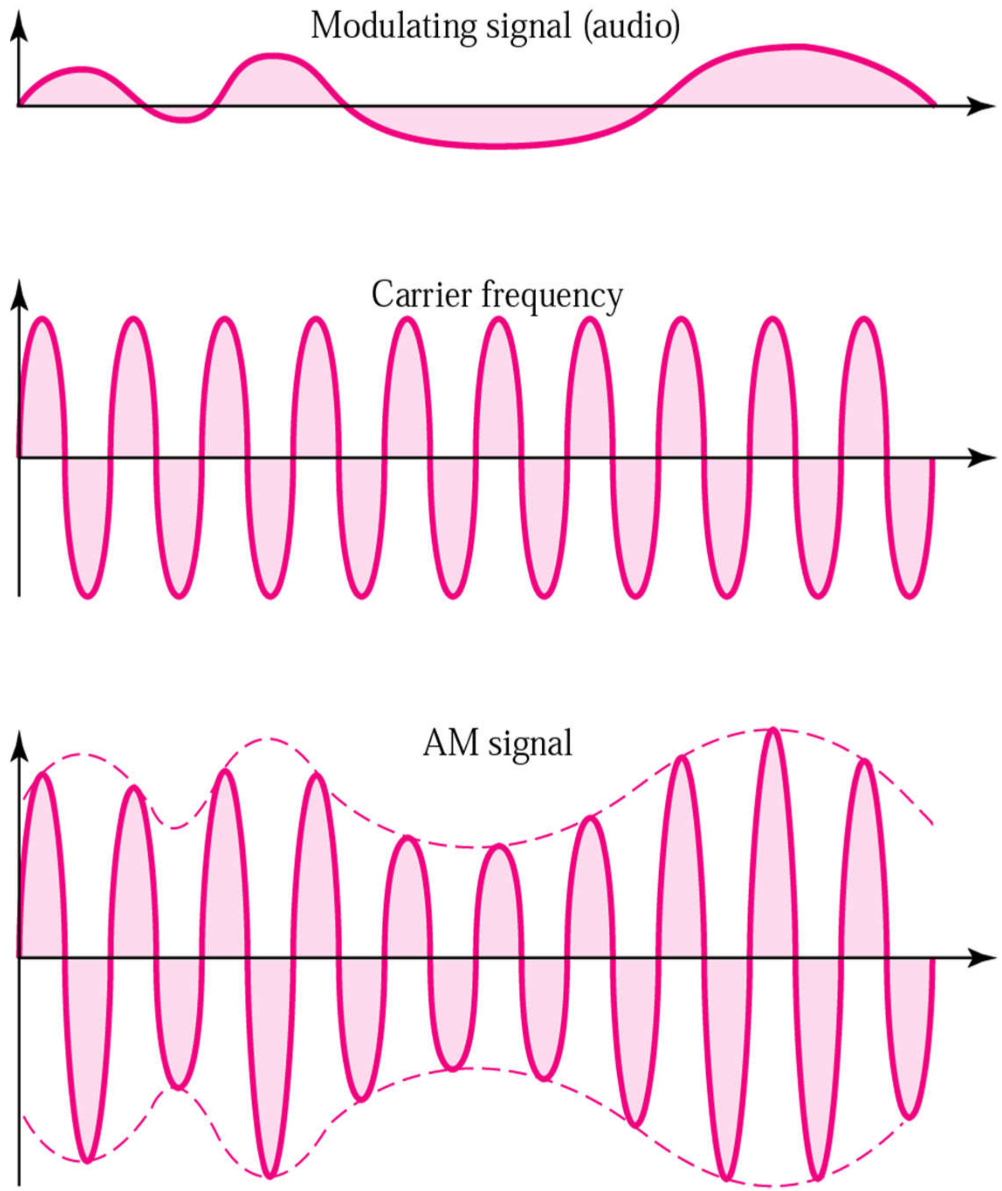
As long as the bandwidth of a noise process at the input of a system is appreciably larger than that of the system itself, then the noise process may be modeled as a white noise.

AMPLITUDE MODULATION

(AM)

Amplitude Modulation

In *Amplitude Modulation (AM)*, the amplitude of the carrier signal A_c is modulated or varied in proportion to the amplitude of the baseband/message/modulating signal $m(t)$ and f_c and ϕ_c are kept constant.



Amplitude Modulation

Different types of amplitude modulation are:

- Double-sideband suppressed carrier (DSB-SC) AM.
- Conventional double-sideband (DSB) AM.
- Single-sideband (SSB) AM.
- Vestigial-sideband (VSB) AM.

Amplitude Modulation

Modulation Index

- **The modulation index (a) is defined as the ratio of the peak amplitude of the modulating signal, A_m, to the peak amplitude of the carrier signal, A_c.**

$$a = A_m / A_c$$

- **It is also referred as percent modulation, modulation factor and depth of modulation.**

- **It is a number lying between 0 and 1 and typically expressed as a percentage.**

Amplitude Modulation

Modulation Carrier and Envelope Detector outputs for various values of α

- **$\alpha<1$: under modulation**
  - **By ensuring the amplitude of the modulating signal to be less than the carrier amplitude, message signal can comfortably be retrieved from the envelope waveform of full AM signal.**

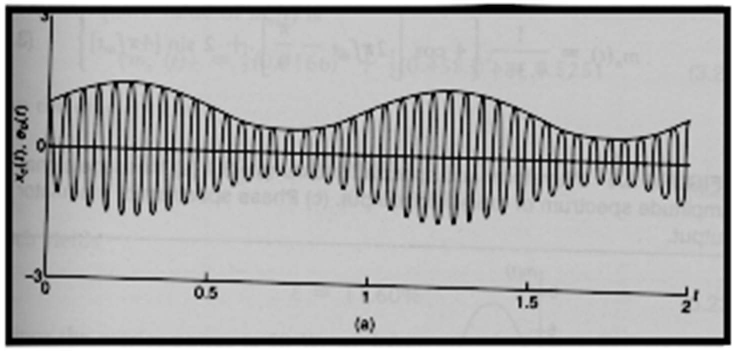
- **$\alpha=1$: ideal modulation**
  - **It ensures successful retrieval of the original transmitted information at the receiver end.**
  - **The ideal condition for amplitude modulation (AM) is when $\alpha=1$ which also means $A_m=A_c$.**
  - **This will give rise to the generation of the maximum message signal outputs at the receiver without distortion.**

- **$\alpha>1$: over modulation**
  - **If the amplitude of the modulating signal is higher than the carrier amplitude, then it implies that the modulation index $\alpha>1$.**
  - **This will cause severe distortion to the modulated signal.**

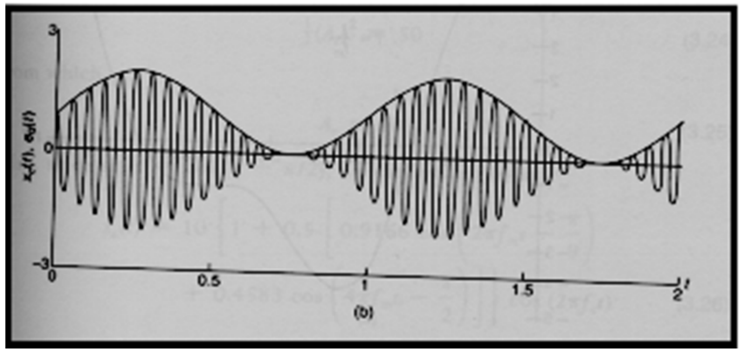
Amplitude Modulation

Modulation Carrier and Envelope Detector outputs for various values of α

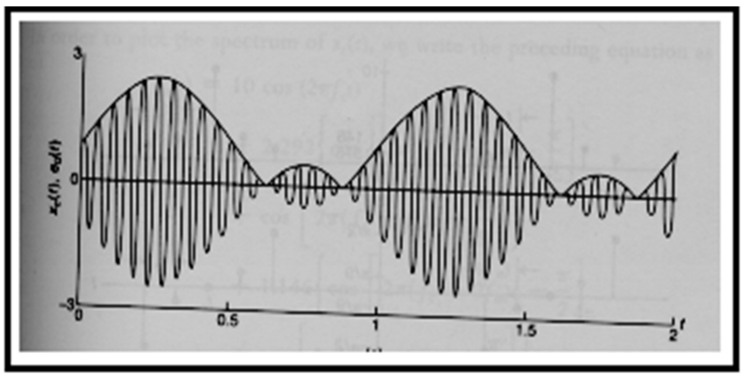
α=0.5



α=1.0



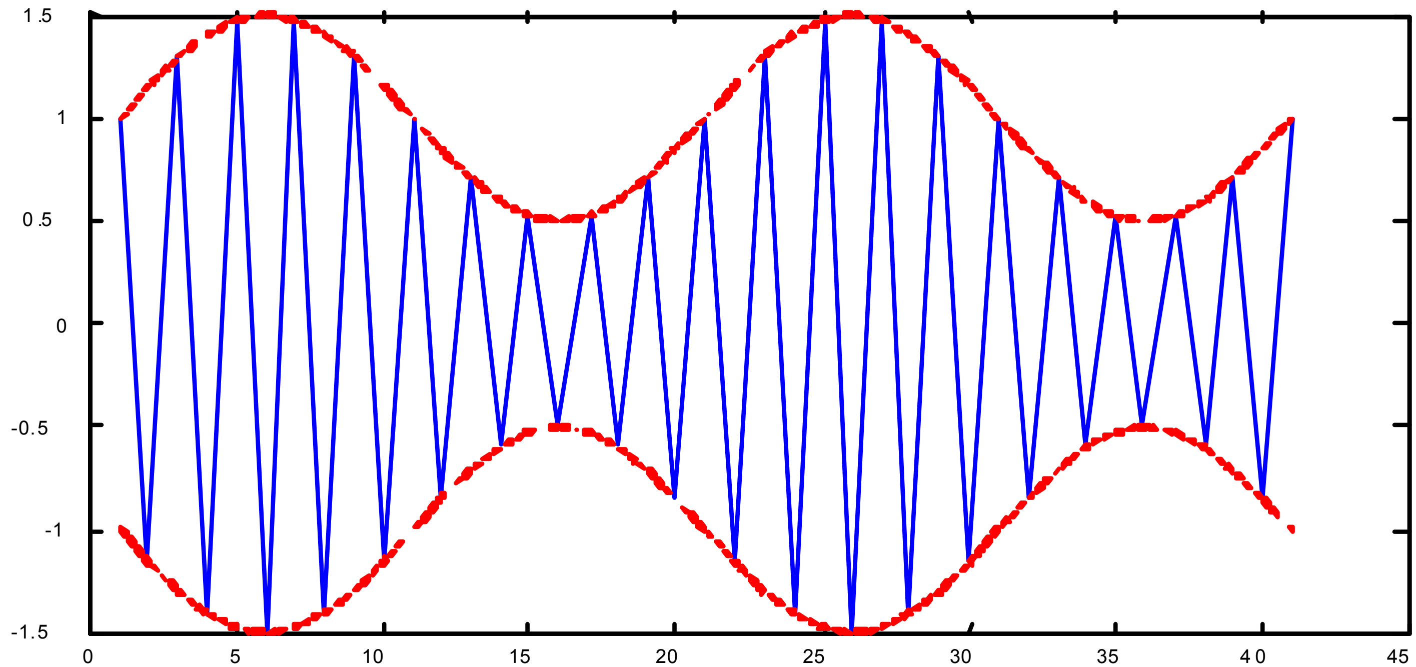
α=1.5



Amplitude Modulation

Modulation Index

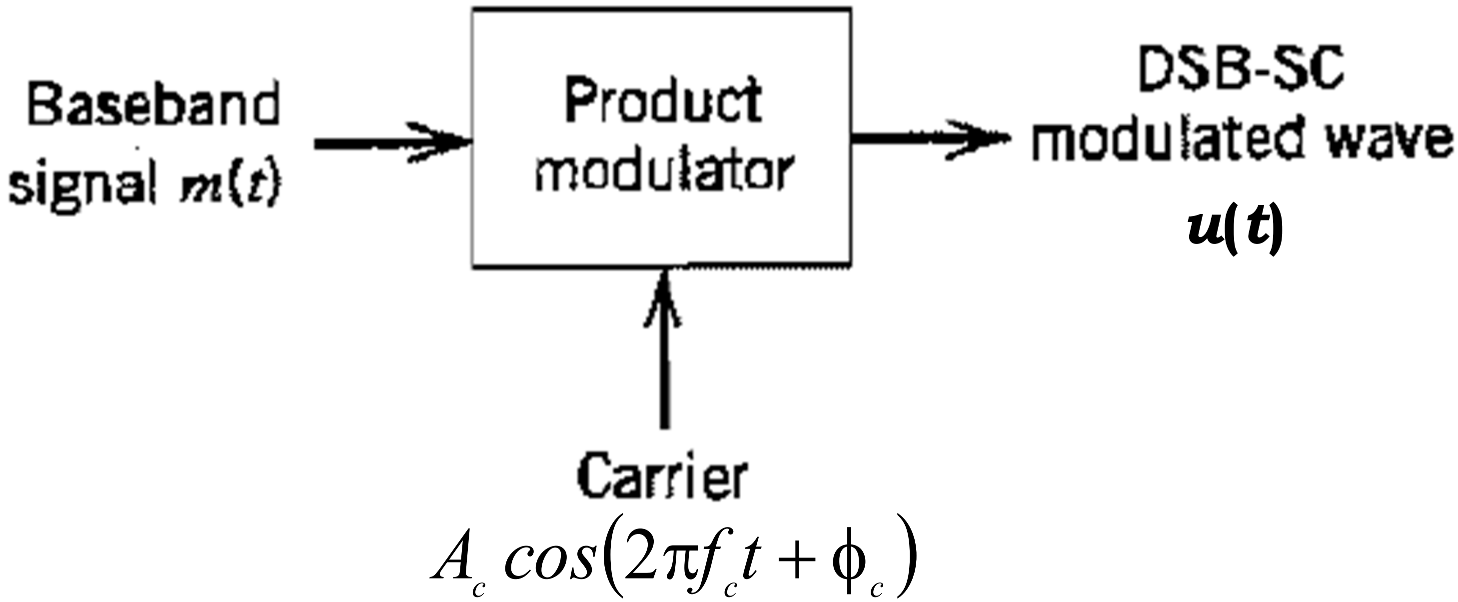
- **In practice, the modulation index of an AM signal can be computed from A_{max} and A_{min} as given below:**



- **A_{max} : half the peak-to-peak value of the max AM signal $A_{max(pk\text{-}pk)}/2$**
- **A_{min} : half the peak-to-peak value of the min AM signal $A_{min(pk\text{-}pk)}/2$**
- **A_m : half the difference of A_{max} and A_{min}.**
- **A_c : half the sum of A_{max} and A_{min}.**

$$\alpha = \frac{A_m}{A_c} = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}$$

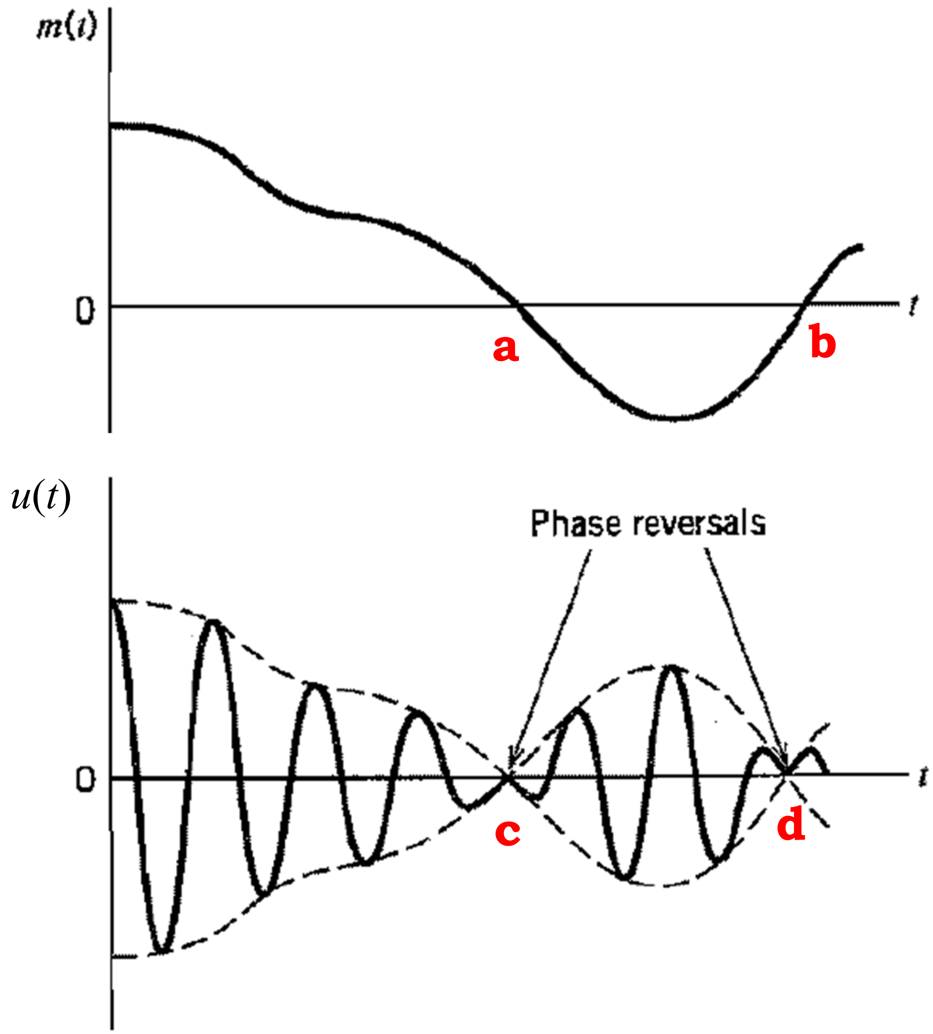
Double-sideband Suppressed Carrier (DSB-SC) AM



$$u(t) = m(t)c(t) = A_c m(t)\cos(2\pi f_c t + \phi_c)$$

***m*(*t*)**: **message signal,** ***c*(*t*)**: **carrier signal,** ***u*(*t*)**: **modulated signal.**

Double-sideband Suppressed Carrier (DSB-SC) AM



Double-sideband Suppressed Carrier (DSB-SC) AM

Using Fourier transform, the spectrum of $u(t)$:

$$U(f) = F[m(t)] * F[A_c \cos(2\pi f_c t + \phi_c)]$$

$$= M(f) * \frac{A_c}{2}\left[e^{j\phi_c}\delta(f - f_c) + e^{-j\phi_c}\delta(f + f_c)\right]$$

$$= \frac{A_c}{2}\left[M(f - f_c)e^{j\phi_c} + M(f + f_c)e^{-j\phi_c}\right]$$

Let us consider $m(t)$ is limited to the interval $-W \le f \le W$. The process of DSB-SC modulation simply translates the magnitude of the spectrum of $m(t)$ by an amount f_c. The phase of $m(t)$ gets translated in frequency and offset by the carrier phase ϕ_c.

Double-sideband Suppressed Carrier (DSB-SC) AM

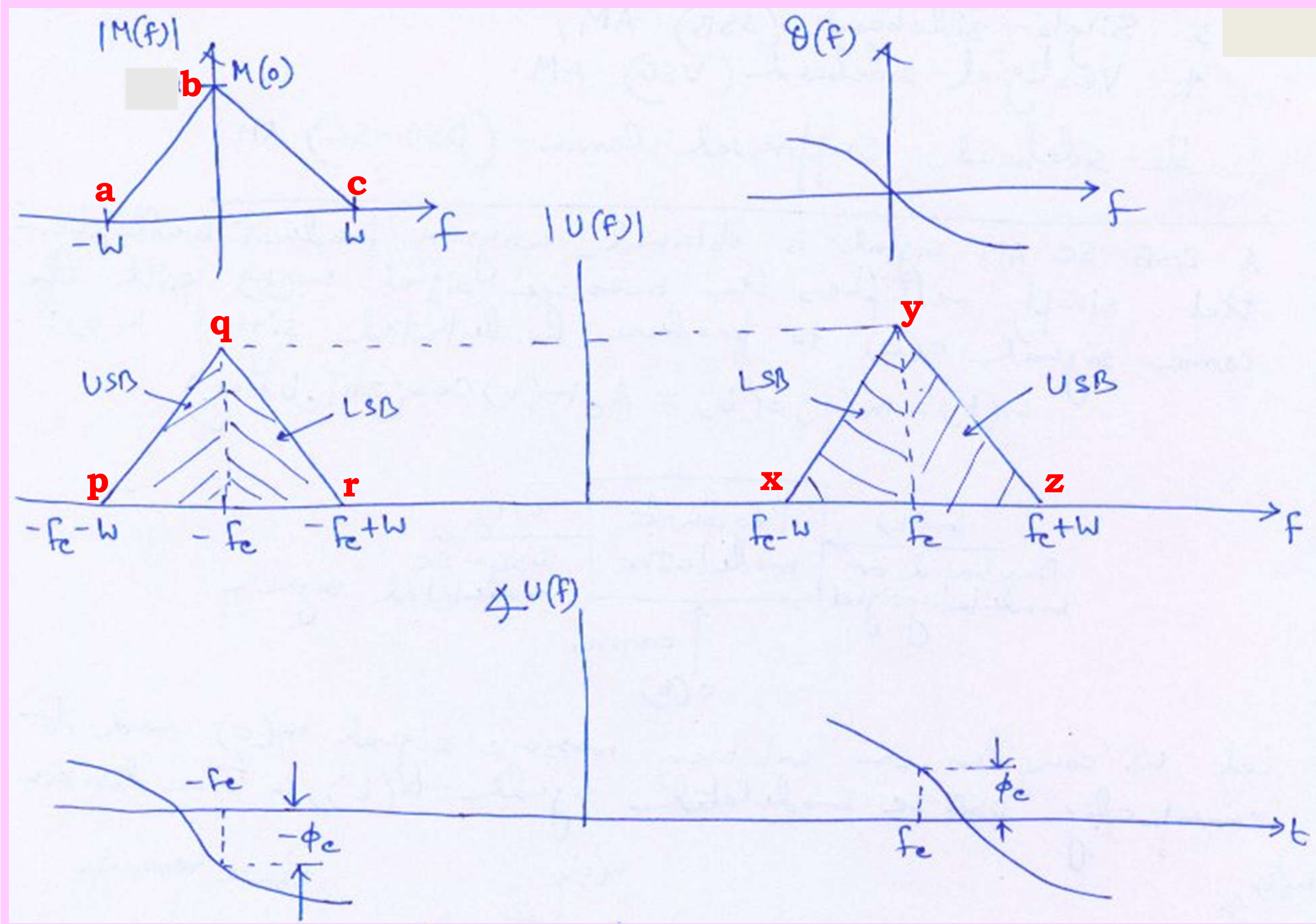
$$U(f) = F[m(t)] * F[A_c \cos(2\pi f_c t + \phi_c)]$$
$$= M(f) * \frac{A_c}{2}\left[e^{j\phi_c}\delta(f - f_c) + e^{-j\phi_c}\delta(f + f_c)\right]$$
$$= \frac{A_c}{2}\left[M(f - f_c)e^{j\phi_c} + M(f + f_c)e^{-j\phi_c}\right]$$

Bandwidth of the message signal = W

Bandwidth occupancy of the modulated signal = $2W$

Channel bandwidth required to transmit $u(t)$ **is** $B_c = 2W$

Double-sideband Suppressed Carrier (DSB-SC) AM



Magnitude and phase spectra of *m*(*t*) and *u*(*t*).

Double-sideband Suppressed Carrier (DSB-SC) AM

Conclusions …

- **The modulated signal spectrum centered at f_c is composed of two parts:**

(i) the portion in the frequency band $|f| > f_c$ is called the upper sideband (USB) of $U(f)$ and

(ii) the portion in the frequency band $|f| < f_c$ is called the lower sideband (LSB) of $U(f)$.

This is why it is called double sideband (DSB) AM signal. The modulated signal does not contain a discrete component of the carrier frequency f_c. Hence it is called a suppressed carrier signal i.e. $u(t)$ is a DSB-SC AM signal.

Note: **One must always maintain $f_c \geq W$.**

Power Content of DSB-SC Signals

Without loss of generality,

we assume ϕ_c = 0.

$$R_u(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} u(t)u(t-\tau)dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} A_c^2 m(t)m(t-\tau)\cos(2\pi f_c t)\cos(2\pi f_c (t-\tau))dt$$

$$= \frac{A_c^2}{2} \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} m(t)m(t-\tau)[\cos(4\pi f_c t - 2\pi f_c \tau) + \cos(2\pi f_c \tau)]dt$$

Power Content of DSB-SC Signals

Now,

$$\int_{-\infty}^{\infty} m(t)m(t-\tau)\cos(4\pi f_c t - 2\pi f_c \tau)dt$$

$$= \int_{-\infty}^{\infty} F[m(t-\tau)]\{F[m(t)\cos(4\pi f_c t - 2\pi f_c \tau)]\}^{*} df$$

$$\left(\text{using Parseval's relation : } \int_{-\infty}^{\infty} x(t)y^{*}(t)dt = \int_{-\infty}^{\infty} X(f)Y^{*}(f)df \right)$$

$$= \int_{-\infty}^{\infty} e^{-j2\pi f\tau} M(f)\left[\frac{M(f-2f_c)e^{-j2\pi f_c\tau}}{2} + \frac{M(f+2f_c)e^{j2\pi f_c\tau}}{2}\right]^{*} df$$

$$= 0$$

This is because ***M*(*f*)** **is limited in [-*W*, *W*] and *W* << f_c. So there is no frequency overlap between *M*(*f*) and *M*($f \pm 2f_c$).**

Power Content of DSB-SC Signals

Then,

$$\int_{-\infty}^{\infty} m(t)m(t-\tau)cos(4\pi f_c t - 2\pi f_c \tau)dt = 0$$

$$R_u(\tau) = \frac{A_c^2}{2} \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} m(t)m(t-\tau)cos(2\pi f_c \tau)dt + 0$$

$$= \frac{A_c^2}{2} R_m(\tau)cos(2\pi f_c \tau)$$

Power Content of DSB-SC Signals

$$R_u(\tau) = \frac{A_c^2}{2} R_m(\tau) \cos(2\pi f_c \tau)$$

Fourier transform ⬇

PSD of the modulated signal:

$$S_u(f) = F[R_u(\tau)] = F\left[\frac{A_c^2}{2} R_m(\tau)\cos(2\pi f_c \tau)\right]$$

$$= \frac{A_c^2}{4}\left[S_m(f - f_c) + S_m(f + f_c)\right]$$

$$\left(\begin{array}{l}\text{From modulation property :} \\ F[x(t)\cos(2\pi f_0 t)] = \frac{1}{2}X(f - f_0) + \frac{1}{2}X(f + f_0)\end{array}\right)$$

Total power in the modulated signal is obtained by putting $\tau = 0$ in $R_u(\tau)$.

Power Content of DSB-SC Signals

Therefore,

$$P_u = \frac{A_c^2}{2} R_m(\tau)\cos(2\pi f_c \tau)\Big|_{\tau=0}$$

$$= \frac{A_c^2}{2} R_m(0)$$

$$= \frac{A_c^2}{2} P_m$$

where $P_m = R_m(0)$ = power in the message signal.

Demodulation of DSB-SC AM Signals

To recover the original signal $m(t)$ from the modulated signal, it is necessary to *retranslate the spectrum to its original position*.

For unique detection of $m(t)$, we assume an ideal channel and assume there is no noise.

$$r(t) = u(t) = A_c m(t) \cos(2\pi f_c t + \phi_c)$$

This $r(t)$ is demodulated by first multiplying $r(t)$ by a locally generated sinusoid $A_c' \cos(2\pi f_c t + \phi)$ and then passing the product signal through an ideal low-pass filter of bandwidth W.

Demodulation of DSB-SC AM Signals

Then,

$$r(t)A_c' \cos(2\pi f_c t + \phi) = v(t) = A_c A_c' m(t)\cos(2\pi f_c t + \phi_c)\cos(2\pi f_c t + \phi)$$

$$= \frac{1}{2} A_c A_c' m(t)\cos(\phi_c - \phi) + \frac{1}{2} A_c A_c' m(t)\cos(4\pi f_c t + \phi + \phi_c)$$

The low-pass filter rejects the double frequency components and passes only the low-pass components.

output

$$y_l(t) = \frac{1}{2} A_c A_c' m(t)\cos(\phi_c - \phi)$$

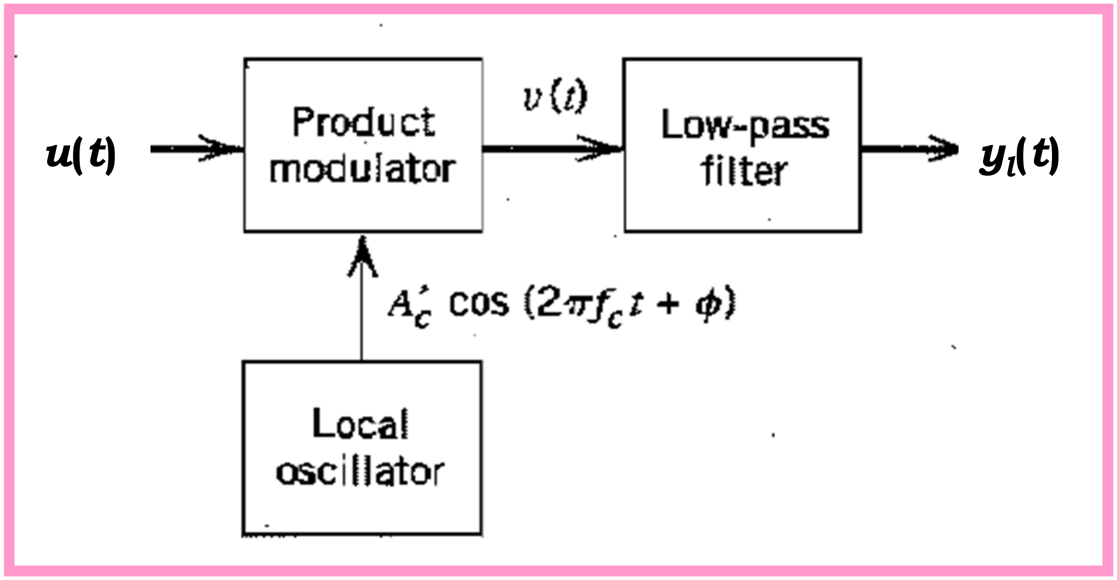
For recovering $m(t)$ from the message signal, we must have $\phi_c = \phi$.

Then, $$y_l(t) = \frac{1}{2} A_c A_c' m(t)$$

Demodulation of DSB-SC AM Signals

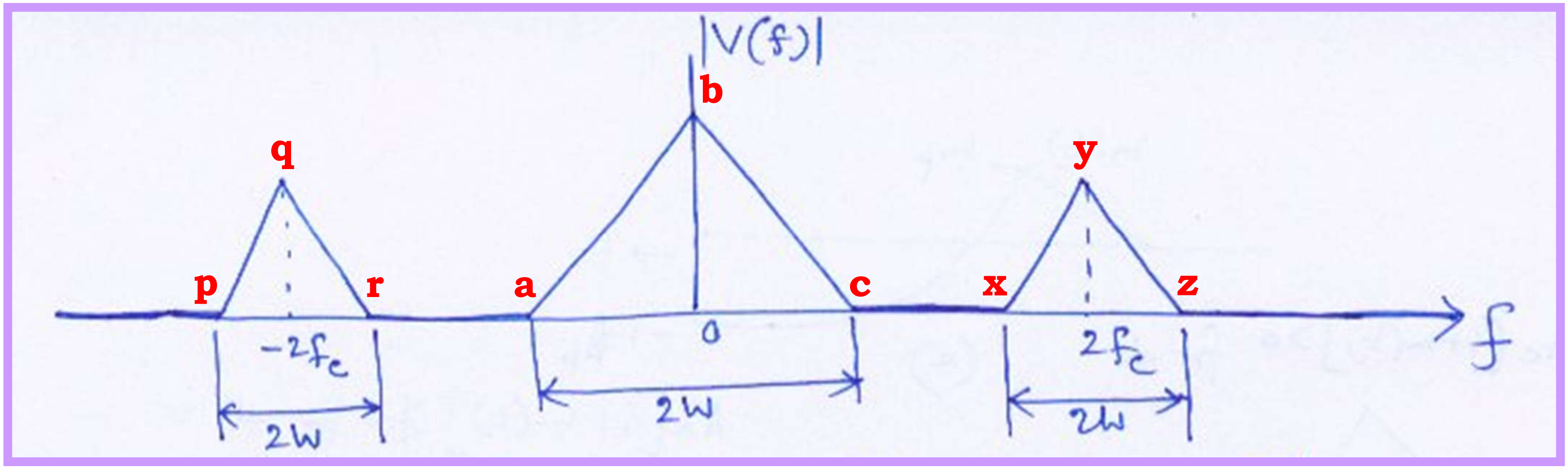
It is required that the phase ϕ of the locally generated sinusoid should ideally be equal to the phase ϕ_c of the received carrier signal.

This method of demodulation is called coherent detection or synchronous demodulation.



Coherent detector for demodulating DSB-SC modulated wave.

Demodulation of DSB-SC AM Signals



The cut off frequency of the low-pass filter is greater than *W* but less than ($2f_c$ – *W*). This requirement is satisfied by choosing $f_c > W$.

If $\phi_c - \phi = 90°$, the demodulated signal becomes zero. This is called quadrature null effect.

Observation: **In coherent detection based system, the system complexity is higher for suppressing the carrier wave to save transmitted power.**

Conventional DSB AM

In conventional AM, the amplitude of the carrier signal $u(t)$ is varied about a mean value, linearly with the baseband signal $m(t)$.

transmitted signal

$$u(t) = A_c\left[1 + m(t)\right]\cos\left(2\pi f_c t + \phi_c\right)$$

$A_c m(t)\cos(2\pi f_c t + \phi_c)$: **a DSB AM signal and**

$A_c \cos(2\pi f_c t + \phi_c)$: **the carrier component.**

Note: **One must always have** $|m(t)| \leq 1$.

Conventional DSB AM

We can express: $m(t) = a m_n(t)$

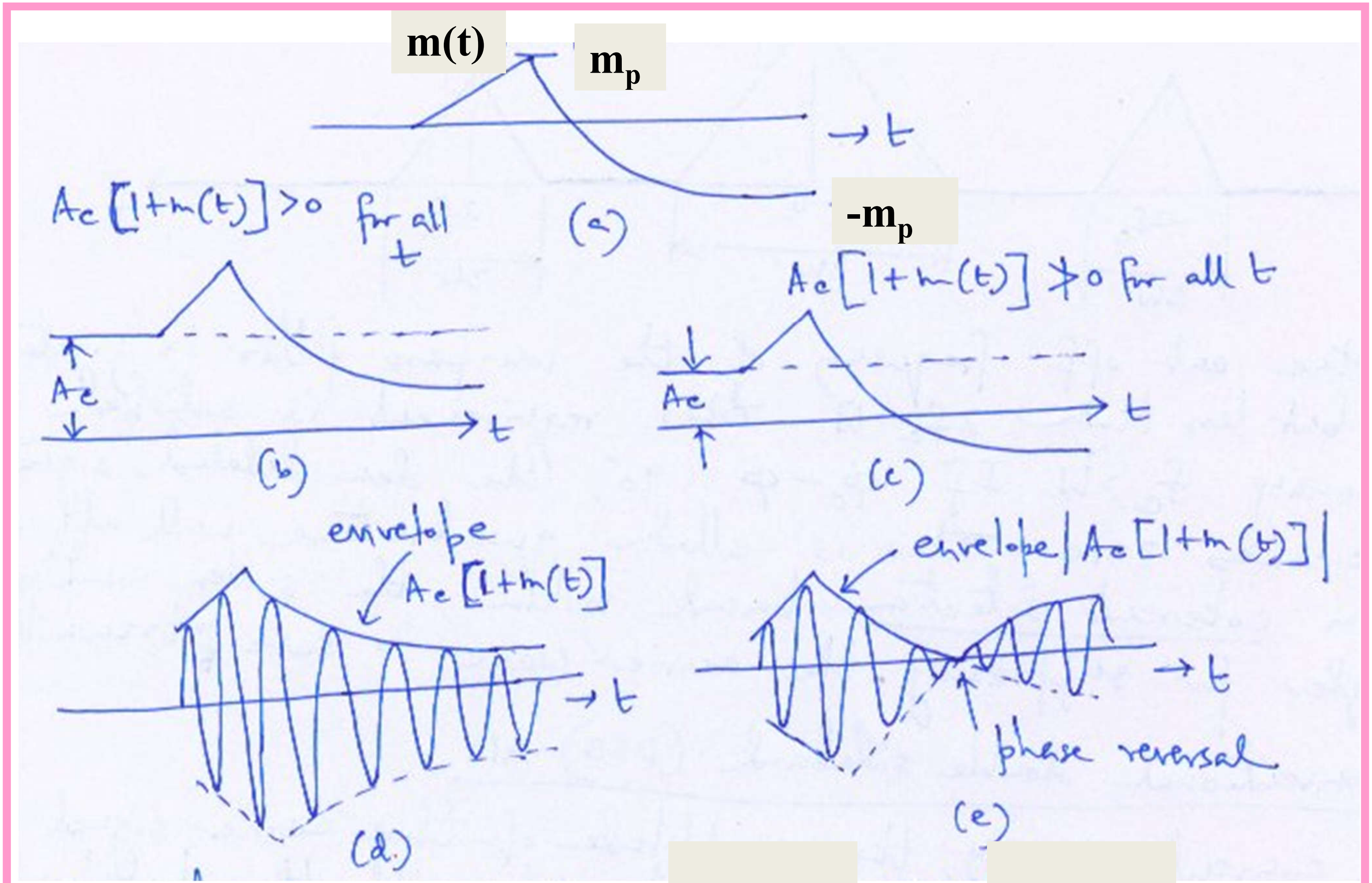
$m_n(t)$ is normalized such that its minimum value is -1.

$$m_n(t) = \frac{m(t)}{\max|m(t)|}$$

a: modulation index

The modulated signal: $u(t) = A_c\left[1 + a m_n(t)\right]\cos(2\pi f_c t + \phi_c)$

Conventional DSB AM



Conventional AM signal and its envelope.

Conventional DSB AM

Conclusion: **The envelope of conventional AM has the information about $m(t)$ only if the AM signal $u(t) = A_c[1+m(t)]\cos(2\pi f_c t + \phi_c)$ satisfies the condition $A_c[1+m(t)] > 0$ for all t.**

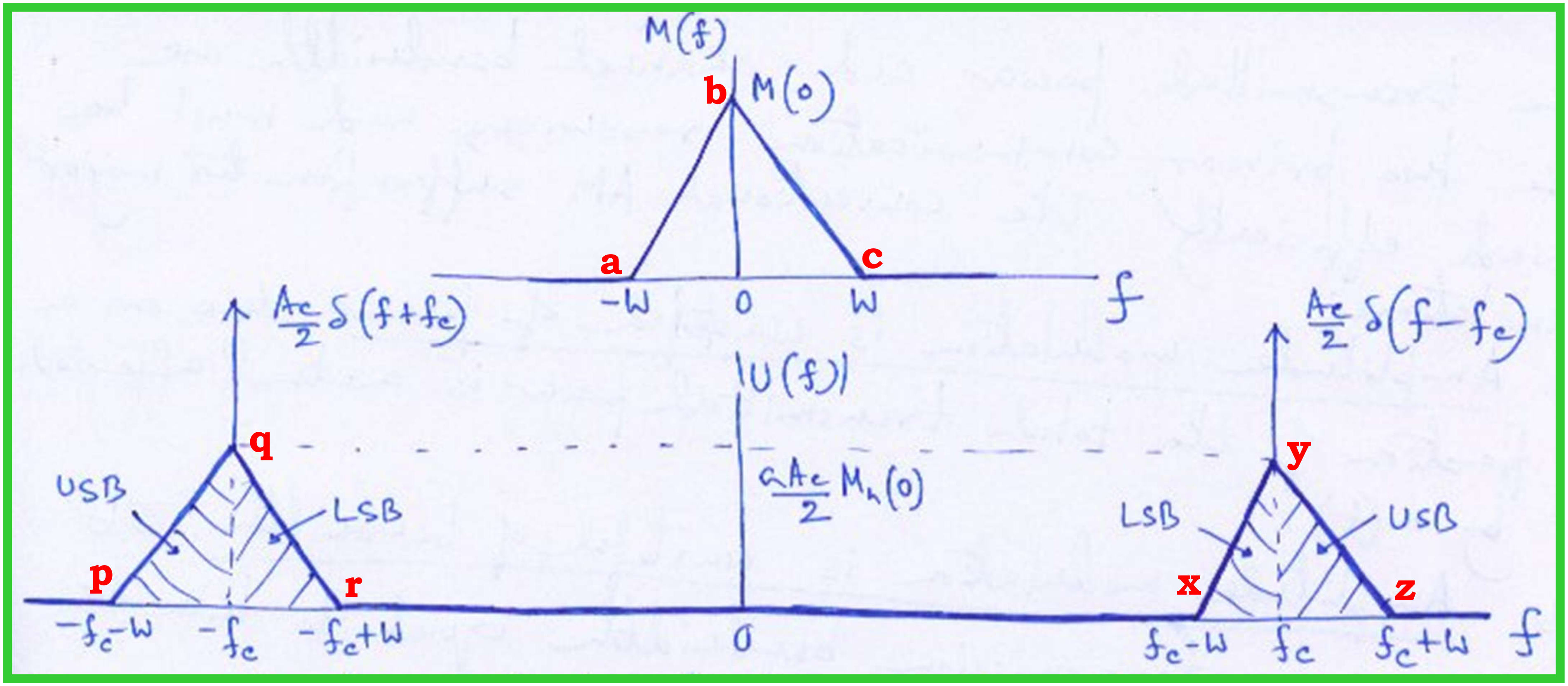
Conventional DSB AM

Spectrum of the modulated signal:

$$U(f) = F(u(t)) = F[A_c(1+m(t))\cos(2\pi f_c t + \phi_c)]$$
$$= F[A_c(1+am_n(t))\cos(2\pi f_c t + \phi_c)]$$
$$= F[am_n(t)] * F[A_c \cos(2\pi f_c t + \phi_c)] + F[A_c \cos(2\pi f_c t + \phi_c)]$$
$$= aM_n(f) * \frac{A_c}{2}\left[e^{j\phi_c}\delta(f-f_c) + e^{-j\phi_c}\delta(f+f_c)\right]$$
$$+ \frac{A_c}{2}\left[e^{j\phi_c}\delta(f-f_c) + e^{-j\phi_c}\delta(f+f_c)\right]$$
$$= \frac{A_c}{2}\begin{bmatrix} e^{j\phi_c} aM_n(f-f_c) + e^{j\phi_c}\delta(f-f_c) \\ + e^{-j\phi_c} aM_n(f+f_c) + e^{-j\phi_c}\delta(f+f_c) \end{bmatrix}$$

Conclusion: The spectrum of a conventional AM signal occupies a bandwidth twice the bandwidth of the message signal.

Conventional DSB AM



Spectrum of baseband signal *m*(*t*) and conventional AM signal *u*(*t*).

Power Content of Conventional AM Signal

Conventional AM signal is similar to DSB when *m*(*t*) is substituted with (1+$am_n(t)$). In DSB-SC signal, power content in the modulated signal:

$$P_u = \frac{A_c^2}{2} P_m$$

P_m: power in the message signal *m*(*t*)

For conventional AM:

$$P_m = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \left[1 + am_n(t)\right]^2 dt = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \left[1 + a^2 m_n^2(t) + 2am_n(t)\right] dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \left[1 + a^2 m_n^2(t)\right] dt \quad (\text{assumption: av. of } m_n(t) = 0)$$

Power Content of Conventional AM Signal

For conventional AM:

$$P_m = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \left[1 + a^2 m_n^2(t)\right] dt = 1 + a^2 P_{m_n}$$

$$P_u = \frac{A_c^2}{2} P_m = \frac{A_c^2}{2} + \frac{A_c^2}{2} a^2 P_{m_n}$$

($A_c^2/2$): exists due to carrier (does not carry any information)

($A_c^2/2$)$a^2 P_{m_n}$: the information carrying component

Conclusion: Conventional AM signals are far less power efficient compared to DSB-SC systems. But conventional AM can be easily demodulated.

Drawbacks of Conventional AM

It is wasteful of power. Here only a fraction of the total transmitted power is actually affected by $m(t)$.

It is wasteful of bandwidth. It requires a transmission bandwidth equal to twice the message bandwidth.

Demodulation of Conventional DSB AM Signals

Here demodulation can be easily carried out. There is no need for a synchronous demodulator.

The received signal is rectified. The rectified signal is $u(t)$ when $u(t) > 0$ and is zero when $u(t) < 0$.

The message signal is recovered by passing the rectified signal through a low-pass filter whose bandwidth matches that of the message signal.

The combination of the rectifier and the low-pass filter is called an envelope detector.

The existence of the extra carrier results in a very simple structure for the demodulator. Hence Conventional AM is a popular choice for AM radio broadcasting.

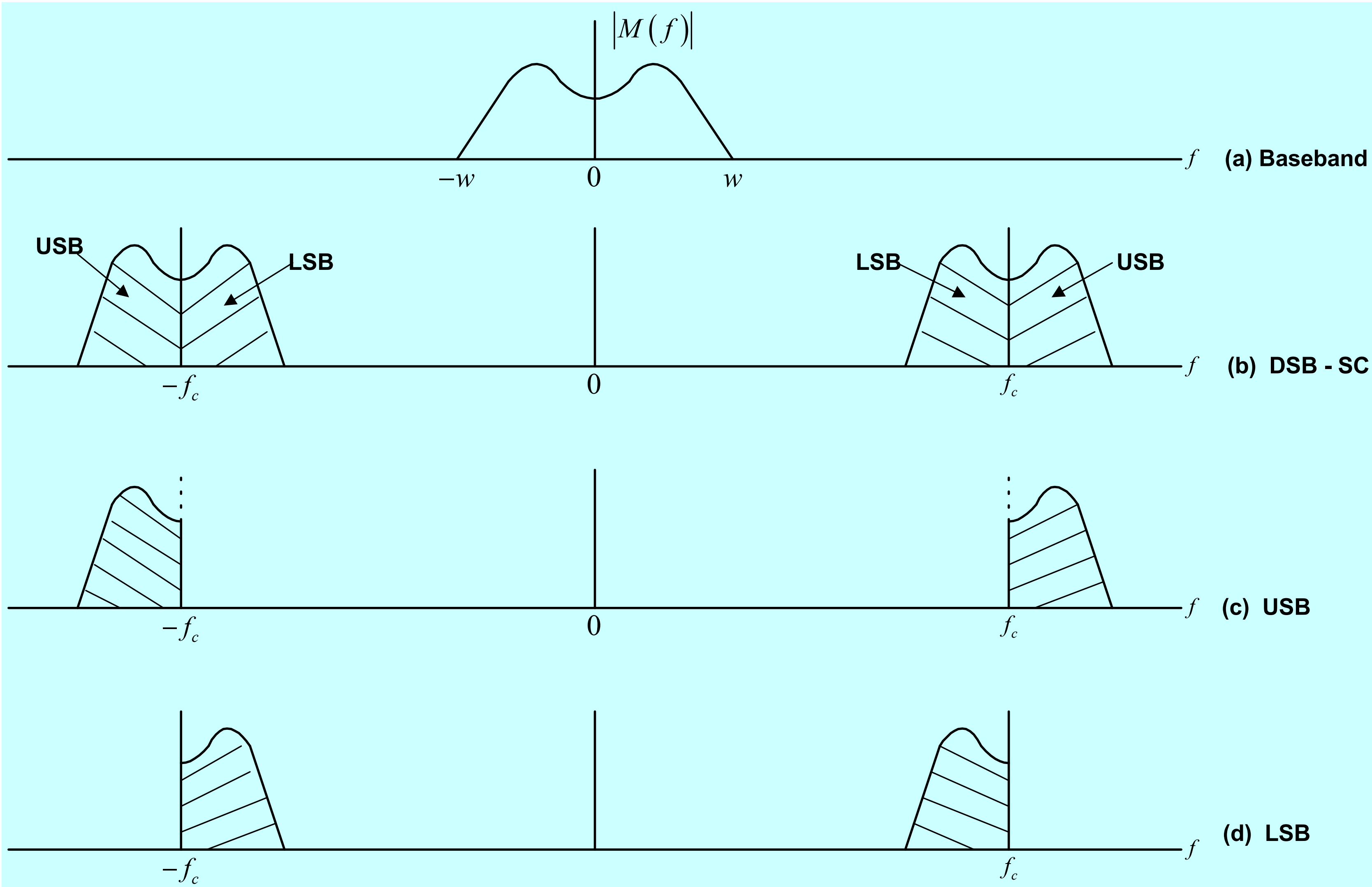
Single-Sideband (SSB) AM

- In **SSB modulation**, only the upper sideband (USB) or the lower sideband (LSB) is transmitted. This is sufficient to reconstruct the message signal $m(t)$ at the receiver.

↓

- This reduces the bandwidth of the transmitted signal to W Hz, that of the baseband signal.

Single-Sideband (SSB) AM



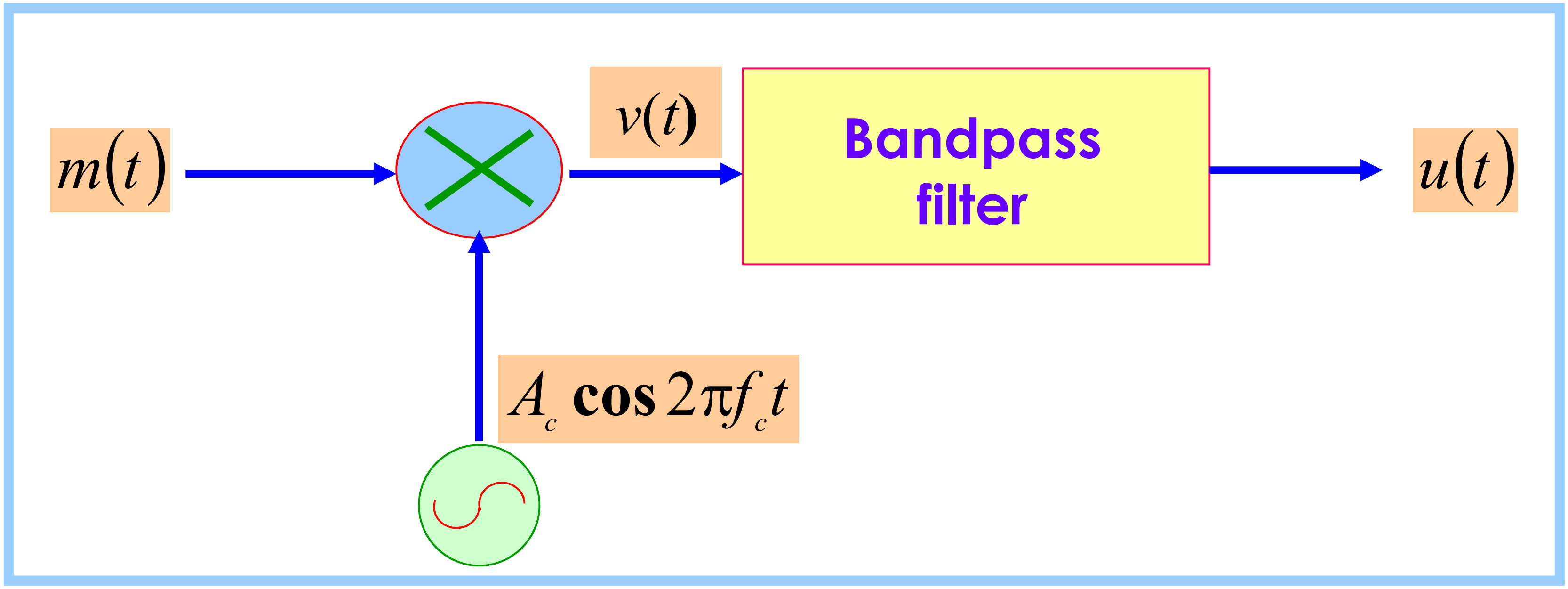
SSB AM spectra.

Generation of SSB AM Signals

Frequency-discrimination method

- **Stage 1:** **Generate a DSB-SC AM signal, using a product modulator.**

- **Stage 2:** **Employ a band-pass filter, designed to pass USB or LSB of the modulated wave.**



Generation of SSB AM signal by frequency-discrimination method.

Generation of SSB AM Signals

Let the DSB-SC AM signal generated be:

$$u_{DSB}(t) = 2A_c m(t)\cos 2\pi f_c t$$

The upper single-sideband AM (USSB AM) is generated by employing a high pass filter with T.F.:

$$H(f) = \begin{cases} 1, & |f| > f_c \\ 0, & \text{otherwise} \end{cases}$$

$$H(f) = u_{-1}(f - f_c) + u_{-1}(-f - f_c)$$

where $u_{-1}(\cdot)$: unit step function

Generation of SSB AM Signals

Spectrum of the USSB AM signal is:

$$U_u(f) = A_c M(f - f_c) u_{-1}(f - f_c) + A_c M(f + f_c) u_{-1}(-f - f_c)$$
$$= A_c M(f) u_{-1}(f)\Big|_{f = f - f_c} + A_c M(f) u_{-1}(-f)\Big|_{f = f + f_c}$$

… (1)

From Fourier transform:

$$F\left[\frac{1}{2}\delta(t) + \frac{j}{2\pi t}\right] = u_{-1}(f)$$
$$F\left[\frac{1}{2}\delta(t) - \frac{j}{2\pi t}\right] = u_{-1}(-f)$$

Generation of SSB AM Signals

Taking inverse Fourier transform of (1) and employing modulation property of Fourier transform:

$$F\left[x(t)e^{j2\pi f_0 t}\right] = X(f - f_0)$$

$$u_u(t) = A_c m(t) * F^{-1}\left[u_{-1}(f)\right]e^{j2\pi f_c t} + A_c m(t) * F^{-1}\left[u_{-1}(-f)\right]e^{-j2\pi f_c t}$$

$$= A_c m(t) * \left[\frac{1}{2}\delta(t) + \frac{j}{2\pi t}\right]e^{j2\pi f_c t} + A_c m(t) * \left[\frac{1}{2}\delta(t) - \frac{j}{2\pi t}\right]e^{-j2\pi f_c t}$$

$$= \frac{A_c}{2}\left[m(t) + j\hat{m}(t)\right]e^{j2\pi f_c t} + \frac{A_c}{2}\left[m(t) - j\hat{m}(t)\right]e^{-j2\pi f_c t}$$

...(2)

This relation used the identities:

$$m(t) * \delta(t) = m(t); \quad m(t) * \frac{1}{\pi t} = \hat{m}(t)$$

$\hat{m}(t)$: Hilbert transform of $m(t)$

Generation of SSB AM Signals

Hilbert transform can be viewed as a linear filter with impulse response $h(t) = (1/\pi t)$ and frequency response:

$$H(f) = \begin{cases} -j, & f > 0 \\ j, & f < 0 \\ 0, & f = 0 \end{cases}$$

From eq. (2):

$$u_u(t) = A_c m(t)\left[\frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2}\right] + jA_c\hat{m}(t)\left[\frac{e^{j2\pi f_c t} - e^{-j2\pi f_c t}}{2}\right]$$
$$= A_c m(t)\cos 2\pi f_c t - A_c\hat{m}(t)\sin 2\pi f_c t$$

$u_u(t)$ is an USSB AM signal in time domain. The LSSB AM signal can be derived from: $u_u(t) + u_l(t) = u_{DSB}(t)$

Generation of SSB AM Signals

Now,

$$u_u(t) + u_l(t) = u_{DSB}(t)$$

$$\text{or, } A_c m(t)\mathbf{cos}\, 2\pi f_c t - A_c \hat{m}(t)\mathbf{sin}\, 2\pi f_c t + u_l(t) = 2A_c m(t)\mathbf{cos}\, 2\pi f_c t$$

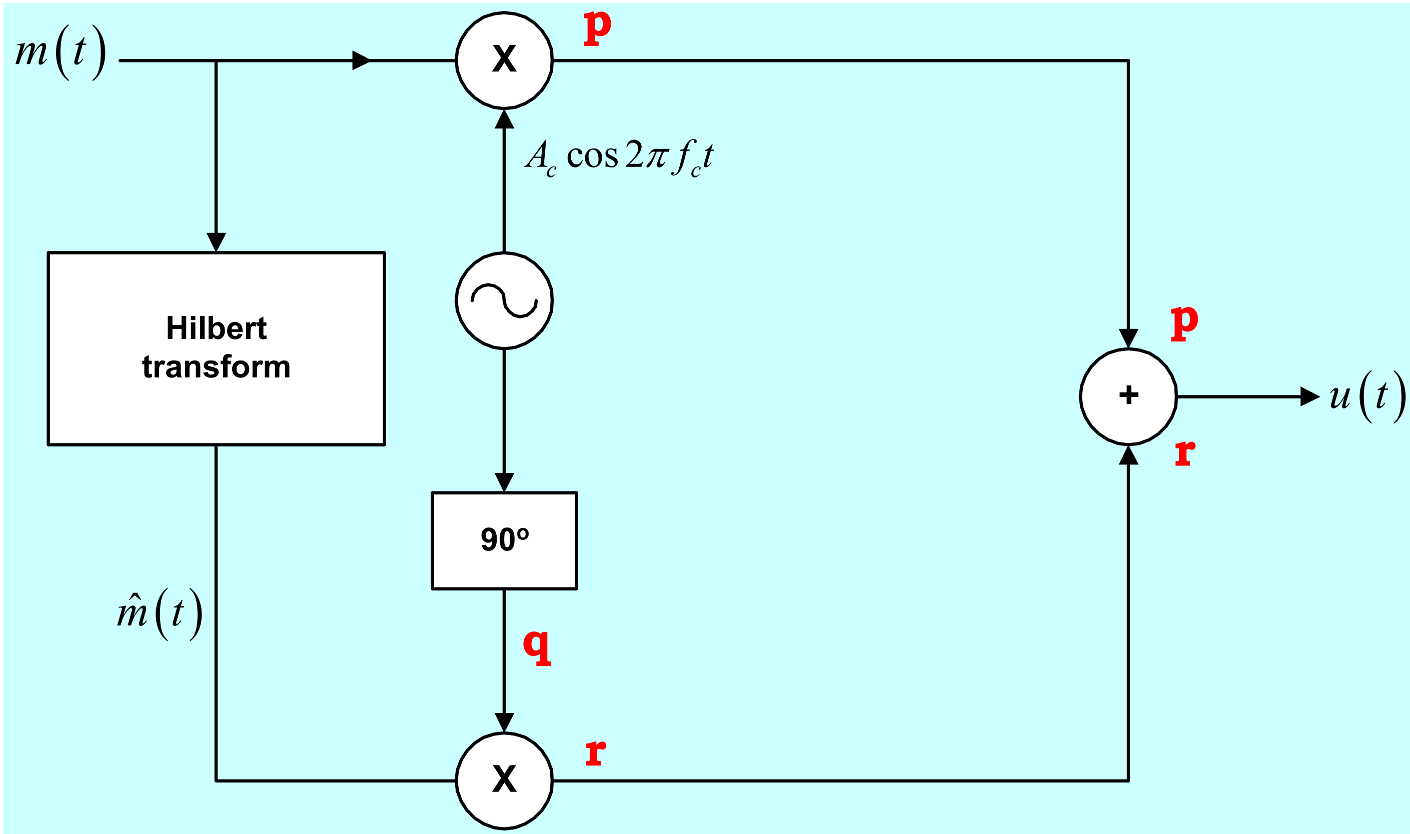
$$\text{or, } u_l(t) = A_c m(t)\mathbf{cos}\, 2\pi f_c t + A_c \hat{m}(t)\mathbf{sin}\, 2\pi f_c t$$

General time domain representation of SSB AM signal:

$$u_{SSB}(t) = A_c m(t)\mathbf{cos}\, 2\pi f_c t \mp A_c \hat{m}(t)\mathbf{sin}\, 2\pi f_c t$$

(- sign: USSB AM signal;
+ sign: LSSB AM signal) ...(3)

Generation of SSB AM Signals



Generation of SSB AM signal.

Demodulation of SSB AM Signals

From the USSB signal in eq. (3):

$$r(t)\cos(2\pi f_c t + \phi) = u(t)\cos(2\pi f_c t + \phi)$$
$$= \frac{1}{2} A_c m(t)\cos\phi + \frac{1}{2} A_c \hat{m}(t)\sin\phi + \text{double frequency terms}$$

This signal is passed through an ideal low-pass filter whose output is:

$$y_l(t) = \frac{1}{2} A_c m(t)\cos\phi + \frac{1}{2} A_c \hat{m}(t)\sin\phi$$

- **The second component contributes to the distortion of the demodulated SSB signal. This was not present in a DSB-SC signal.**

Demodulation of SSB AM Signals

- **For phase coherent demodulation, $\phi = 0$ and $y_l(t) = (1/2)A_c m(t)$.**

- **This can be done by multiplying $u(t)$ by a locally generated carrier and then low-pass filtering the product.**

How to achieve perfect synchronism ??

- **A low power pilot carrier or pilot tone is transmitted at the carrier frequency, in addition to the selected sideband.**

> ***Note:*** **SSB-AM is very popular in voice communication over telephone channels (wires and cables).**

Demodulation of SSB AM Signals

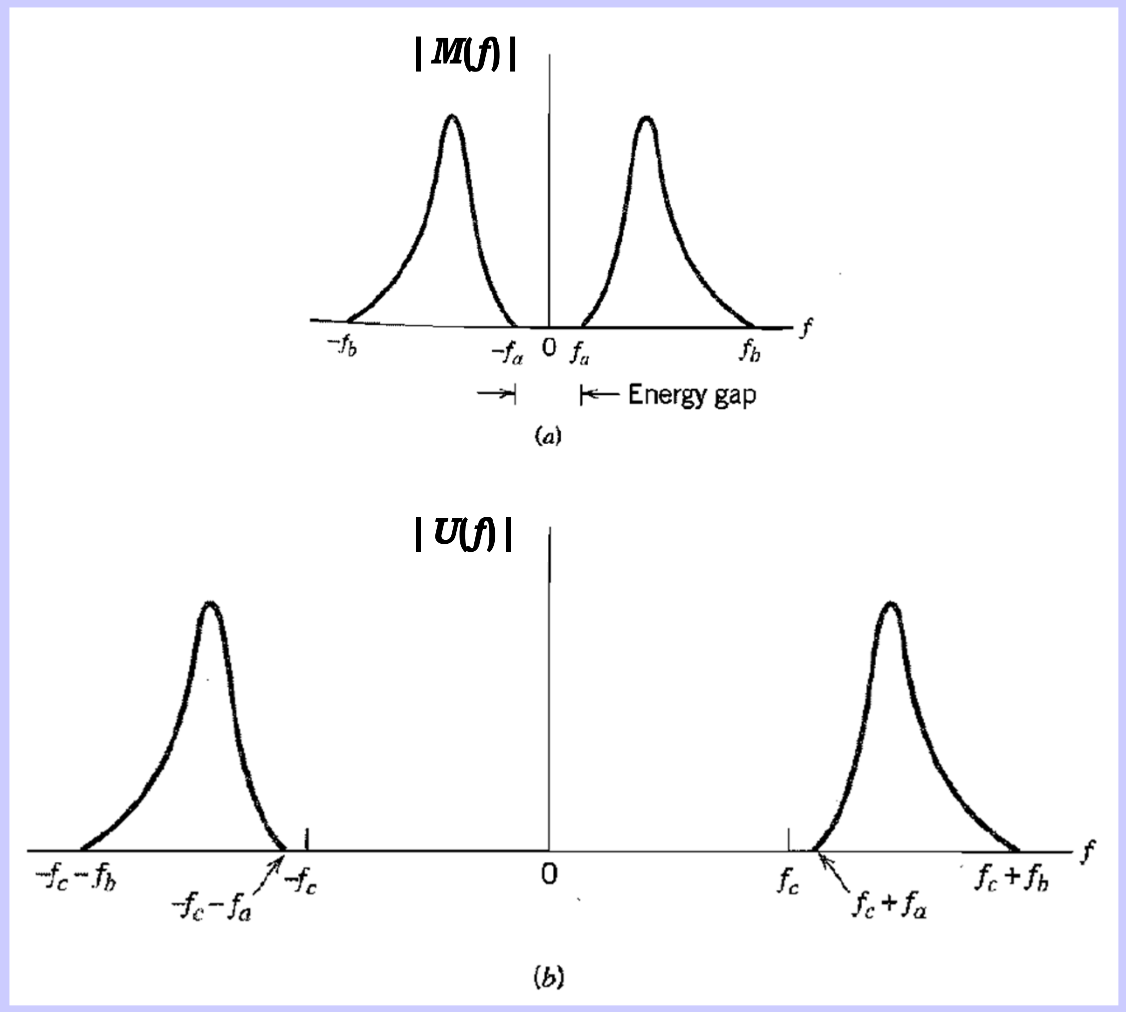
Any practical constraint ??

- **YES. It is difficult to eliminate one sideband if the message signal has large power concentrated in the vicinity of $f = 0$.**

What is an ideal situation ??

- **For useful generation of SSB-AM signal, the message spectrum must have an *energy gap* centered at the origin.**

Demodulation of SSB AM Signals



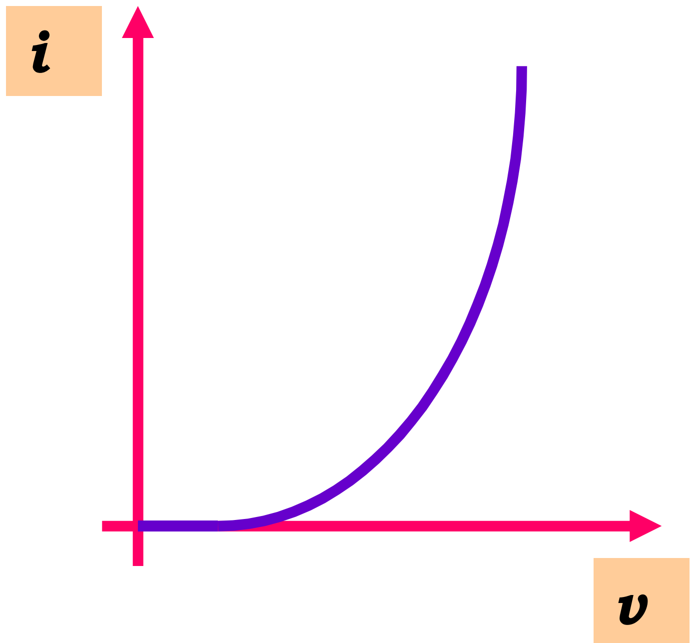
(a) Spectrum of a message signal $m(t)$ with an energy gap of width $2f_a$ centered on the origin. (b) Spectrum of corresponding SSB signal containing the upper sideband.

Implementation of AM Modulators and Demodulators

Conventional DSB AM Modulators

Power-law Modulators

✓ **A nonlinear device like a P-N diode, with a voltage-current characteristic shown below, can be used for modulation.**



Voltage-current characteristic of P-N diode.

The input-output characteristic of the nonlinear device may be governed by a square law:

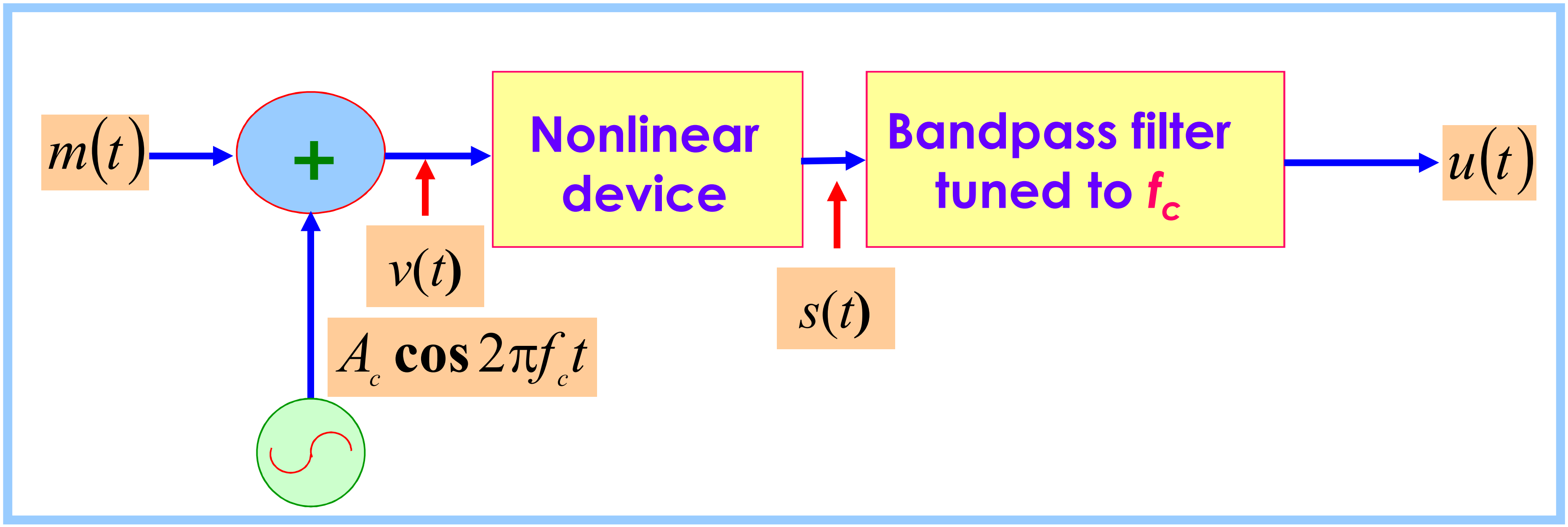
$$v_o(t) = a_1 v_i(t) + a_2 v_i^2(t)$$

$v_i(t)$: input; $v_o(t)$: output;

a_1, a_2 : constant parameters

Conventional DSB AM Modulators

Power-law Modulators



Block-diagram of power-law AM modulator.

Conventional DSB AM Modulators

Power-law Modulators

The input to the nonlinear device: $v_i(t) = m(t) + A_c \cos 2\pi f_c t$

$$v_o(t) = a_1\left[m(t) + A_c \cos 2\pi f_c t\right] + a_2\left[m(t) + A_c \cos 2\pi f_c t\right]^2$$

$$= a_1 m(t) + a_2 m^2(t) + a_2 A_c^2 \cos^2 2\pi f_c t + A_c a_1\left[1 + \frac{2a_2}{a_1} m(t)\right]\cos 2\pi f_c t$$

The output of the bandpass filter with bandwidth $2W$ and centered at $f = f_c$ is:

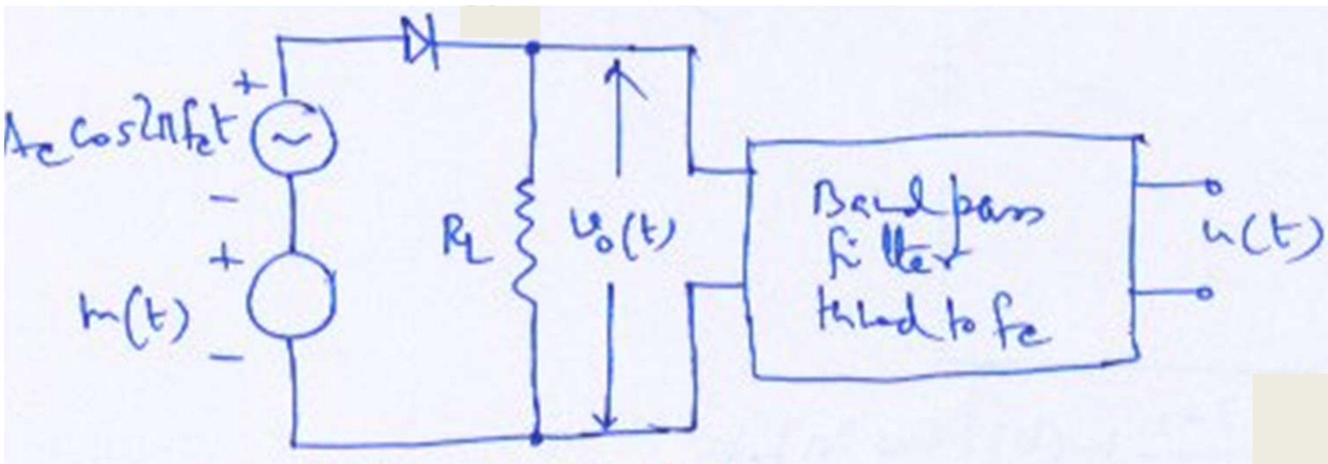
$$u(t) = A_c a_1\left[1 + \frac{2a_2}{a_1} m(t)\right]\cos 2\pi f_c t$$

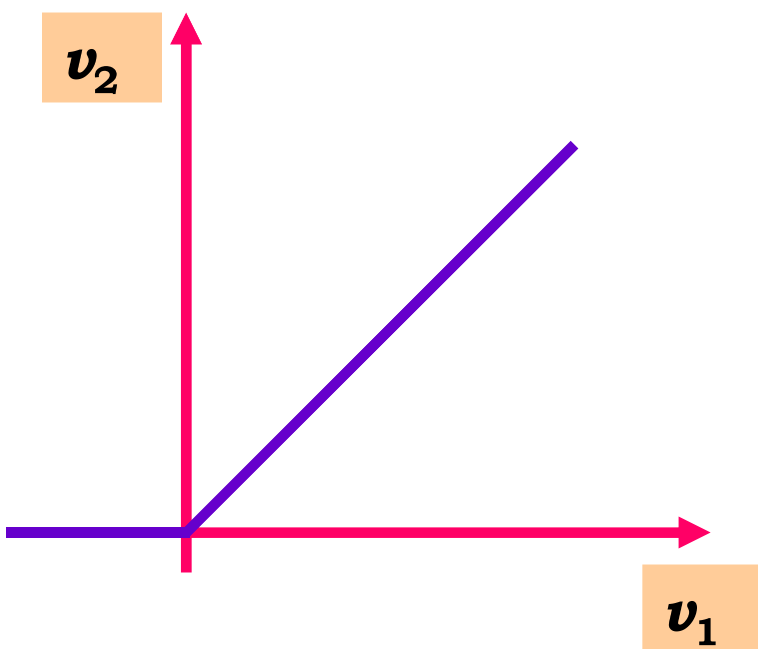
By design, one maintains:

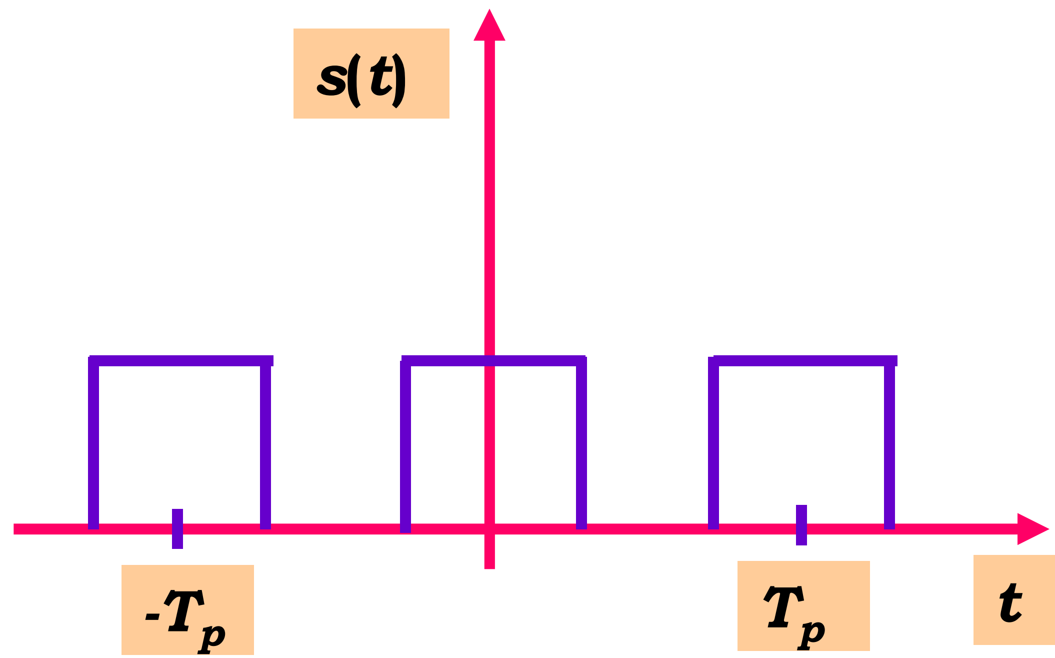
$$\frac{2a_2}{a_1}\left|m(t)\right| < 1$$

Conventional DSB AM Modulators

Switching Modulators







Conventional DSB AM Modulators

Switching Modulators

The input to the diode: $v_i(t) = m(t) + A_c \cos 2\pi f_c t$

The input output characteristic of the diode is shown by the v_2-v_1 curve, with $A_c >> m(t)$. Output across the load resistor:

$$v_o(t) = \begin{cases} v_i(t), & c(t) > 0 \\ 0, & c(t) < 0 \end{cases}$$

This can be viewed as:

$$v_o(t) = v_i(t)s(t) = [m(t) + A_c \cos 2\pi f_c t]s(t)$$

$s(t)$: switching function shown in the figure.

For a periodic function $s(t)$, its Fourier series representation:

$$s(t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos \omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \cdots\right)$$

Conventional DSB AM Modulators

Switching Modulators

Then,

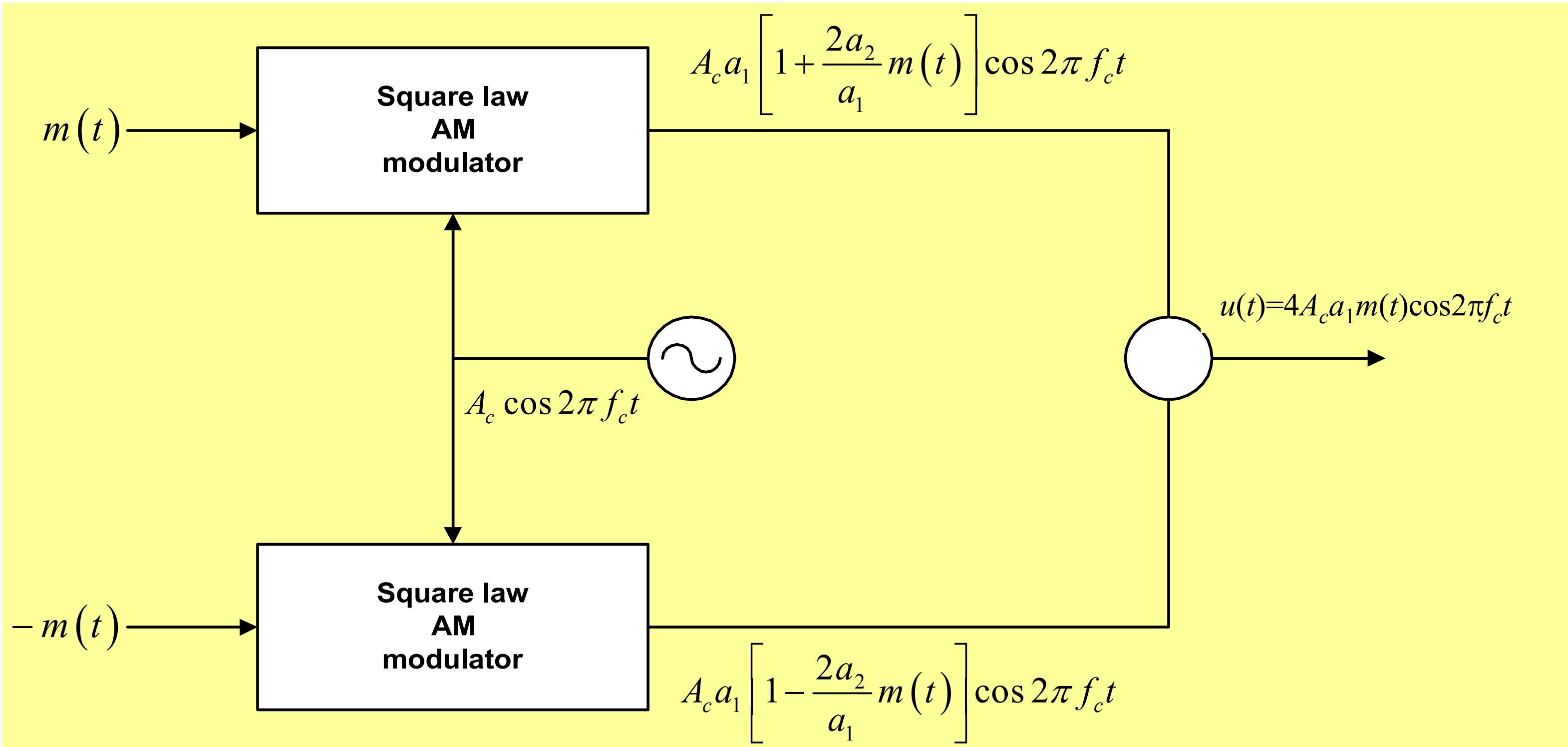
$$v_o(t) = \left[m(t) + A_c \cos 2\pi f_c t\right] s(t)$$
$$= \left[m(t) + A_c \cos 2\pi f_c t\right]$$
$$\left[\frac{1}{2} + \frac{2}{\pi}\left(\cos \omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \cdots\right)\right]$$
$$= \frac{A_c}{2}\left[1 + \frac{4}{\pi A_c} m(t)\right]\cos 2\pi f_c t + \text{other terms}$$

The bandpass filter tuned at f_c (i.e. ω_c) suppresses other terms and the output is the desired conventional DSB AM signal:

$$u(t) = \frac{A_c}{2}\left[1 + \frac{4}{\pi A_c} m(t)\right]\cos 2\pi f_c t$$

DSB-SC AM Modulators

Balanced Modulators

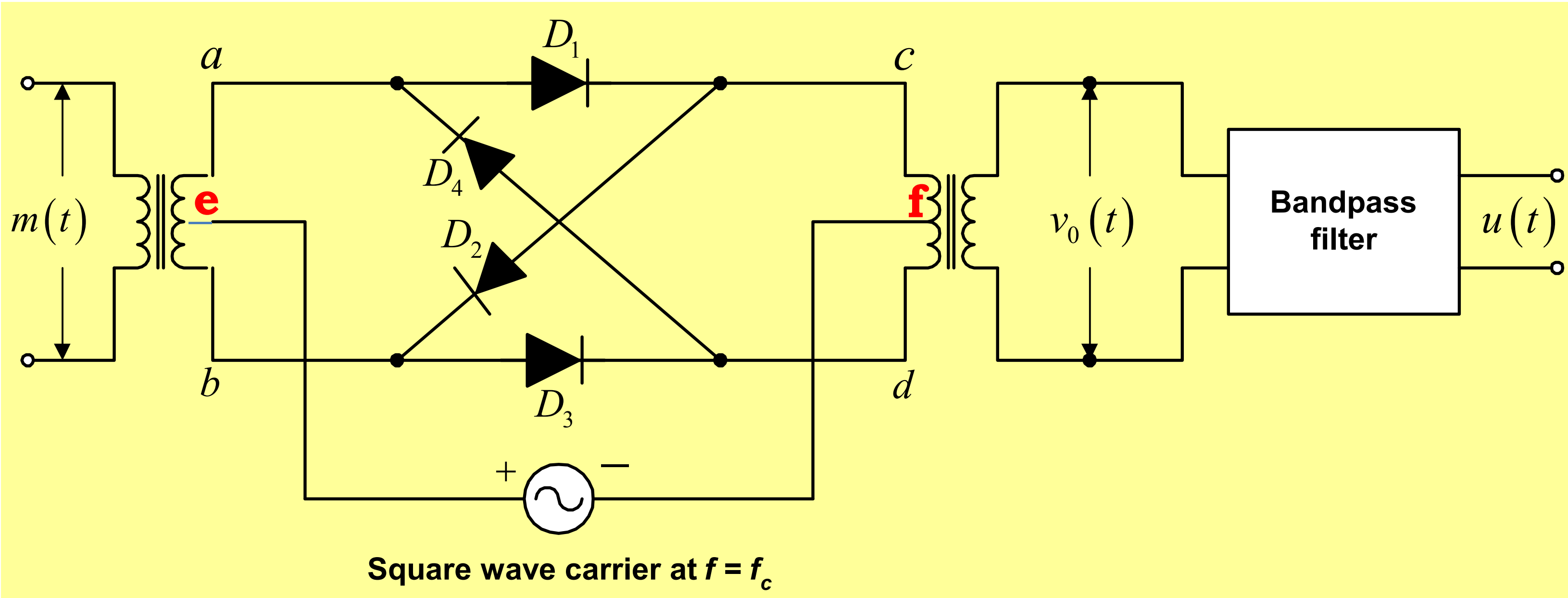


Block-diagram of a balanced modulator.

Note: This is called a single balanced modulator. A circuit balanced with respect to both inputs is called a double balanced modulator e.g. a ring modulator.

DSB-SC AM Modulators

Ring Modulator



A ring modulator.

Mathematically speaking, $v_o(t) = m(t)c(t)$ where $c(t)$ is a square wave of frequency f_c.

DSB-SC AM Modulators

Ring Modulator

As $c(t)$ is a periodic function, its Fourier series representation is:

$$c(t) = \frac{4}{\pi}\left(\cos \omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \cdots \right)$$

$$\therefore v_o(t) = m(t)c(t)$$

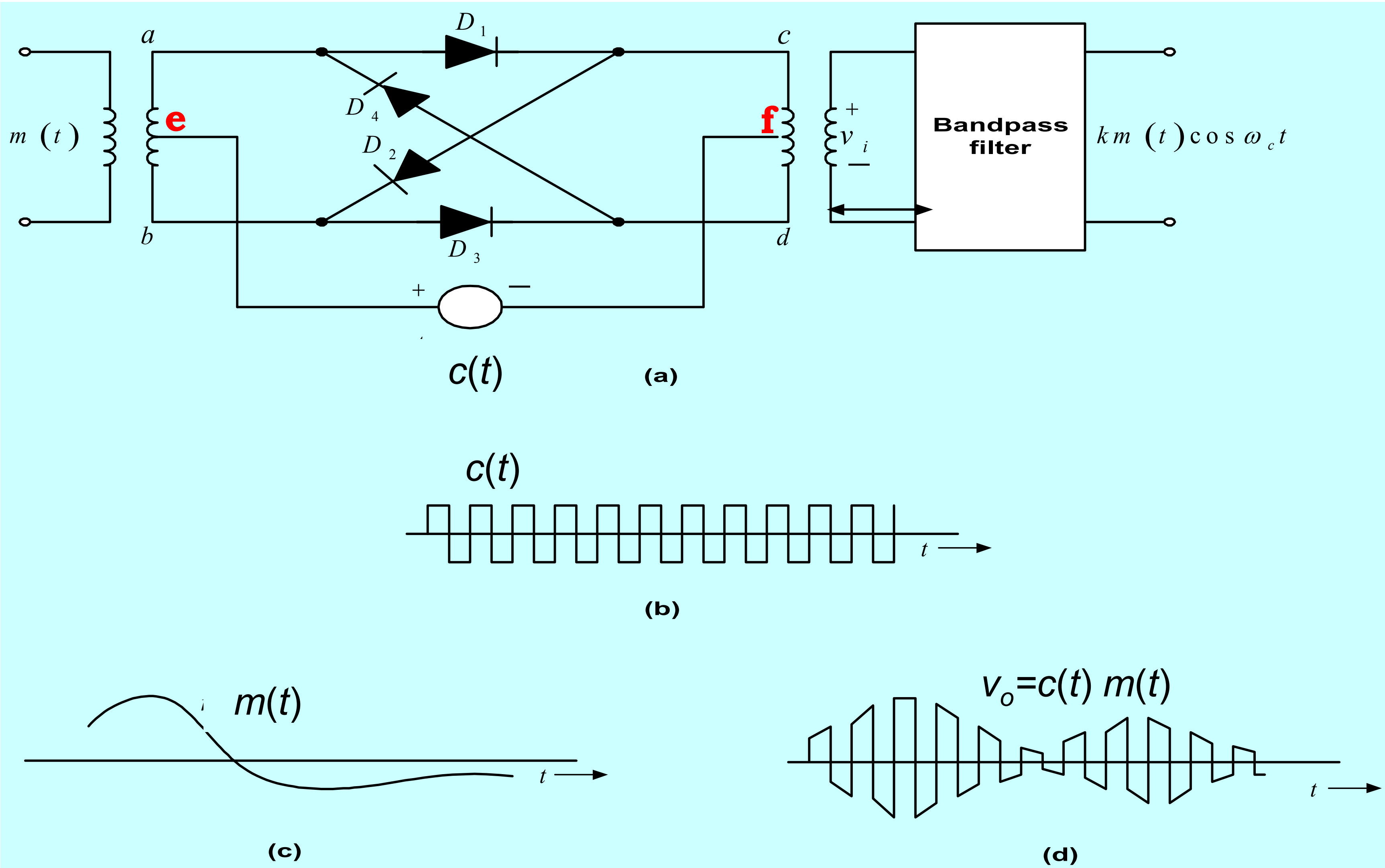
$$= \frac{4}{\pi}\left(m(t)\cos \omega_c t - \frac{1}{3}m(t)\cos 3\omega_c t + \frac{1}{5}m(t)\cos 5\omega_c t - \cdots \right)$$

When this $v_o(t)$ is passed through a bandpass filter with center frequency f_c and bandwidth $2W$, the output:

$$u(t) = \frac{4}{\pi} m(t)\cos 2\pi f_c t$$

DSB-SC AM Modulators

Ring Modulator



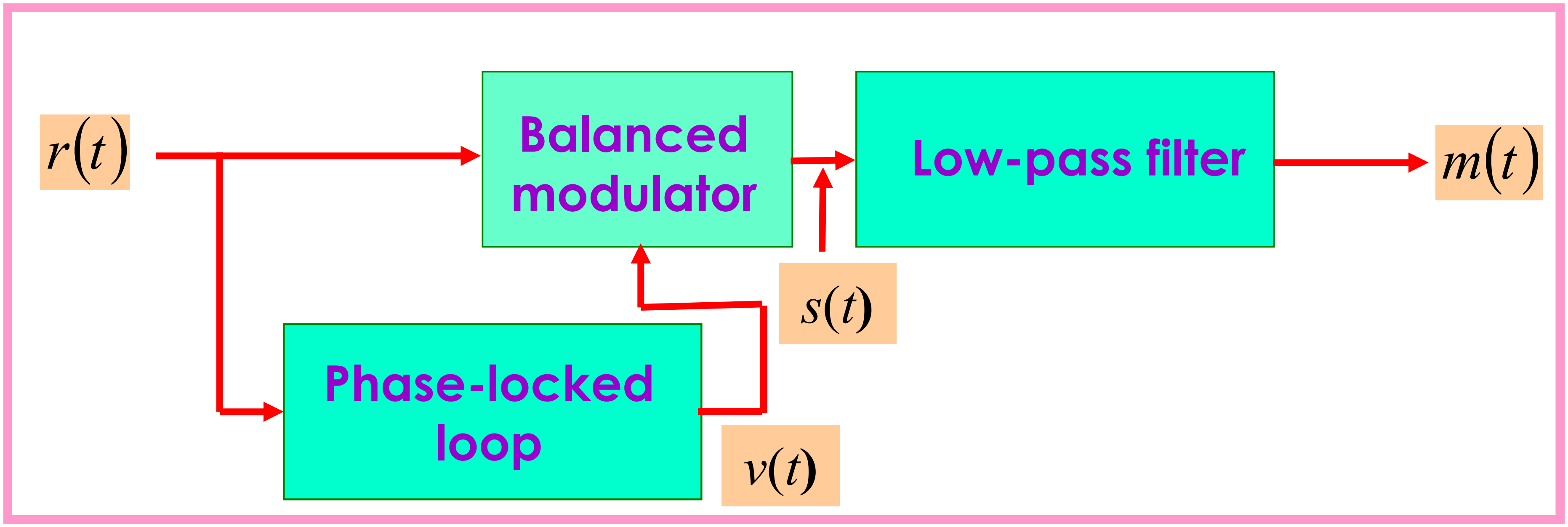
SSB AM Modulators

- **The Hilbert transform based method uses two mixers i.e. two balanced modulators in addition to a Hilbert transformer.**

- **The Frequency discrimination based method uses a single balanced modulator and a sideband filter.**

DSB-SC AM Demodulators

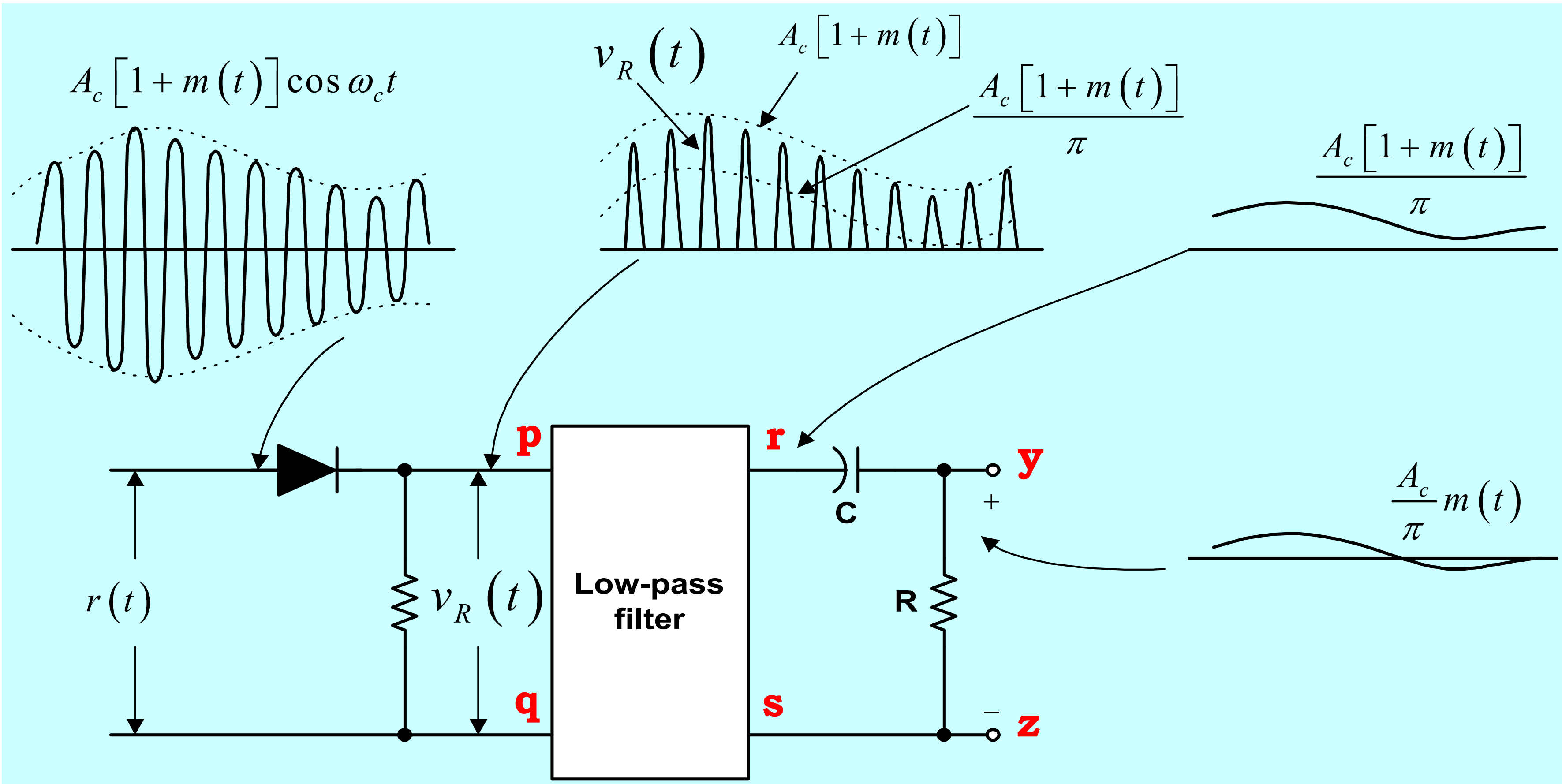
Synchronous or Coherent or Homodyne Demodulator



The low pass filter is of bandwidth W Hz. It passes the desired signal and rejects all signal and noise components above W Hz.

Conventional DSB AM Demodulators

Rectifier Detector



Without loss of generality, we assume $\phi_c = 0$. Here,

$$r(t) = A_c\left[1+m(t)\right]\cos 2\pi f_c t$$

Conventional DSB AM Demodulators

Rectifier Detector

$$r(t) = A_c[1+m(t)]\cos 2\pi f_c t$$

The output across resistor in the input side can be viewed as the AM signal multiplied by the periodic function *s*(*t*):

$$v_R(t) = \{A_c[1+m(t)]\cos\omega_c t\}s(t)$$

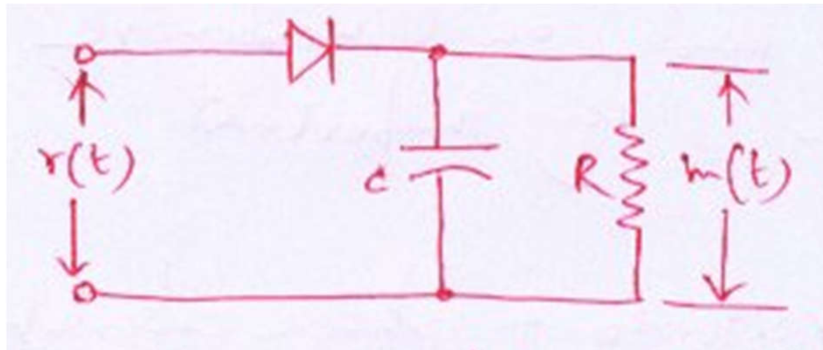
$$= \{A_c[1+m(t)]\cos\omega_c t\}\left[\frac{1}{2}+\frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \cdots\right)\right]$$

$$= \frac{A_c[1+m(t)]}{\pi} + \text{other terms of higher frequencies}$$

The output of the low-pass filter (with cut-off frequency *W* Hz): → $\frac{A_c[1+m(t)]}{\pi}$ → **The final output:** → $\frac{A_c m(t)}{\pi}$

Conventional DSB AM Demodulators

Envelope Detector



The time constant must be so selected that $(1/f_c) << RC << (1/W)$, where W = highest frequency in $m(t)$. The envelope detector output = $A_c[1+m(t)]$.

Conventional DSB AM Demodulators

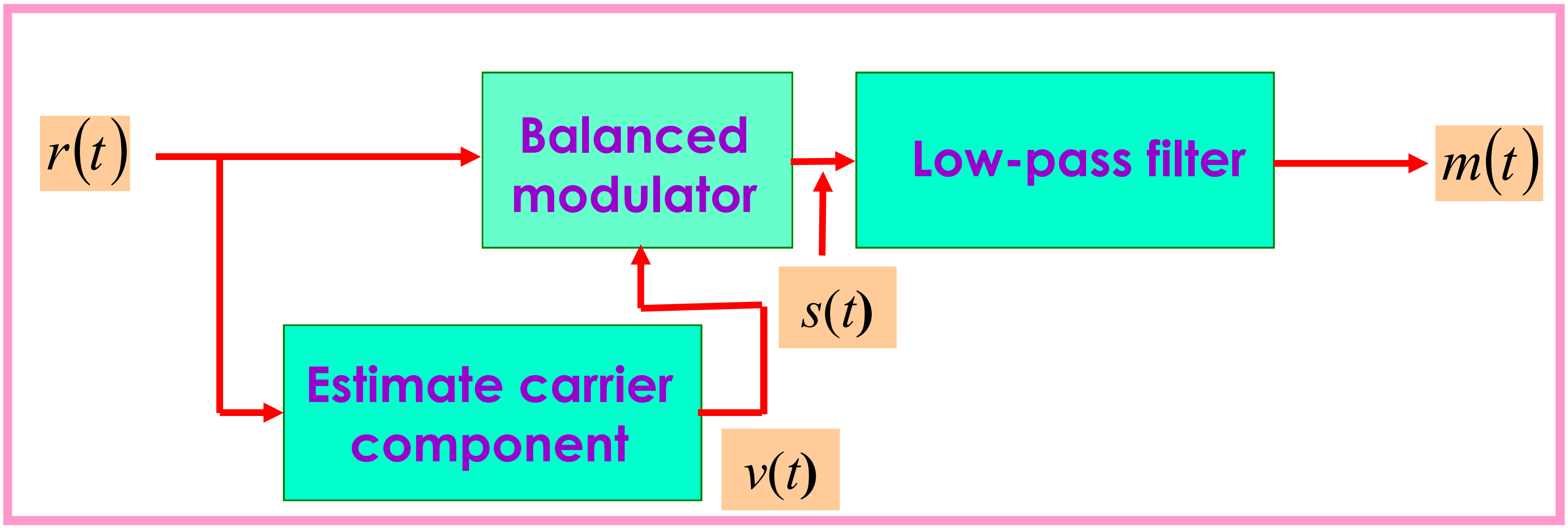
Envelope Detector

- **The simplicity of this demodulator has made conventional DSB AM a practical choice for AM radio broadcasting.**

- **There are literally billions of radio receivers and an inexpensive implementation of the demodulator is very important.**

SSB AM Demodulator

- **Like DSB-SC scheme, this also requires a phase coherent reference. For signals like speech, which have relatively little or no power content at d.c., one can use a demodulator scheme like this.**



Here the balanced modulator is used for frequency conversion of the bandpass signal to low-pass or baseband signal.

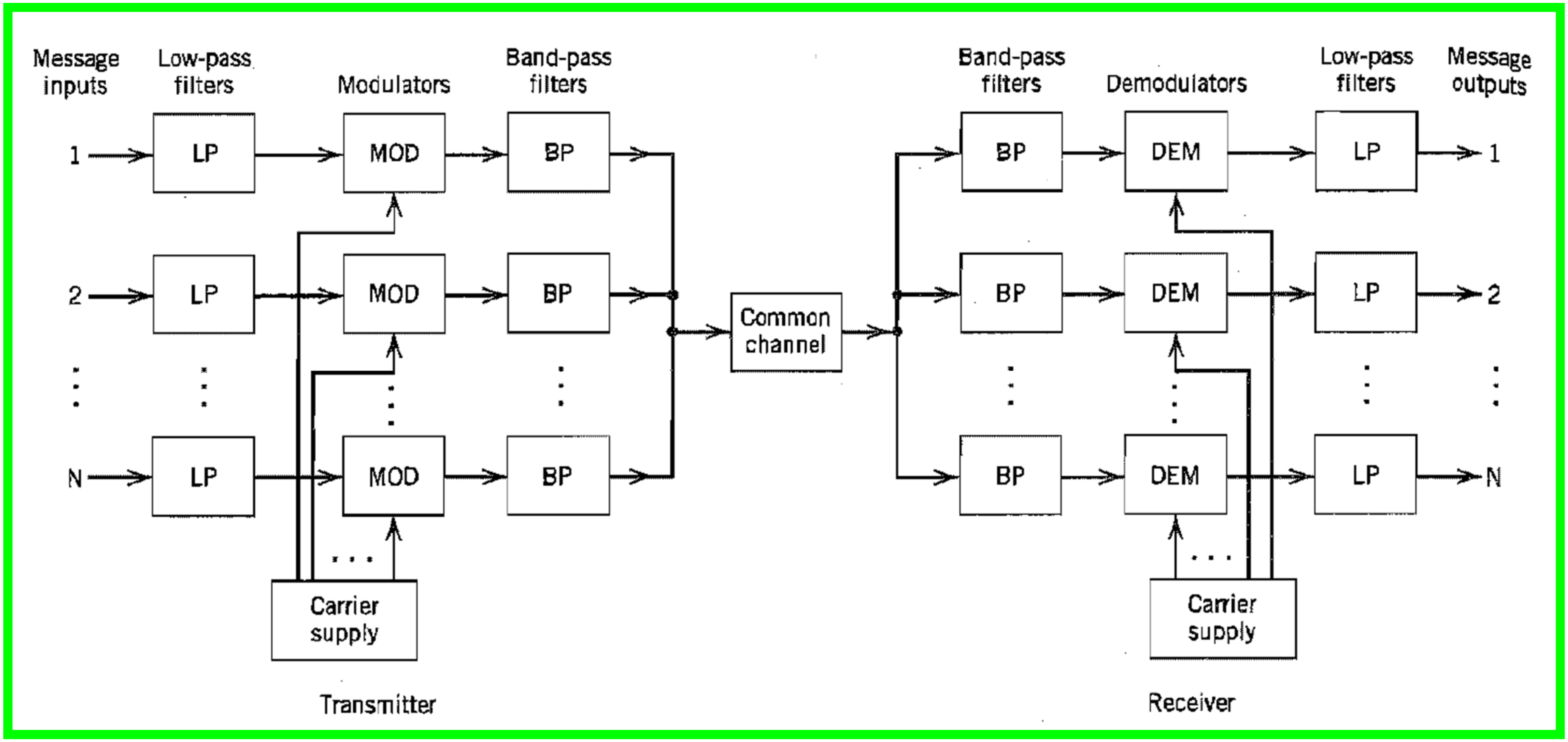
Frequency Division Multiplexing (FDM)

- In multiplexing, a number of independent signals can be combined into a composite signal suitable for transmission over a common channel.

- TDM is used in digital transmission only. But FDM is used both in analog and digital transmission.

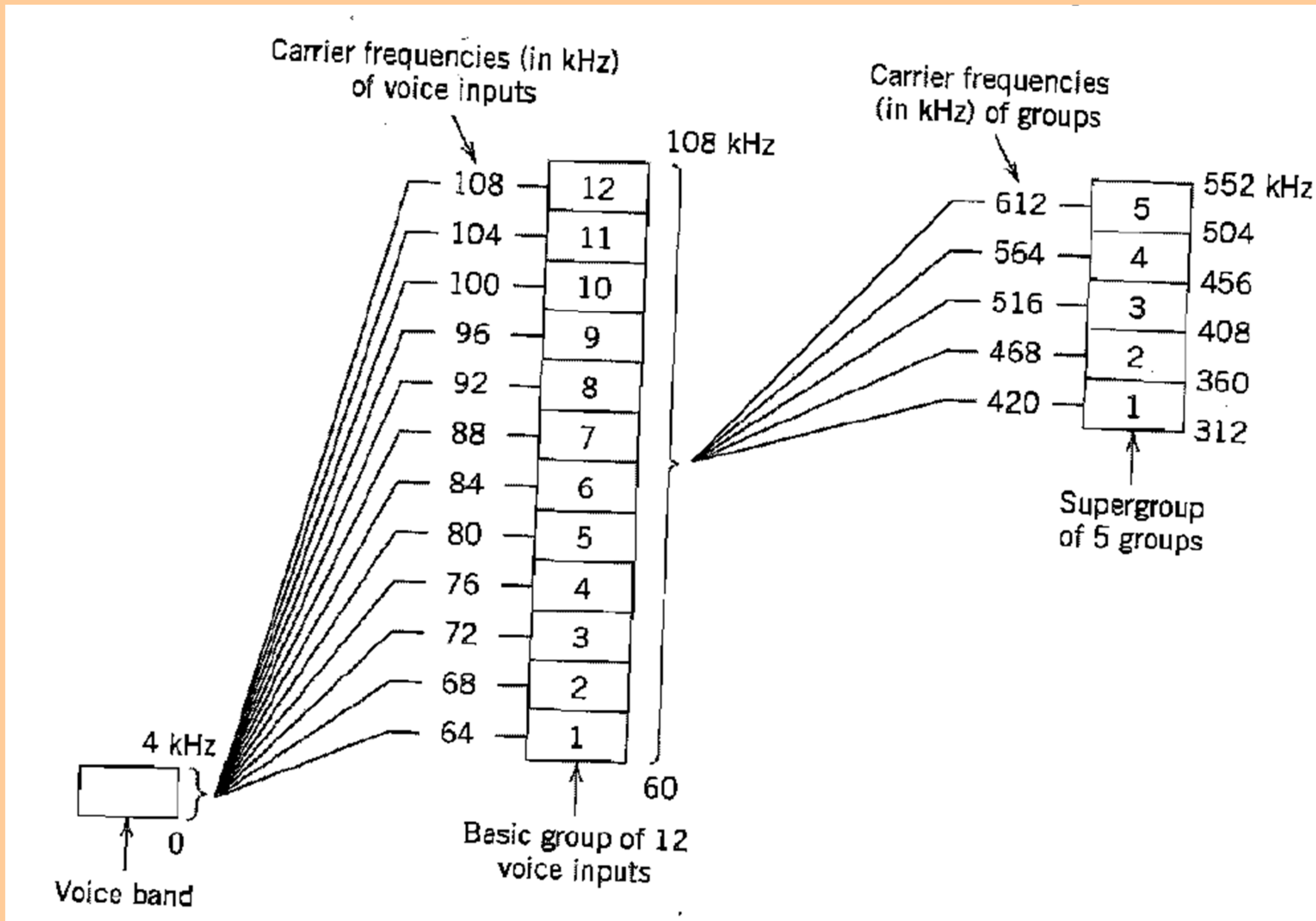
- In FDM, each message signal modulates a carrier of a different frequency, where the minimum separation between two adjacent carriers is either $2W$ (DSB AM) or W (SSB AM) (W = bandwidth of each message signal).

Frequency Division Multiplexing (FDM)



Block diagram of FDM System.

Frequency Division Multiplexing (FDM)



Practical implementation of an FDM System.

ANGLE MODULATION

Angle Modulation

In Angle Modulation, the angle of the modulating sinusoidal carrier wave is varied according to the baseband signal.

Angle Modulation Techniques

Frequency Modulation (FM)

Phase Modulation (PM)

Angle Modulation

Merits and Demerits …

Angle Modulation methods are more complex to implement and more difficult to analyze. In many cases, only an approximate analysis can be done.

Angle Modulation possesses a bandwidth expansion property. FM and PM generally expand the bandwidth such that the effective bandwidth of the modulated signal is usually many times the bandwidth of the message signal.

So increased transmission bandwidth and increased complexity are the principle demerits of angle modulation methods.

However, the major benefit is that these systems provide high degree of noise immunity i.e. better discrimination against noise and interference than AM signals.

Angle Modulation

Let $\theta_i(t)$ be the **'angle'** of a modulated sinusoidal carrier and $\theta_i(t)$ be a function of the message signal $m(t)$.

Implication

The angle modulated wave is: $u(t) = A_c \cos[\theta_i(t)]$.

A_c = **amplitude of the carrier wave.**

A ***complete oscillation*** occurs whenever $\theta_i(t)$ changes by 2π radians. If $\theta_i(t)$ increase **monotonically with time**, the **average frequency in Hertz** over an interval from t to $t+\Delta t$ is:

$$f_{\Delta t}(t) = \frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi \Delta t}$$

Angle Modulation

Then one can define 'instantaneous frequency' of the angle modulated wave:

$$f_i(t) = \lim_{\Delta t \to 0} f_{\Delta t}(t) = \lim_{\Delta t \to 0} \left[\frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi \Delta t} \right] = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}$$

Implication

Hence we can say $u(t)$ is a rotating phasor of length A_c and angle $\theta_i(t)$. Angular velocity of such a phasor = $(d\theta_i(t)/dt)$ (in rad/sec.).

In the simple case of an *unmodulated carrier*, $\theta_i(t) = 2\pi f_c t + \phi_c$. This phasor rotates with a constant angular velocity of $2\pi f_c$. ϕ_c = value of $\theta_i(t)$ when $t = 0$.

Angle Modulation

There are two common methods of varying $\theta_i(t)$.

1. Phase Modulation (PM): Here $\theta_i(t)$ is varied linearly with $m(t)$.

$$\theta_i(t) = 2\pi f_c t + k_p m(t)$$

$$2\pi f_c t = \text{angle of the unmodulated carrier}$$

$$k_p = \text{phase sensitivity of the modulator (rad/volt)}$$

Assumptions: $m(t)$ is a voltage waveform. Also, angle of the unmodulated carrier is chosen zero at $t = 0$ (for convenience).

Therefore, $u(t) = A_c \cos\left[2\pi f_c t + k_p m(t)\right]$

Angle Modulation

2. Frequency Modulation (FM): Here instantaneous frequency $f_i(t)$ is varied linearly with $m(t)$.

$$f_i(t) = f_c + k_f m(t)$$

f_c = frequency of the unmodulated carrier

k_f = frequency sensitivity of the modulator (Hz/volt)

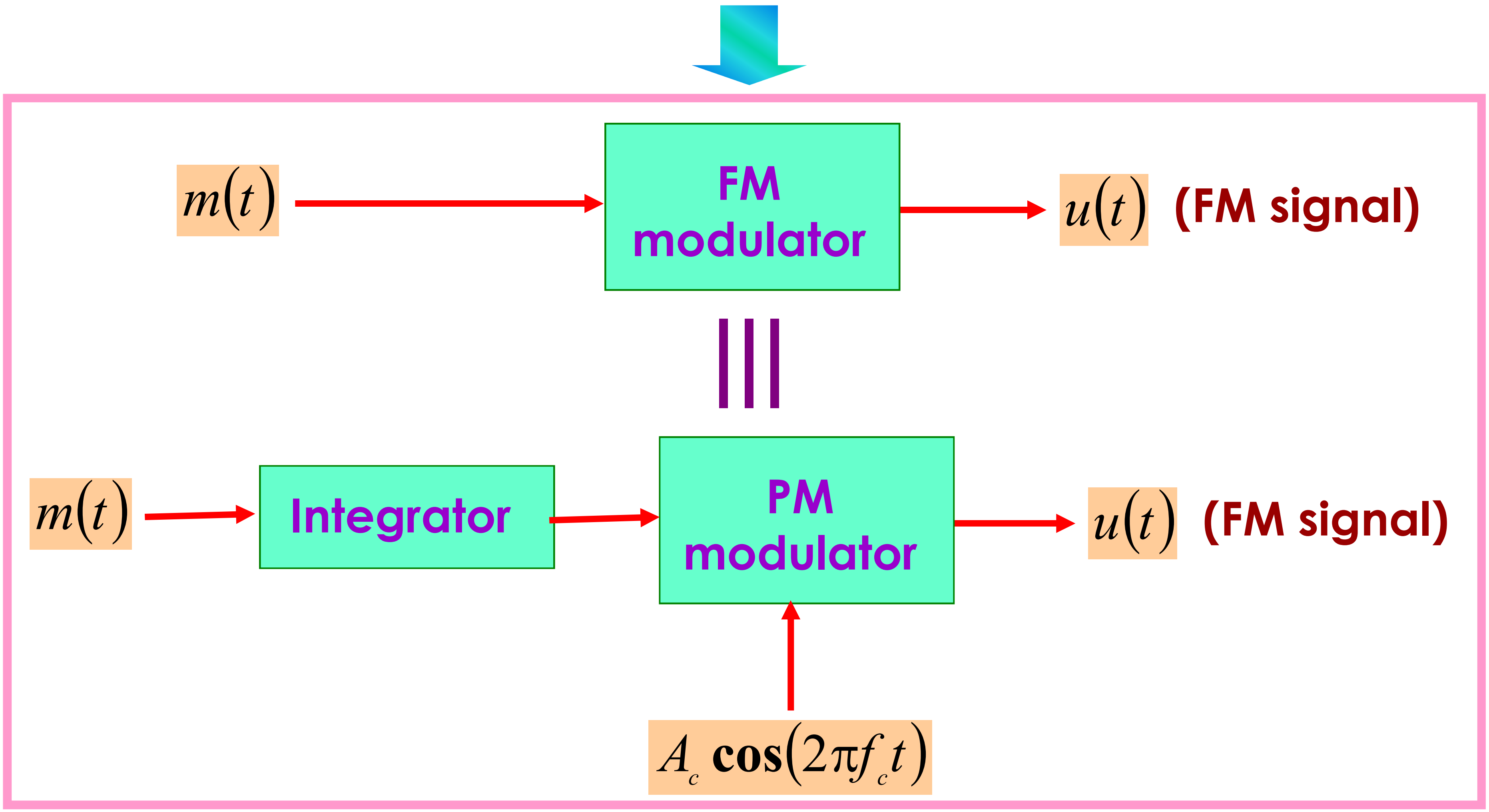
$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau$$

Here we have made the same assumptions as before.

Therefore, $u(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right]$

Angle Modulation

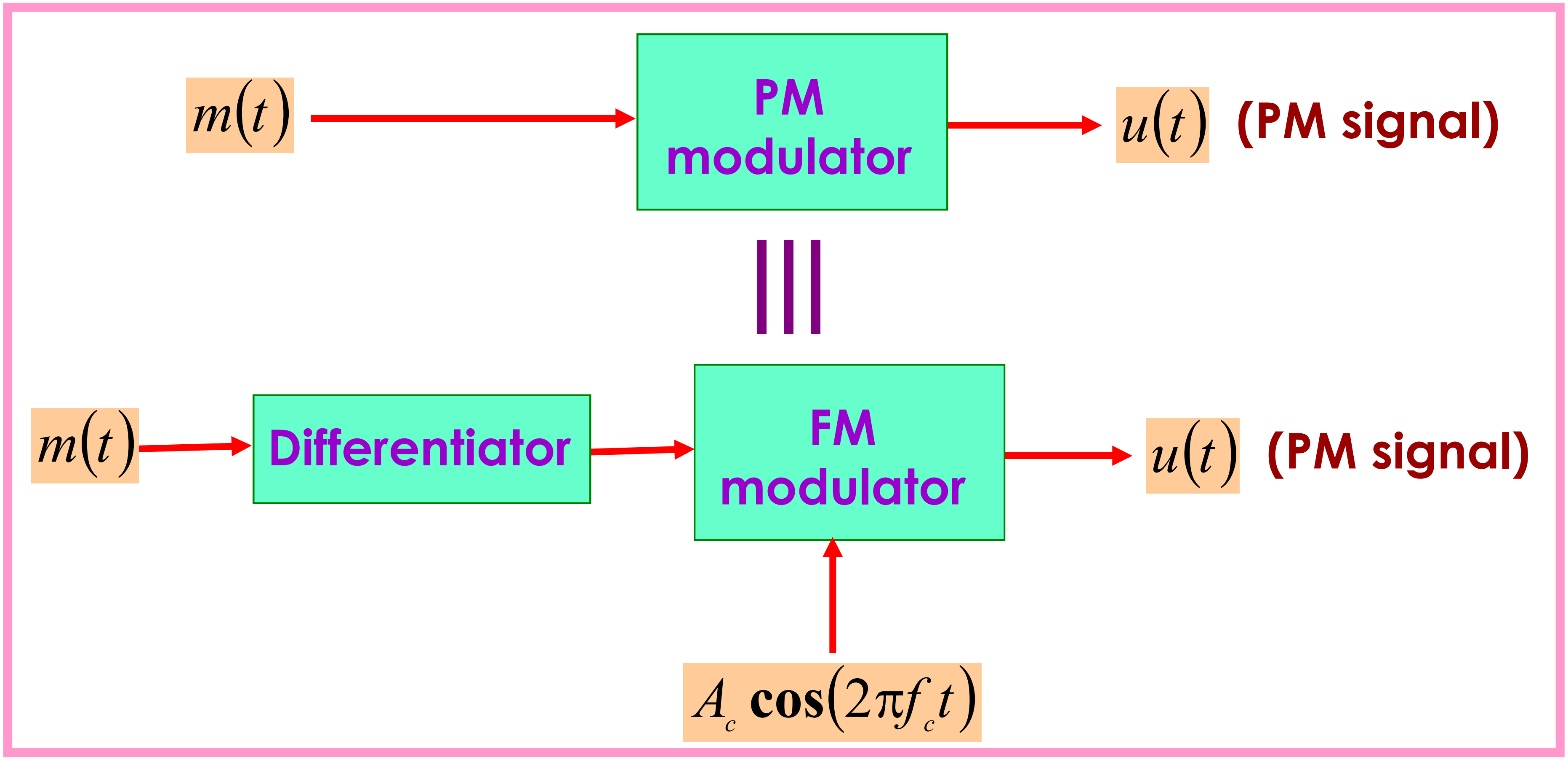
An FM signal may be regarded as a PM signal whose modulating wave is $\int_0^t m(\tau)d\tau$ in place of $m(t)$.



Generation of FM signal using a Phase Modulator.

Angle Modulation

Similarly, a PM signal can be generated by first differentiating $m(t)$ and then using the result as an input to a frequency modulator.



Generation of PM signal using a Frequency Modulator.

Angle Modulation

Note: **Since $\theta_i(t)$ is dependent on the message signal $m(t)$ or on its integral, hence zero-crossings of PM or FM signal no longer have a perfect regularity in their spacing. These zero crossings refer to the instants of time at which a waveform changes from a negative to a positive value or vice-versa. This characteristics of angle modulated signal distinguishes them from AM signals.**

Frequency Modulation

- **FM signal $u(t)$ is a nonlinear function of $m(t)$. Hence frequency modulation is a nonlinear modulation process. Hence spectrum of FM signal is not related in a simple manner to $m(t)$ as in the case of AM.**

One can consider two situations.

Situation I: In the simplest case, we consider a single-tone modulation that produces a Narrowband FM signal.

Situation II: In the general case, we consider a single-tone modulation where the FM signal is wideband.

Frequency Modulation

Let the sinusoidal modulating signal be:

$$m(t) = A_m \cos(2\pi f_m t)$$

The instantaneous frequency of the resulting FM signal:

$$f_i(t) = f_c + k_f A_m \cos(2\pi f_m t)$$
$$= f_c + \Delta f \cos(2\pi f_m t)$$

$\Delta f = k_f A_m$ = **frequency deviation = maximum departure of f_i from the carrier frequency f_c.**

The angle of the FM signal:

$$\theta_i(t) = 2\pi\int_0^t f_i(\tau)d\tau = 2\pi\int_0^t [f_c + \Delta f \cos(2\pi f_m \tau)]d\tau$$
$$= 2\pi f_c t + \frac{\Delta f}{f_m}\sin(2\pi f_m t)$$

Frequency Modulation

The angle of the FM signal:

$$\theta_i(t) = 2\pi\int_0^t f_i(\tau)d\tau = 2\pi\int_0^t \left[f_c + \Delta f\,\mathbf{cos}(2\pi f_m\tau)\right]d\tau$$

$$= 2\pi f_c t + \frac{\Delta f}{f_m}\,\mathbf{sin}(2\pi f_m t)$$

We denote:

$$\beta = \frac{\Delta f}{f_m} = \text{modulation index of FM signal.}$$

$$\theta_i(t) = 2\pi f_c t + \beta\,\mathbf{sin}(2\pi f_m t)$$

∴ FM signal:

$$u(t) = A_c\,\mathbf{cos}\left[2\pi f_c t + \beta\,\mathbf{sin}(2\pi f_m t)\right]$$

- **For narrowband FM, β is small compared to 1 radian.**
- **For wideband FM, β is large compared to 1 radian.**

Frequency Modulation

Note: **The value β represents the phase deviation of the FM signal, i.e. the maximum departure of the angle $\theta_i(t)$ from the angle $2\pi f_c t$ of the unmodulated carrier. Hence, β is measured in radians.**

Narrowband Frequency Modulation

Let us consider FM signals generated using a sinusoidal modulating signal:

$$u(t) = A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right]$$

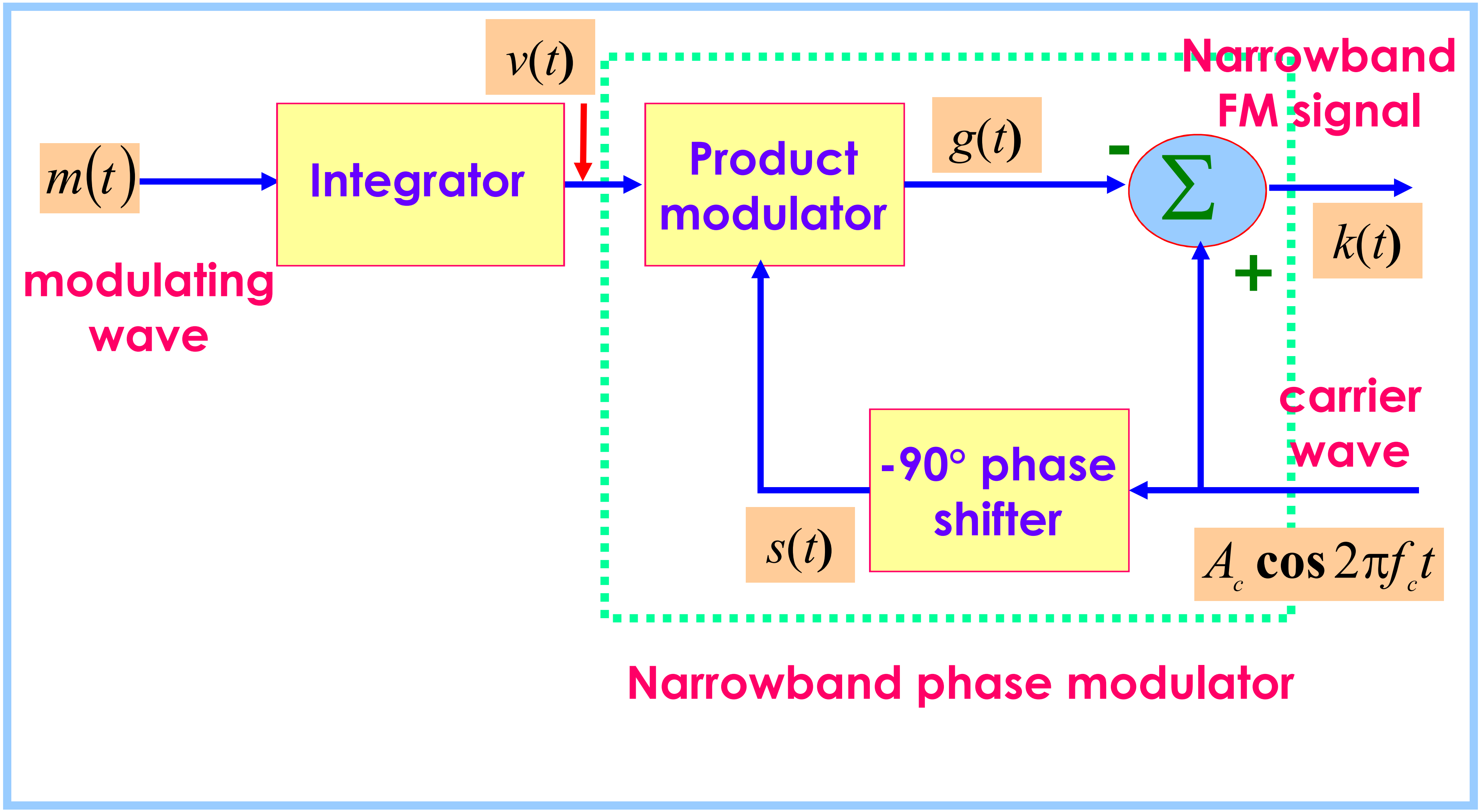
$$u(t) = A_c \cos(2\pi f_c t)\cos\left[\beta \sin(2\pi f_m t)\right] - A_c \sin(2\pi f_c t)\sin\left[\beta \sin(2\pi f_m t)\right]$$

for β *small compared to 1 radian*

$$u(t) \approx A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t)\sin(2\pi f_m t)$$

This equation defines the approximate form of a narrowband FM signal produced by a sinusoidal modulating signal $A_m \cos(2\pi f_m t)$.

Narrowband Frequency Modulation



Block diagram for generation of narrowband FM signal.

Narrowband Frequency Modulation

- Ideally, an FM signal has a constant envelope and, for the case of a sinusoidal modulating frequency f_m, $\theta_i(t)$ is also sinusoidal with same frequency.

- But the modulated signal produced by the narrowband modulator differs from the ideal condition in two fundamental aspects:

I. The envelope contains a *residual* amplitude modulation and, therefore, varies with time.

II. For a sinusoidal modulating wave, the angle $\theta_i(t)$ contains harmonic distortion in the form of third- and higher-order harmonics of the modulation frequency f_m.

Narrowband Frequency Modulation

The signal *u*(*t*) can be expanded as:

$$u(t) = A_c \mathbf{cos}(2\pi f_c t) + \frac{1}{2}\beta A_c \{\mathbf{cos}[2\pi(f_c + f_m)t] - \mathbf{cos}[2\pi(f_c - f_m)t]\}$$

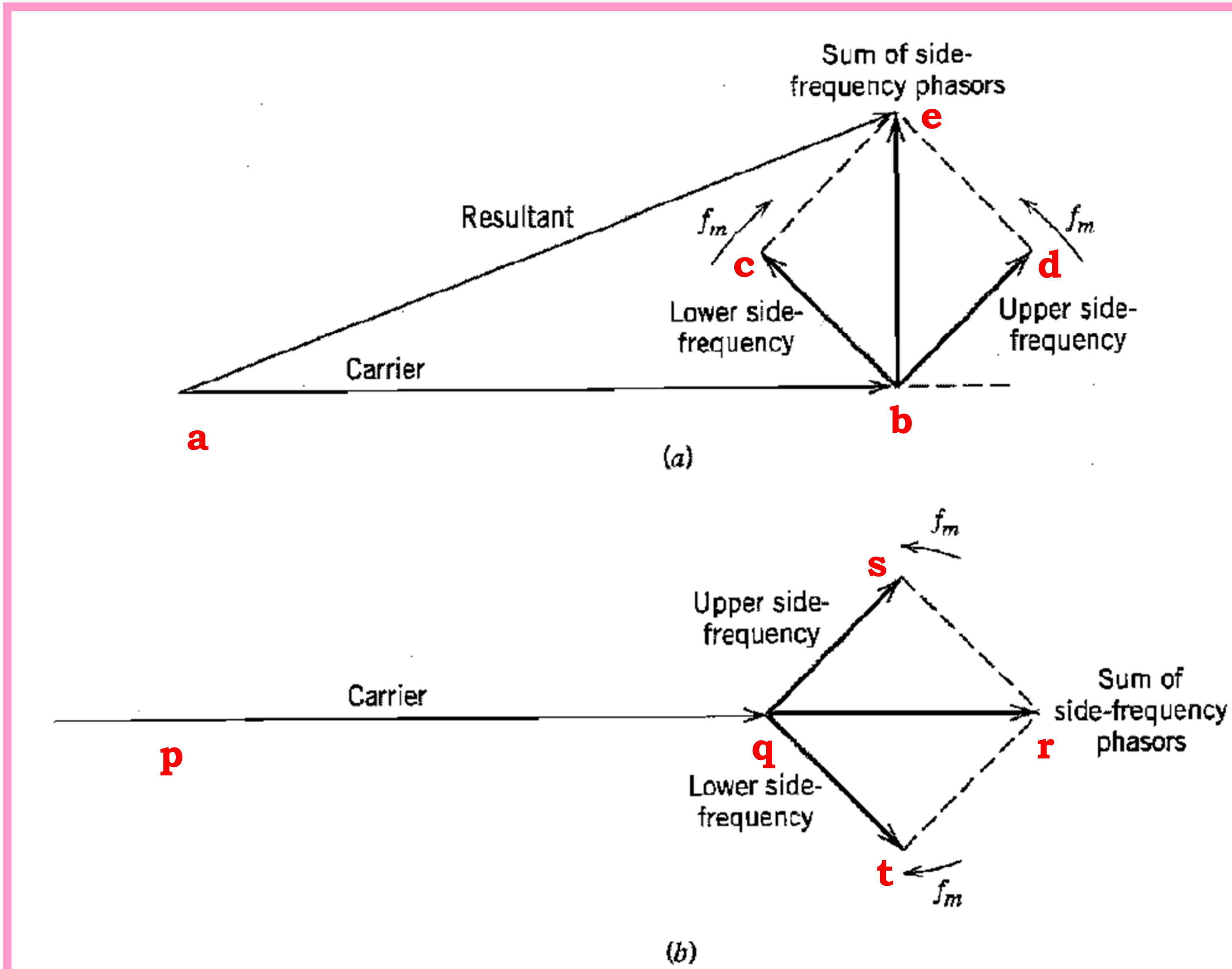
This is similar to an AM signal:

$$u_{AM}(t) = A_c\, cos(2\pi f_c t) + \frac{1}{2}\mu A_c \{cos[2\pi(f_c + f_m)t] + cos[2\pi(f_c - f_m)t]\}$$

μ = modulation factor of an AM signal.

The only difference is the *sign of the lower side frequency* in the narrowband FM which is reversed. Hence a narrowband FM requires essentially the same bandwidth of transmission ($2f_m$) as the AM signal. Also it can be noted that *narrowband frequency modulation* method does not provide better noise immunity than a conventional AM system.

Narrowband Frequency Modulation



Phasor comparison of (a) narrowband FM signal and (b) AM signal.

Wideband Frequency Modulation

Let us consider the single-tone FM signal once more:

$$u(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$$

for an arbitrary value of β

Let us assume that f_c is large enough compared to the bandwidth of the FM signal:

$$u(t) = \mathrm{Re}[A_c \exp(j2\pi f_c t + j\beta \sin(2\pi f_m t))]$$

$$= \mathrm{Re}[\tilde{u}(t)\exp(j2\pi f_c t)]$$

$\tilde{u}(t)$: complex envelope of the FM signal $u(t)$

$$\tilde{u}(t) = A_c \exp[j\beta \sin(2\pi f_m t)]$$

Wideband Frequency Modulation

$$\tilde{u}(t) = A_c \exp[j\beta \sin(2\pi f_m t)]$$

Unlike the original FM signal $u(t)$, the complex envelope $\tilde{u}(t)$ is a periodic function of time with a fundamental frequency of f_m.

Expanding $\tilde{u}(t)$ in form of a complex Fourier series:

$$\tilde{u}(t) = \sum_{n=-\infty}^{\infty} c_n \exp[j2\pi n f_m t]$$

$$c_n = f_m \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} \tilde{u}(t) \exp(-j2\pi n f_m t)dt = f_m A_c \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} \exp[j\beta \sin(2\pi f_m t) - j2\pi n f_m t]dt$$

Wideband Frequency Modulation

$$\tilde{u}(t) = \sum_{n=-\infty}^{\infty} c_n \exp[j2\pi n f_m t]$$

$$c_n = f_m \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} \tilde{u}(t)\exp(-j2\pi n f_m t)dt = f_m A_c \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} \exp[j\beta \sin(2\pi f_m t) - j2\pi n f_m t]dt$$

Let $x = 2\pi f_m t$. Then c_n can be rewritten as:

$$c_n = \frac{A_c}{2\pi} \int_{-\pi}^{\pi} \exp[j(\beta \sin x - nx)]dx$$

This expression contains *nth order Bessel function of the first kind and argument β*. This is commonly denoted by the symbol $J_n(\beta)$.

Wideband Frequency Modulation

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} exp[j(\beta \sin x - nx)]dx$$

Then $c_n = A_c J_n(\beta)$. Hence,

$$\tilde{u}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) exp(j2\pi n f_m t) \quad \text{and}$$

$$u(t) = A_c . Re\left[\sum_{n=-\infty}^{\infty} J_n(\beta) exp\; j2\pi(f_c + nf_m)t\right]$$

Interchanging the order of summation and evaluation of the real part in the right-hand side of this equation:

$$u(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \mathbf{cos}[2\pi(f_c + nf_m)t]$$

Wideband Frequency Modulation

$$u(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos[2\pi(f_c + nf_m)t]$$

This is the desired form for the Fourier series representation of the single-tone FM signal *u*(*t*) for an arbitrary β. The discrete spectrum of *u*(*t*) is obtained by taking the Fourier transforms of both sides of the last equation.

$$U(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta)[\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m)]$$

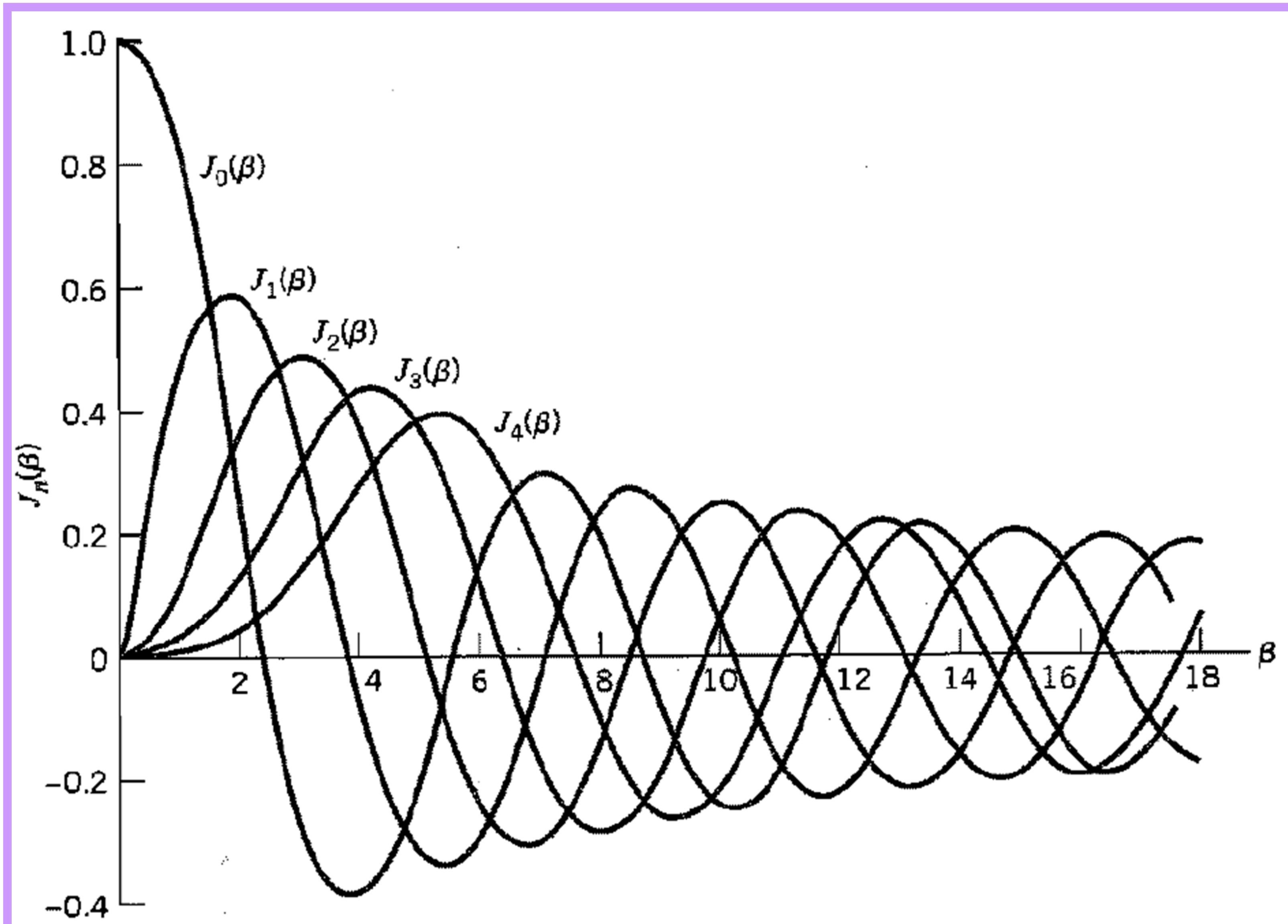
Wideband Frequency Modulation

$$u(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos\left[2\pi\left(f_c + nf_m\right)t\right]$$

This relation shows that, even in this very simple case where the modulation signal is a pure sinusoid of frequency f_m, the angle modulated signal contains all frequencies of the form f_c+nf_m for $n=0,\pm1,\pm2\ldots$ Therefore, the actual bandwidth of the modulated signal is infinite!

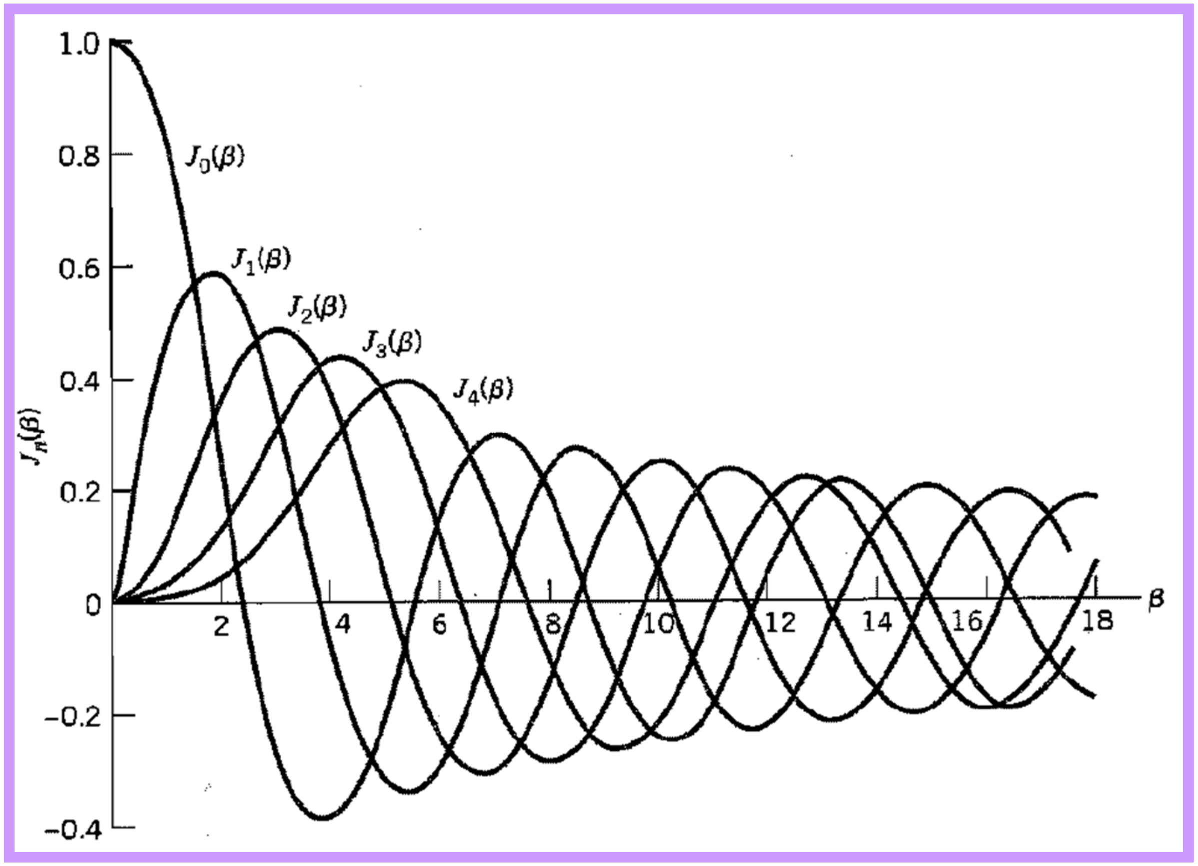
For small β, a good approximation is $J_n(\beta) \approx \beta^n/(2^n n!)$.

Wideband Frequency Modulation



Plot of Bessel functions of first kind for varying order.

Wideband Frequency Modulation



- **The properties of Bessel function $J_n(\beta)$:**
- $J_n(\beta) = (-1)^n J_{-n}(\beta)$ **for all n both +ve and –ve.**
- **For small β, $J_0(\beta) \approx 1$, $J_1(\beta) \approx (\beta/2)$, $J_n(\beta) \approx 0$ (n > 2)**

$$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$$

Wideband Frequency Modulation

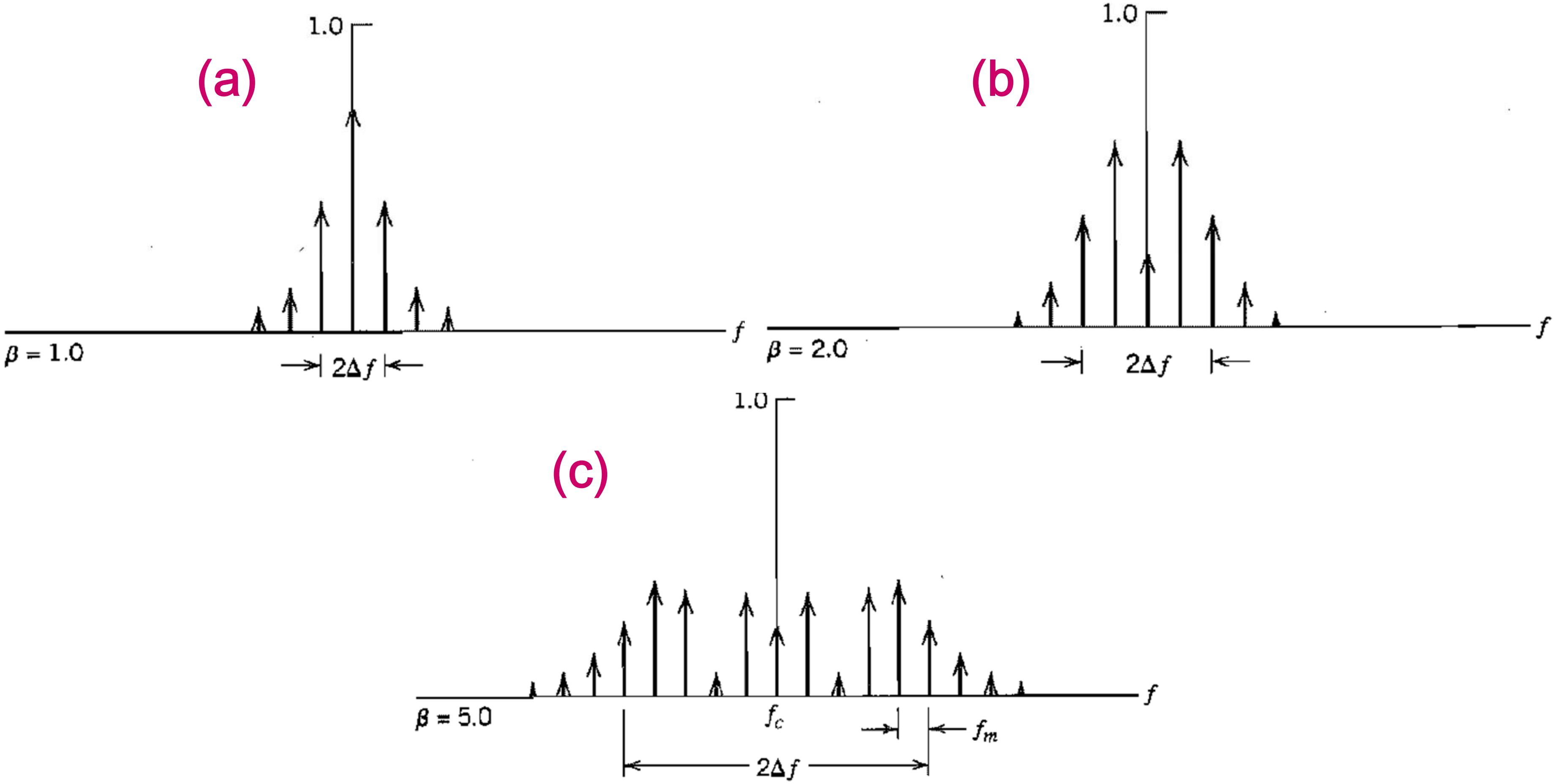
Observations and Conclusions…

- **The spectrum of an FM signal contains a carrier component and an infinite set of side frequencies localted symmetrically on either side of the carrier at frequency separations of f_m, $2f_m$, $3f_m$ … This is unlike AM where a sinusoidal modulating signal gives rise to only one pair of side frequencies.**

- **For the special case of β small compared to unity, only Bessel coefficients $J_0(\beta)$ and $J_1(\beta)$ have significant values, so that the FM signal is effectively composed of a carrier and single pair of side frequencies at $f_c \pm f_m$. This situation corresponds to the special case of narrowband FM.**

Wideband Frequency Modulation

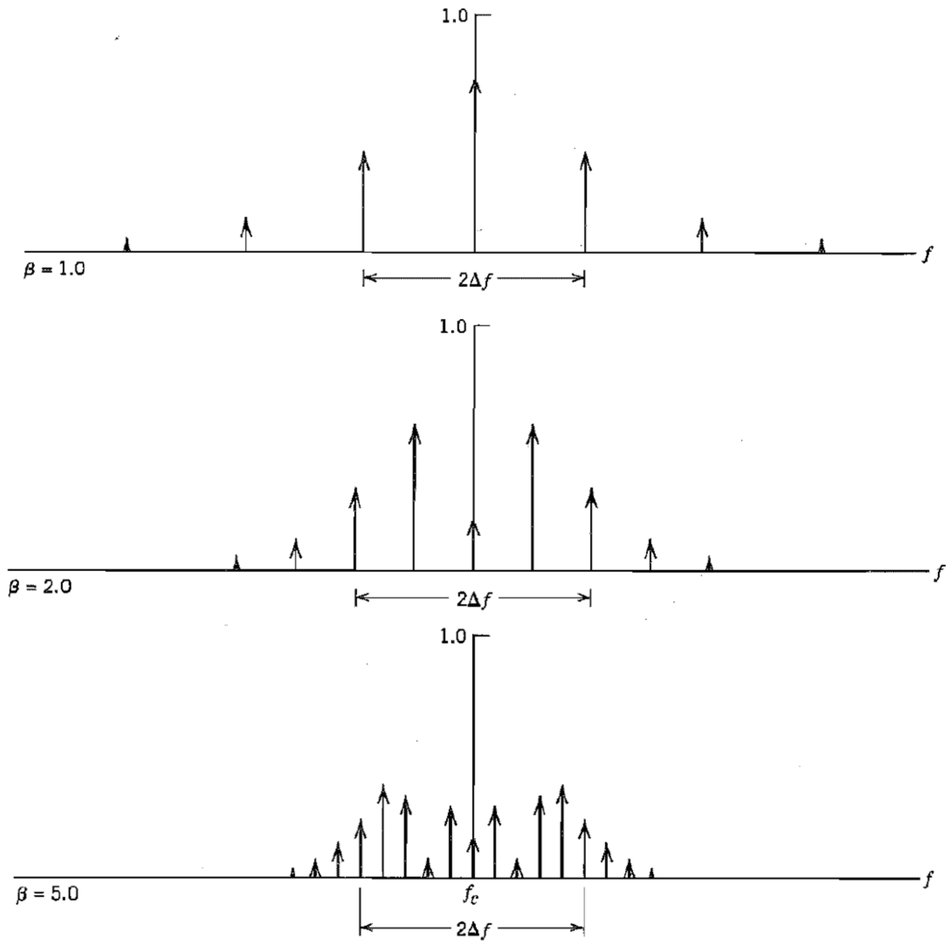
Case I: Spectra of the FM signal, when frequency of the modulating signal is fixed, but its amplitude is varied.



Discrete amplitude spectra of an FM signal, normalized w.r.t. the carrier amplitude (only positive frequencies are shown).

Wideband Frequency Modulation

Case II: Spectra of the FM signal, when amplitude of the modulating signal is fixed, but its frequency f_m is varied.



Discrete amplitude spectra of an FM signal, normalized w.r.t. the carrier amplitude (only positive frequencies are shown).

Transmission Bandwidth of FM Signals

An effective bandwidth can be specified for the transmission of an FM signal.

For large β, the bandwidth approaches, and is only slightly greater than, the total frequency excursion $2\Delta f$.

For small β, the spectrum of the FM signal is effectively limited to f_c and one pair of side frequencies at $f_c \pm f_m$, so that the bandwidth approaches $2f_m$.

Carson's Rule:

The transmission bandwidth of an FM signal generated by a single-tone modulating signal of frequency f_m:

$$B_C \approx 2\Delta f + 2f_m = 2\Delta f\left(1 + \frac{1}{\beta}\right) = 2(\beta + 1)f_m$$

Transmission Bandwidth of FM Signals

Generally speaking, the effective bandwidth of the modulated signal, is given as, by Carson's rule, as:

$$B_C = 2(\beta + 1)W$$

W **= bandwidth of the message signal.**

Note: **In wideband FM, usually $\beta \geq 5$.**

conclusion

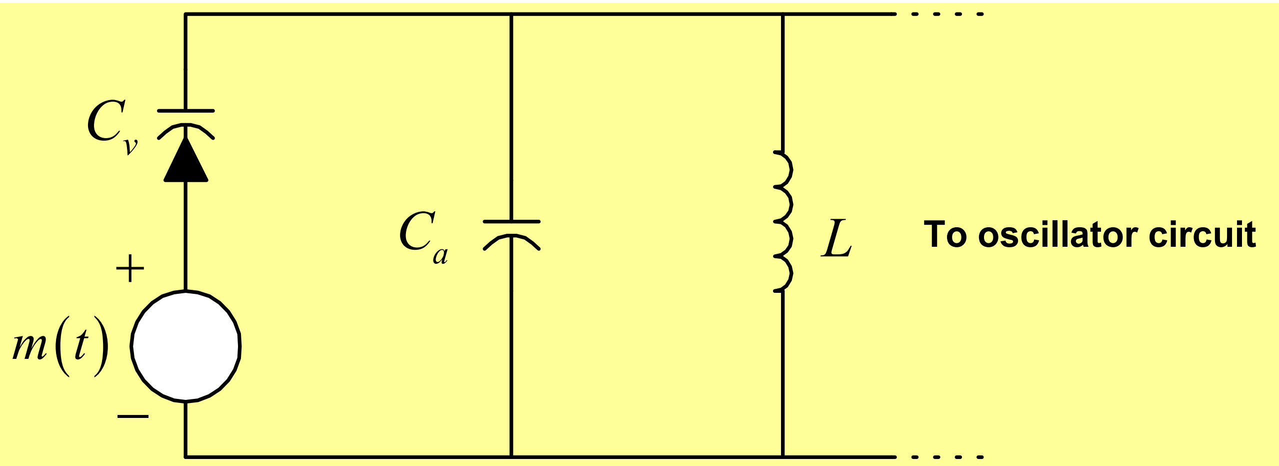
The bandwidth of an angle-modulated signal is much greater than the bandwidth of various AM schemes, which is either *W* (in SSB) or 2*W* (in DSB or conventional AM).

Implementation of Frequency Modulators and Demodulators

Frequency Modulators

i) Direct FM

- **In the direct method, the carrier frequency is directly varied in accordance with the input baseband signal. This is accomplished using a VCO and a VCO can be popularly designed using a varactor diode.**

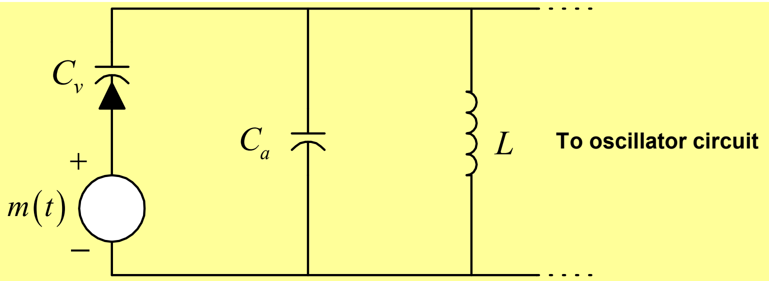


Varactor diode implementation of Direct FM modulator.

Implementation of Frequency Modulators and Demodulators

Frequency Modulators

i) Direct FM



- Let the **inductance** of the inductor in the tuned condition be L_0 and the **capacitance of the varactor diode** is: $C(t) = C_0 + k_0 m(t)$

- **When $m(t) = 0$, the frequency of the tuned circuit is:**

$$f_c = \frac{1}{2\pi\sqrt{L_0 C_0}}$$

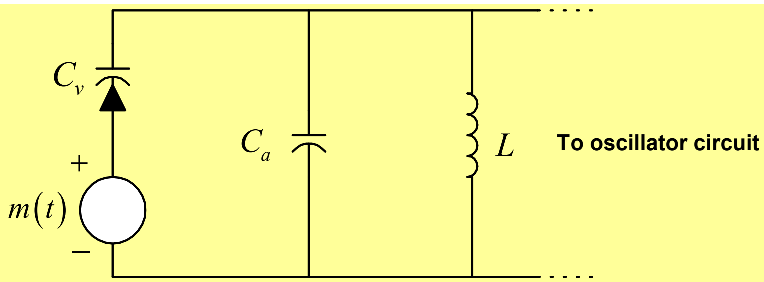
- **Generally, when $m(t) \neq 0$:**

$$f_i(t) = \frac{1}{2\pi\sqrt{L_0\left(C_0 + k_0 m(t)\right)}} = \frac{1}{2\pi\sqrt{L_0 C_0}} \frac{1}{\sqrt{1 + \frac{k_0}{C_0} m(t)}} = f_c \frac{1}{\sqrt{1 + \frac{k_0}{C_0} m(t)}}$$

Implementation of Frequency Modulators and Demodulators

Frequency Modulators

i) Direct FM



- **Assuming that:**

$$\varepsilon = \frac{k_0}{C_0} m(t) << 1$$

- **Then, using approximations:**

$$\sqrt{(1+\varepsilon)} \approx 1 + \frac{\varepsilon}{2} \quad \text{and} \quad \frac{1}{1+\varepsilon} \approx 1 - \varepsilon$$

- **Then, we obtain:**

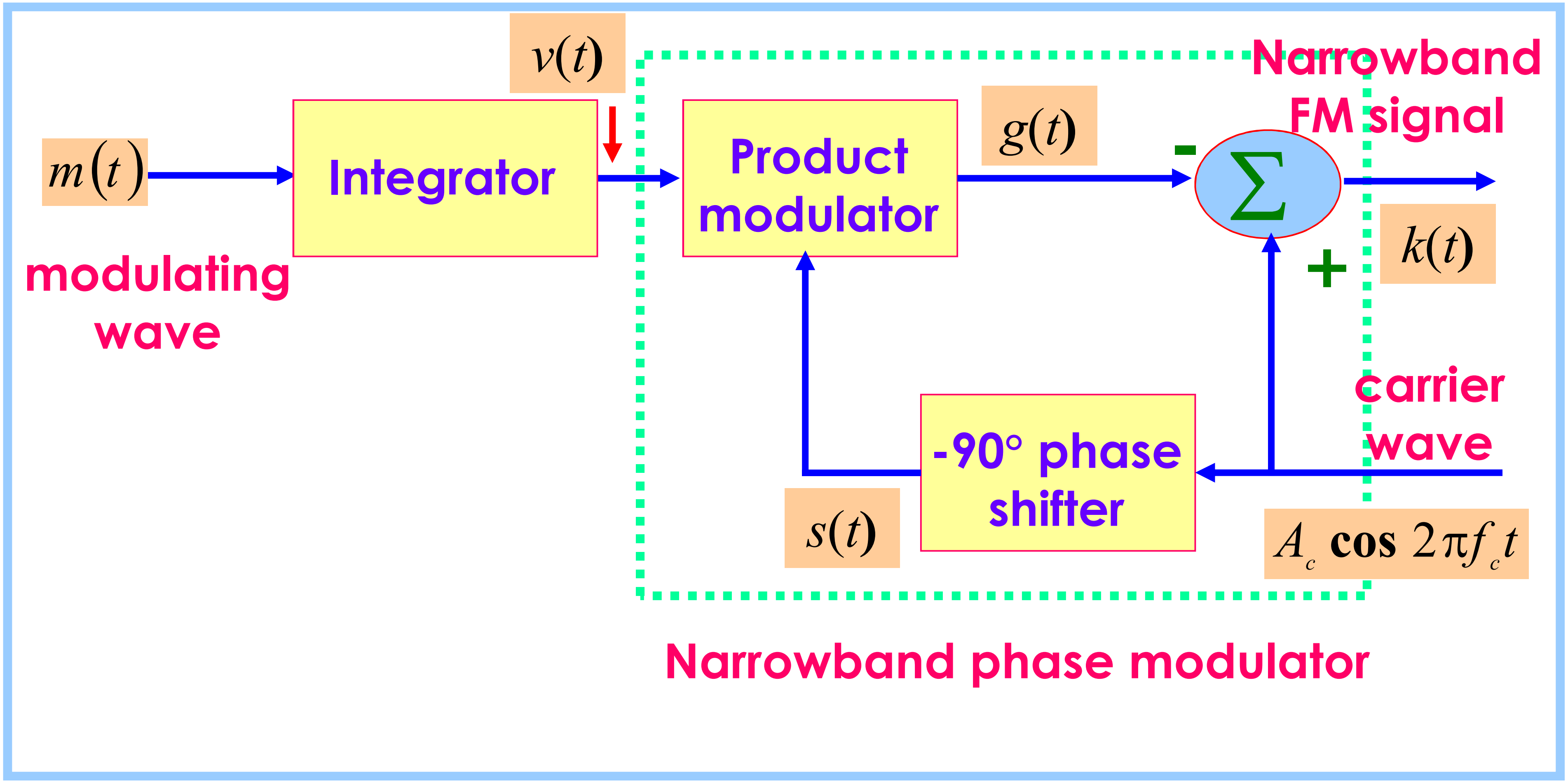
$$f_i(t) \approx f_c\left[1 - \frac{k_0}{2C_0} m(t)\right]$$

This is the relation for the frequency modulated signal.

Implementation of Frequency Modulators and Demodulators

Frequency Modulators

ii) Indirect FM

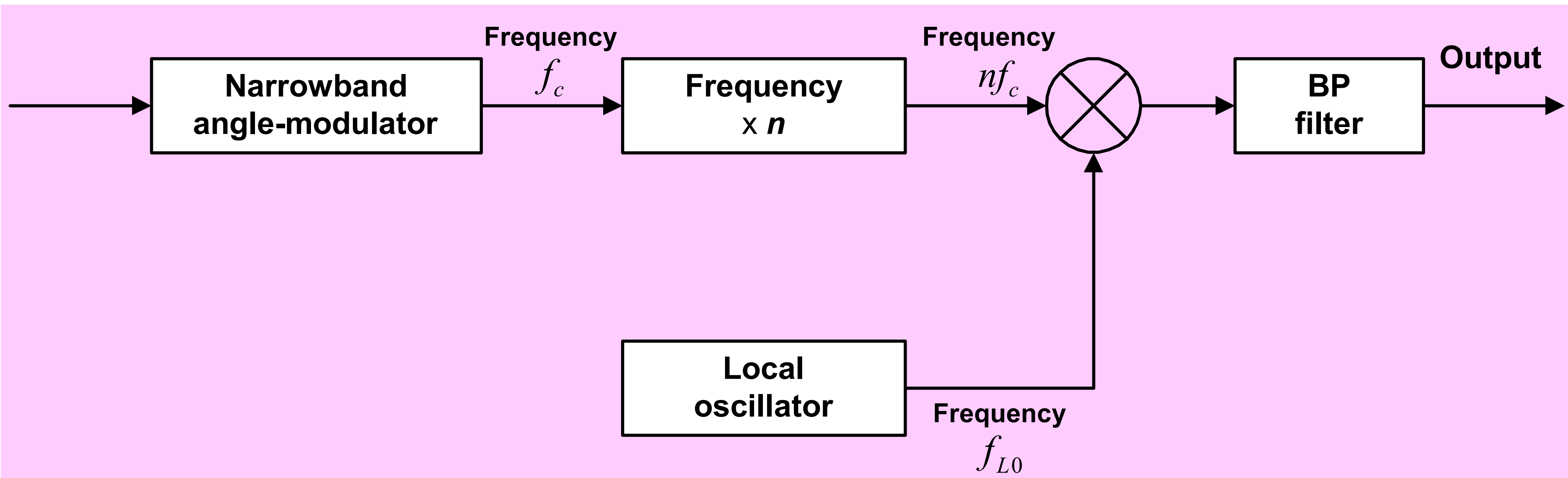


Generation of narrowband FM signal.

Implementation of Frequency Modulators and Demodulators

Frequency Modulators

ii) Indirect FM

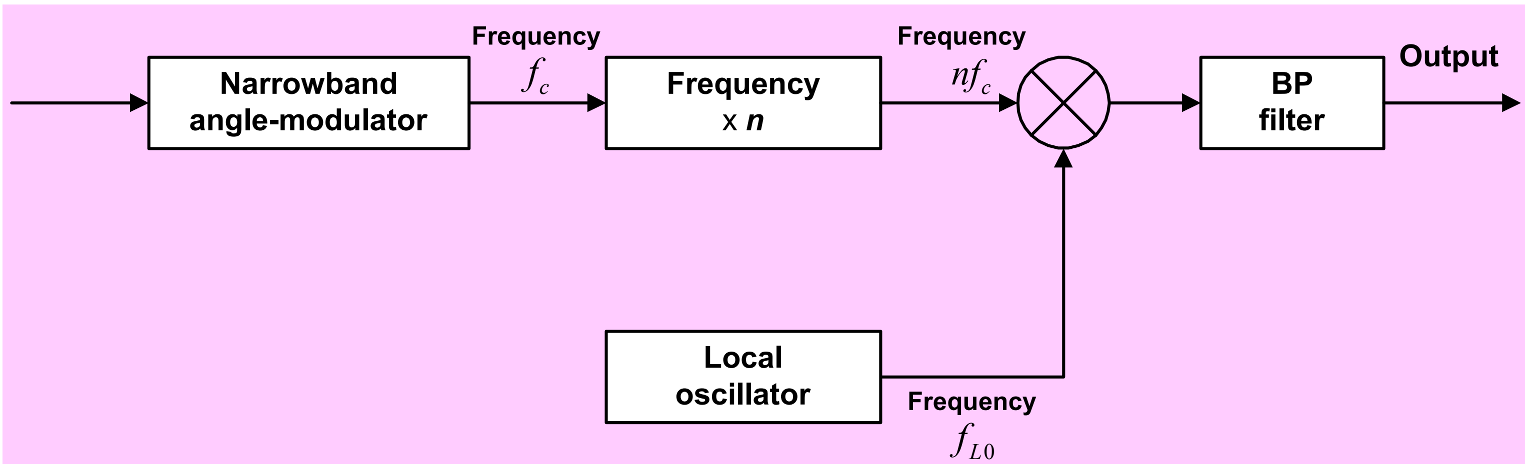


Indirect generation of FM signals.

Implementation of Frequency Modulators and Demodulators

Frequency Modulators

ii) Indirect FM



Indirect generation of FM signals.

Let the narrowband signal be:

$$u_n(t) = A_c \cos[2\pi f_c t + \phi(t)]$$

Then the output of the frequency multiplier:

$$y(t) = A_c \cos[2\pi n f_c t + n\phi(t)]$$

The final wideband FM signal is:

$$u(t) = A_c \cos[2\pi[nf_c - f_{L0}]t + n\phi(t)]$$

FM Demodulators

FM Demodulator –Scheme I:

- **FM demodulators are implemented by generating an AM signal whose amplitude is proportional to the instantaneous frequency of the FM signal.**

- **To achieve the first step to transform theFM signal into an AM signal, the FM signal is passed through an LTI system whose frequency response is approximately a straight line in the frequency band of the FM signal.**

Let the frequency response of such a system be:

$$|H(f)| = V_0 + k(f - f_c) \quad \text{for } |f - f_c| < \frac{B_C}{2}$$

and if the input to the system is:

$$u(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right)$$

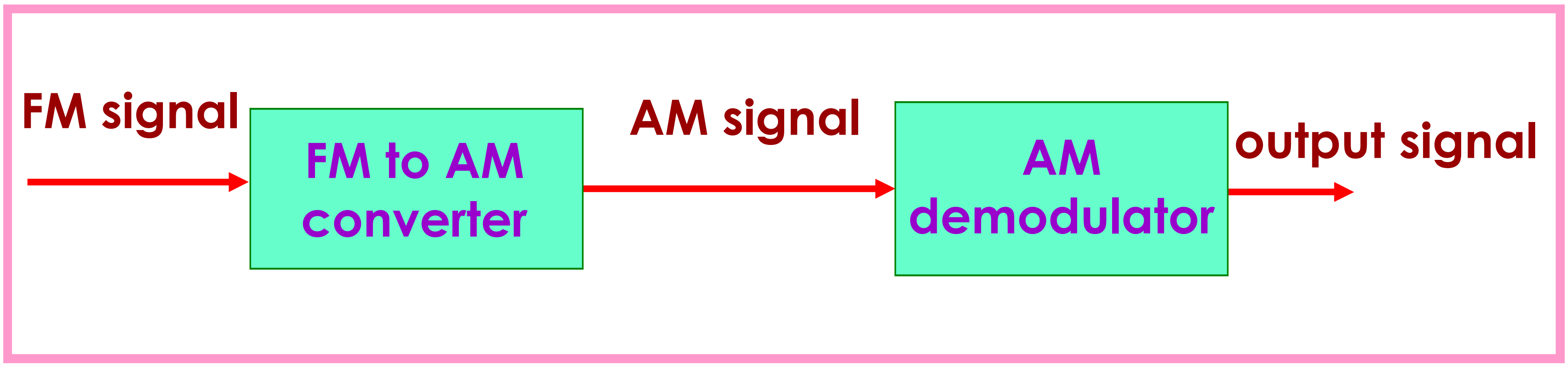
Then the output will be:

$$v_o(t) = A_c\left(V_0 + kk_f m(t)\right)\cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right)$$

FM Demodulators

FM Demodulator –Scheme I:

- **The next step is to demodulate this signal so that the message signal $m(t)$ can be recovered.**



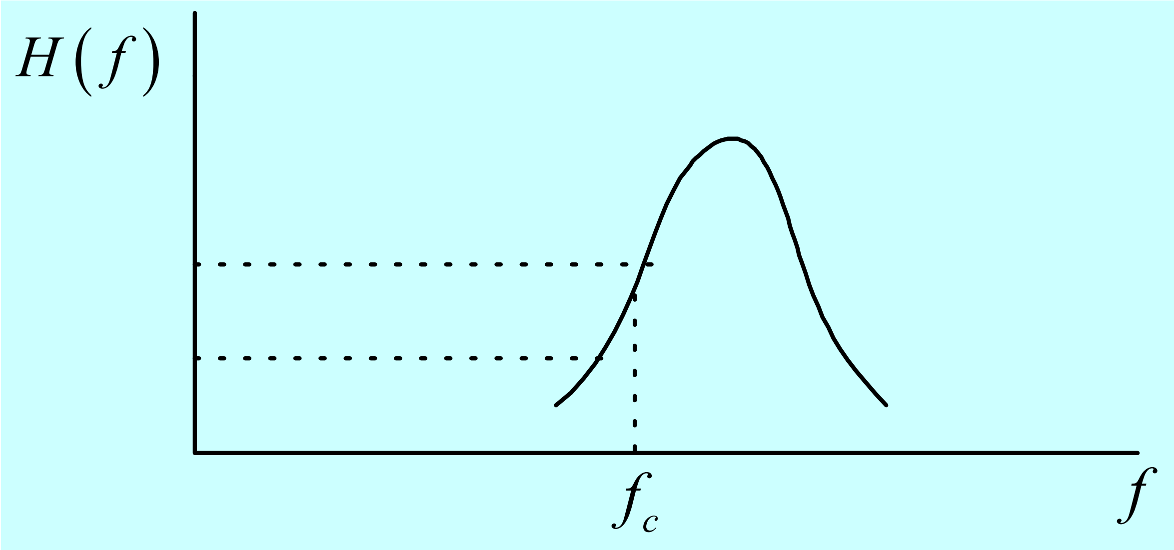
A general FM demodulator.

- **One candidate for implementation of FM to AM conversion is a simple differentiator with $|H(f)| = 2\pi f$.**

FM Demodulators

FM Demodulator –Scheme I:

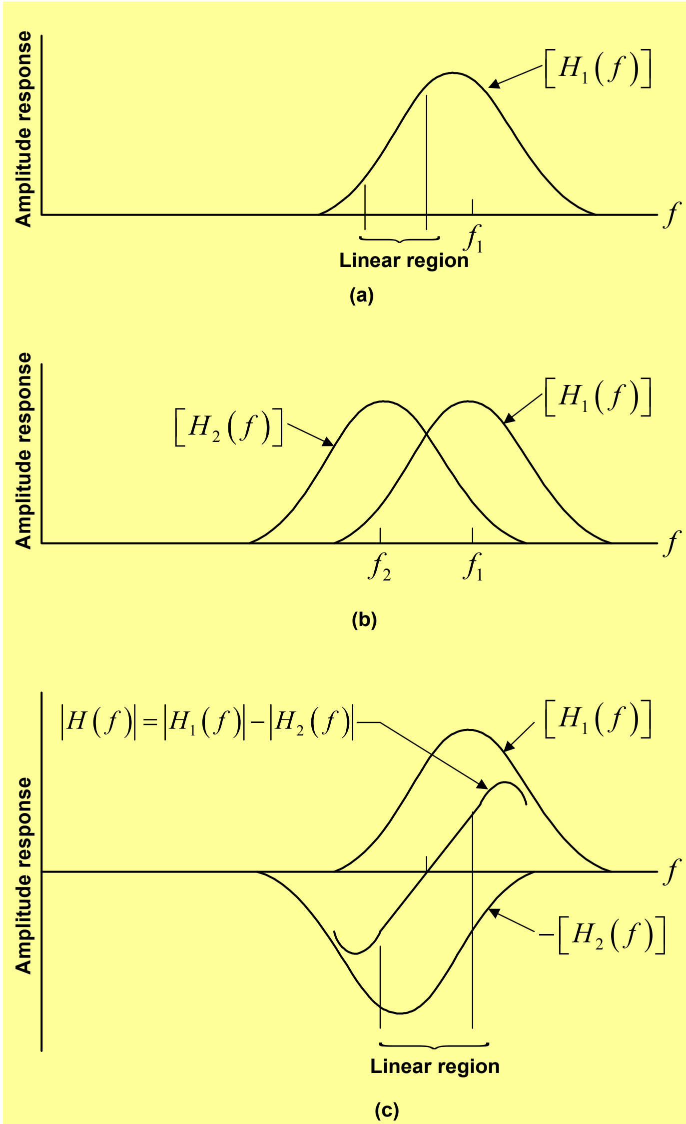
- **Another candidate for implementation of FM to AM conversion is the rising half of the frequency characteristic of a tuned circuit.**



To obtain a linear characteristic over a wide range of frequencies, usually two circuits tuned at two frequencies, f_1 and f_2, are connected in a configuration called a balanced discriminator.

FM Demodulators

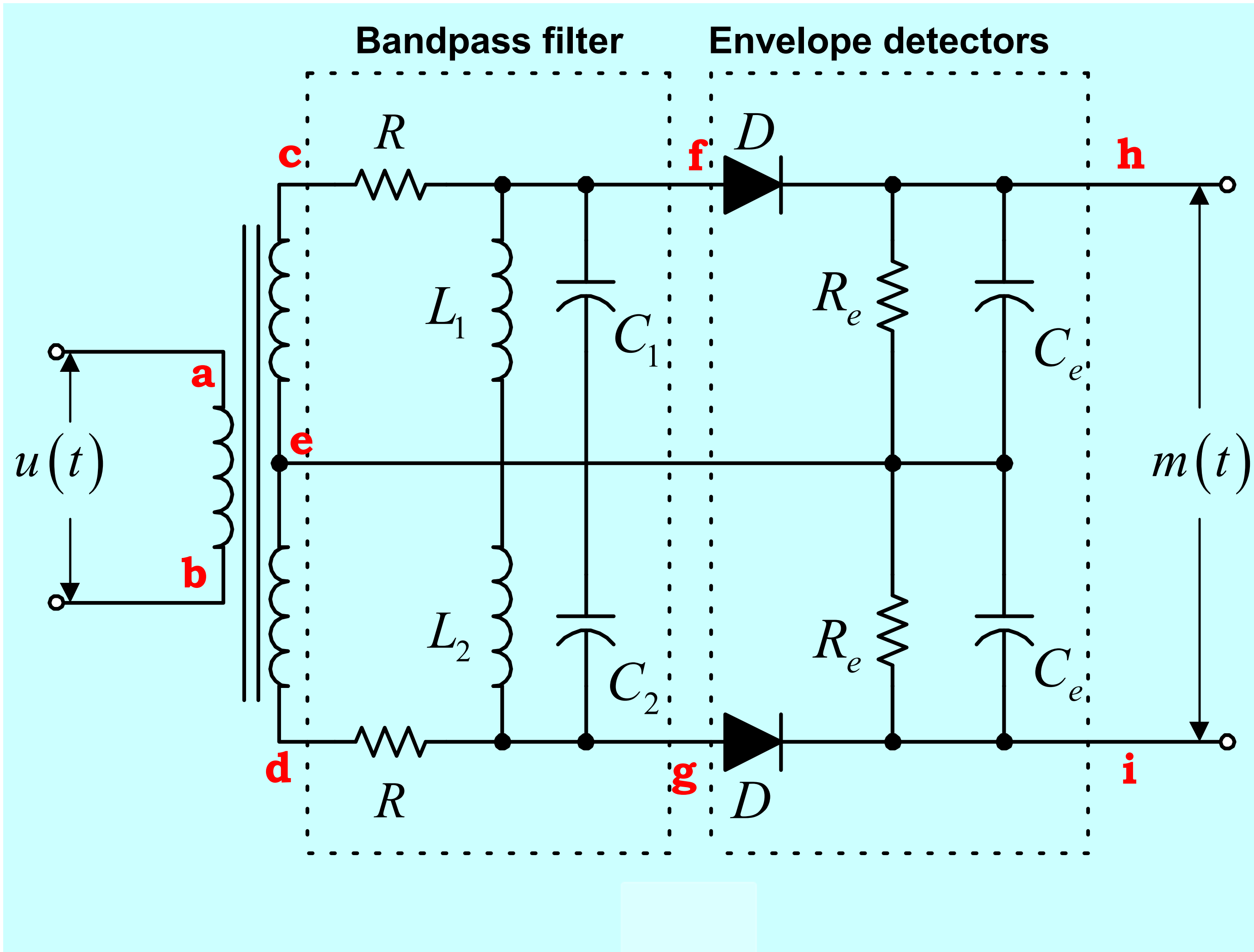
FM Demodulator –Scheme I:



The frequency response of a balanced discriminator circuit.

FM Demodulators

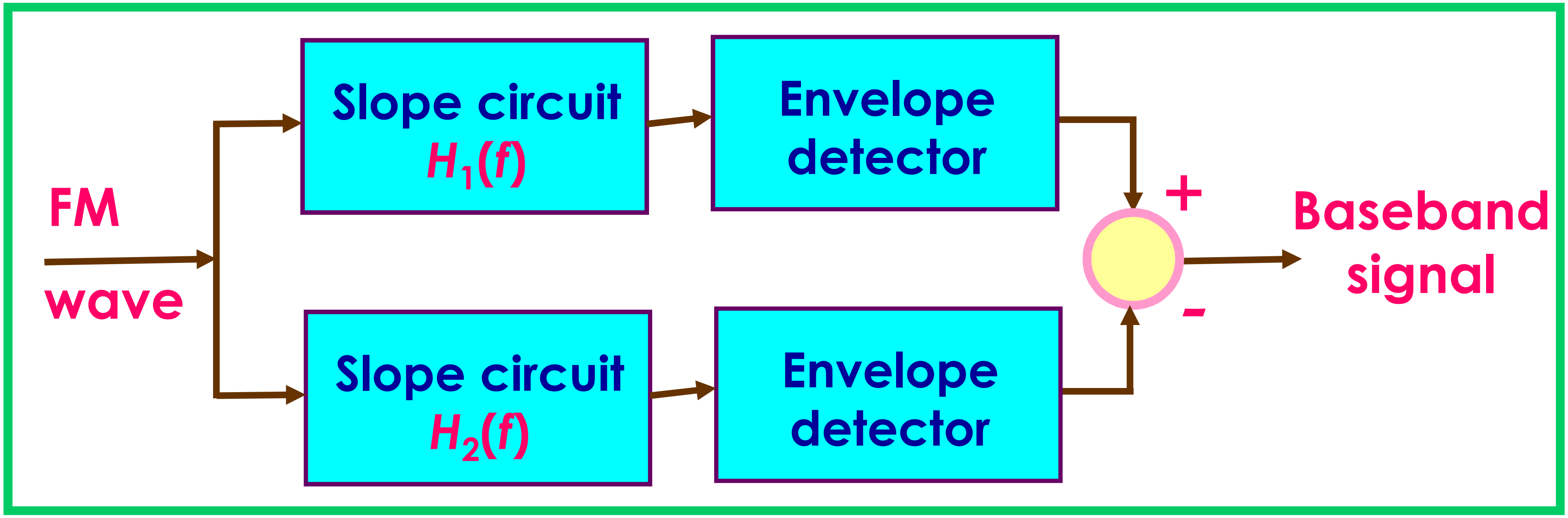
FM Demodulator –Scheme I:



A balanced discriminator circuit.

FM Demodulators

The Frequency Discriminator:



The incoming FM signal:

$$u(t) = A_c \cos\left(2\pi f_c t + 2\pi k_v \int_0^t m(\tau)d\tau\right)$$

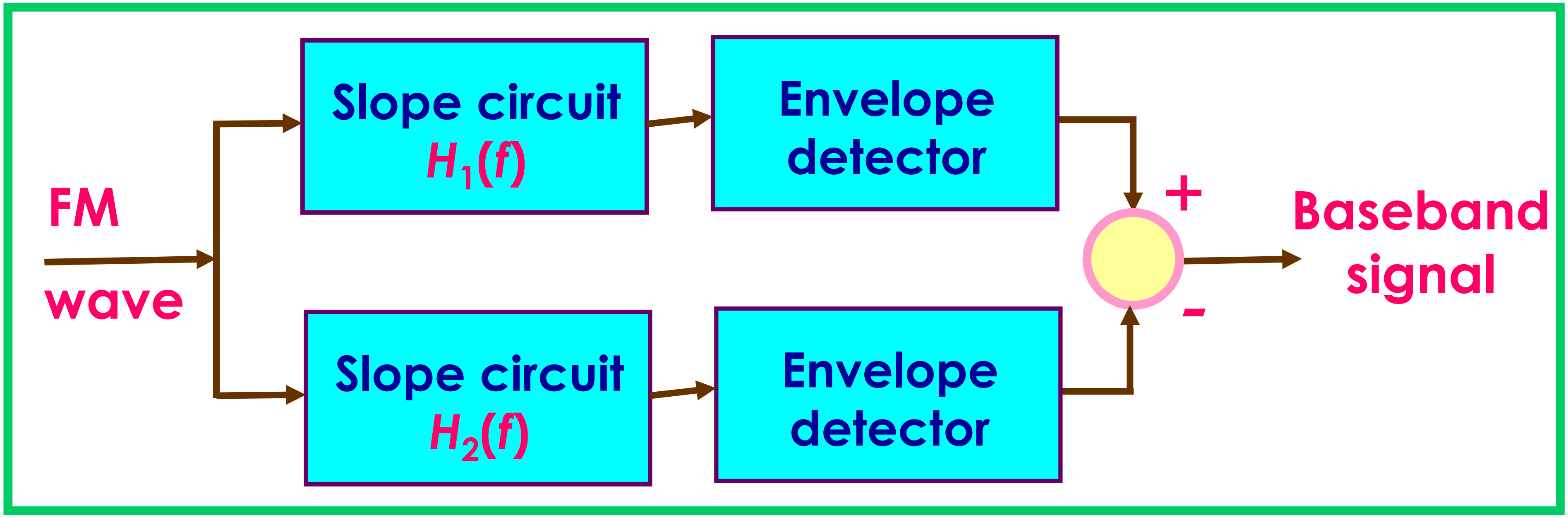
The complex envelope of $u(t)$:

$$\tilde{u}(t) = A_c \exp\left[j2\pi k_f \int_0^t m(\tau)d\tau\right]$$

- **Let $\tilde{u}_1(t)$ denote the complex envelope of the slope circuit $H_1(f)$ response, due to $\tilde{u}(t)$. The desired response of the slope circuit $H_1(f)$ i.e. $u_1(t)$ is a hybrid modulated signal.**

FM Demodulators

The Frequency Discriminator:



The slope circuit response:

$$u_1(t) = A_c\left(V_0 + kk_f m(t)\right)\cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right)$$

Let us choose, for all *t*,

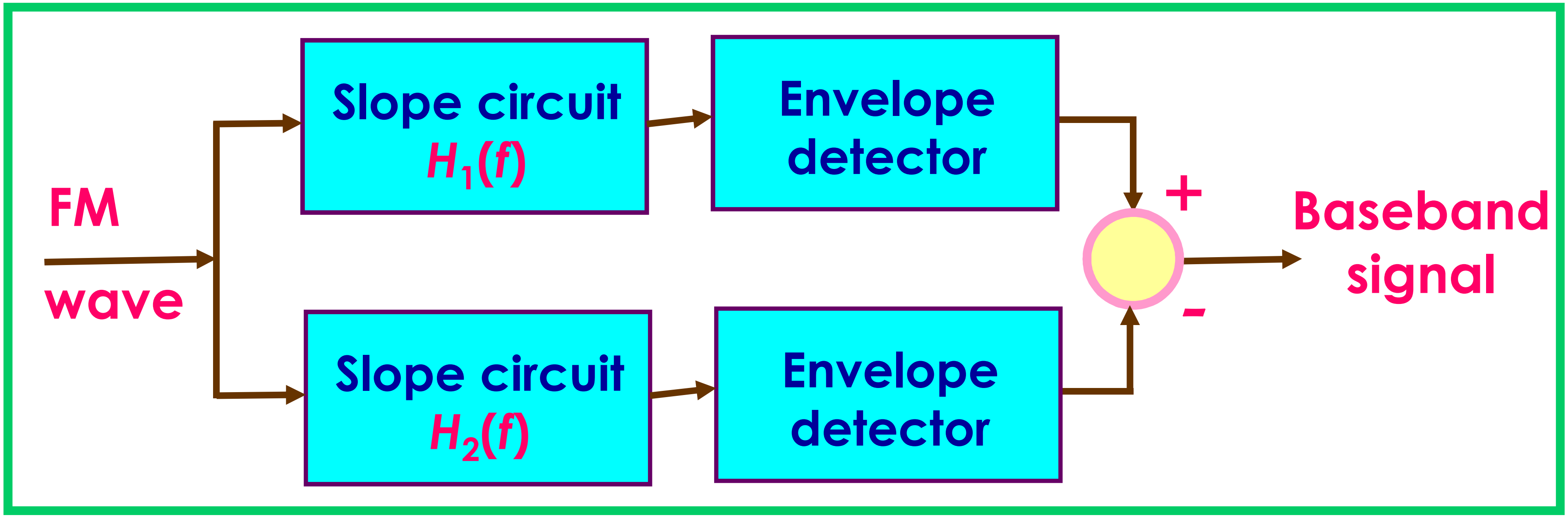
$$\left|\frac{kk_f}{V_0} m(t)\right| < 1$$

The resulting envelope detector output:

$$\left|\tilde{u}_1(t)\right| = A_c V_0\left(1 + \frac{kk_f}{V_0} m(t)\right)$$

FM Demodulators

The Frequency Discriminator:



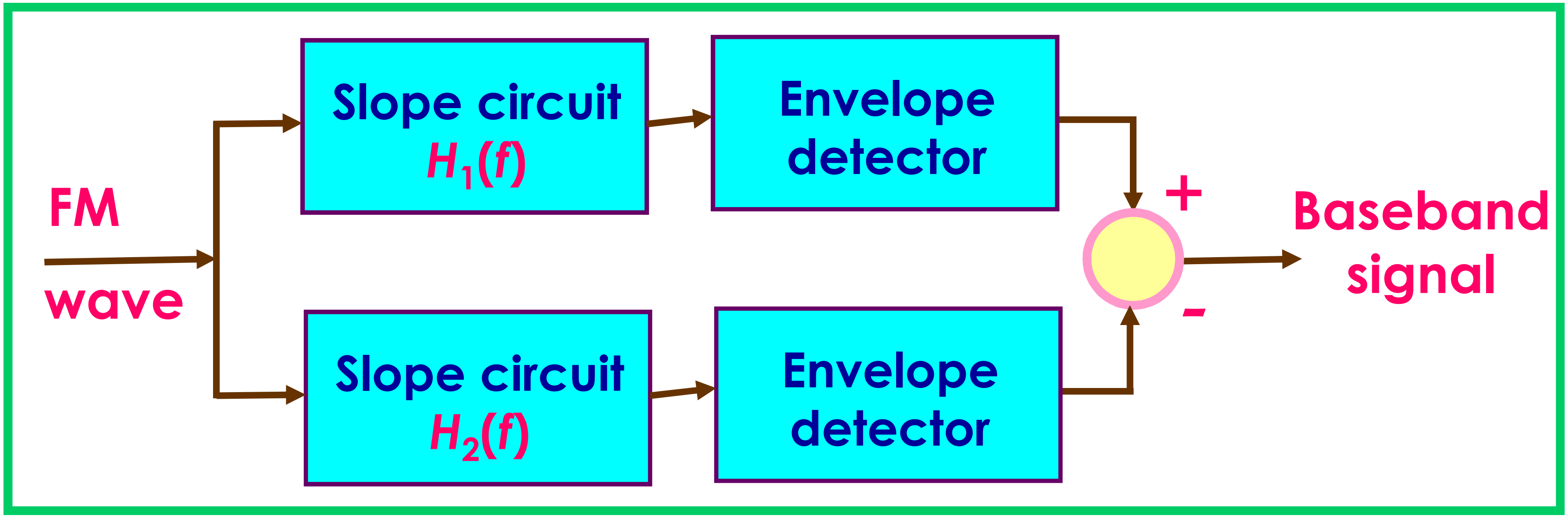
- **The complementary slope circuit $H_2(f)$ is so designed that $\hat{H}_2(f) = \hat{H}_1(-f)$ where $\hat{H}_1(f)$ is an equivalent low pass filter with a frequency response that can be used to replace the band pass filter with frequency response $H_1(f)$.**

- **Let $u_2(t)$ denote the response of the complementary slope circuit produced by $u(t)$. Then it's envelope detector output:**

$$\left|\tilde{u}_2(t)\right| = A_c V_0 \left(1 - \frac{k k_f}{V_0} m(t)\right)$$

FM Demodulators

The Frequency Discriminator:



The final output: $$|\tilde{u}_1(t)| - |\tilde{u}_2(t)| = 2A_c k k_f m(t)$$

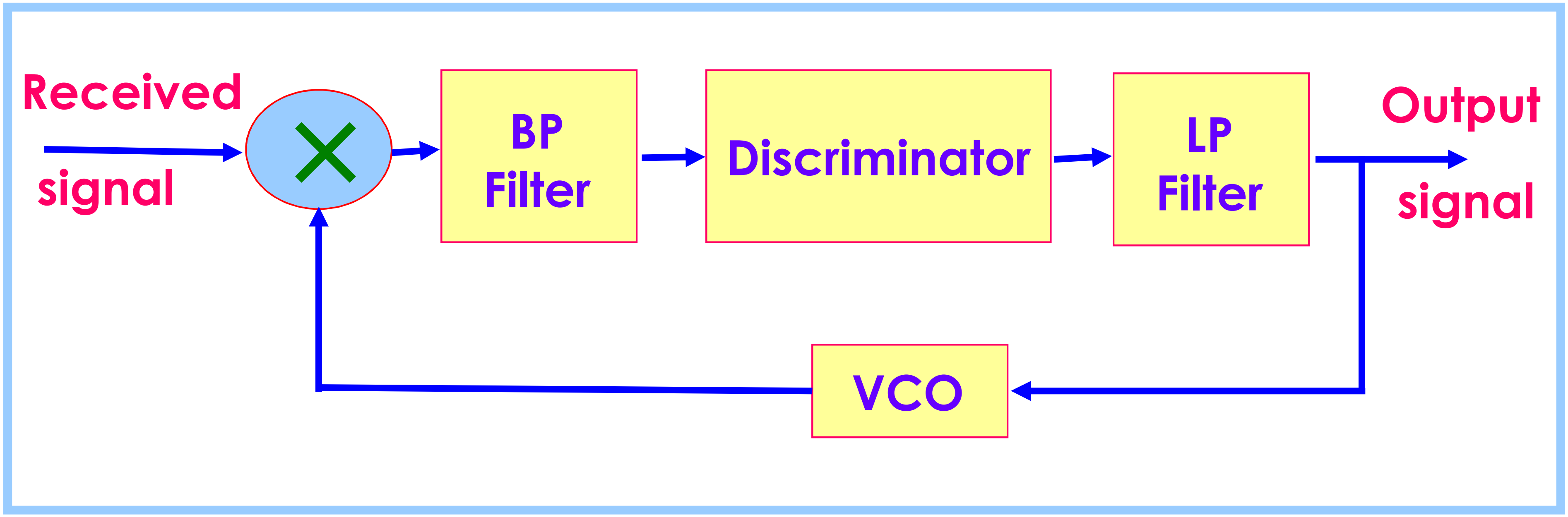
This output is a scaled version of the original message signal $m(t)$ and free from bias.

FM Demodulators

FM Demodulator –Scheme II:

- **Another FM signal demodulator uses feedback in the demodulator to narrow the bandwidth of the FM detector.**

- **This reduces the noise power at the output of the demodulator. An FM demodulator with feedback (FMFB) employs a VCO in the feedback branch.**

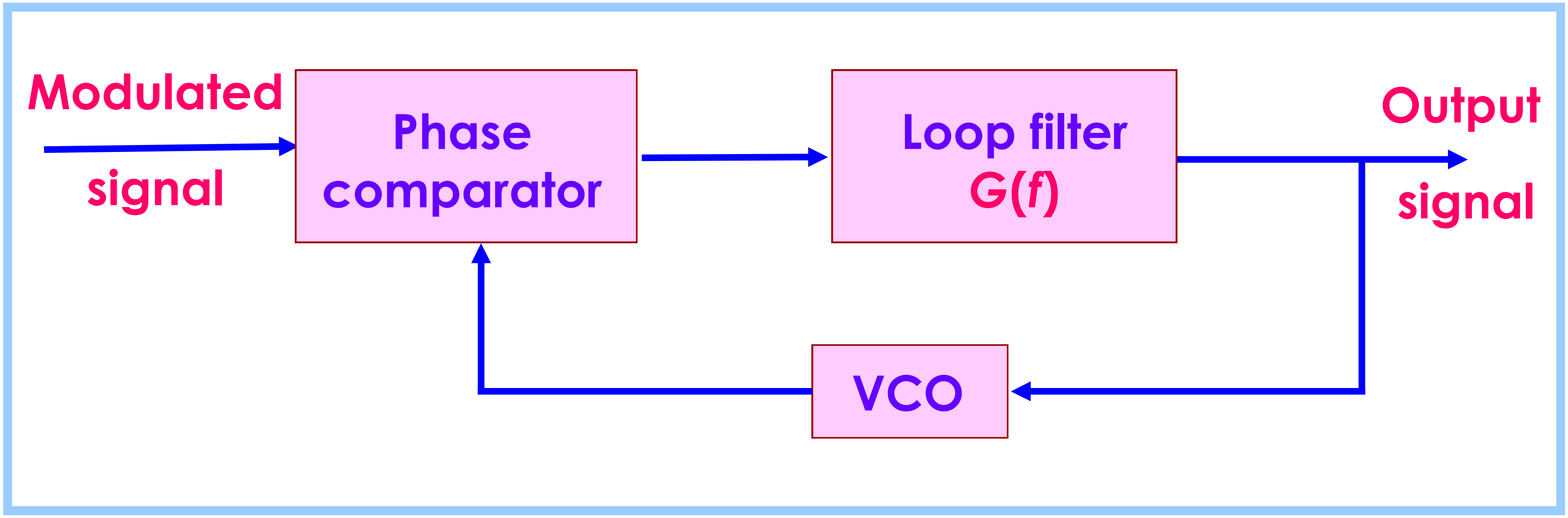


FMFB demodulator.

FM Demodulators

FM Demodulator –Scheme II:

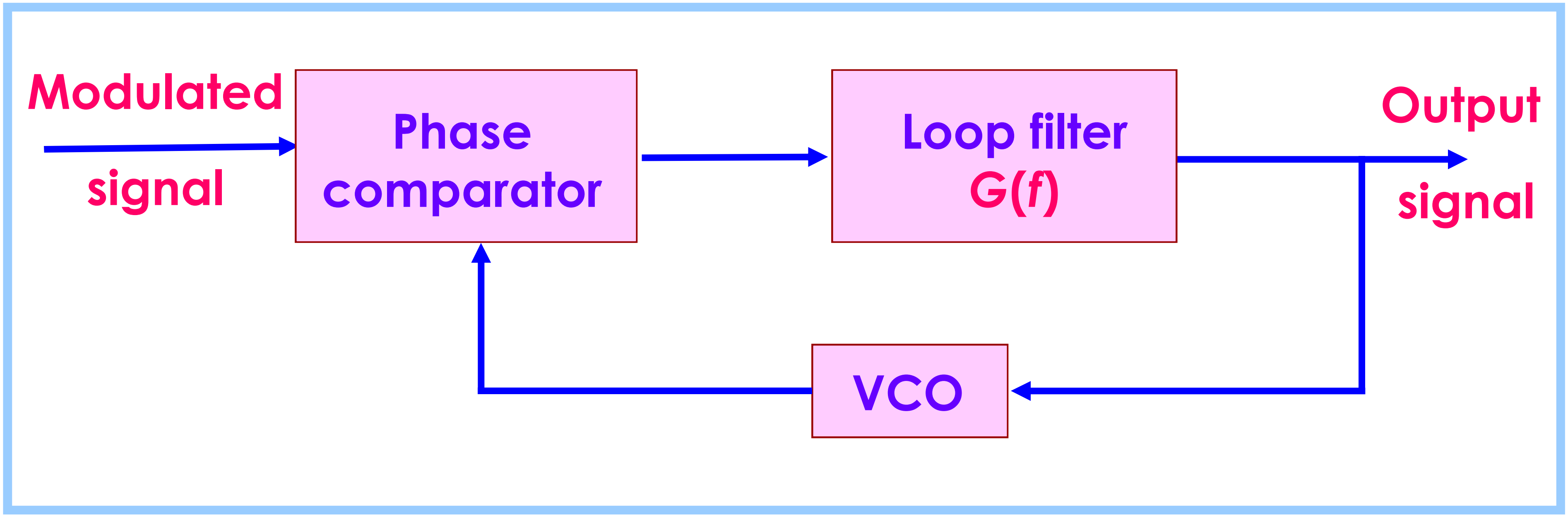
- **An alternative to FMFB demodulator is to use PLL.**



PLL-FM demodulator.

FM Demodulators

PLL-FM Demodulator:



- **The input to the PLL is the angle-modulated wave:**

$$u(t) = A_c \cos(2\pi f_c t + \phi(t))$$

For FM:

$$\phi(t) = 2\pi k_f \int_{-\infty}^{t} m(\tau) d\tau$$

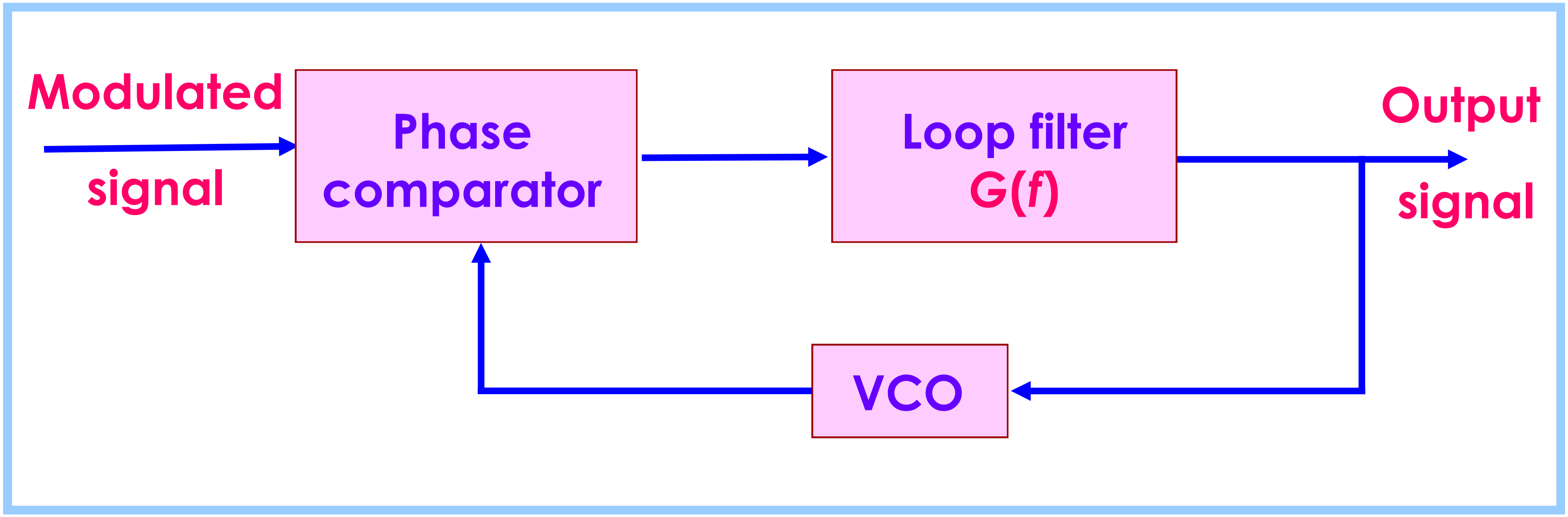
- **Instantaneous frequency of the VCO:**

$$f_v(t) = f_c + k_v v(t)$$

$v(t)$ = control voltage of the VCO; k_v = a deviation constant

FM Demodulators

PLL-FM Demodulator:



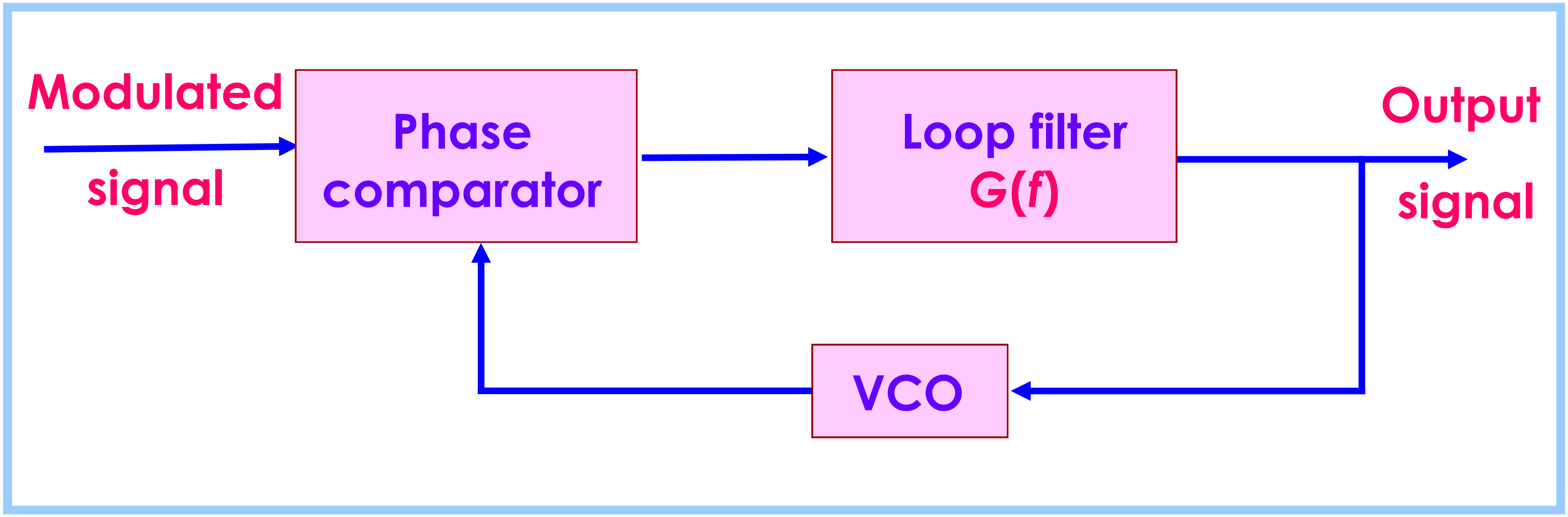
- **Therefore, the VCO output:**

$$y_v(t) = A_v \sin\left(2\pi f_c t + \phi_v(t)\right) \Rightarrow \phi_v(t) = 2\pi k_v \int_0^t v(\tau)d\tau$$

- **The phase comparator is basically a multiplier and a filter that rejects the signal component centered at $2f_c$.**

FM Demodulators

PLL-FM Demodulator:



- **The phase comparator output:**

$$e(t) = \frac{1}{2} A_v A_c \sin[\phi(t) - \phi_v(t)] \Rightarrow \phi(t) - \phi_v(t) = \text{phase error}$$

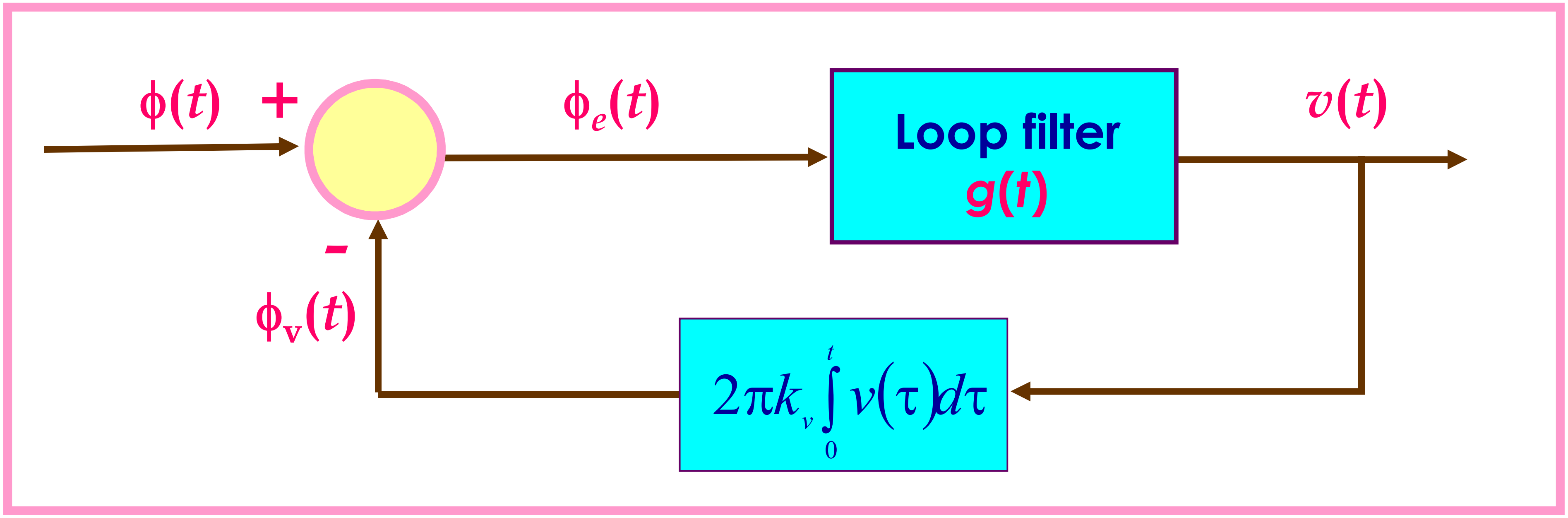
- **The signal $e(t)$ is the input to the loop filter.**

If the PLL is *in lock*, the phase error is small:

$$\sin[\phi(t) - \phi_v(t)] \approx \phi(t) - \phi_v(t) = \phi_e(t)$$

FM Demodulators

PLL-FM Demodulator:



Linearized PLL.

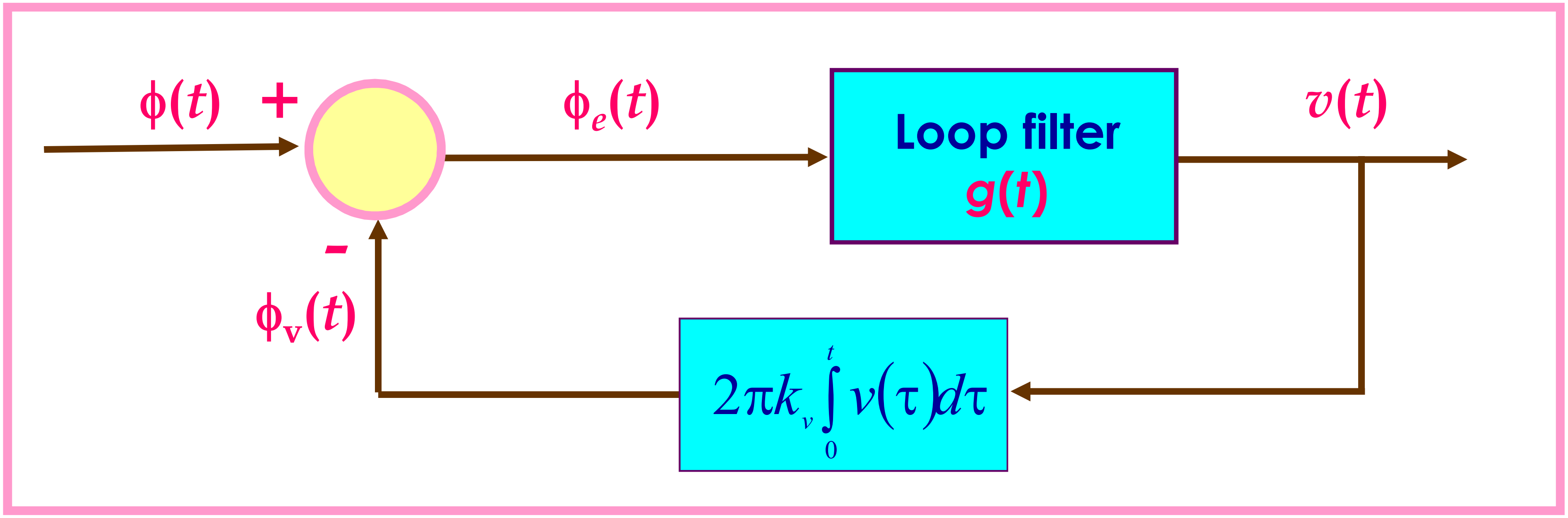
The phase error is:

$$\phi_e(t) = \phi(t) - 2\pi k_v \int_0^t v(\tau)d\tau$$

$$\therefore \frac{d}{dt}\phi_e(t) + 2\pi k_v v(t) = \frac{d}{dt}\phi(t) \text{ or } \frac{d}{dt}\phi_e(t) + 2\pi k_v \int_0^\infty \phi_e(\tau)g(t-\tau)d\tau = \frac{d}{dt}\phi(t)$$

FM Demodulators

PLL-FM Demodulator:



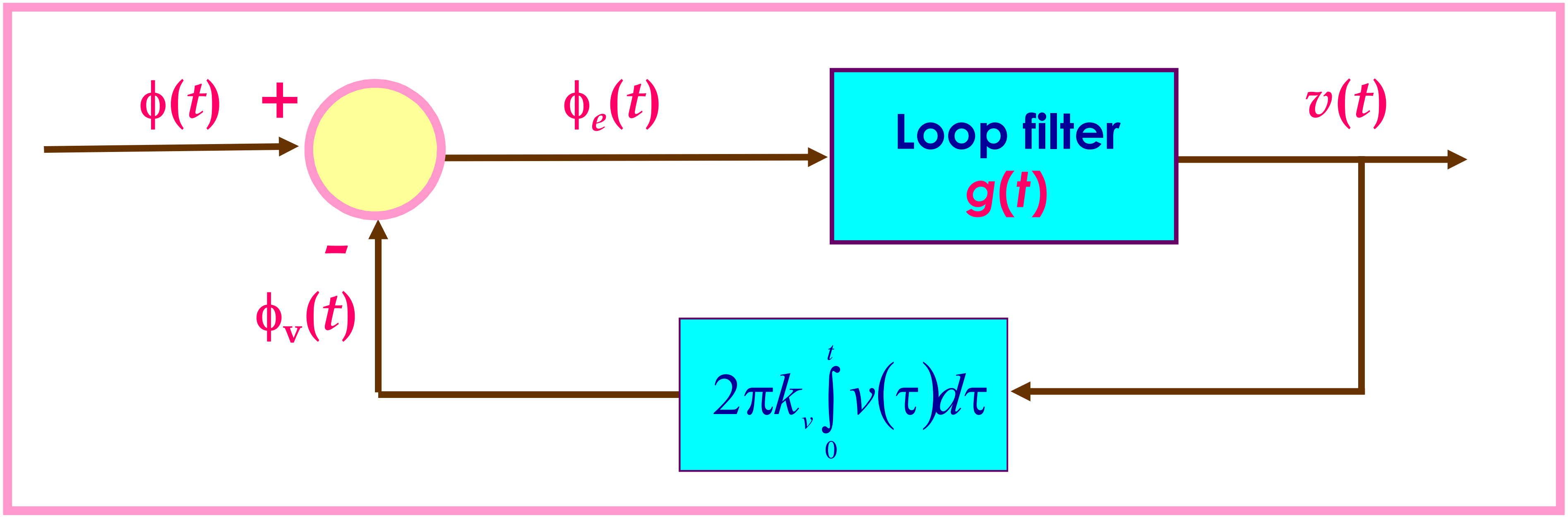
The Fourier transform of this integro-differential equation:

$$(j2\pi f)\Phi_e(f) + 2\pi k_v \Phi_e(f)G(f) = (j2\pi f)\Phi(f)$$

$$\text{Hence, } \Phi_e(f) = \frac{1}{1 + \left(\frac{k_v}{jf}\right)G(f)}\Phi(f)$$

FM Demodulators

PLL-FM Demodulator:



Then control voltage of the VCO is:

$$V(f) = \Phi_e(f) G(f) = \frac{G(f)}{1 + \left(\frac{k_v}{jf}\right) G(f)} \Phi(f)$$

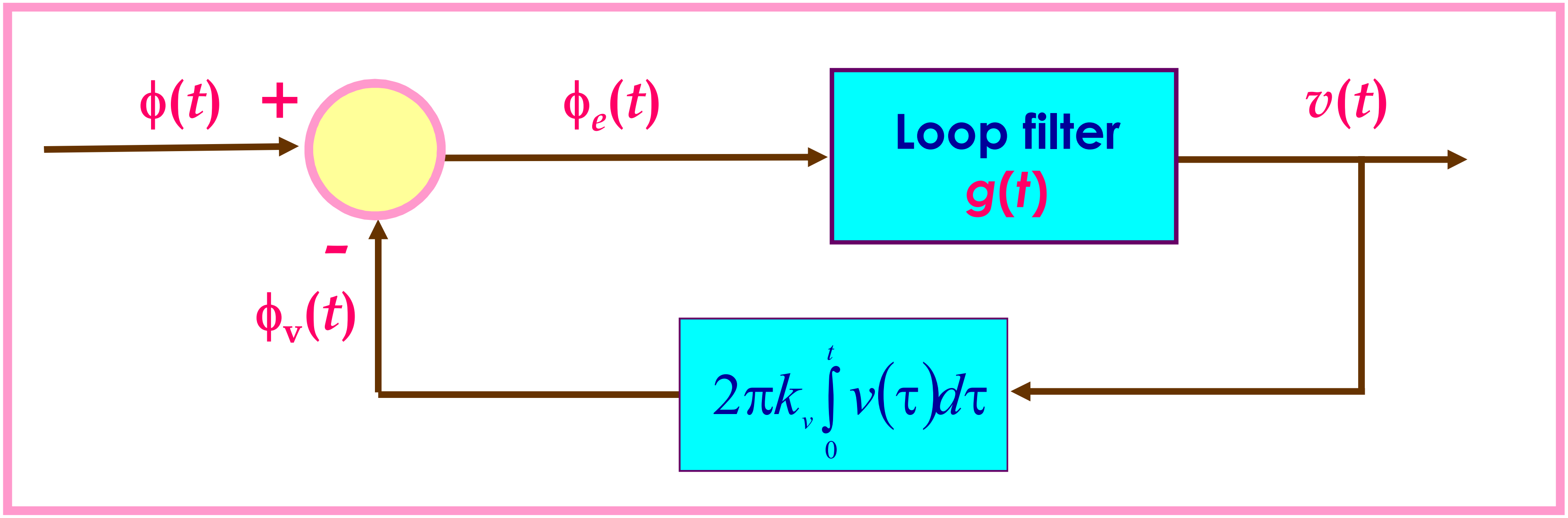
Now $G(f)$ is designed such that:

$$\left| k_v \frac{G(f)}{jf} \right| >> 1$$

in the frequency band $|f| < W$ of the message signal.

FM Demodulators

PLL-FM Demodulator:



Hence:

$$V(f) = \frac{j2\pi f}{j2\pi k_v}\Phi(f)$$

$$\text{Equivalently, } v(t) = \frac{1}{2\pi k_v}\frac{d}{dt}\phi(t) = \frac{k_f}{k_v}m(t)$$

***Conclusion*: As the control voltage of the VCO is proportional to the message signal, *v*(*t*) is the demodulated signal.**

AM Radio Broadcasting

In a broadcasting system, the receiver performs some system functions in addition to demodulating the incoming modulated signal.

- **Carrier-frequency tuning:** to select the desired signal.

- **Filtering:** required to separate the desired signal from other modulating signals that may be picked up along the way.

- **Amplification:** intended to compensate for the loss of signal power incurred in the course of transmission.

AM Radio Broadcasting

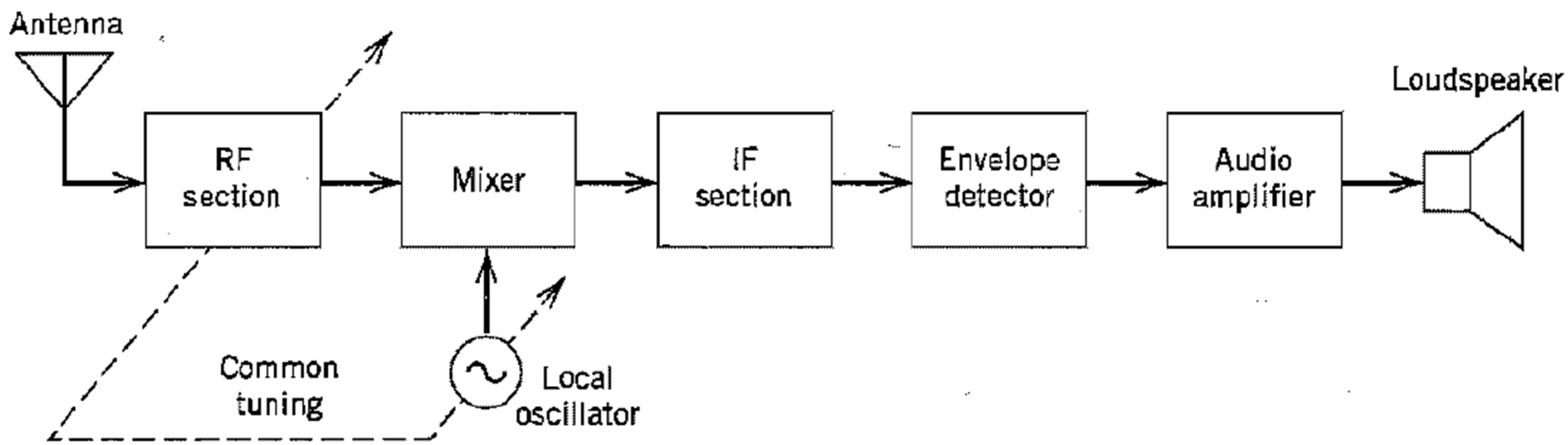
	AM radio	FM radio
RF carrier range	535-1605 kHz	88-108 MHz
Midband frequency of IF section	455 kHz	10.7 MHz
IF bandwidth	10 kHz	200 kHz

Typical frequency parameters of commercial AM and FM radio receivers.

The use of conventional AM for broadcast is justified from an economic standpoint. The major objective is to reduce the cost of implementing the receiver.

Superheterodyne AM Receiver

The receiver most commonly used in AM radio broadcast is called superheterodyne receiver.



Superheterodyne AM receiver.

The common IF frequency chosen is: $f_{IF} = 455\,\text{kHz}$

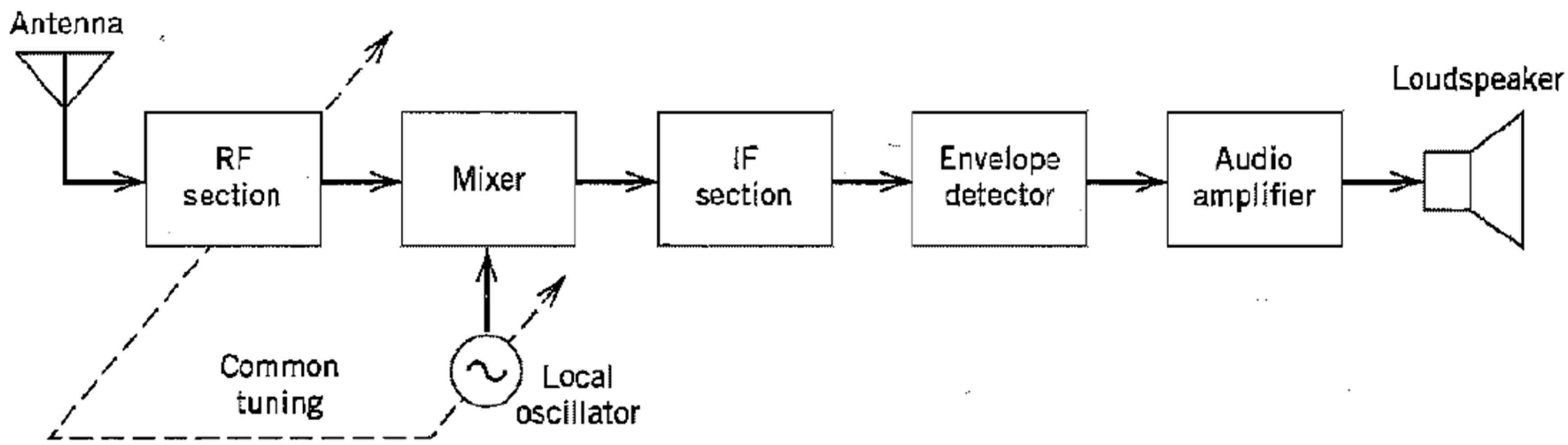
The bandwidth of the IF amplifier is designed as 10 kHz.

The frequency of the local oscillator is:

$$f_{LO} = f_c + f_{IF};$$

f_c : carrier frequency of the desired AM radio signal.

Superheterodyne AM Receiver



The tuning range of the local oscillator: 995 - 2055 kHz.

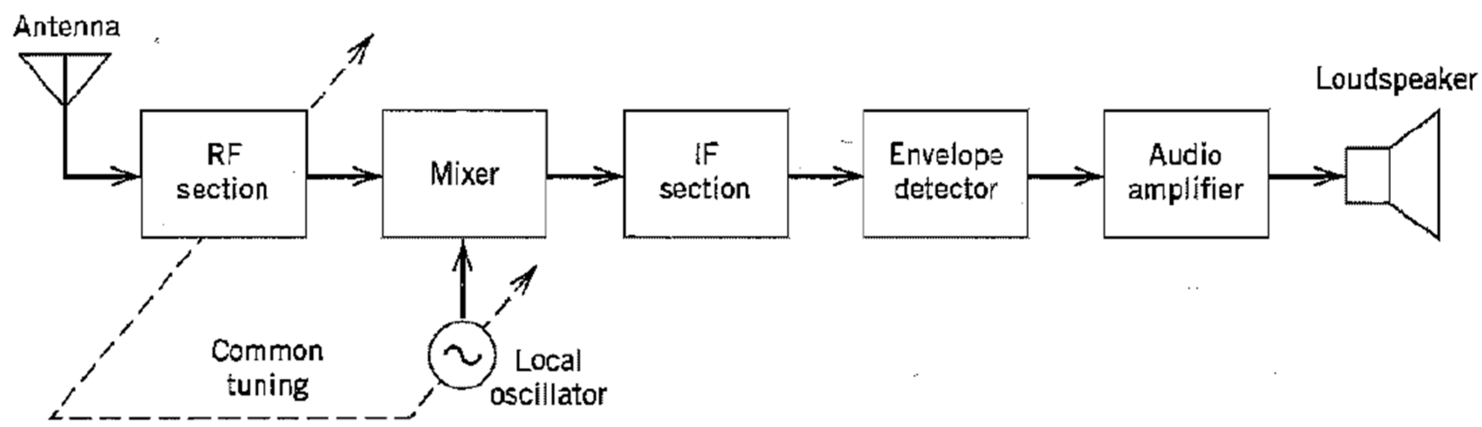
The RF amplifier is tuned to f_c and its output is mixed with the local oscillator frequency f_{LO}.

The bandwidth of the RF amplifier is limited to the range:

$$B_C < B_{RF} < 2 f_{IF}$$

(B_c = Bandwidth of the AM radio signal (10 kHz)).

Superheterodyne AM Receiver



The local oscillator output $\cos 2\pi f_{L0} t$ is mixed with the received signals:

$$r_1(t) = A_c[1 + m_1(t)]\cos 2\pi f_c t; \quad r_2(t) = A_c[1 + m_2(t)]\cos 2\pi f_c' t$$

$$f_c = f_{L0} - f_{IF} \quad \text{and} \quad f_c' = f_{L0} + f_{IF}$$

The mixer output consists of two signals:

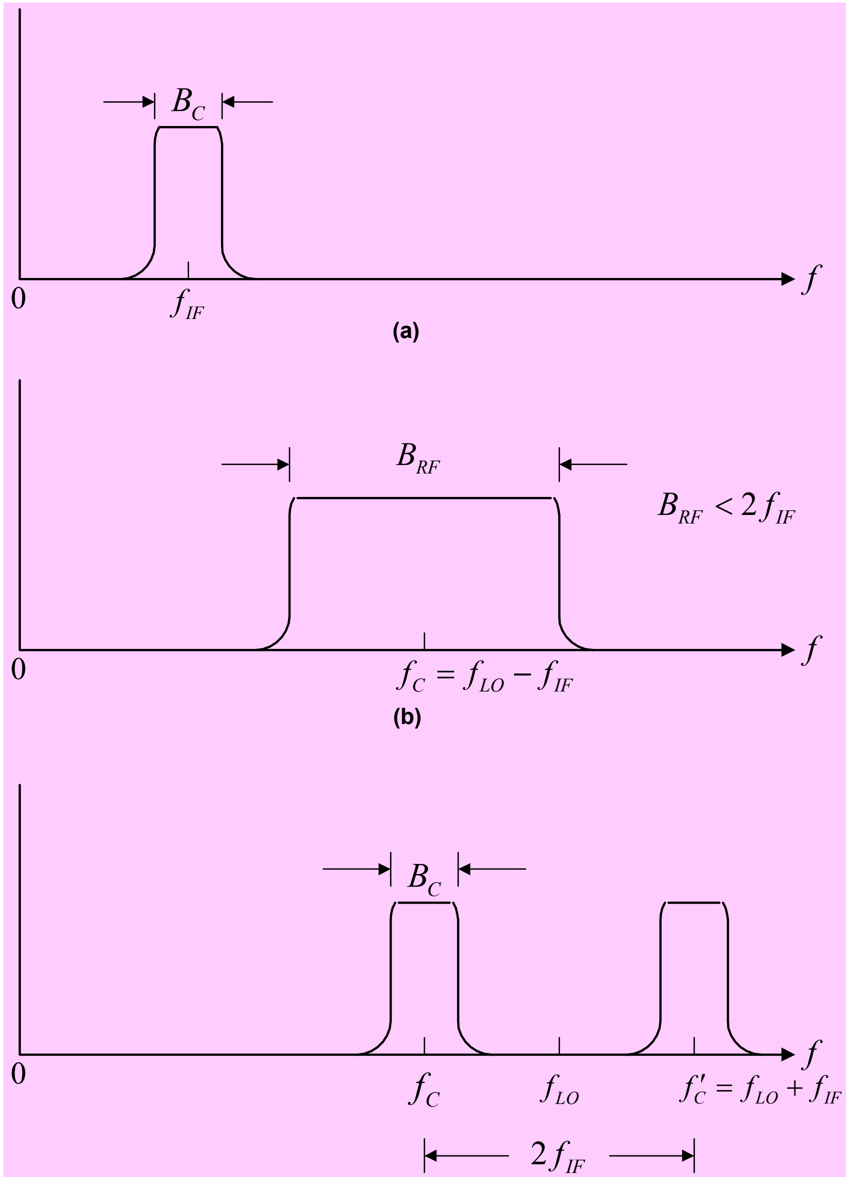
$$y_1(t) = A_c[1 + m_1(t)]\cos 2\pi f_{IF} t + \text{ double frequency term}$$

$$y_2(t) = A_c[1 + m_2(t)]\cos 2\pi f_{IF} t + \text{ double frequency term}$$

$m_1(t)$ = the desired signal and

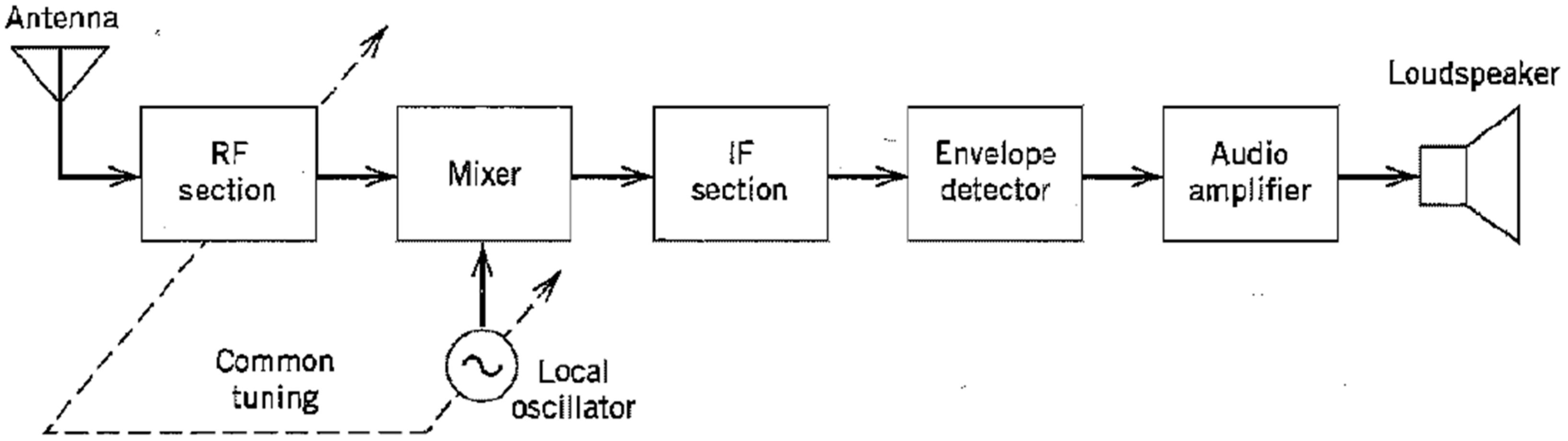
$m_2(t)$ = the signal transmitted by the radio station transmitting at the carrier frequency $f_c' = f_{L0} + f_{IF}$.

Superheterodyne AM Receiver



Frequency response characteristics of IF and RF amplifiers.

Superheterodyne AM Receiver



The output of the IF amplifier is passed through an envelope detector which produces the desired audio-band message signal $m(t)$.

Finally, the output of the envelope detector is amplified and the amplified signal drives a loudspeaker.

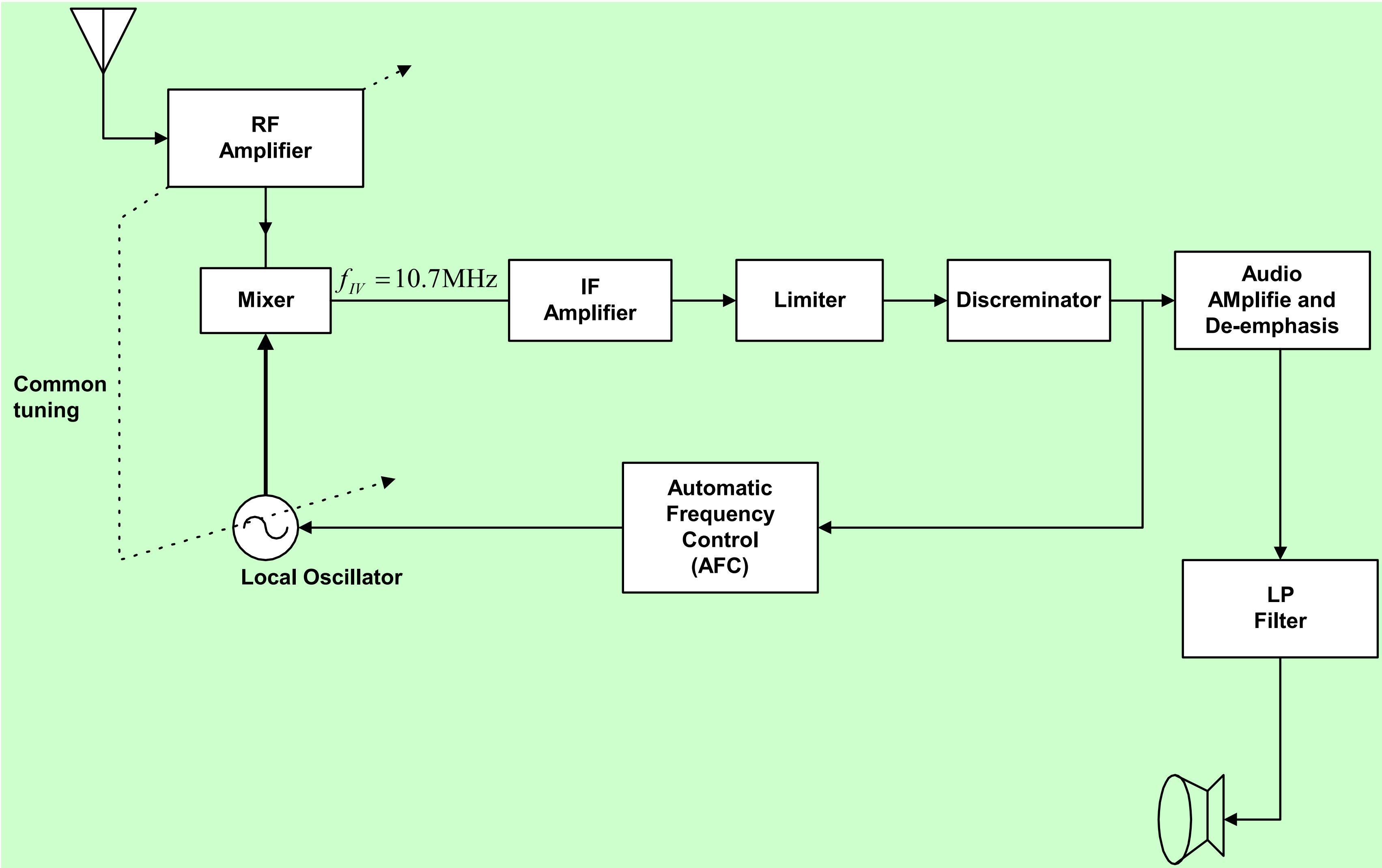
FM Radio Broadcasting

Commercial FM radio broadcasting utilizes the frequency band 88 – 108 MHz for transmission of voice and music channels.

The carrier frequencies are separated by 200 kHz and the peak frequency deviation is fixed at 75 kHz.

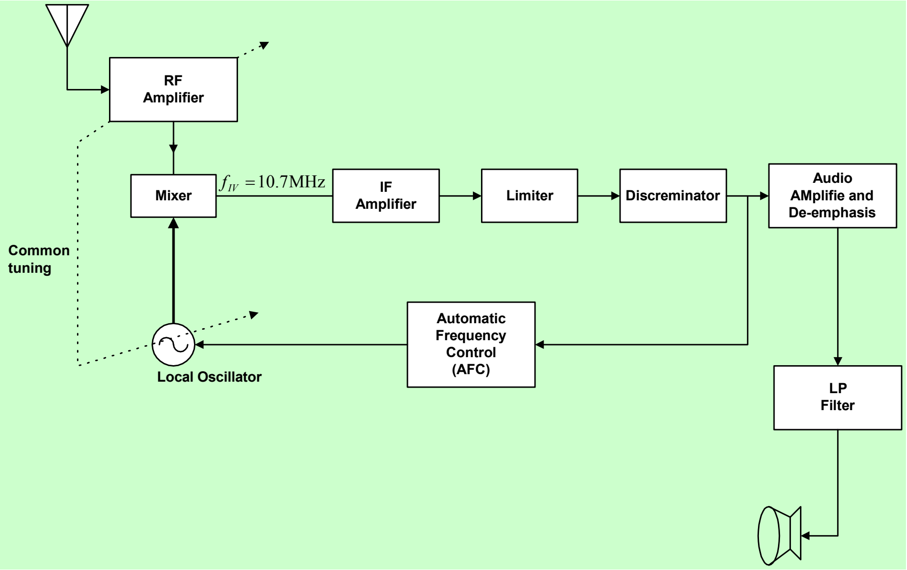
- **The receiver most commonly used in FM radio broadcast is a superheterodyne type.**

FM Radio Broadcasting



Superheterodyne FM receiver.

FM Radio Broadcasting

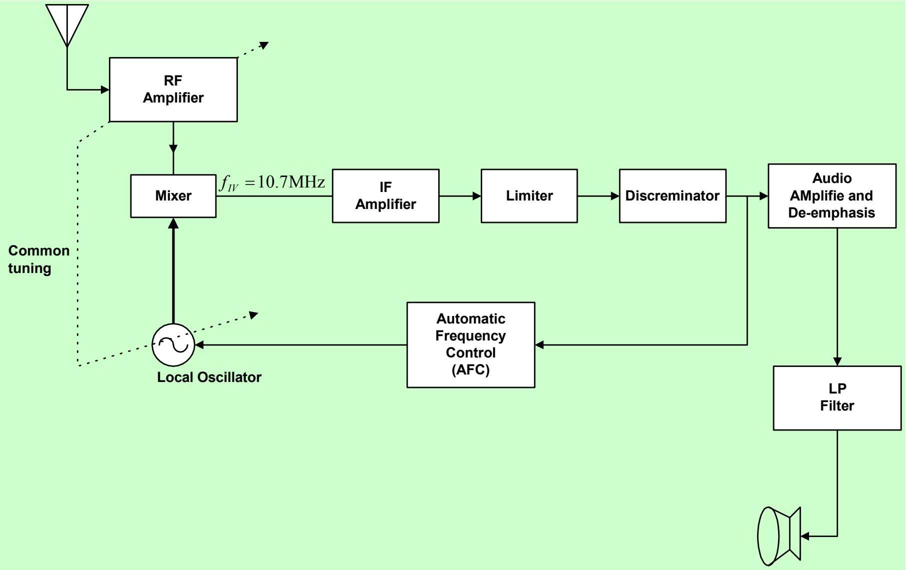


The mixer brings all FM radio signals to a common IF bandwidth of 200 kHz, centered at f_{IF} = 10.7 MHz.

The amplitude limiter removes any amplitude variations in the received signal at the output of the IF amplifier by band-limiting the signal.

A band-pass filter centered at f_{IF} = 10.7 MHz with a bandwidth of 200 kHz is included in the limiter to remove higher order frequency components.

FM Radio Broadcasting



A balanced frequency discriminator is used for frequency demodulation.

The output of the audio amplifier is filtered by a low pass filter to remove out-of-band noise.

The output of the low pass filter is used to drive a loudspeaker.

References

- **John G. Proakis and Masoud Salehi, *Communication Systems Engineering*. 2nd Edition, Pearson Education, 2008.**

- **Simon Haykin, *Communication Systems*. 4th Edition, Wiley India Edition, 2008.**

- **B. P. Lathi, *Modern Digital and Analog Communication Systems*. 3rd Edition, Oxford University Press, 2000.**

Thank You ...