Linearization of Sensors

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Topics to be covered

- Linearization Techniques for RTDs
- Linearization Techniques for Thermistors

Resistance Temperature Detectors (RTDs)

- ✓**Resistance Temperature Detectors (RTDs) are temperature transducers which produce an output resistance (R) in response to an input temperature (t).**

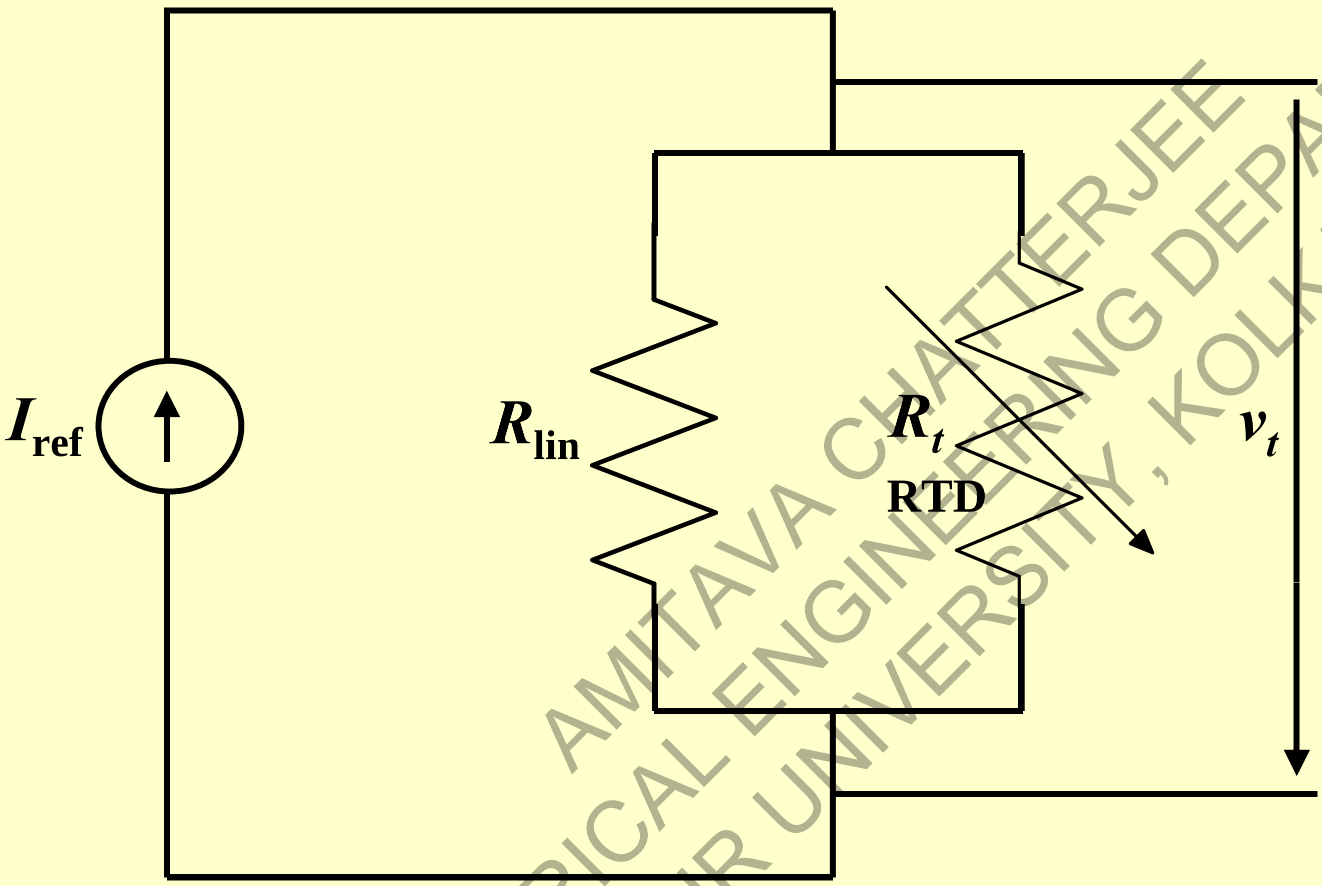
$$R = R_0\left(1+\alpha_1 t+\alpha_2 t^2+\cdots+\alpha_n t^n\right)$$

- ✓**Most commonly used materials for construction of RTDs: Platinum, Nickel and Copper.**

- ✓**Usually 2 or 3 of the α constants are good enough for highly accurate representation.**

Linearization of RTDs

Scheme 1



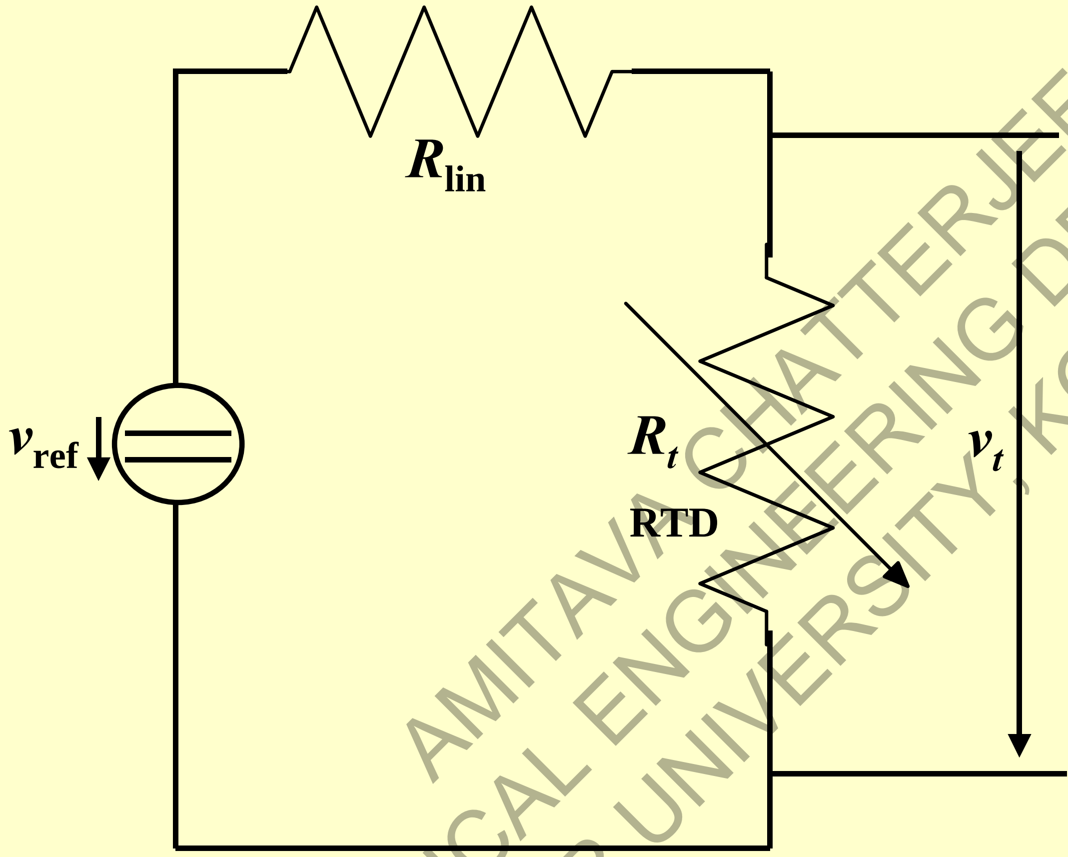
Linearization employing a constant current source

$$v_t = I_{ref} \cdot R_{lin} \cdot \frac{R_t}{R_t + R_{lin}}$$

✓ **A suitable fixed resistor R_{lin} is connected in parallel with the RTD and the arrangement is fed from a constant current source.**

Linearization of RTDs (contd…)

Scheme 2

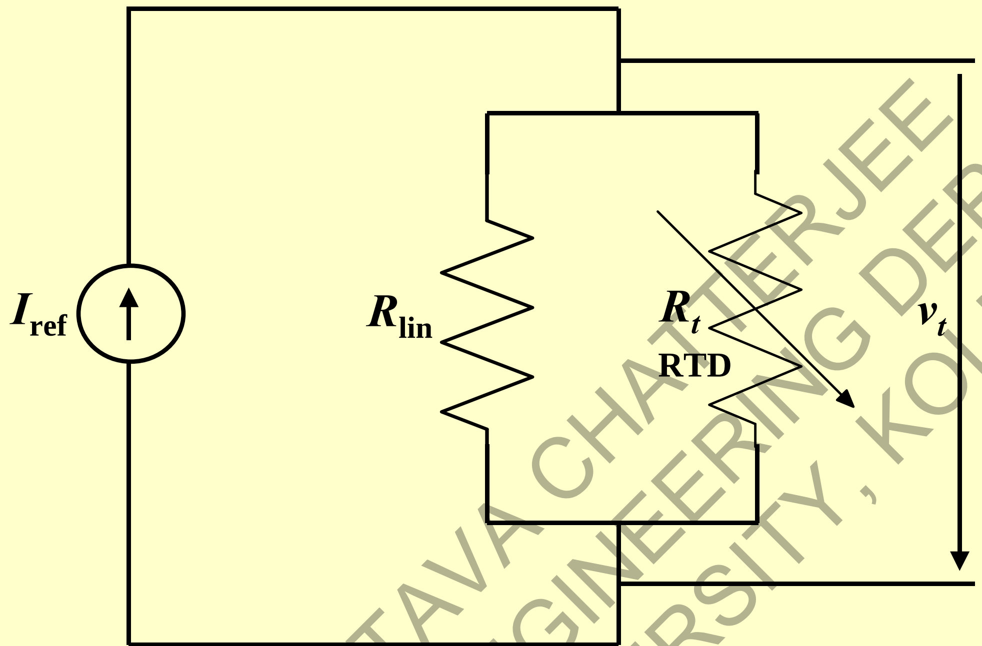


Linearization employing a voltage source

$$v_t = v_{ref} \cdot \frac{R_t}{R_t + R_{lin}}$$

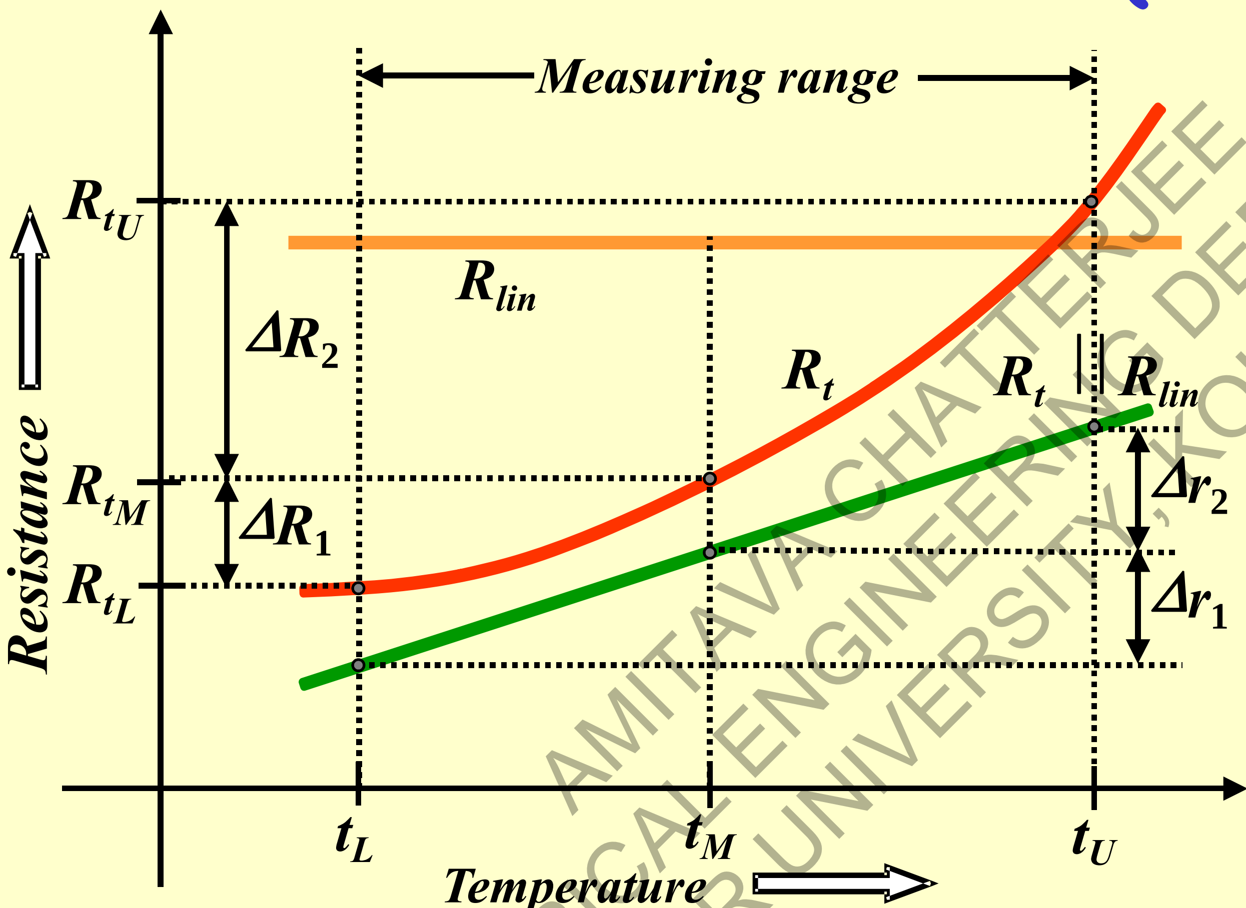
✓ **A suitable fixed resistor R_{lin} is connected in series with the RTD and the arrangement is fed from a constant voltage source.**

Analysis of the Circuit employing Constant Current Source



- ✓ If $R_{eq} = \left(R_t \parallel R_{lin}\right) = \dfrac{R_t R_{lin}}{R_t + R_{lin}}$ vary linearly with t, v_t will also vary linearly with t.
- ✓ The design is based on the resistance values of the sensor at three temperatures: R_{t_L} (lower), R_{t_U} (upper), and R_{t_M} (mid-point).

Analysis of the Circuit employing Constant Current Source (contd…)



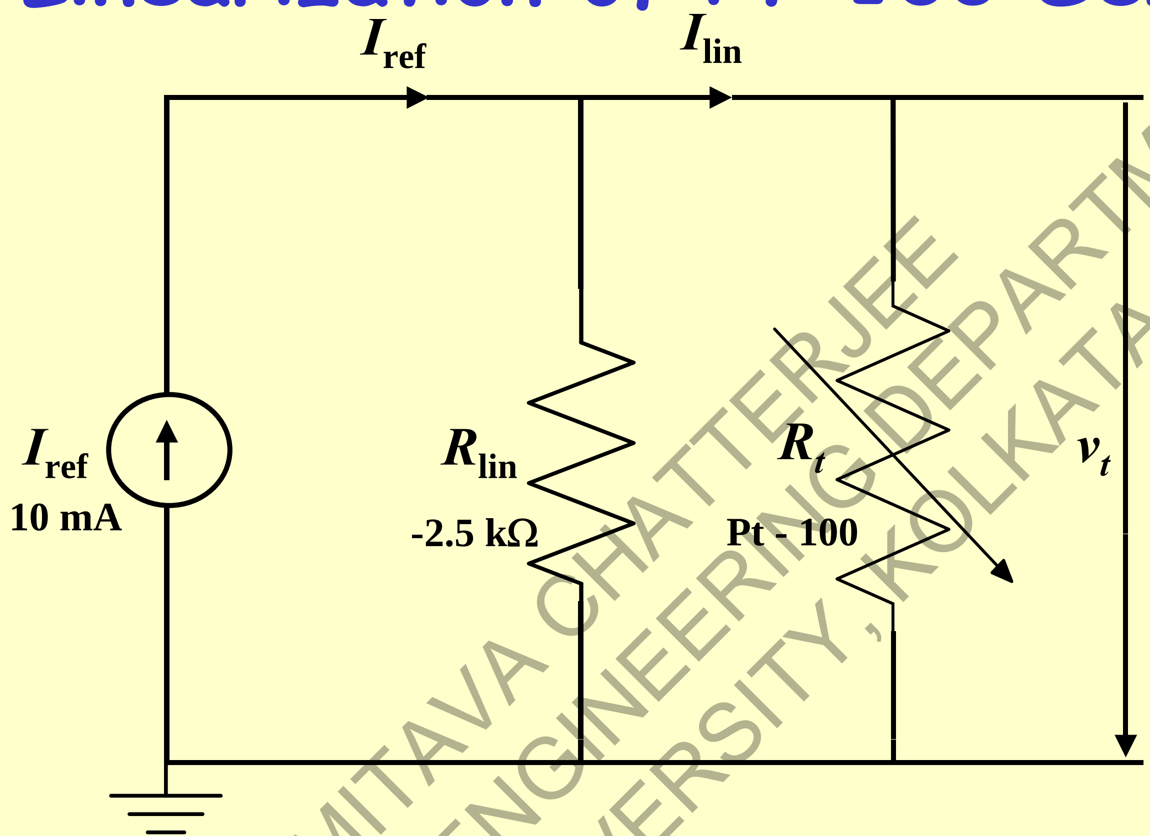
Linearization of RTD employing a parallel resistor

linearization condition

$$R_{eq}(t_M) - R_{eq}(t_L) = R_{eq}(t_U) - R_{eq}(t_M)$$

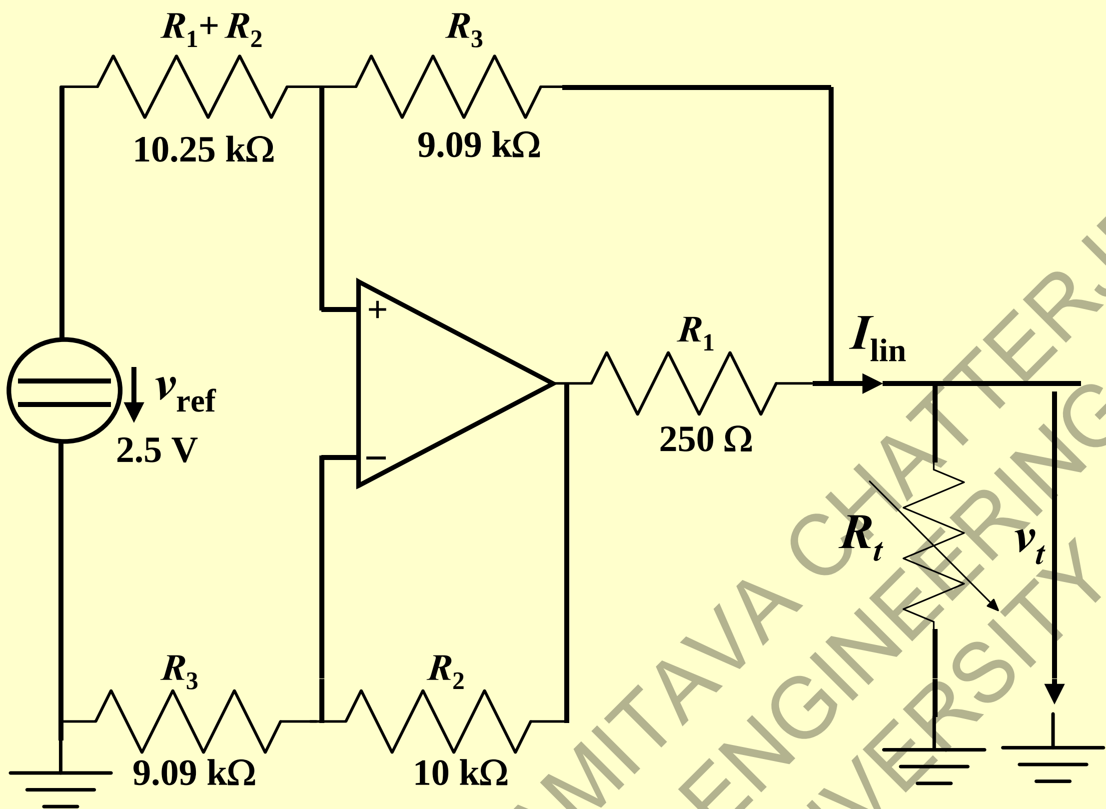
$$R_{lin} = \frac{R_{t_M}\left(R_{t_L} + R_{t_U}\right) - 2R_{t_L}R_{t_U}}{R_{t_L} + R_{t_U} - 2R_{t_M}}$$

Linearization of Pt-100 Sensor



- ✓ For a Pt-100 sensor, operated in the temperature range between 0°C and 400°C, a linearizing resistor R_{lin} = -2.5 KΩ is required.
- ✓ A current source of negative internal resistance must be used in this case.

Linearization of Pt-100 Sensor (contd…)



$$r_o = -\frac{\Delta v_t}{\Delta I} = \frac{R_1 R_3 \left(R_2 + R_3\right)}{R_3^2 - R_2^2} = R_{lin}$$

Implementation of current source having a negative output resistance

✓ **If we make R_1 = 250 Ω, R_2 = 10 KΩ and R_{lin} = -2.5 KΩ, we obtain R_3 = 9.09 KΩ.**

Thermistors

✓Thermistors are semiconductor type temperature transducers with a negative temperature co-efficient of resistance.

$$R_T = R_0 \cdot exp\left[\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$$

R_0 : resistance at the reference temperature T_0,

β : an experimentally determined constant, varies between 3500K and 4600K ,

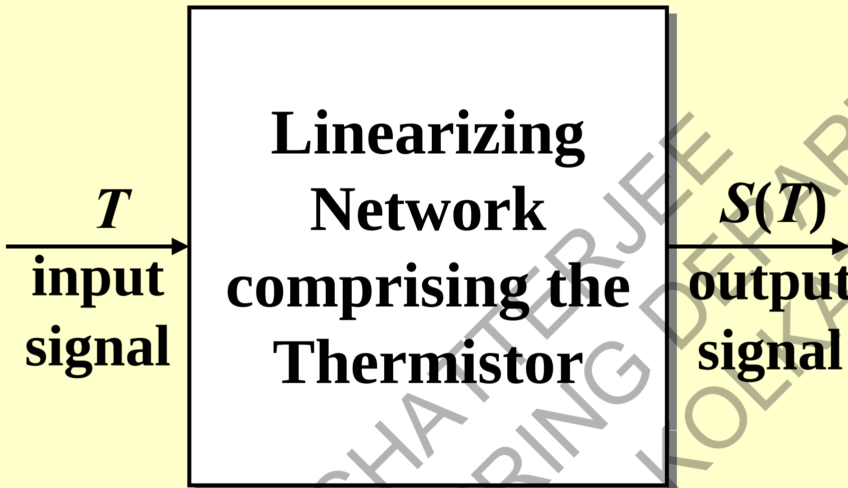
T and T_0: measured in Kelvin.

✓Thermistors have large temperature co-efficients (-3 to –5% per °C .

✓More accurate temperature-resistance relation:

$$\frac{1}{T} = \frac{1}{T_0} + \frac{1}{\beta} ln\frac{R_T}{R_0} + \frac{1}{C}\left(ln\frac{R_T}{R_0}\right)^3$$

Linearization of Thermistors



Schematic representation of linearizing networks for the thermistors

$$S(T) = S(T_r) + h \cdot S'(T_r) + \frac{h^2}{2!} \cdot S''(T_r) + \frac{h^3}{3!} \cdot S'''(T_r) + \cdots$$

- ✓ **$h = T \sim T_r$ is the increment or decrement in temperature about T_r. $S^{/}(T_r)$, $S^{//}(T_r)$,… are derivatives of $S(T)$ with respect to T, at $T = T_r$.**

Linearization of Thermistors (contd…)

Considerations

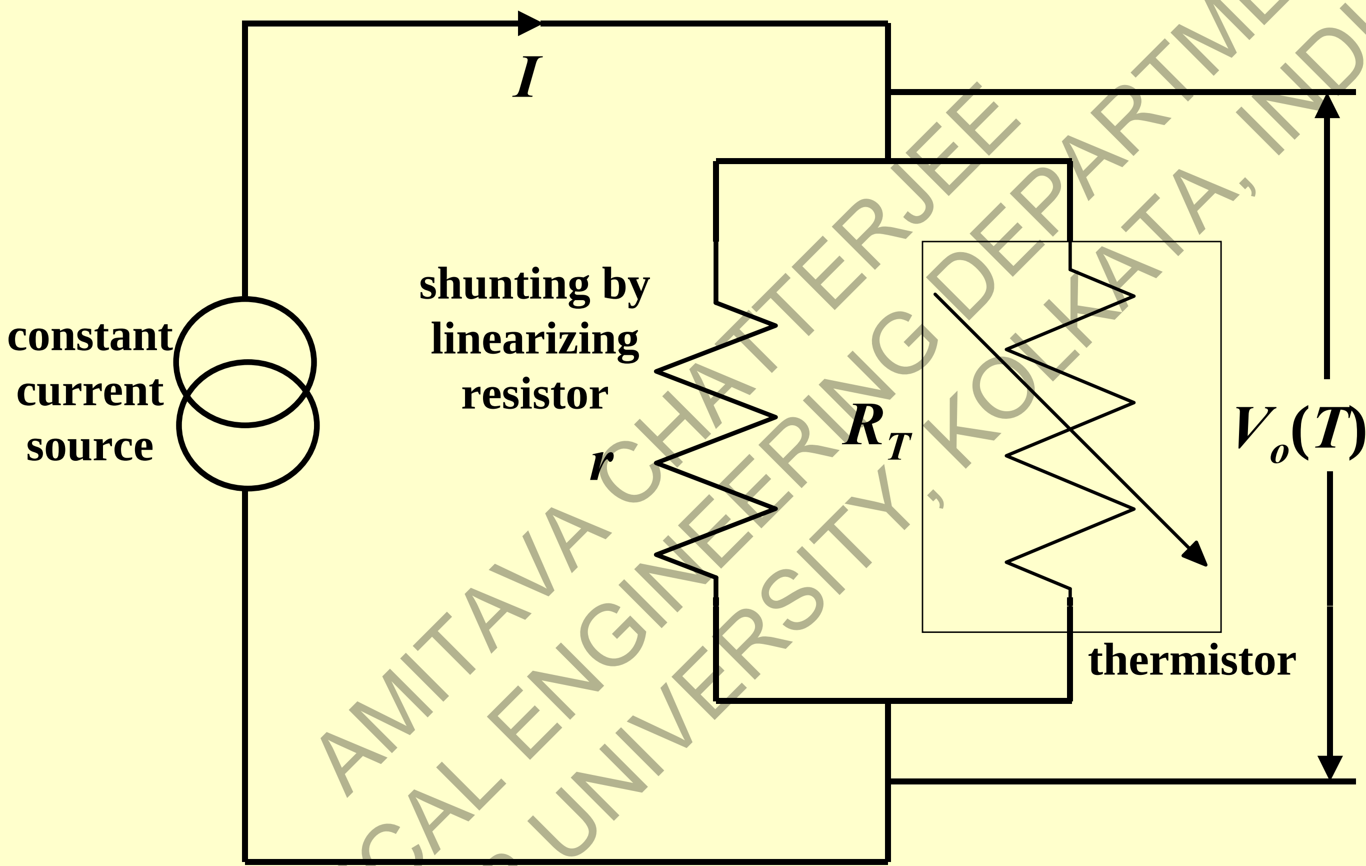
- ✓ **The major contribution to non-linearity comes from the h^2 term containing $S^{//}(T_r)$. This term can be made zero by proper choice of circuit components.**

- ✓ **Under this condition, the $S(T)$ vs. T characteristic can be assumed to be linear over a wide span of temperature, as long as the h^3 term remains negligibly small.**

- ✓ **The condition $S^{//}(T_r) = 0$ implies that the $S(T)$ vs. T curve should have a point of inflection at $T = T_r$.**

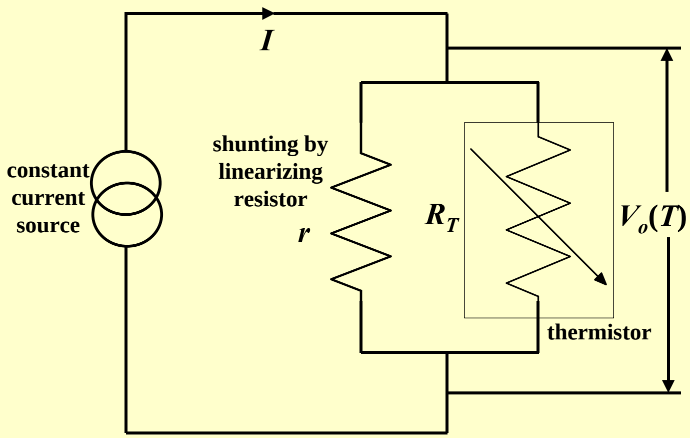
- ✓ **In practice, by proper selection of components, the point of inflection is located at the midpoint T_M of the range of temperature over which linearization is to be carried out.**

Linearization of Thermistors by a Shunt Resistor



A simple linearization circuit for thermistors

Linearization of Thermistors by a Shunt Resistor (contd…)



output signal = voltage across the thermistor

$$S(T)=V_o(T)=I\cdot R_{eq}(T)=I\cdot\frac{rR_T}{r+R_T}$$

as *I* is const., equivalent $S(T)=R_{eq}(T)=\dfrac{rR_T}{r+R_T}$

$$S'(T)\approx R'_{eq}(T)=\frac{r\cdot R'_T(r+R_T)-r\cdot R_T\cdot R'_T}{(r+R_T)^2}=\frac{r^2\cdot R'_T}{(r+R_T)^2}$$

similarly $S''(T)\approx R''_{eq}(T)=\dfrac{r^2\cdot R''_T(r+R_T)^2-2r^2\cdot(R'_T)^2(r+R_T)}{(r+R_T)^4}$

Design Procedure: make $S''(T_M)=0 \Rightarrow R''_{eq}(T_M)=0 \Rightarrow r=\dfrac{2\left(R'_{T_M}\right)^2}{R''_{T_M}}-R_{T_M}$

Now, $R'_{T_M}=R_{T_M}\cdot\left(-\dfrac{\beta}{T_M^2}\right)=R\cdot e^{\frac{\beta}{T_M}}\cdot\left(-\dfrac{\beta}{T_M^2}\right) \Rightarrow R''_{T_M}=R_{T_M}\cdot\left(\dfrac{\beta}{T_M^3}\right)\left(2+\dfrac{\beta}{T_M}\right)$

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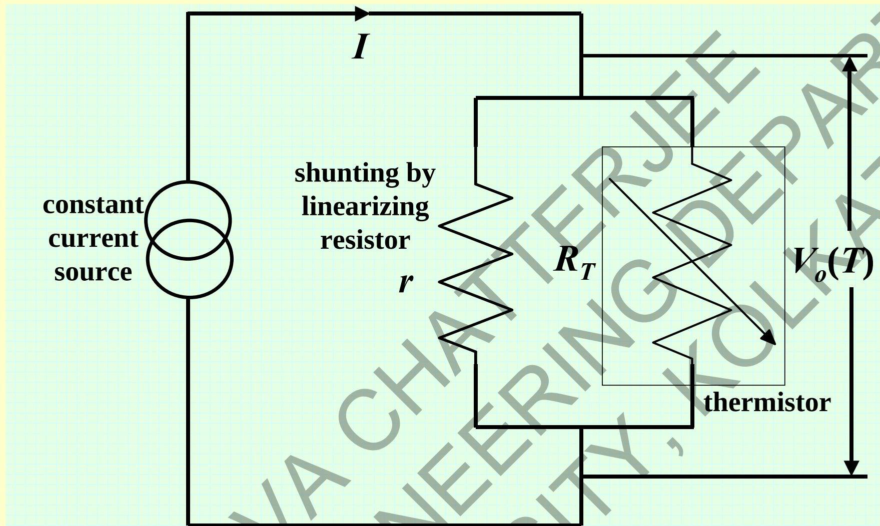
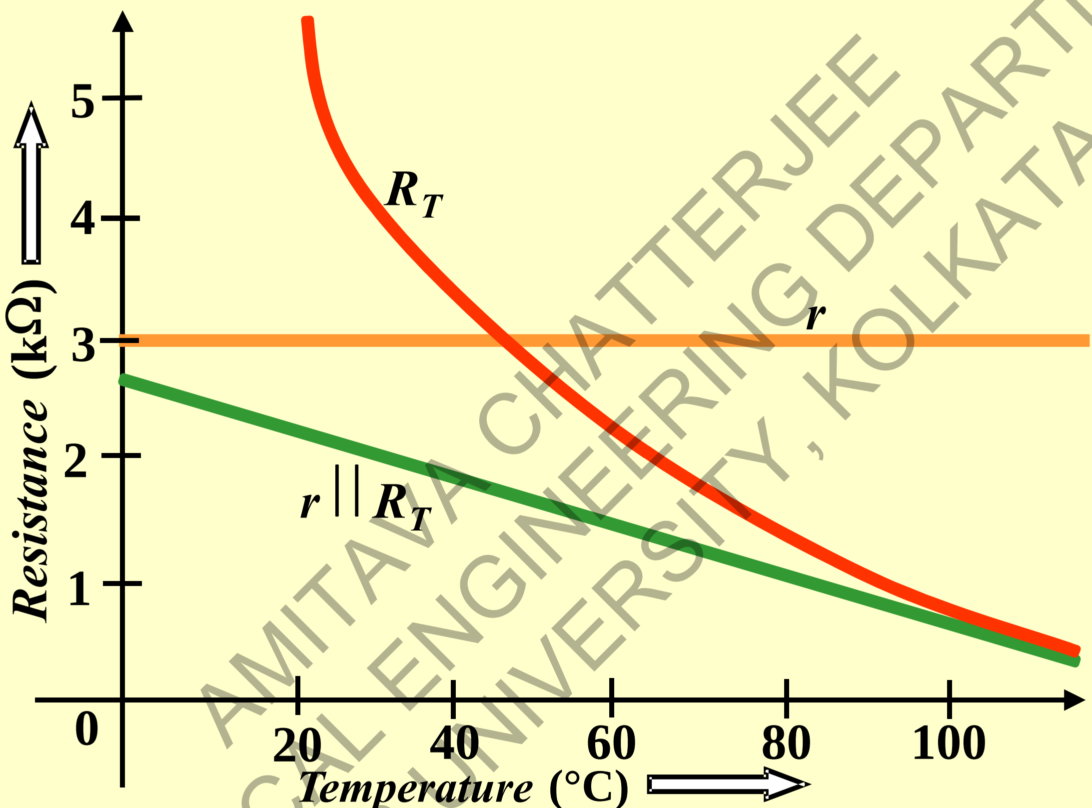
Linearization of Thermistors by a Shunt Resistor (contd…)



Final expression for the linearizing shunt resistance *r*:

$$r = R_{T_M}\left[\frac{\beta - 2T_M}{\beta + 2T_M}\right]$$

Linearization of Thermistors by a Shunt Resistor (contd…)



Linearization of NTC thermistor employing a parallel resistor

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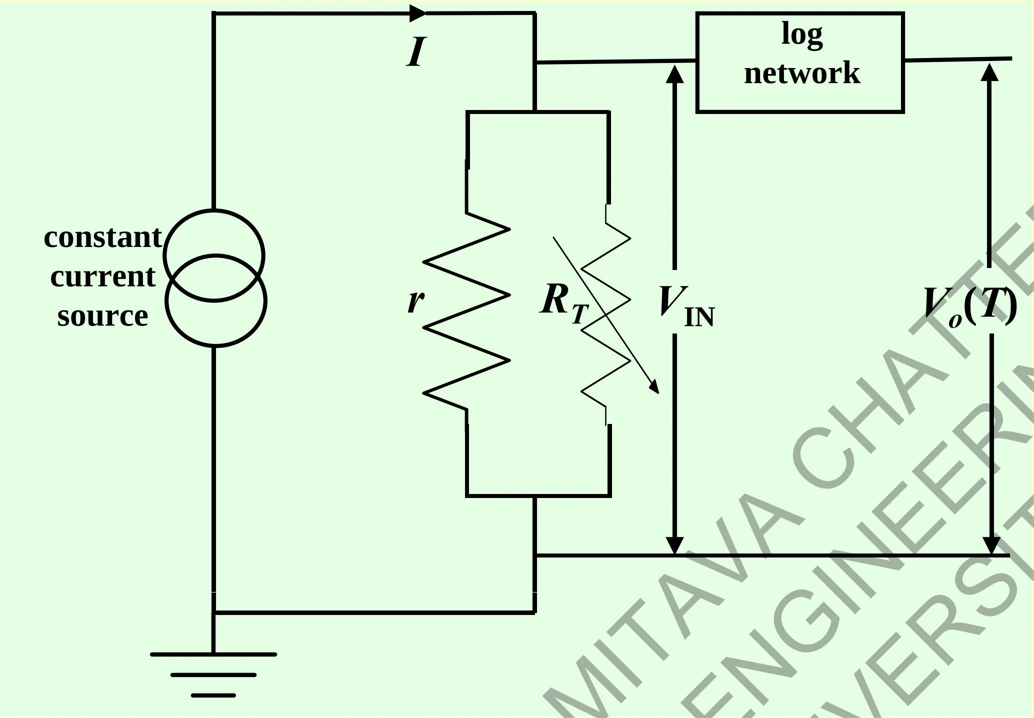
Linearization of Thermistors by a Logarithmic Network



Thermistor linearization using logarithmic network

$$V_{IN} = I \cdot R_{eq} \quad \textbf{and}$$

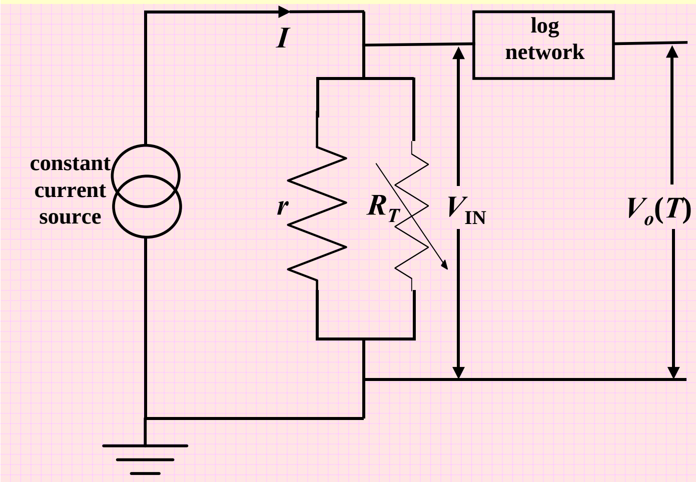
$$V_o(T) = -K \log_{10} \frac{V_{IN}}{V_{ref}}$$

K = scale factor of the logarithmic network,
V_{ref} = effective internally generated voltage in the log-network,
R_T = thermistor resistance at T K

$$R_{eq} = \frac{rR_T}{r + R_T}$$

- ✓ **This scheme can display a linear voltage-temperature relation over a much wider range of temperature (-25°C to +100°C) with reasonably good response linearity.**

Linearization of Thermistors by a Logarithmic Network (contd…)



$$R_T = R_0 \cdot \exp\left[\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right] \;\&\; V_o(T) = -K\log_{10}\left[\left(\frac{I}{V_{ref}}\right)\cdot\frac{rR_T}{r+R_T}\right]$$

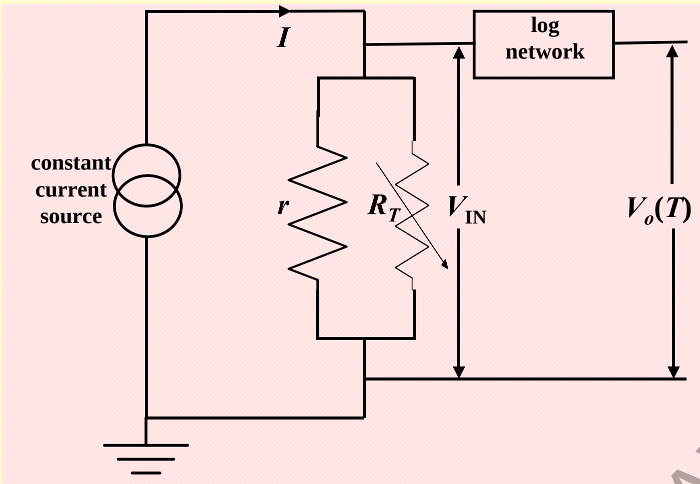
$$V_o(T) = V_o(T_r + h) = V_o(T_r) + h\cdot V_o'(T_r) + \frac{h^2}{2!}\cdot V_o''(T_r) + \frac{h^3}{3!}\cdot V_o'''(T_r) + \cdots$$

h = $T \sim T_r$ is the increment or decrement in temperature about the reference temperature T_r. $V_o'(T_r)$, $V_o''(T_r)$, $V_o'''(T_r)$ etc. are the first, second, third, etc. derivatives of $V_o(T)$ w.r.t. t temperature T, at $T = T_r$.

Design Procedure: make the h^2 term zero ⇒ $V_o''(T_r) = 0$

T_r is considered at the midpoint T_M of the range of temperature over which linearization is to be carried out

Linearization of Thermistors by a Logarithmic Network (contd…)



$$V_o(T) = -K\log_{10}\left[\left(\frac{I}{V_{ref}}\right)\cdot\frac{rR_T}{r+R_T}\right] = -K\log_{10}\left[Y\cdot\frac{rR_T}{r+R_T}\right] \quad \left(Y = \frac{I}{V_{ref}}\right)$$

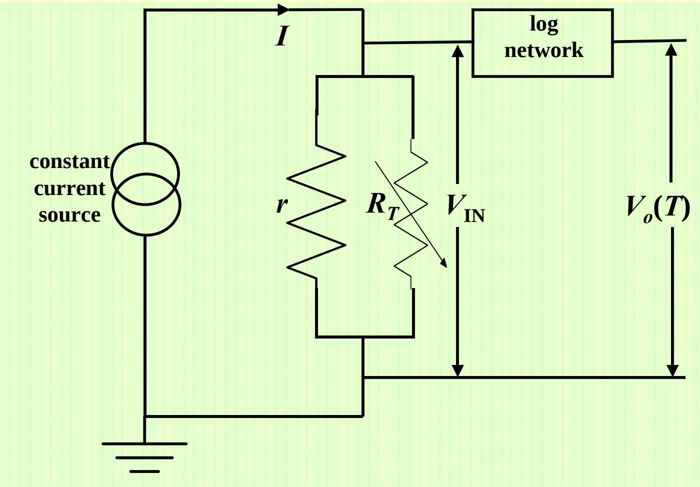
$$V_o'(T) = \frac{-K}{Y}\cdot\frac{r^2 R_T'}{rR_T(r+R_T)} = -K_1\cdot\frac{R_T'}{R_T(r+R_T)} \quad \left(K_1 = \frac{Kr}{Y}\right)$$

$$V_o''(T) = -K_1\cdot\frac{\left[rR_T R_T'' + R_T^2 R_T'' - R_T'^2(r+2R_T)\right]}{R_T^2(r+R_T)^2}$$

Design condition: $V_o''(T_M) = 0$ ⇒

$$r = \frac{-R_{T_M}\left[2R_{T_M}'^{\,2} - R_{T_M}R_{T_M}''\right]}{\left[R_{T_M}'^{\,2} - R_{T_M}R_{T_M}''\right]}$$

Linearization of Thermistors by a Logarithmic Network (contd…)



$$R'_T = R \cdot e^{\frac{\beta}{T}} \cdot \left(-\frac{\beta}{T^2}\right) = R_T \cdot \left(-\frac{\beta}{T^2}\right)$$

$$R''_T = R \cdot e^{\frac{\beta}{T}} \cdot \left(\frac{\beta^2}{T^4}\right) + R \cdot e^{\frac{\beta}{T}} \cdot \left(\frac{2\beta}{T^3}\right) = R \cdot e^{\frac{\beta}{T}} \cdot \left(\frac{\beta}{T^3}\right)\left(2 + \frac{\beta}{T}\right)$$

$$r = \frac{-R_{T_M}\left[2\left(R \cdot e^{\frac{\beta}{T_M}}\right)^2\left(\frac{\beta^2}{T_M^4}\right) - \left(R \cdot e^{\frac{\beta}{T_M}}\right)^2\left(\frac{\beta}{T_M^3}\right)\left(2 + \frac{\beta}{T_M}\right)\right]}{\left[\left(R \cdot e^{\frac{\beta}{T_M}}\right)^2\left(\frac{\beta^2}{T_M^4}\right) - \left(R \cdot e^{\frac{\beta}{T_M}}\right)^2\left(\frac{\beta}{T_M^3}\right)\left(2 + \frac{\beta}{T_M}\right)\right]}$$

$$r = \frac{R_{T_M}\left[\beta - 2T_M\right]}{2T_M}$$

Logarithmic Conversion with an Inherently Logarithmic Device

Shockley's first order theory for a single p-n junction:

$$I = I_0\left(\exp\frac{qV}{kT} - 1\right)$$

I = current through the junction (A)

I_0 = the theoretical reverse saturation current (A)

V = the voltage across the junction

q = magnitude of the electronic charge (1.6×10^{-19} C)

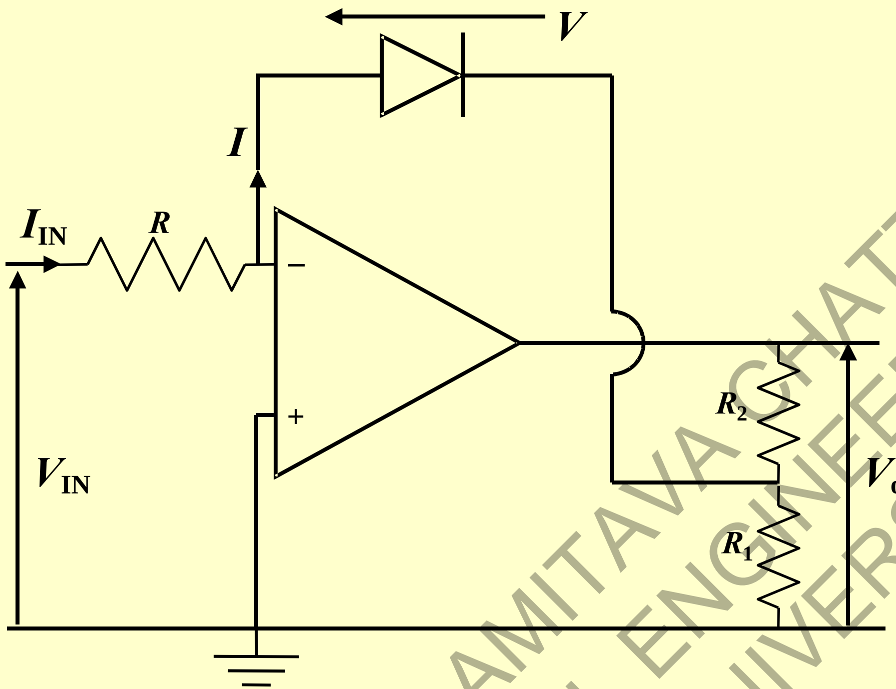
k = Boltzmann's constant (1.38×10^{-23} J/K) and

T = the temperature in K

$\frac{kT}{q} \approx 26\,\text{mV}$ **at 27°C. Hence, for V > 100 mV,** $I \approx I_0 \exp\frac{qV}{kT}$ ⇒ $V = 2.3\frac{kT}{q}\log_{10}\frac{I}{I_0}$

$\log_{10} I$ varies linearly with V, with a slope of $2.3\frac{kT}{q}$ **Volts/decade of current change.**

Logarithmic Conversion with an Inherently Logarithmic Device (contd…)



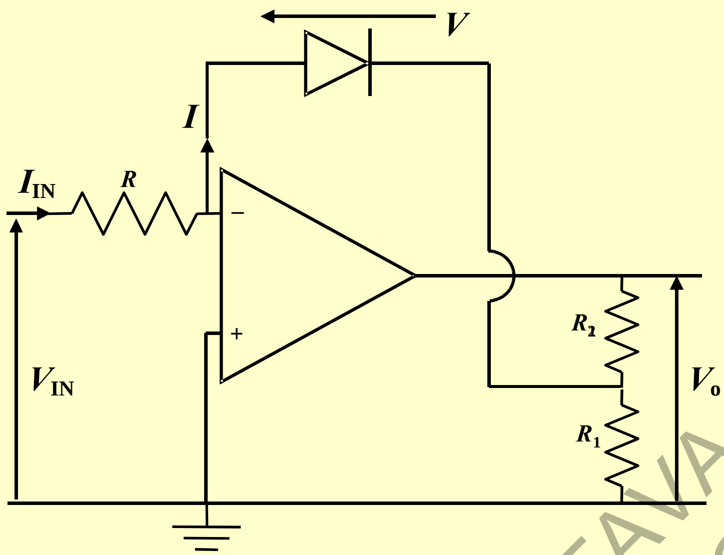
An op-amp based log amplifier with a diode as a log element

$$V = -\frac{R_1}{R_1 + R_2} V_o = 2.3\frac{kT}{q}\log_{10}\frac{I_{IN}}{I_o}$$

$$V_o = -\left(\frac{R_1 + R_2}{R_1}\right) 2.3\frac{kT}{q} log_{10}\frac{I_{IN}}{I_o}$$

$$= -\left(\frac{R_1 + R_2}{R_1}\right) 2.3\frac{kT}{q} log_{10}\left(\frac{V_{IN}}{R \cdot I_o}\right)$$

Logarithmic Conversion with an Inherently Logarithmic Device (contd…)



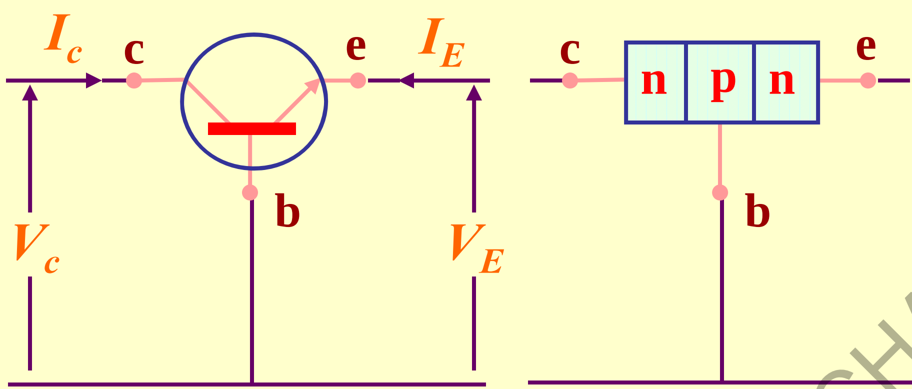
Limitations

- ✓ The problem of temperature dependence of scaling factor, E_o.
- ✓ Marked nonlinear temperature dependence exhibited by I_o.
- ✓ Diodes, used as ideal log elements, do not actually obey Shockley's relation accurately.

$$I_j = I_{0j}\left(\exp\frac{qV}{m_j kT} - i\right)$$

Conclusion: **Transistors appear to be better candidates than diodes as logarithmic elements.**

Logarithmic Conversion with an Inherently Logarithmic Device (contd…)



A simple n-p-n transistor model, along with its sign conventions

$$I_C = \alpha_F I_{ES}\left(\exp\frac{-qV_E}{kT} - 1\right) - I_{CS}\left(\exp\frac{-qV_C}{kT} - 1\right) - \sum I_{CS_j}\left(\exp\frac{-qV_C}{m_j kT} - 1\right)$$

α_F = current transfer ratio between emitter and collector (≈ 1)

I_{CS} = collector reverse saturation current with the emitter shorted to the base

I_{ES} = emitter reverse saturation current with the collector shorted to the base

m_j = a constant, between 1 and 4

If $V_C = 0$ ⇒ $$I_C = \alpha_F I_{ES}\left(\exp\frac{-qV_E}{kT} - 1\right)$$

When $I_C >> I_{ES}$ ⇒ $$-V_E = 2.3\frac{kT}{q}\log_{10}\left(\frac{I_C}{I_o}\right) \quad \text{where} \quad I_o = \alpha_F I_{ES}$$

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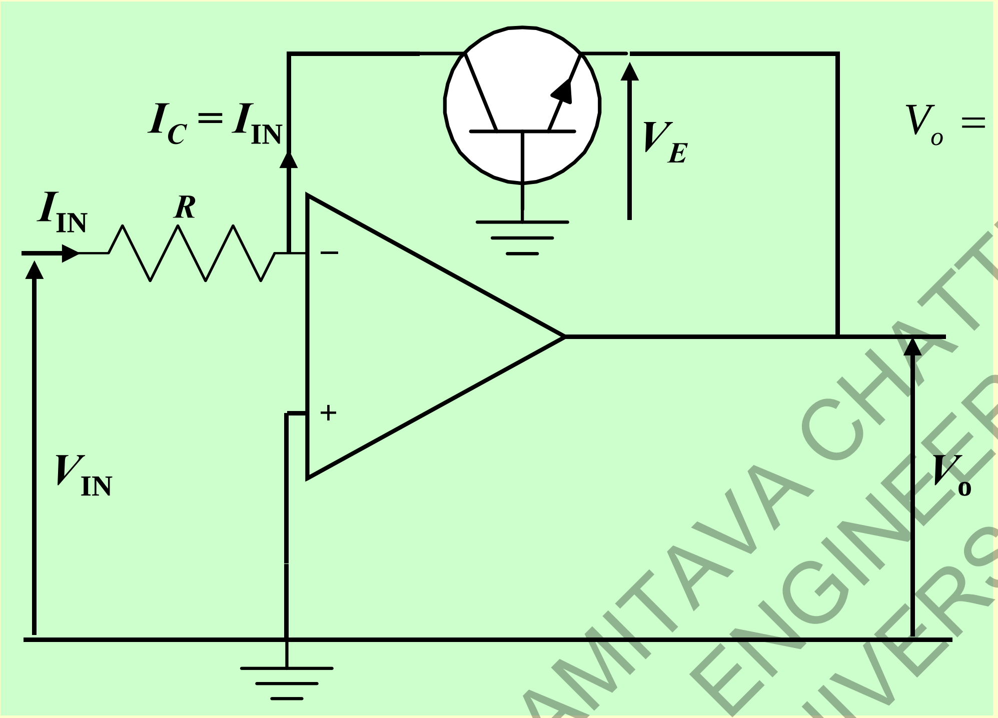
Logarithmic Conversion with an Inherently Logarithmic Device (contd…)



The transdiode configuration of a logarithmic amplifier

$$V_o = -2.3\frac{kT}{q}\log_{10}\left(\frac{I_{IN}}{\alpha_F I_{ES}}\right) = -2.3\frac{kT}{q}\log_{10}\left(\frac{V_{IN}}{R\cdot\alpha_F I_{ES}}\right)$$

Practical considerations

- ✓ **Temperature dependent errors.**

Sources:

$I_0 = \alpha_F I_{ES}$ **and** $E_o = 2.3\frac{kT}{q}$

⇩

Solution??

- ✓ **Use matched transistors to enable cancellation of the I_0 terms.**

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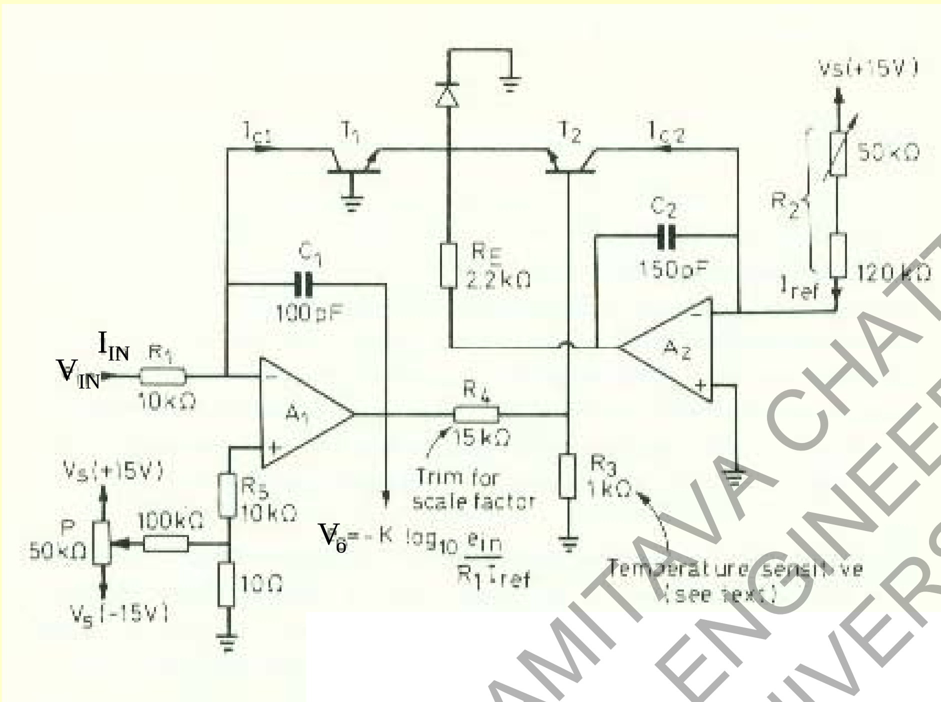
Logarithmic Conversion with an Inherently Logarithmic Device (contd…)



A temperature compensated log converter

(with 2 op-amps and 2 logging transistors)

$$V_{E_1} - V_{E_2} = V_o \frac{R_3}{R_3 + R_4}$$

We can write:

$$V_{E_1} = -E_o \log_{10}\left(\frac{I_{C_1}}{I_{o_1}}\right)$$

$$V_{E_2} = -E_o \log_{10}\left(\frac{I_{C_2}}{I_{o_2}}\right)$$

Hence: $V_{E_2} - V_{E_1} = E_o \log_{10} \frac{I_{C_1}}{I_{C_2}} \cdot \frac{I_{0_2}}{I_{0_1}}$

$$V_o = -\frac{R_3 + R_4}{R_3} 2.3 \frac{kT}{q} \log_{10} \frac{I_{IN}}{I_{ref}}$$

$$= -\frac{R_3 + R_4}{R_3} 2.3 \frac{kT}{q} \log_{10} \frac{V_{IN} \cdot R_2}{V_s \cdot R_1}$$

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Thank You