Process Controllers

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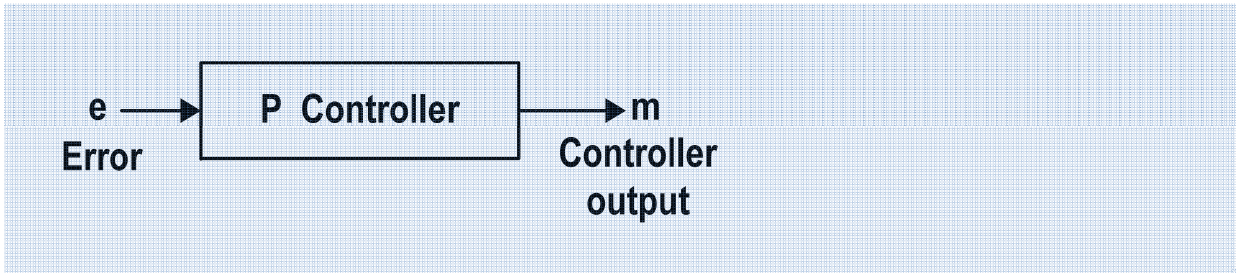
Realization of Electronic Proportional (P) controller



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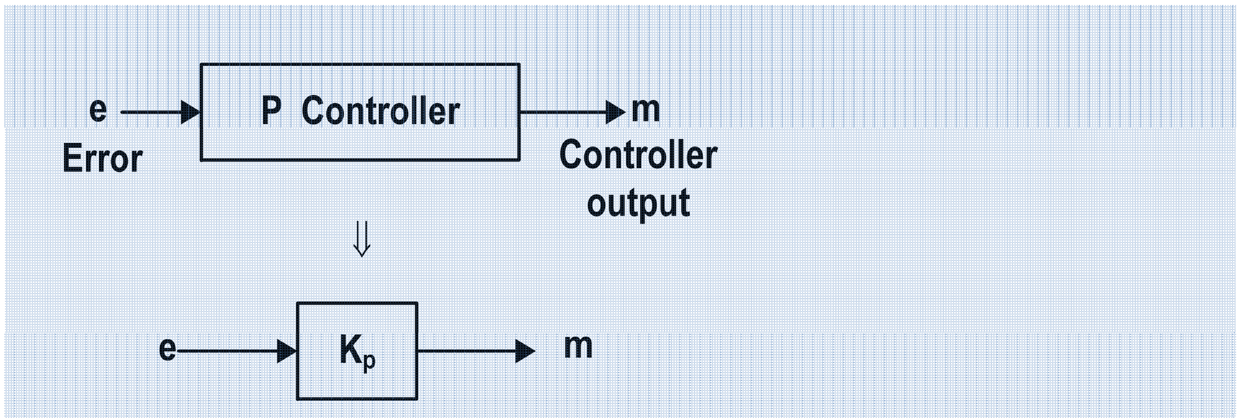

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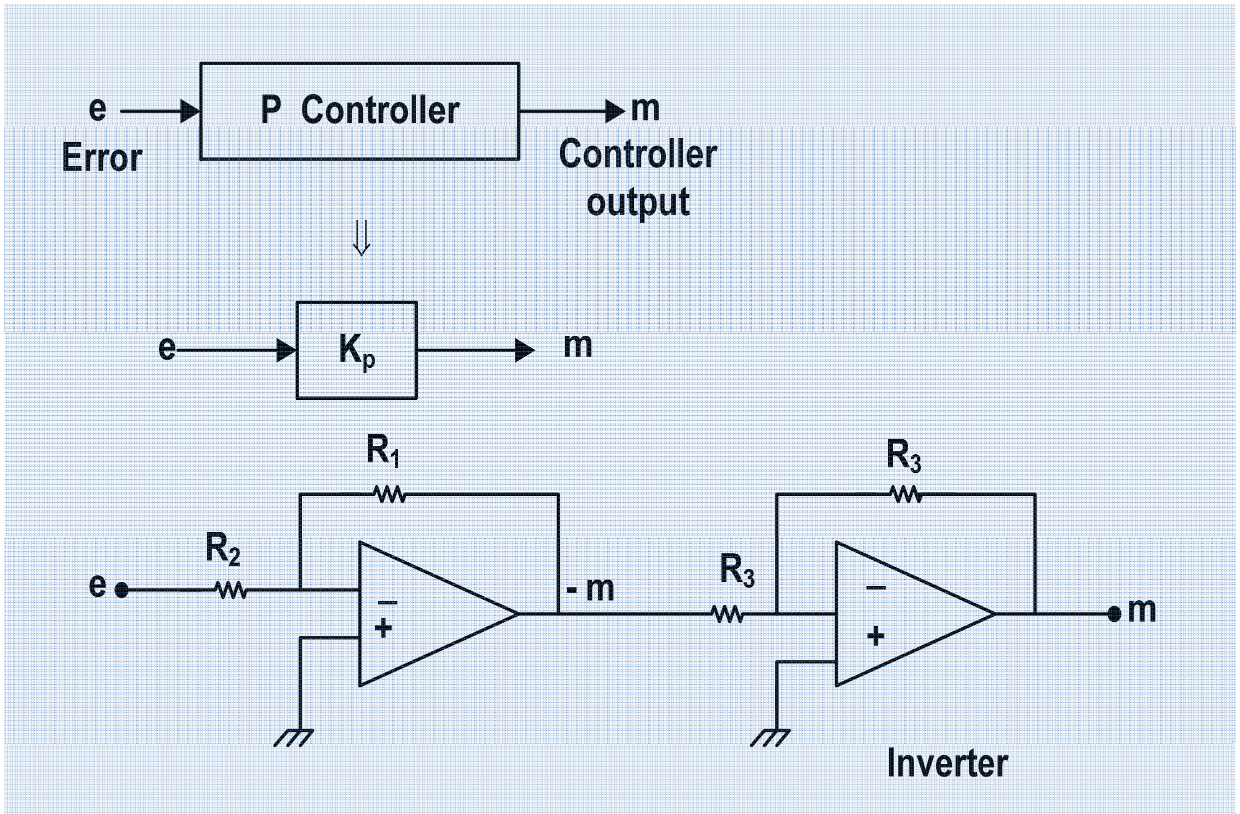

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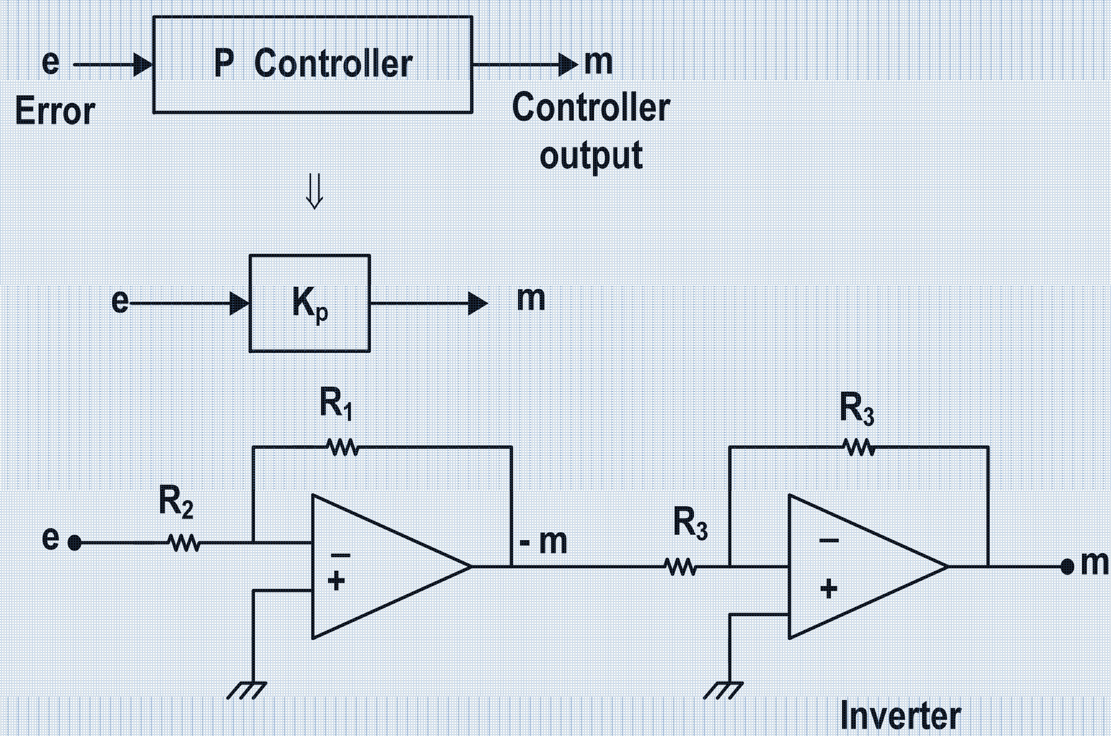

The controller output may be expressed as:

$m = K_p \cdot e$

where $K_p = R_1/R_2$

and P = % proportional band = $100 / K_p$

Realization of Electronic PID controller
(P, PI, & PD are special cases of PID)

The controller output may be expressed as:

$$m = K_p \left(e + \frac{1}{T_i} \int_0^t e\,dt + T_D \frac{de}{dt} \right)$$

where T_i = integral time constant
T_D = derivative time constant

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where T_i = integral time constant
T_D = derivative time constant

Or,

$$m = K_p e + \frac{K_p}{T_i}\int_0^t e\,dt + K_p T_D\frac{de}{dt}$$

Taking Laplace transform with zero initial conditions,

$$M(s) = K_p E(s) + \frac{K_p E(s)}{sT_i} + sK_p T_D E(s)$$

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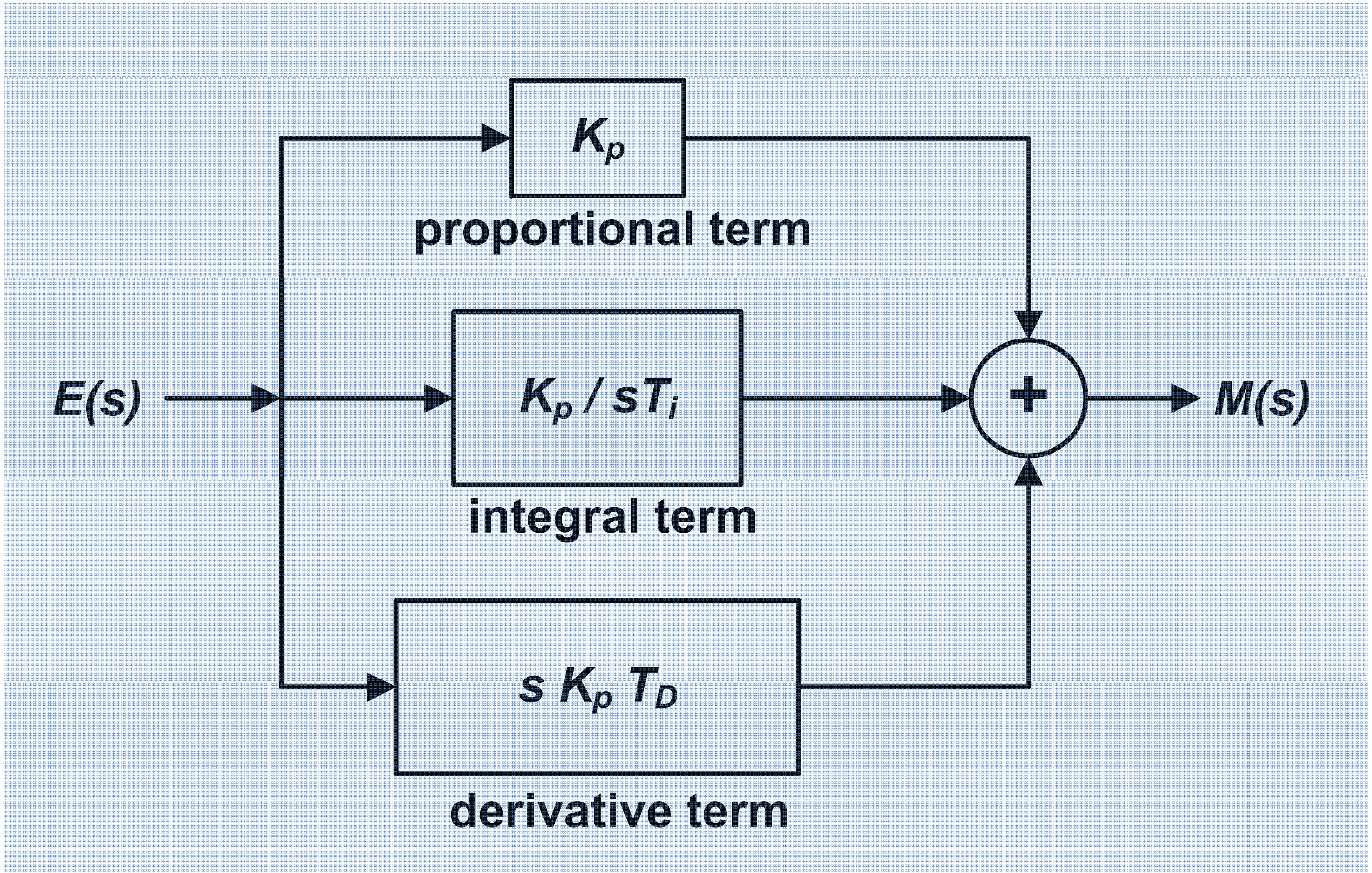
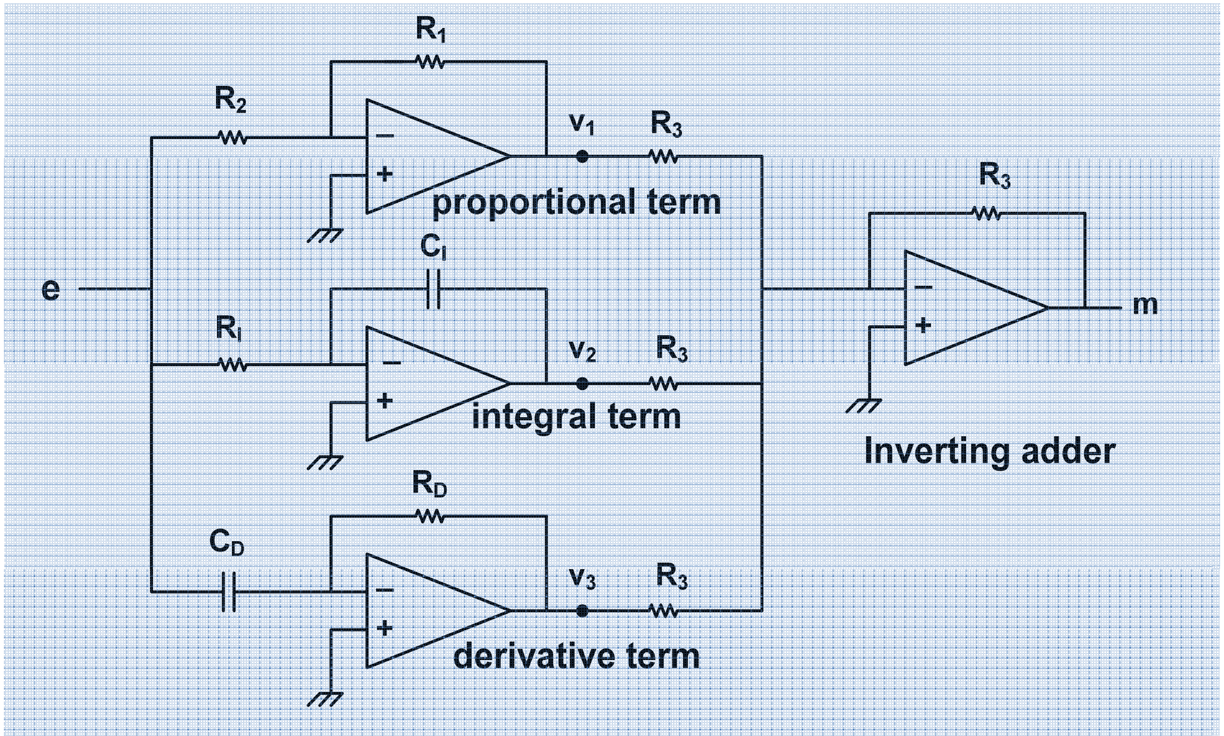
Or, $$\frac{M(s)}{E(s)} = K_p\left(1 + \frac{1}{sT_i} + sT_D\right)$$

the transfer function of PID controller

Parallel Realization of PID Controller



Circuit realization



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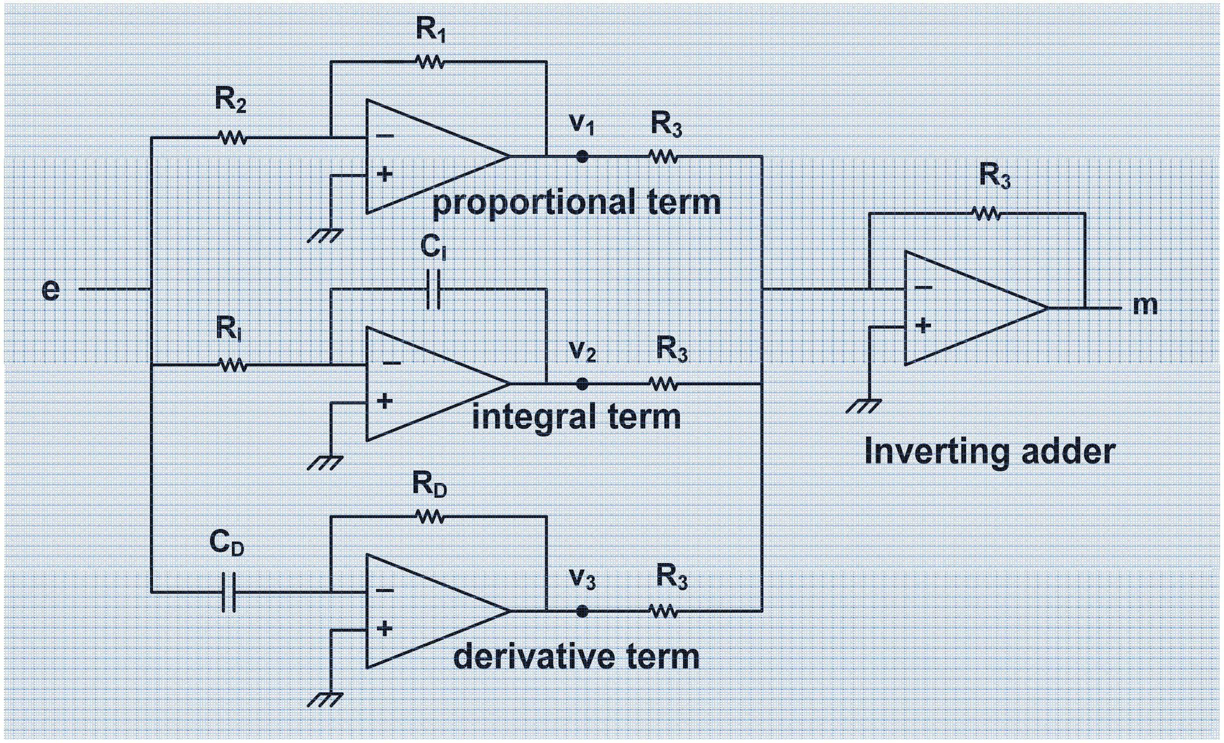
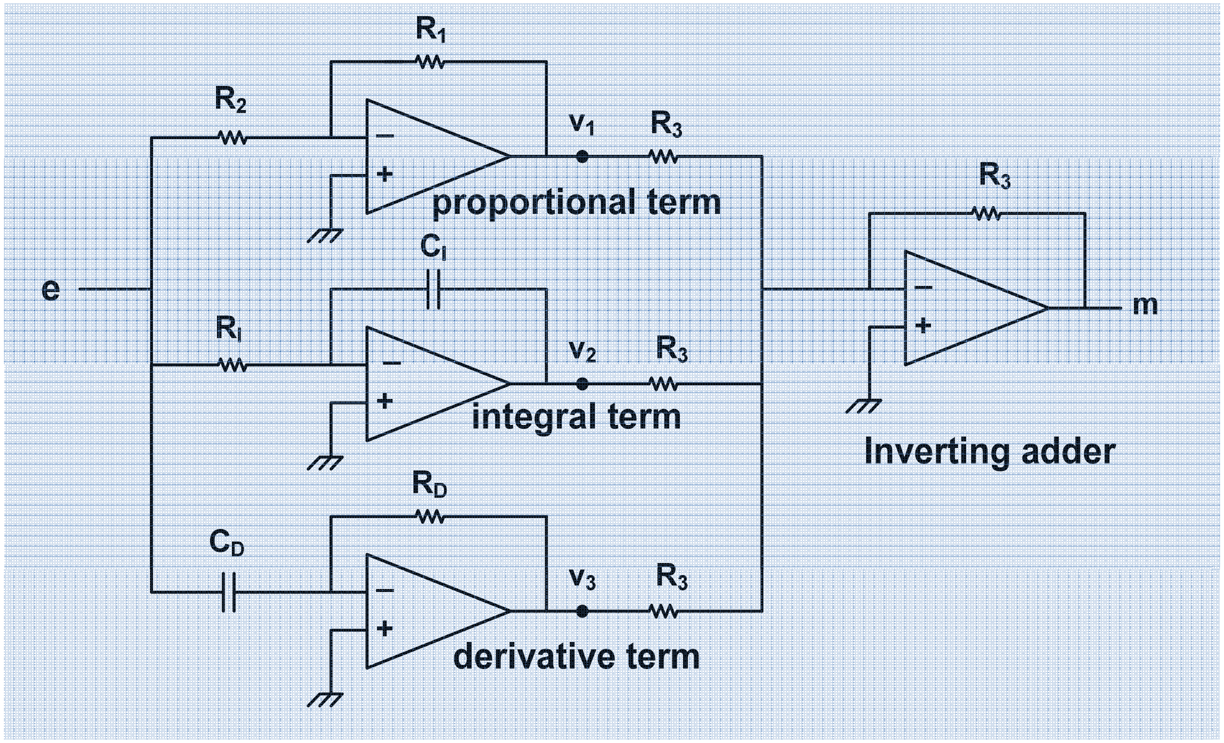


$$M(s) = -\left(V_1(s) + V_2(s) + V_3(s)\right)$$

Circuit realization



$$M(s) = -\left(V_1(s) + V_2(s) + V_3(s)\right)$$

Or,

$$M(s) = E(s)\left[\frac{R_1}{R_2} + \frac{1}{sR_I C_I} + sR_D C_D\right]$$

Circuit realization

Therefore, transfer function of PID controller becomes

$$\frac{M(s)}{E(s)} = \left[\frac{R_1}{R_2} + \frac{1}{sR_I C_I} + sR_D C_D \right]$$

Here,

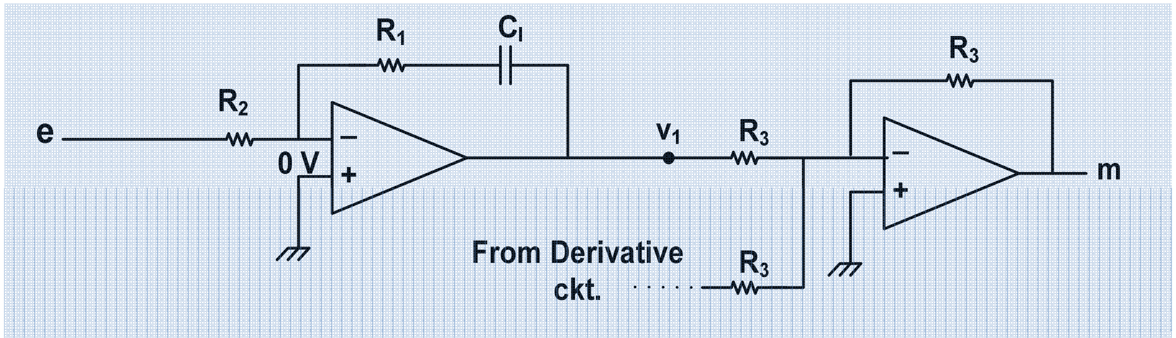
$$K_p = \frac{R_1}{R_2}$$

$$\frac{K_p}{T_i} = \frac{1}{R_I C_I}$$

$$K_p T_D = R_D C_D$$

Circuit realization

Proportional and integral terms may be combined as:



$$\frac{E(s)}{R_2} + \frac{V_1(s)}{R_1 + \frac{1}{sC_I}} = 0$$

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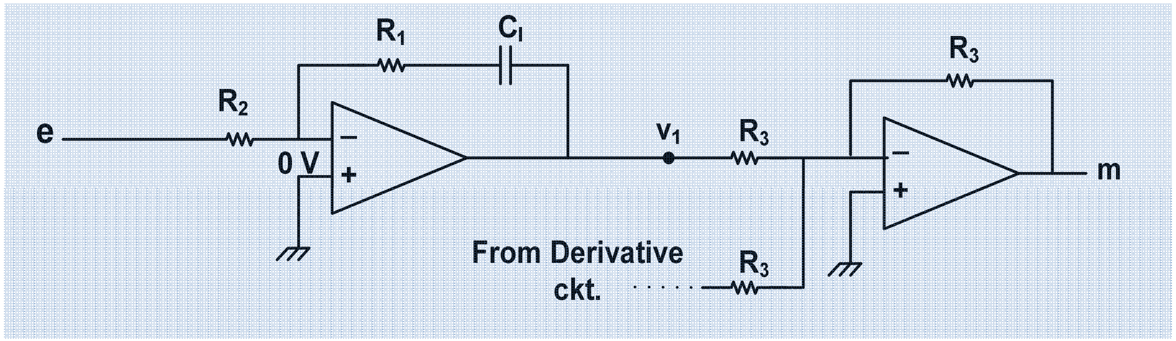

$$\frac{E(s)}{R_2} + \frac{V_1(s)}{R_1 + \frac{1}{sC_I}} = 0$$

Or,

$$\frac{E(s)}{R_2} = -\frac{V_1(s)}{R_1 + \frac{1}{sC_I}} = -\frac{V_1(s)sC_I}{1 + sR_1C_I}$$

Circuit realization

Or, $$\frac{V_1(s)}{E(s)} = -\frac{1 + sR_1C_I}{sR_2C_I}$$

Circuit realization

Or,

$$\frac{V_1(s)}{E(s)} = -\frac{1 + sR_1C_I}{sR_2C_I}$$

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$$= -\left(\frac{R_1}{R_2} + \frac{1}{sR_2C_I}\right)$$

$$= -\frac{R_1}{R_2}\left(1 + \frac{1}{sR_1C_I}\right)$$

Circuit realization

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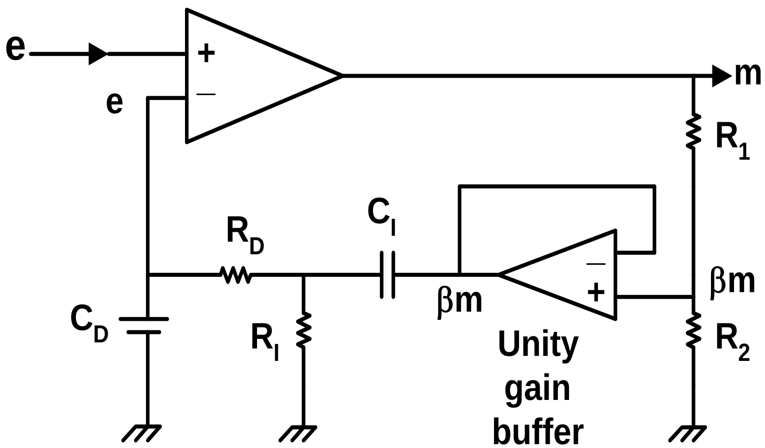
$$= -\frac{R_1}{R_2}\left(1 + \frac{1}{sR_1C_I}\right)$$

$$= -K_p\left(1 + \frac{1}{sT_i}\right)$$

Where

$$K_p = \frac{R_1}{R_2} \quad \text{and} \quad T_i = R_1C_I$$

A simple PID Controller with two op-amps



Here, $$\beta = \frac{R_2}{R_I + R_2}$$

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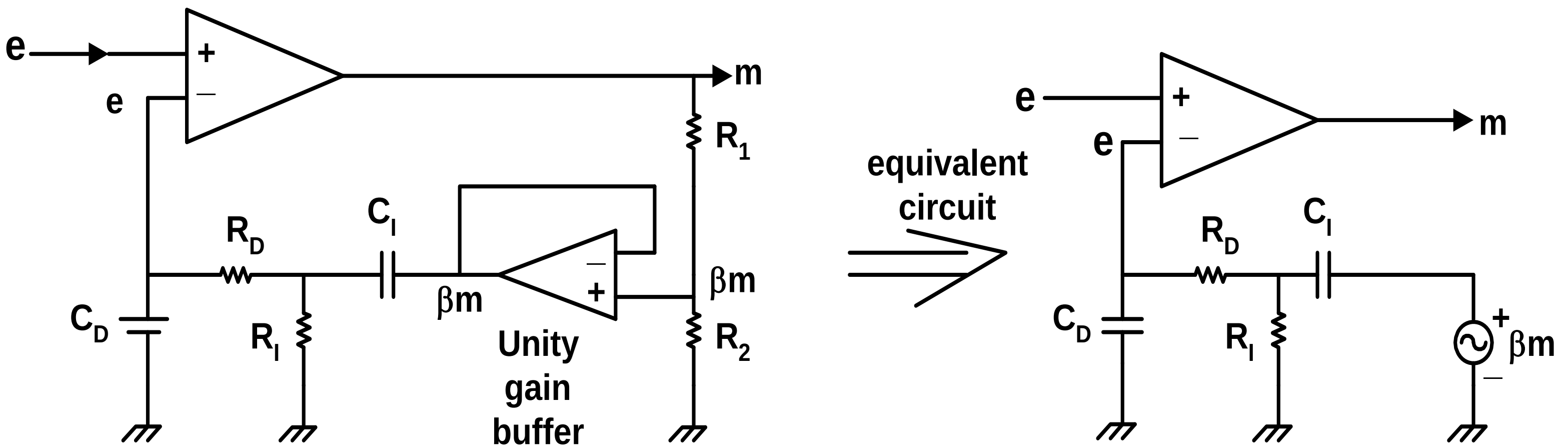

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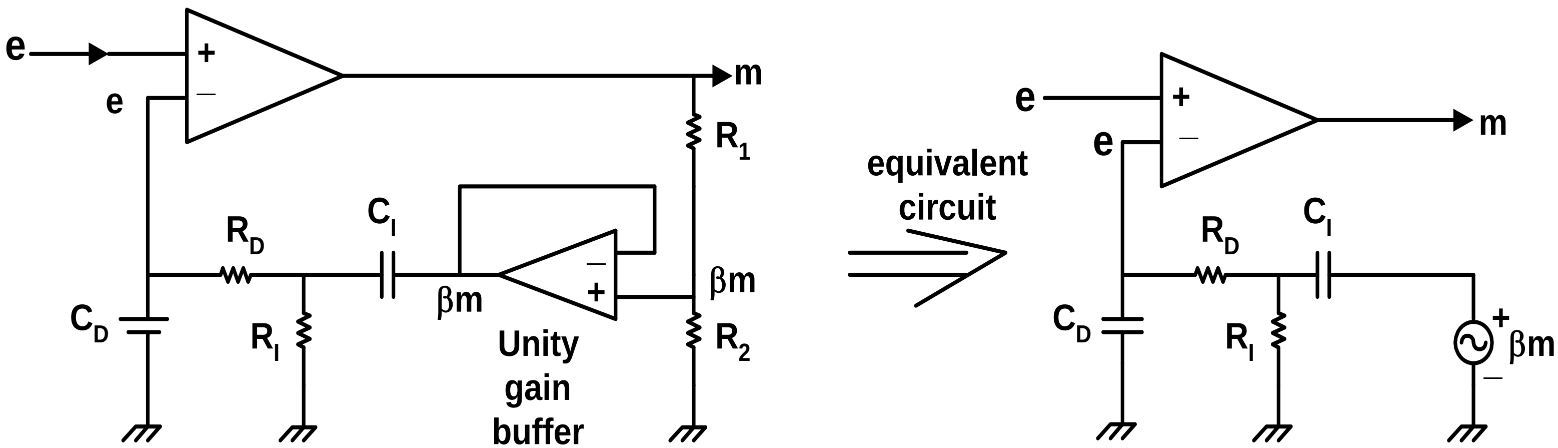
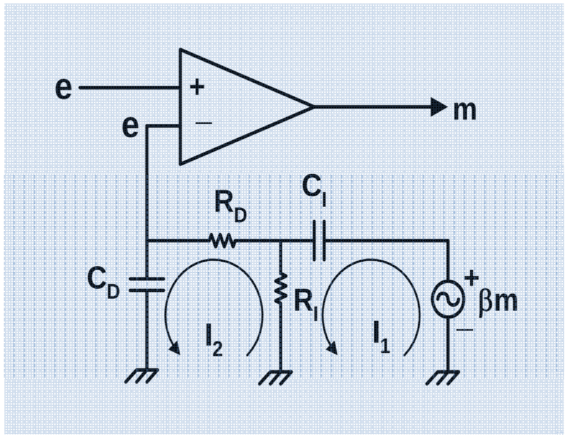


Here, $$\beta = \frac{R_2}{R_I + R_2}$$

The unity gain buffer amplifier is required to avoid the loading effect of the feedback network

Circuit solution



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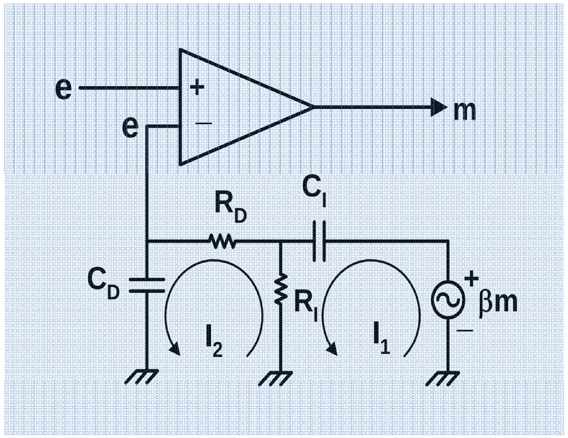
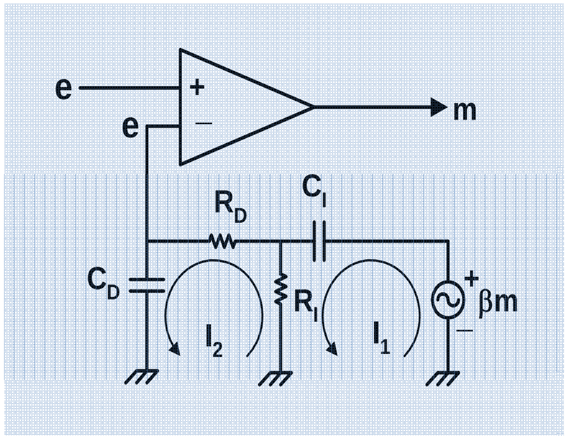


Loop equations:

$$\beta M(s) = I_1\left(R_I + \frac{1}{sC_I}\right) - I_2 R_I$$

$$0 = -I_1 R_I + I_2\left(R_I + R_D + \frac{1}{sC_D}\right)$$

Circuit solution



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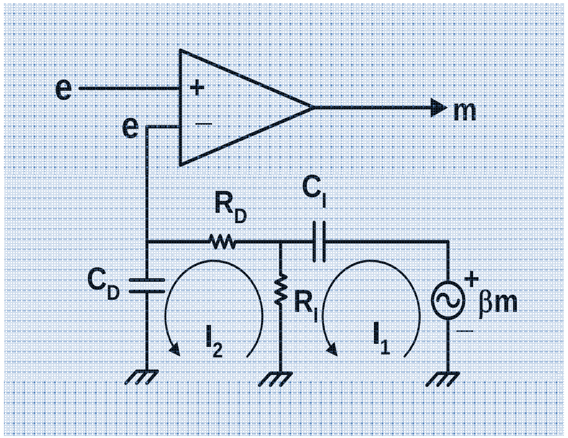
In matrix form:

$$\begin{bmatrix} R_I + \frac{1}{sC_I} & -R_I \\ -R_I & R_I + R_D + \frac{1}{sC_D} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \beta M(s) \\ 0 \end{bmatrix}$$

Circuit solution



By Cramer's rule:

$$I_2 = \frac{\begin{vmatrix} R_I + \dfrac{1}{sC_I} & \beta M(s) \\ -R_I & 0 \end{vmatrix}}{\begin{vmatrix} R_I + \dfrac{1}{sC_I} & -R_I \\ -R_I & R_I + R_D + \dfrac{1}{sC_D} \end{vmatrix}}$$

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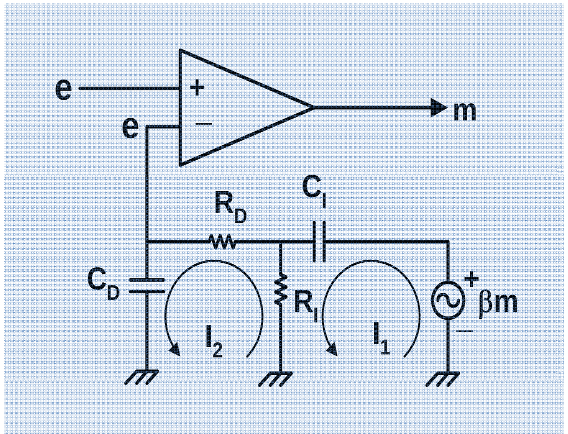

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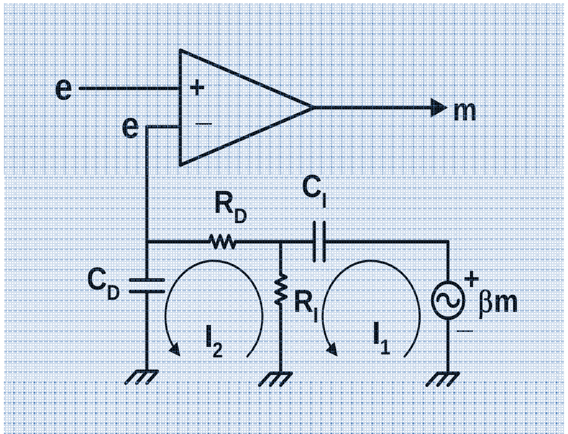
Or, $$I_2 = \frac{\beta M(s) R_I}{\left(R_I + \dfrac{1}{sC_I}\right)\left(R_I + R_D + \dfrac{1}{sC_D}\right) - R_I^2}$$

Circuit solution



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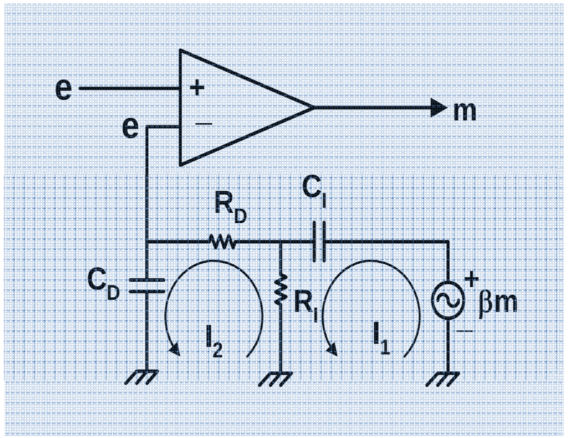
$$\therefore E(s) = \frac{I_2}{sC_D} = \frac{\beta M(s) R_I \big/ (sC_D)}{\left(R_I + \dfrac{1}{sC_I}\right)\left(R_I + R_D + \dfrac{1}{sC_D}\right) - R_I^2}$$

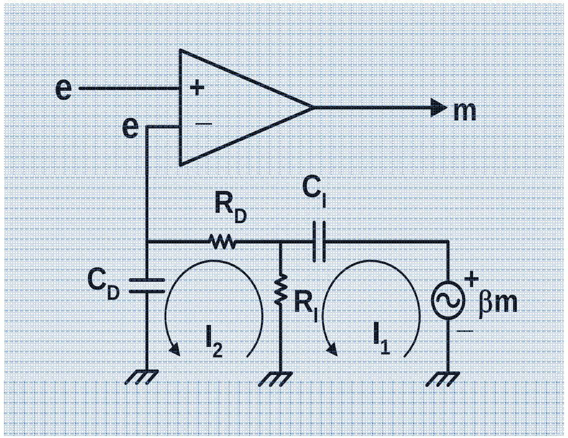
Circuit solution



Then, the T.F. becomes:

$$\frac{M(s)}{E(s)} = \frac{1}{\beta}\left[\frac{\left\{\left(R_I + \frac{1}{sC_I}\right)\left(R_I + R_D + \frac{1}{sC_D}\right) - R_I^2\right\} sC_D}{R_I}\right]$$

Circuit solution



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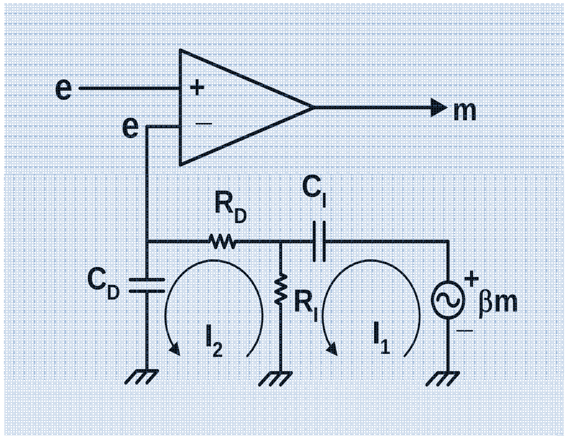
Simplifying,

$$\frac{M(s)}{E(s)} = \frac{1}{\beta}\left[\left(\frac{T_1 + T_2 + R_I C_D}{T_1}\right) + \frac{1}{sT_1} + sT_2\right]$$

where $T_1 = R_I C_I$ **and** $T_2 = R_D C_D$

Circuit solution



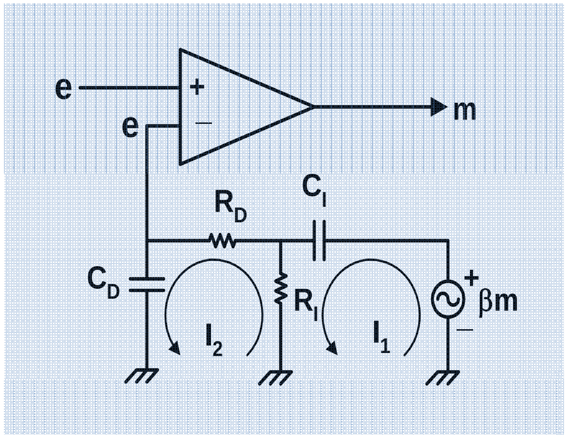
By substituting

$$A = \frac{T_1 + T_2 + R_I C_D}{T_1}$$

Or,

$$A = 1 + \frac{C_D}{C_I} + \frac{T_2}{T_1}$$

Circuit solution



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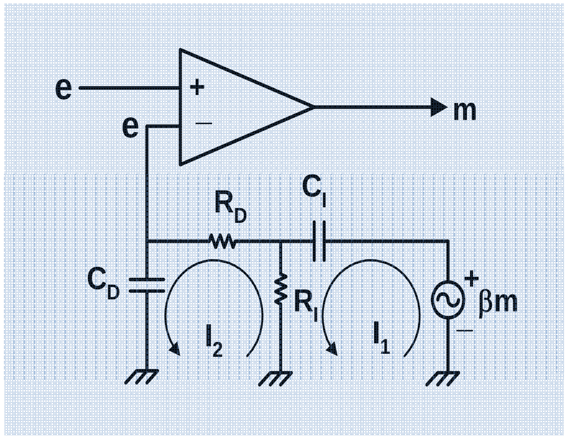
This expression of A presents the problem of interaction and hence, the controller developed is called an **interacting controller.**

Circuit solution



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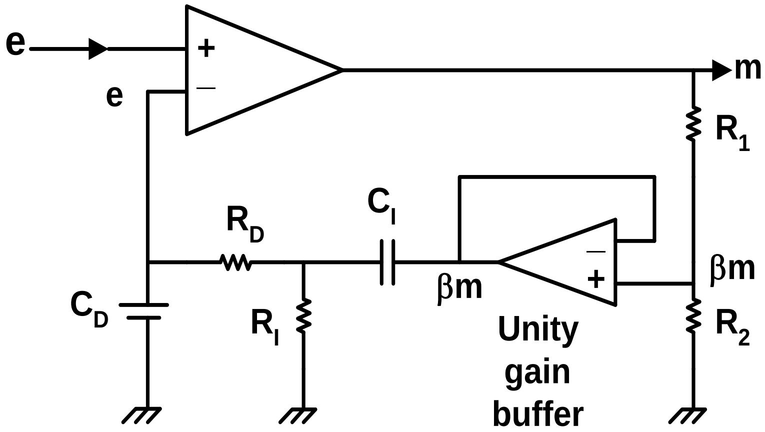
The T.F. becomes:

$$\frac{M(s)}{E(s)} = \frac{A}{\beta}\left[1 + \frac{1}{sT_1 A} + \frac{sT_2}{A}\right]$$

$$= K_p\left[1 + \frac{1}{sT_I} + sT_D\right]$$

$$Where\ K_p = \frac{A}{\beta}, T_I = T_1 A, and\ T_D = T_2/A$$

A simplified PID Controller with one op-amp



Original circuit with two op-amps

The unity gain buffer amplifier may be omitted if the resistances are chosen such that

$$\left(R_I \,\|\, R_D\right) >> \left(R_1 \,\|\, R_2\right)$$

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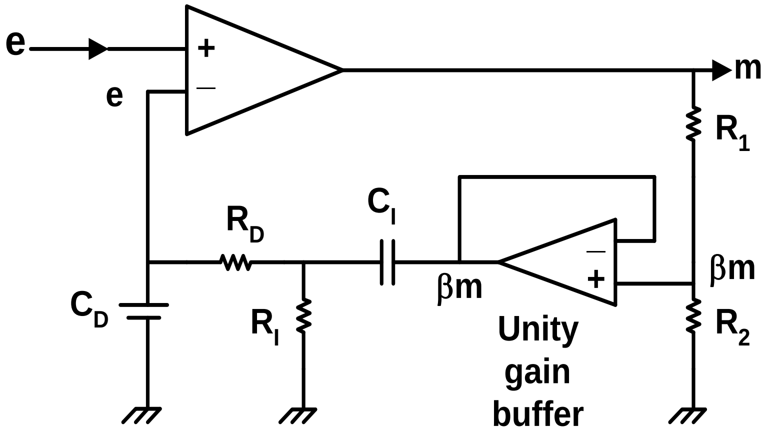

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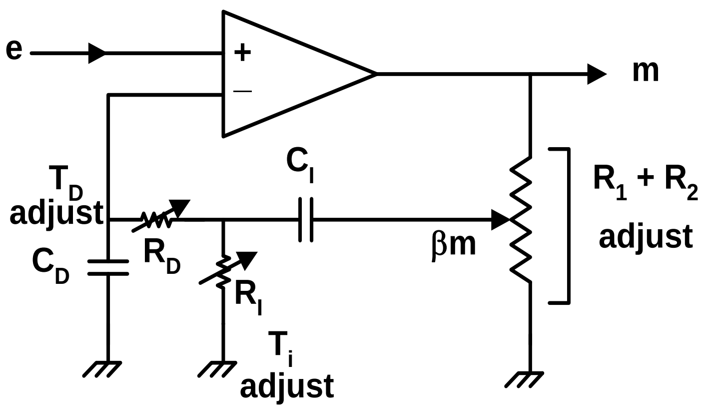

Final circuit with one op-amp

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- Provision for **Local/Remote** modes of operation

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Bias may be properly adjusted for zero steady state error with P and PD controllers, when load is constant. Proper bias can also ensure efficient operation under start-up condition.

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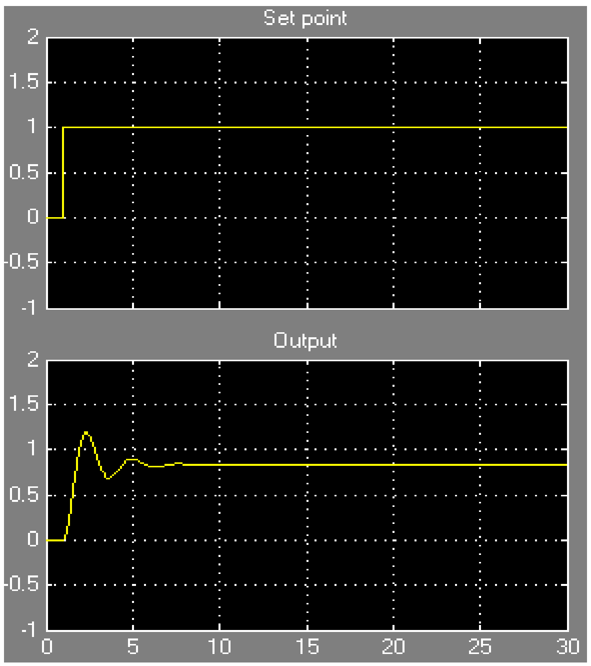

Non-zero steady state error

P-control without bias

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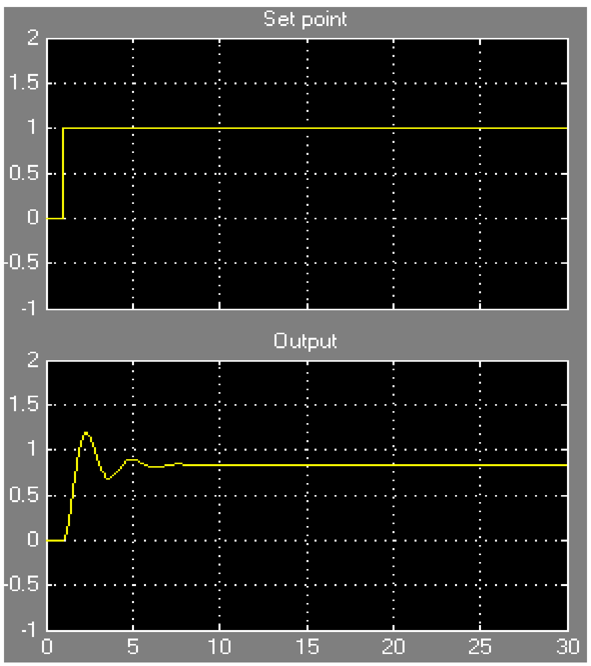

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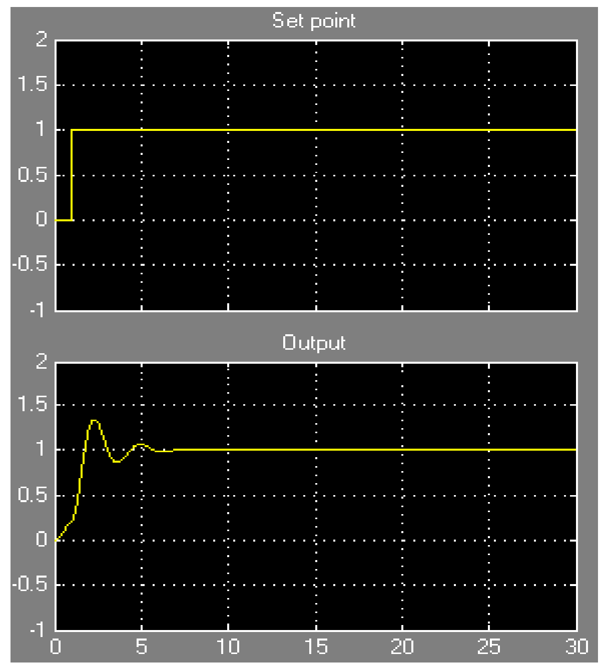

P-control with bias

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Derivative action may produce an unwanted kick at the controller output when there is a step change in the set-point. This effect may be eliminated if the derivative term is computed from the measured variable, instead of the error (anti-derivative kick feature).

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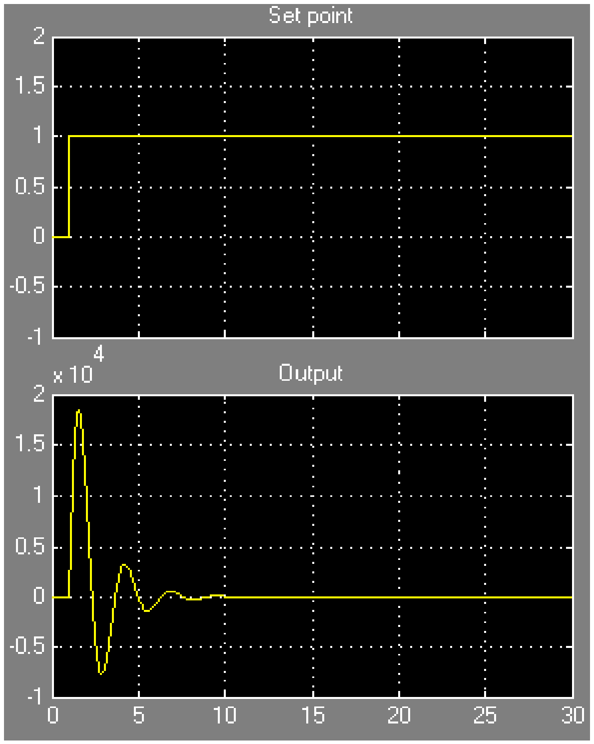

PD-control with derivative kick

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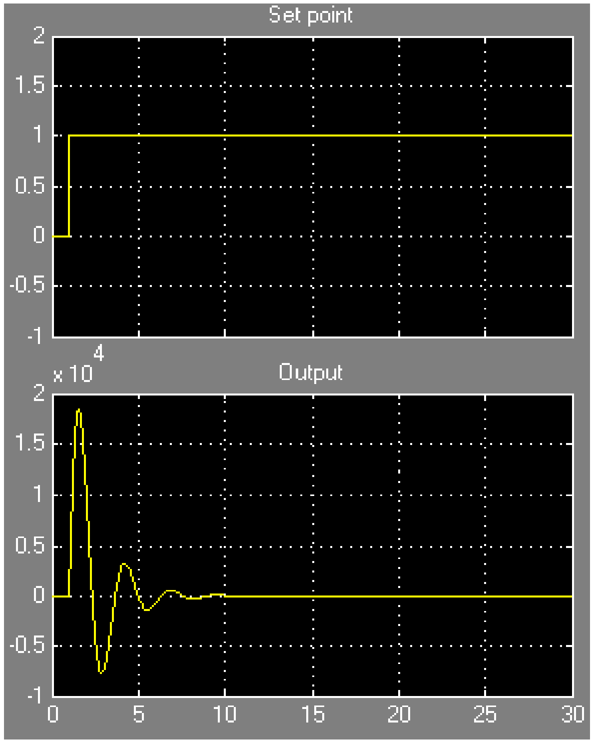

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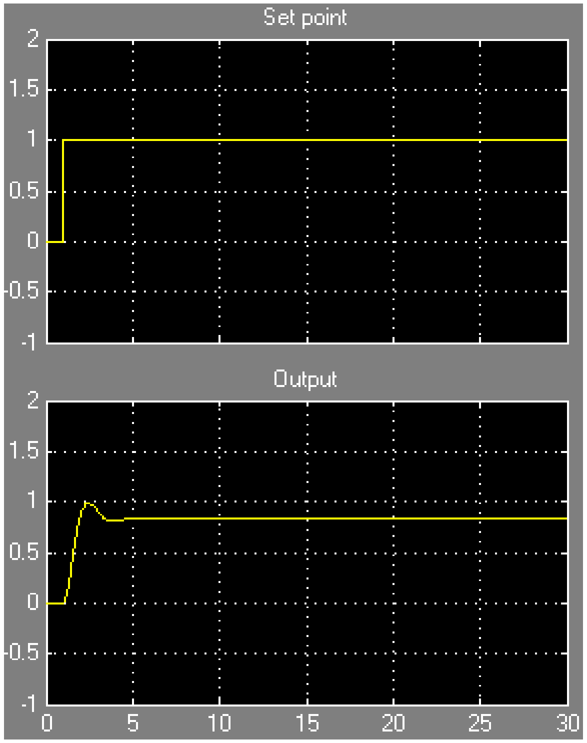

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Provision for anti-integral wind-up

Integral action in PI and PID controllers may produce a large integral error term when a non-zero error persists for a long time. Integration action may be switched off to combat this situation, otherwise, a long time may be required to come back to normal working range.

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In some controllers, when the integral term is present, the integral amount during T_i is summed with each pass through the calculation and becomes the controller bias. This technique is known as automatic reset and this is done to avoid long duration operation of the integral action – thus error due to drift etc. during integration may be avoided.

Output must be limited (saturation)

It is desirable to limit the controller output (say between 0% and 100%) so that control valves or other final control elements may operate safely within their working limits.

Provision for Auto/Manual modes of control

A change-over switch is normally provided for configuring the controller as an automatic controller (AUTO) for closed loop operation or as a manual controller (MAN) for open-loop operation.

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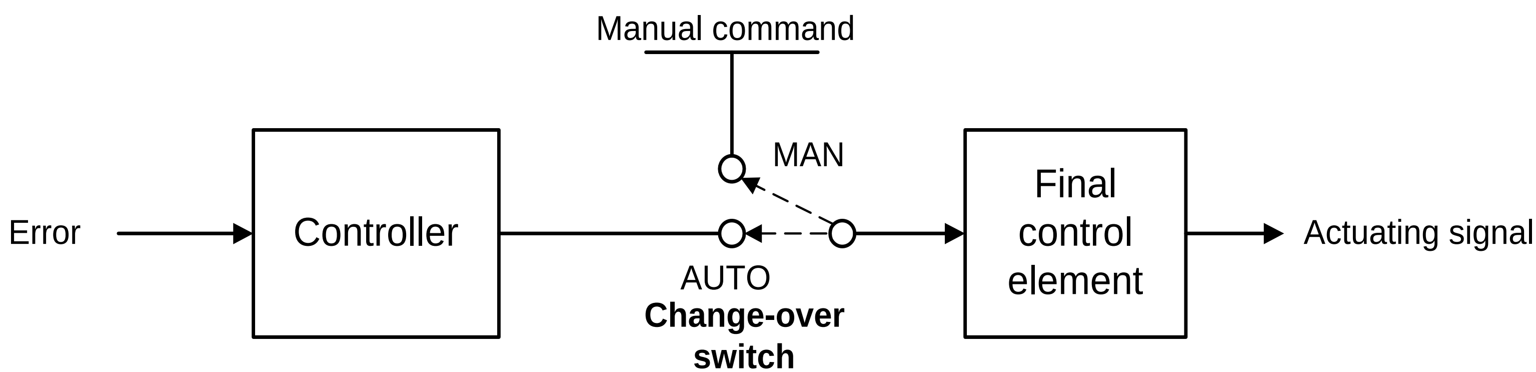

Provision for Local/Remote modes of operation

A provision is made for the set point input, such that, the set point may be changed either locally or from a remote link.

Electronic Process Controllers

Advantages

- No transmission lag (as compared to pneumatic systems)

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- Compatibility with other electrical components

Electronic Process Controllers

Disadvantages

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Electronic Process Controllers

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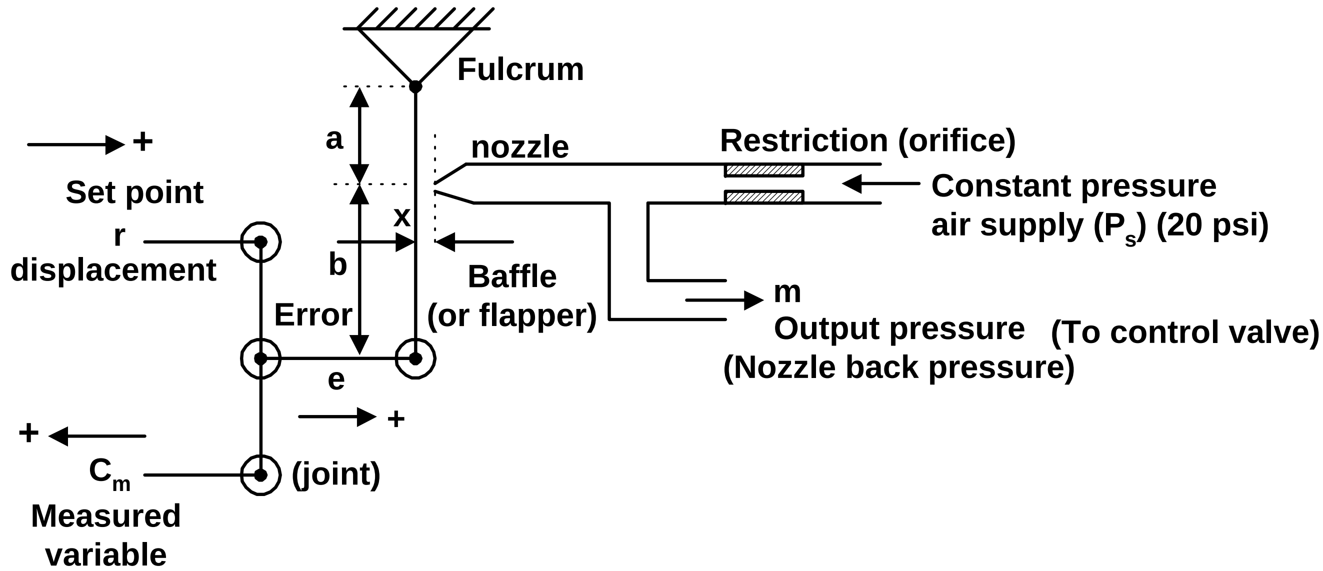
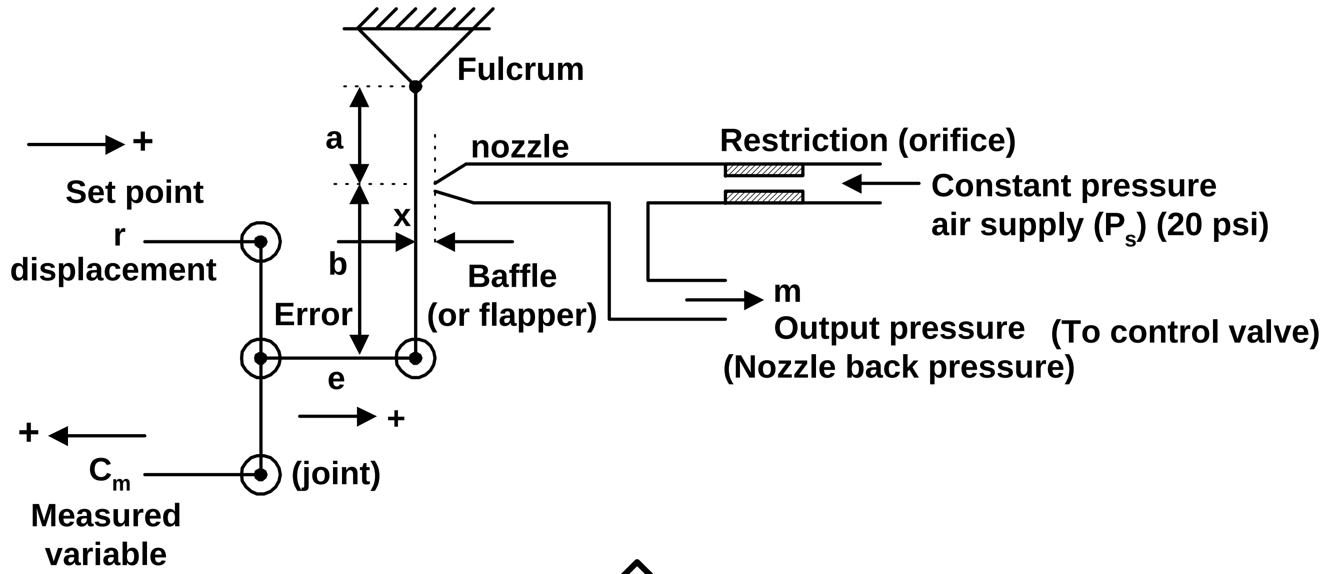
- Prone to electrical hazards (in inflammable atmosphere)
- Analog integrators are not very reliable
- Conversion equipments are necessary to interface pneumatic and hydraulic devices

Pneumatic Baffle-Nozzle or Flapper-Nozzle amplifier



$e = (r - C_m)/2$

Pneumatic Baffle-Nozzle or Flapper-Nozzle amplifier



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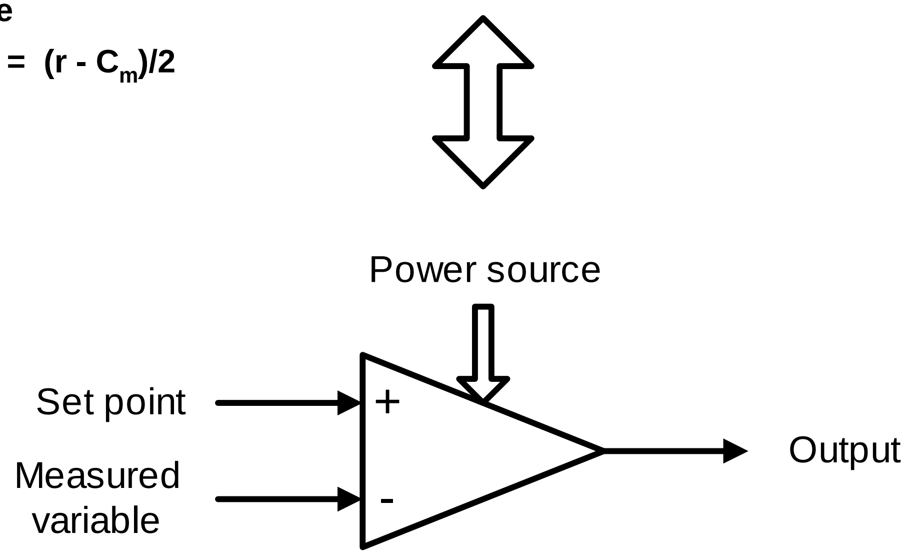
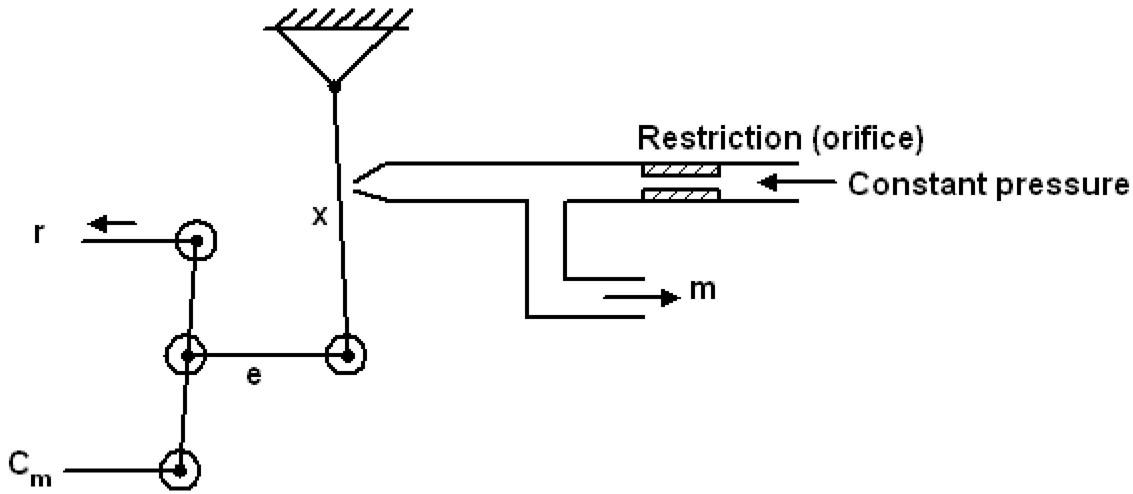
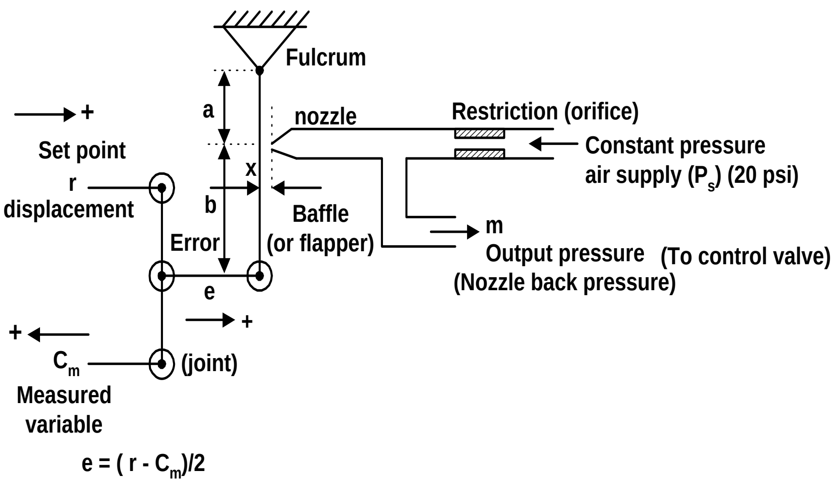
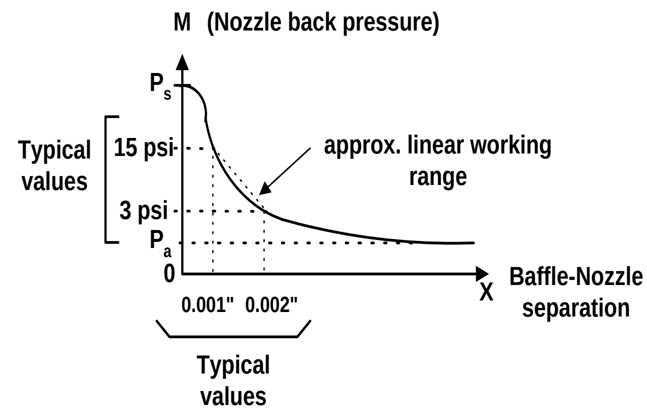



Restriction (orifice)
Constant pressure
x
r
m
e
Cm

Pneumatic Baffle-Nozzle or Flapper-Nozzle amplifier

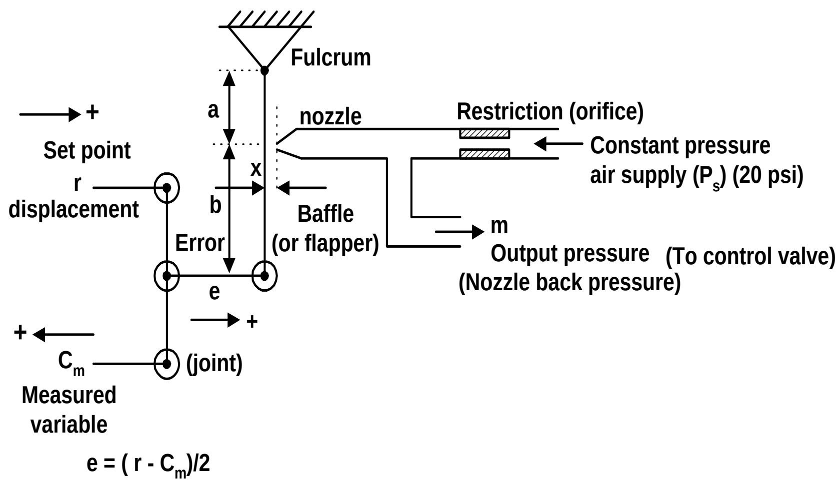


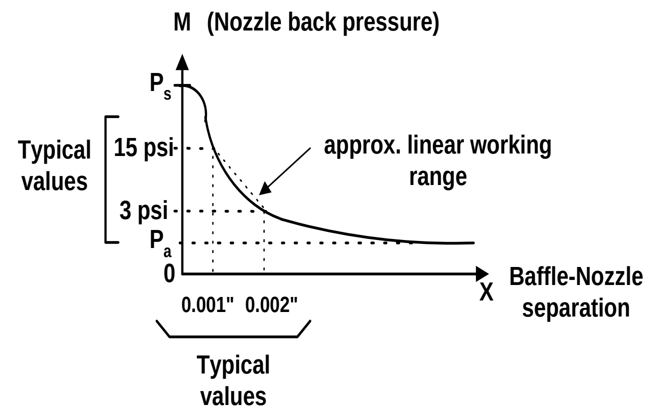


A typical curve relating nozzle back pressure *M* to Baffle-Nozzle separation *X*

x: **change in baffle-nozzle separation.**
m: **change in back pressure.**
P_a : the lowest possible pressure (= ambient pressure).

Pneumatic Baffle-Nozzle or Flapper-Nozzle amplifier





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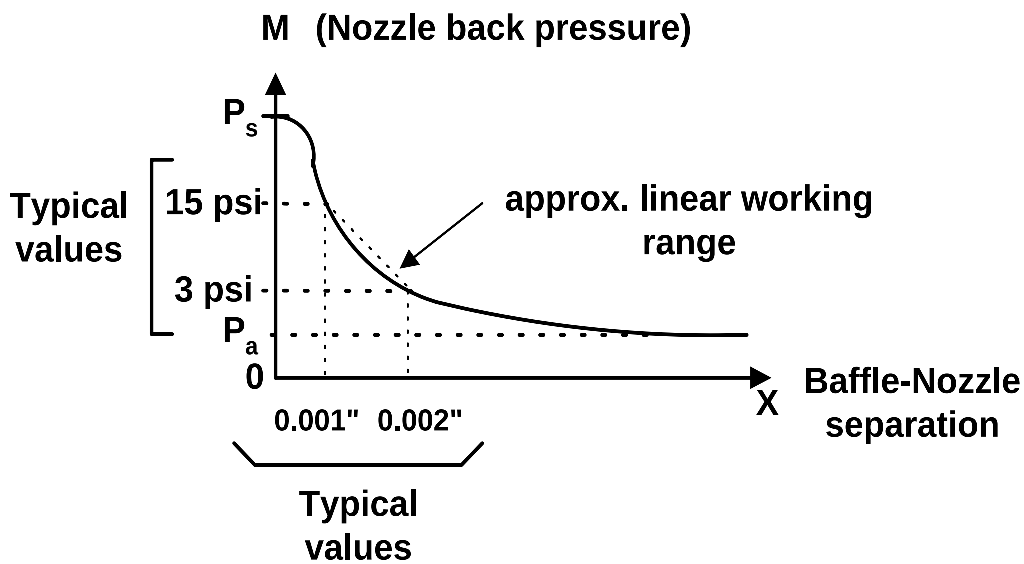
$X = X` - x$ and $M = M` + m$, $X`$: baffle-nozzle separation with zero error and $M`$: output pressure with zero error.

Pneumatic Baffle-Nozzle or Flapper-Nozzle amplifier



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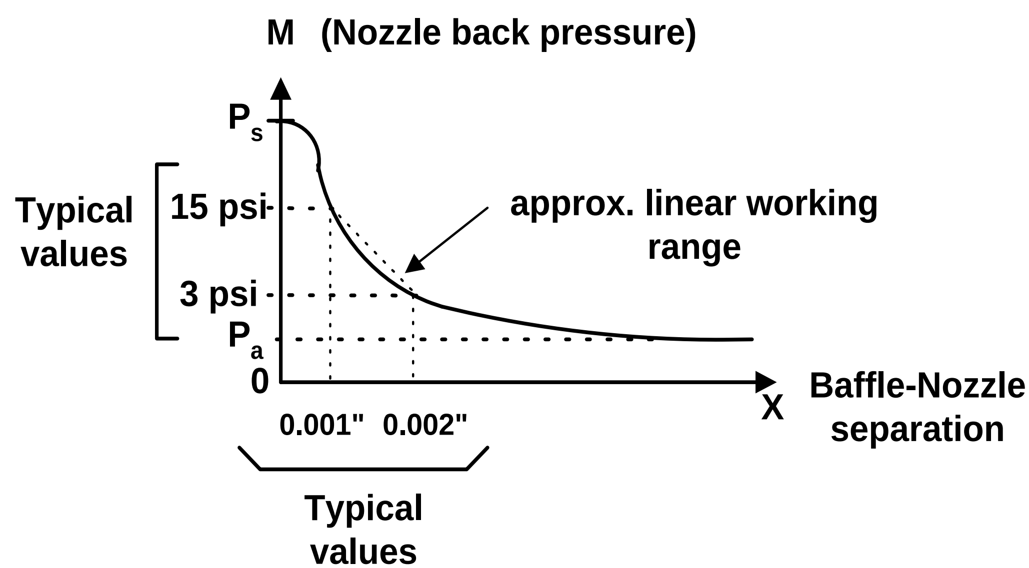
Pneumatic Baffle-Nozzle or Flapper-Nozzle amplifier



A typical curve relating nozzle back pressure *M* to Baffle-Nozzle separation *X*

For approximately linear working range:
$M = K_n X + C$ and $M` = K_n X` + C$, K_n being the slope

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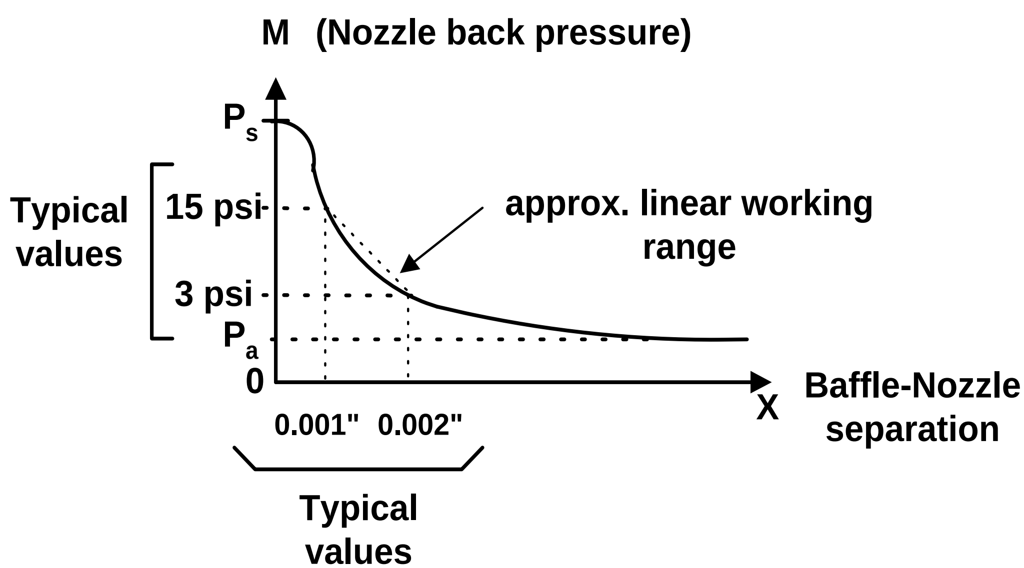
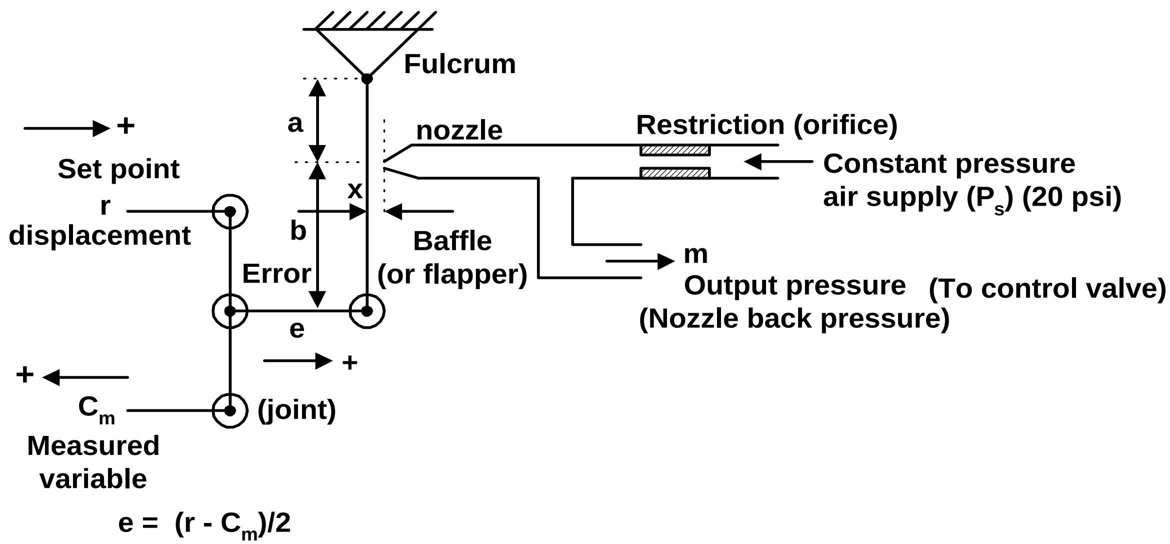


A typical curve relating nozzle back pressure *M* to Baffle-Nozzle separation *X*

For approximately linear working range:
$M = K_n X + C$ and $M` = K_n X` + C$, K_n being the slope
Subtracting, $M - M` = K_n(X – X`)$, i.e., $m = - K_n.x$

Pneumatic Baffle-Nozzle or Flapper-Nozzle amplifier

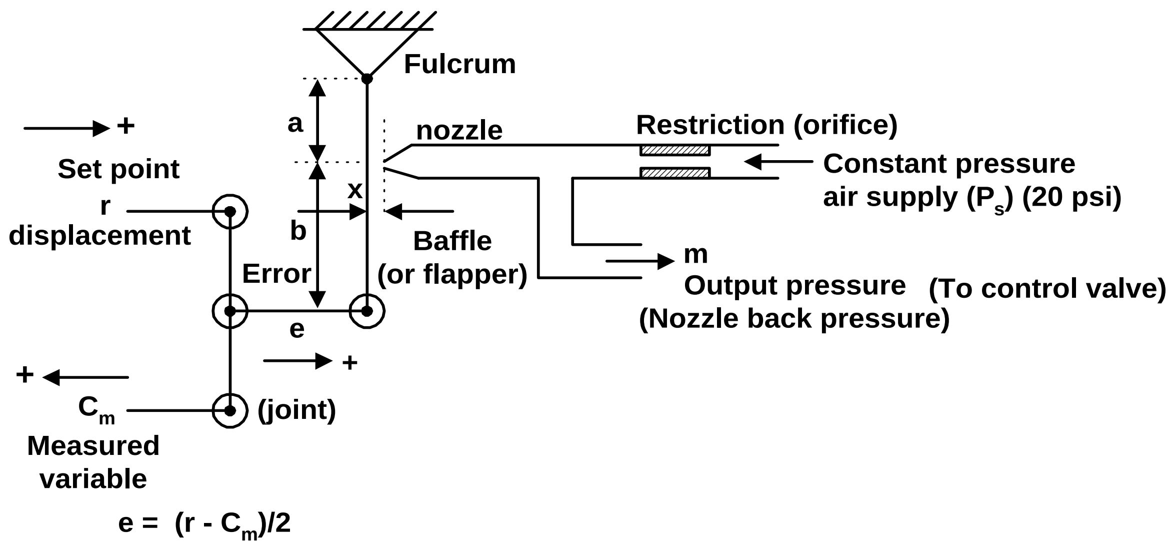


Under steady state condition, change in output pressure may be expressed as:

$m = -K_n \cdot x$, where $-K_n$ is the nozzle gain

Here, $-x = \left(\frac{a}{a+b}\right)e$

Pneumatic Baffle-Nozzle or Flapper-Nozzle amplifier



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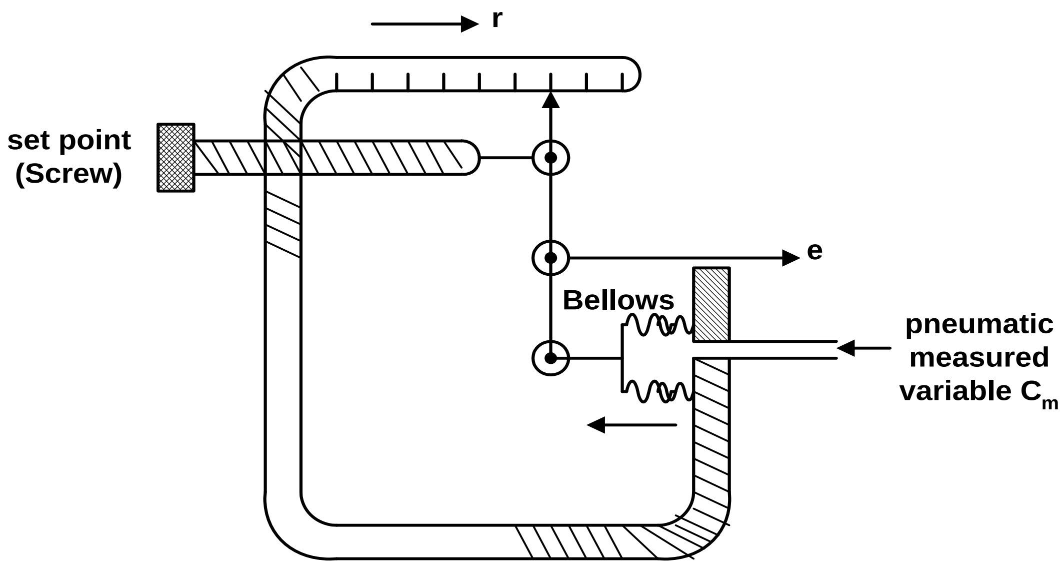
Here, $-x = \left(\frac{a}{a+b}\right)e$

Thus, $m = K_n \cdot e\left(\frac{a}{a+b}\right) = K \cdot e$, K is called the amplifier gain and $K = K_n\left(\frac{a}{a+b}\right)$

Arrangement for mechanical set-point and pneumatic measured variable



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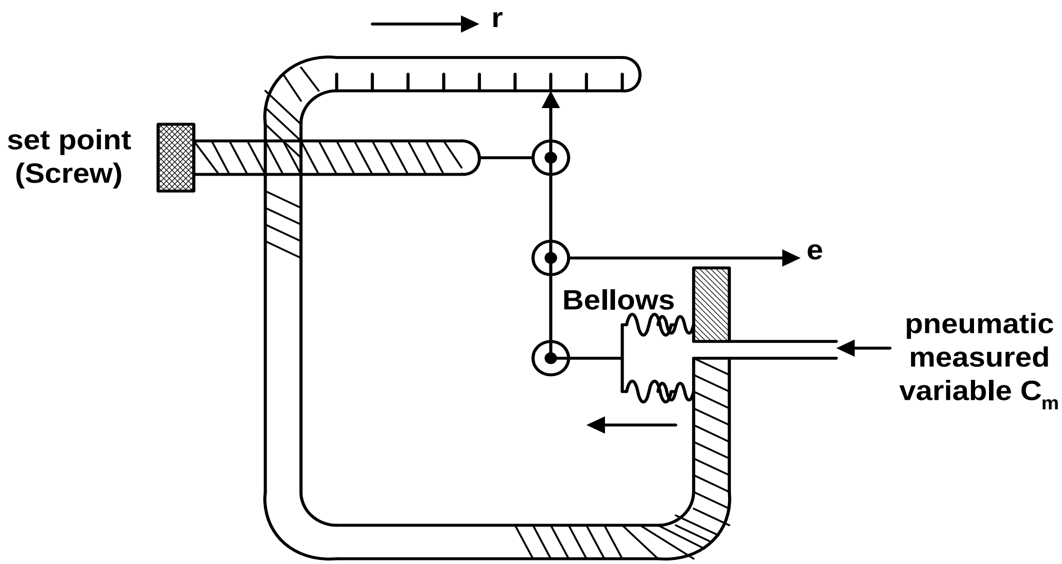
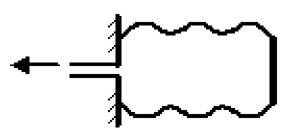


Bellows



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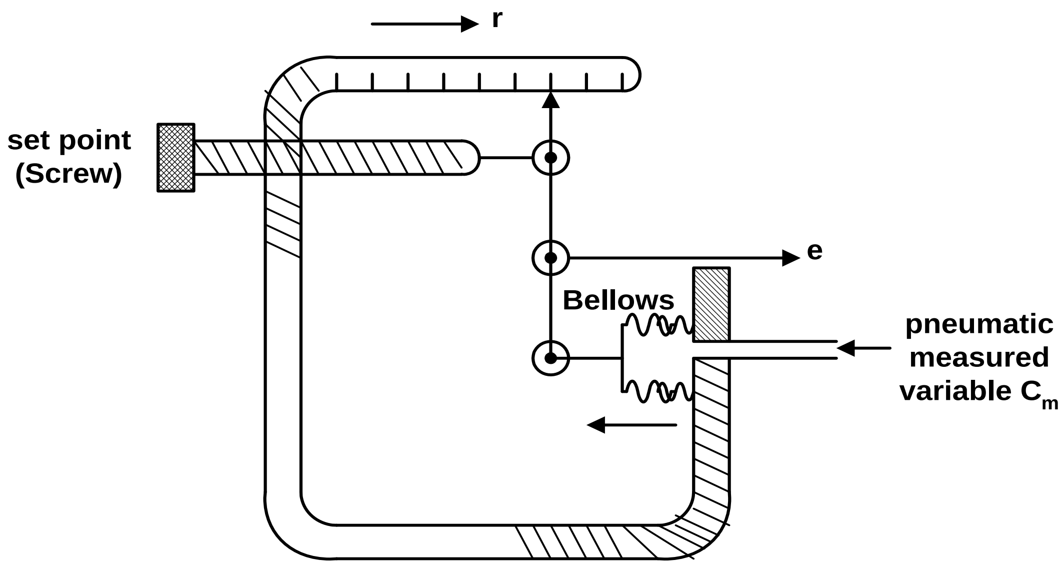
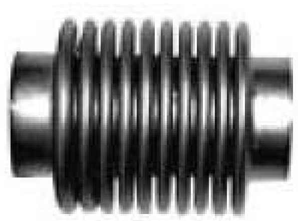


<u>Bellows</u>



Relay valve or pilot valve

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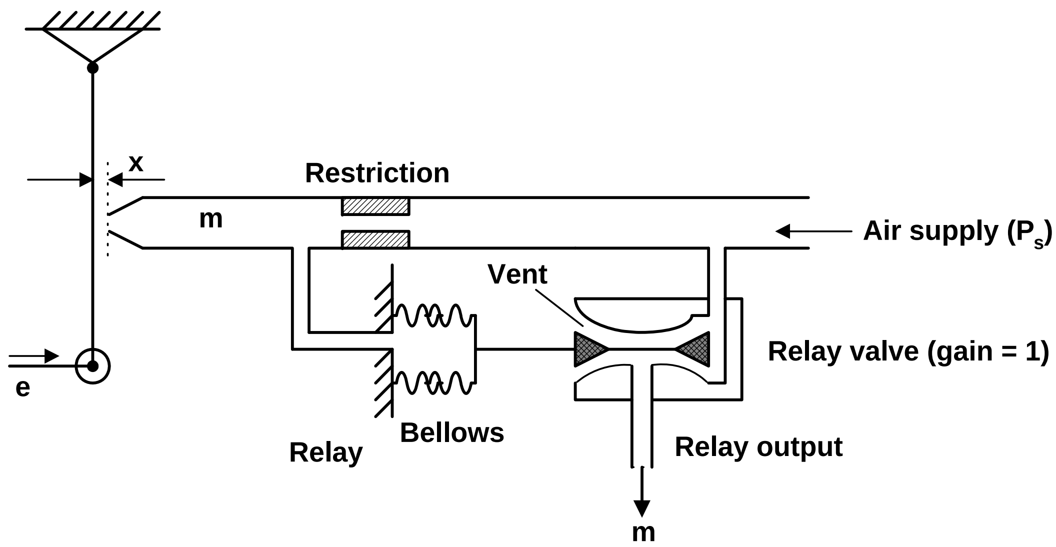
A relay valve may be *direct-acting* (positive gain) or *reverse-acting* (negative gain).

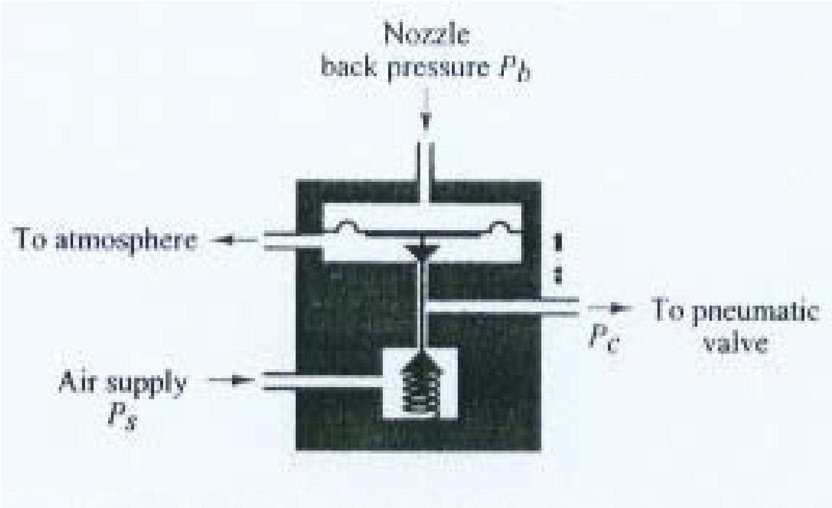
Direct-acting relay valve with a Baffle-Nozzle amplifier



$$m = - K_n . x$$

As the nozzle back pressure *m* increases, the relay output pressure also increases

Direct-acting relays

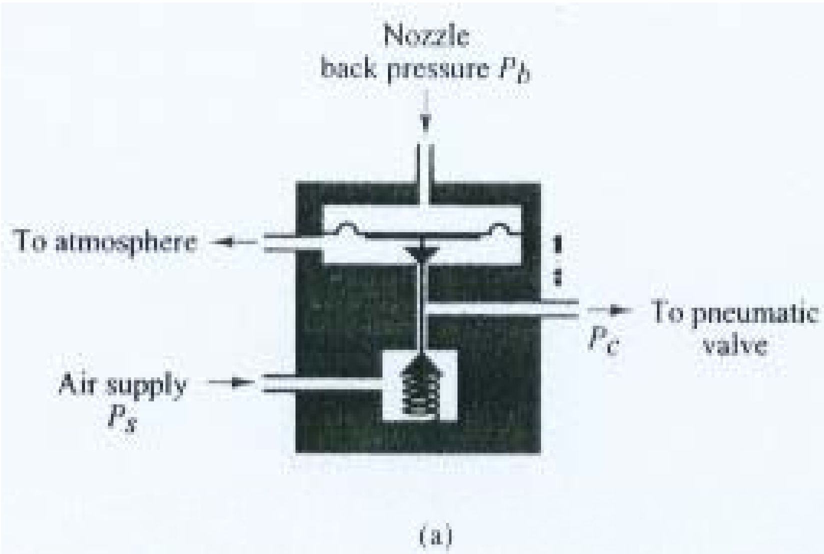


(a)

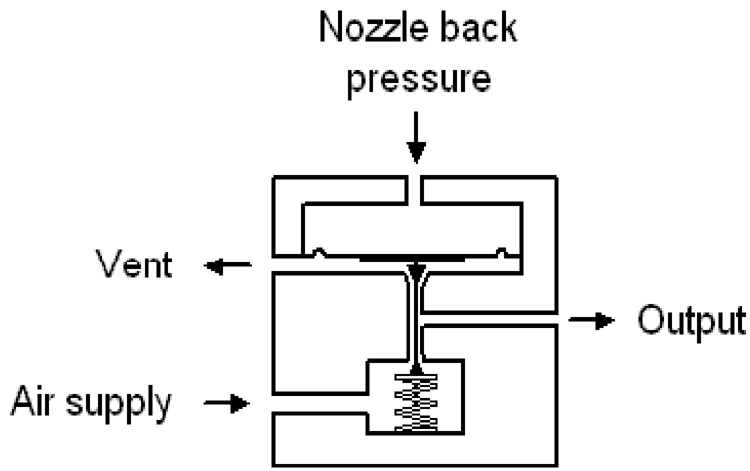
Bleed type relay

In all positions of the valve, except at the position to shut off the air supply, air continues to bleed into the atmosphere.

Direct-acting relays



Bleed type relay



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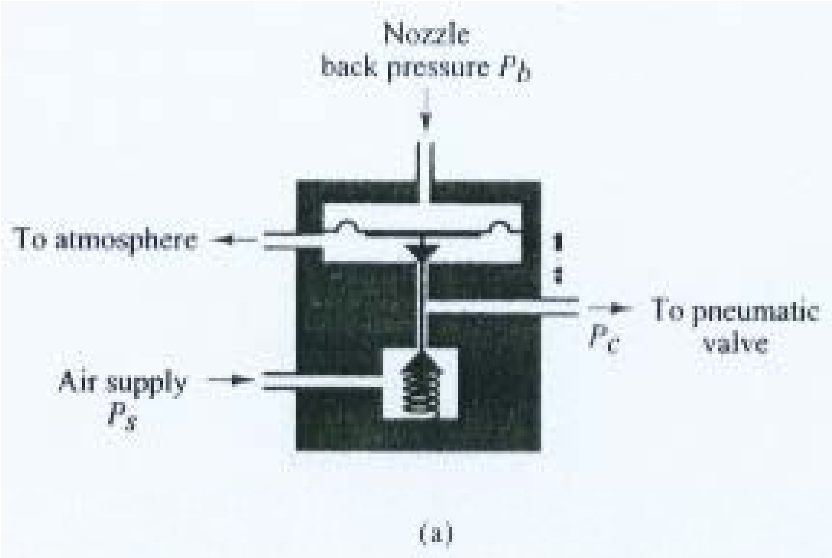
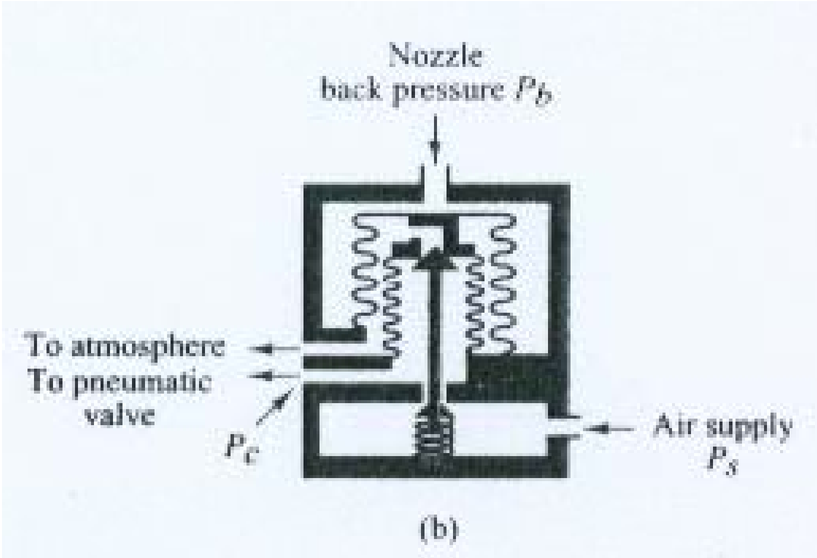


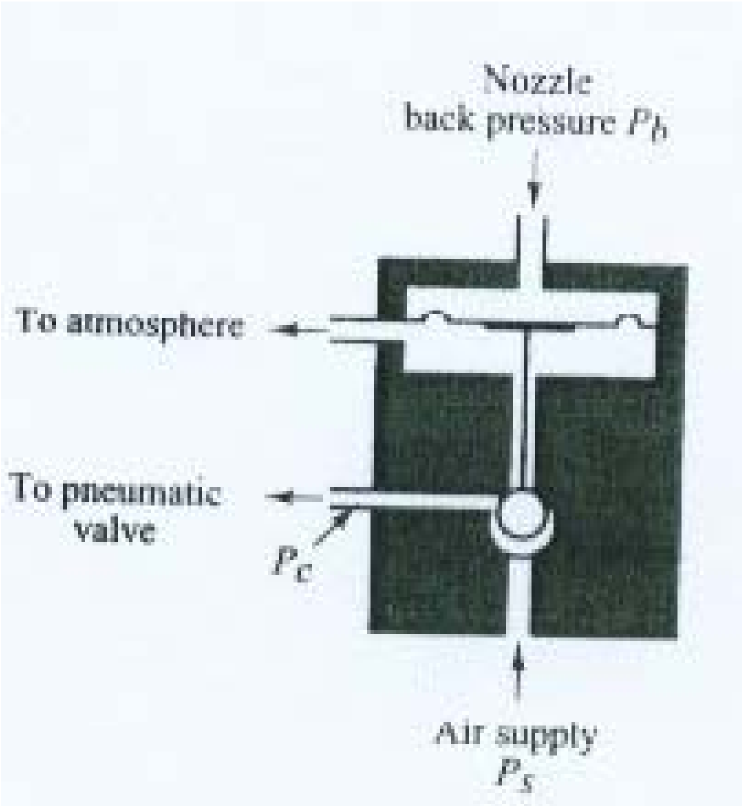
Bleed type relay



Non-bleed type relay

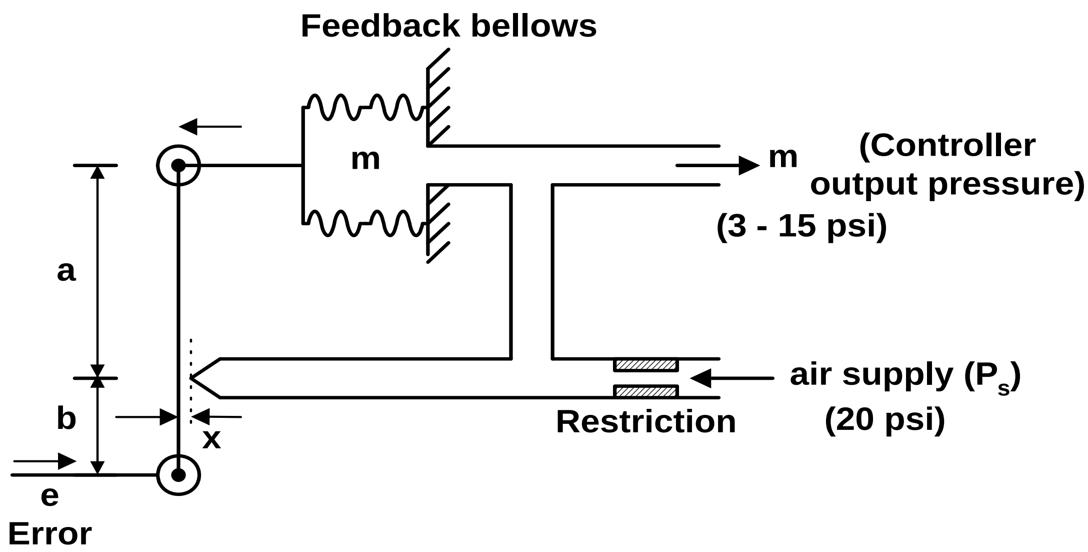
The air bleed stops when the equilibrium condition is obtained, and, therefore, there is no loss of pressurized air at steady-state operation.

Reverse-acting relay

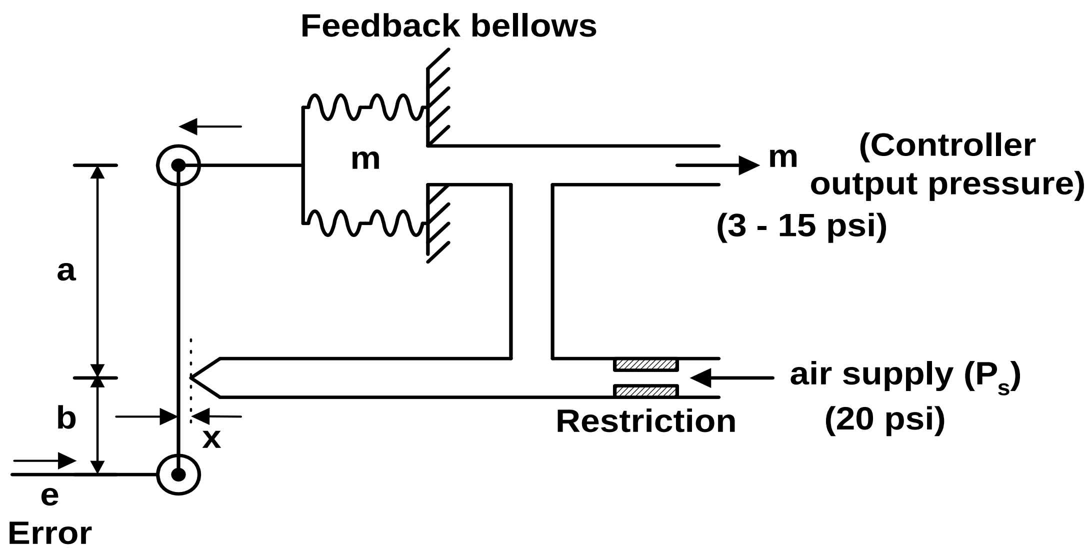


As the nozzle back pressure increases, the ball valve is forced towards the lower seat, thereby decreasing the output pressure

Pneumatic Proportional Controller



Pneumatic Proportional Controller

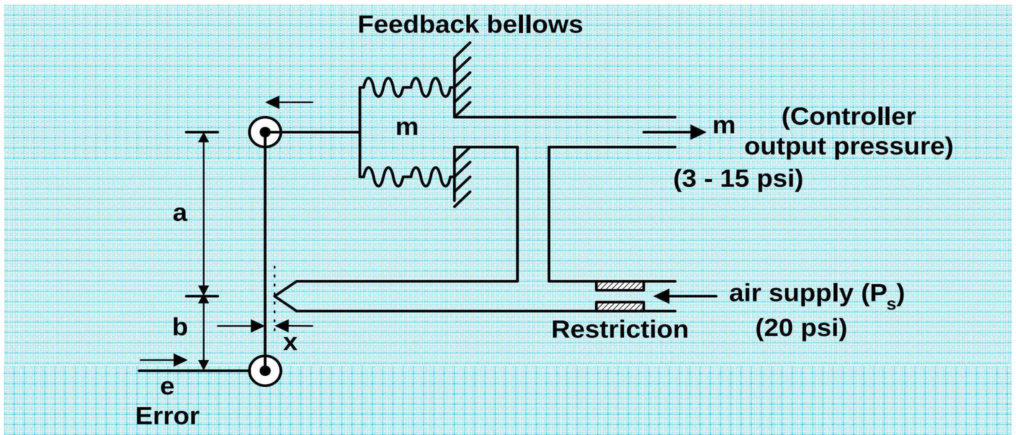


The Baffle-Nozzle separation may be expressed as:

$$-(x) = \left(\frac{a}{a+b}\right)e - \left(\frac{b}{a+b}\right)K_b m$$

where K_b is the bellows stiffness factor and m is the change in output pressure.

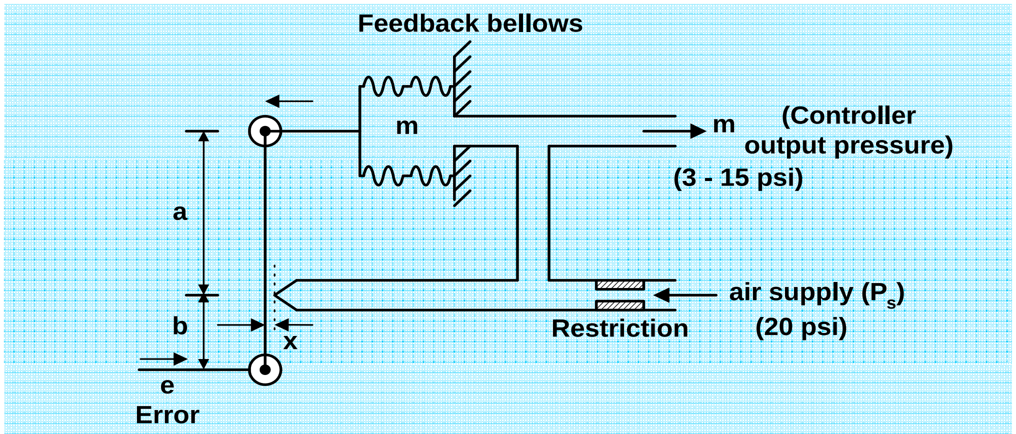
Pneumatic Proportional Controller



$$-(x) = \left(\frac{a}{a+b}\right)e - \left(\frac{b}{a+b}\right)K_b m$$

Now m = $-K_n .x$, where K_n is the nozzle gain.

Pneumatic Proportional Controller



$$-(x) = \left(\frac{a}{a+b}\right)e - \left(\frac{b}{a+b}\right)K_b m$$

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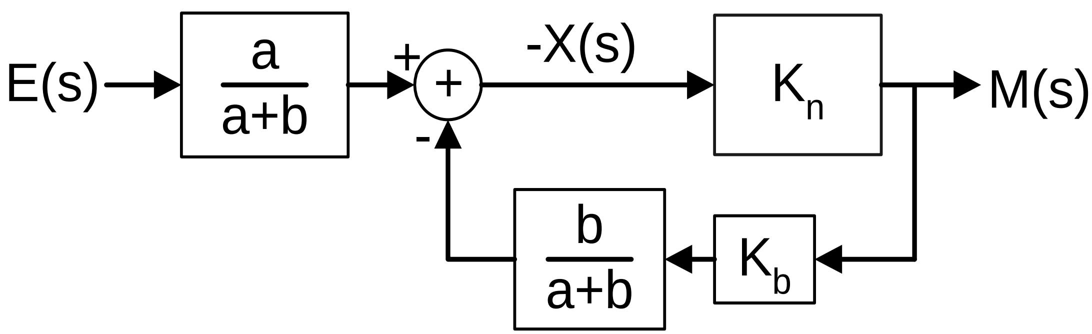
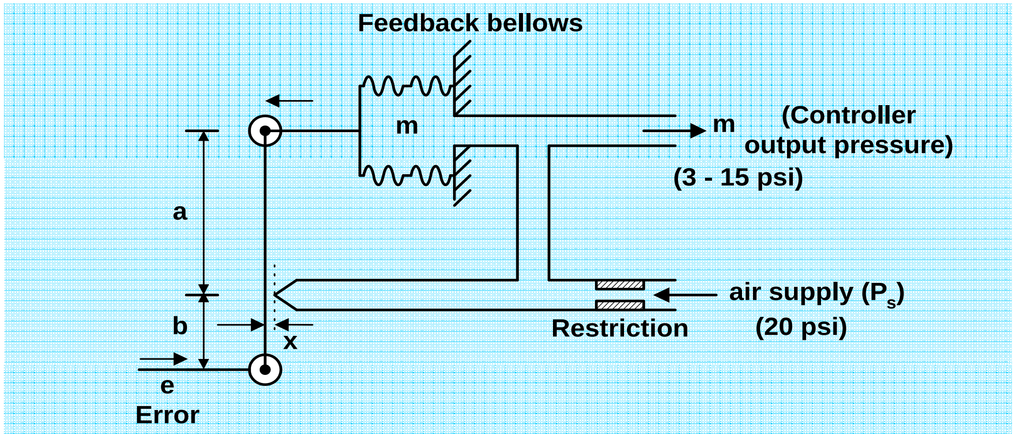
Block diagram of the controller



Pneumatic Proportional Controller



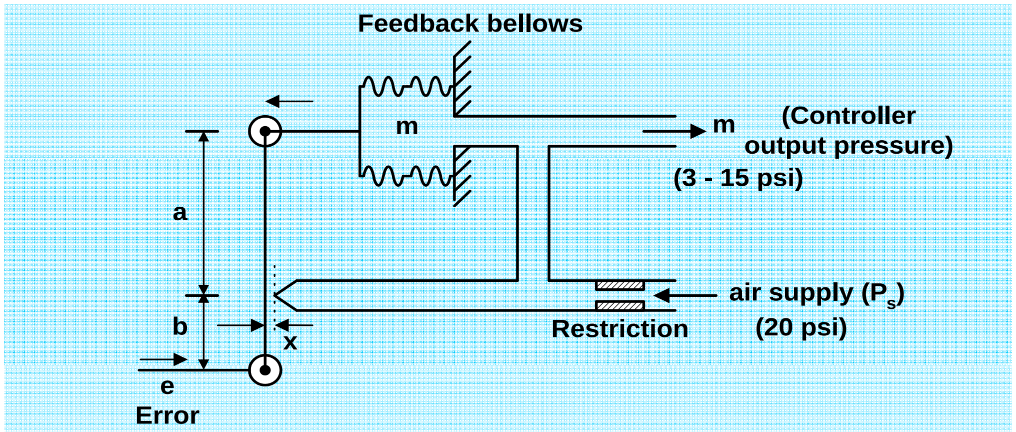
$$-(x) = \left(\frac{a}{a+b}\right)e - \left(\frac{b}{a+b}\right)K_b m$$

Now m = $-K_n . x$, where K_n is the nozzle gain.

Thus,

$$m/K_n = \left(\frac{a}{a+b}\right)e - \left(\frac{b}{a+b}\right)K_b m$$

Pneumatic Proportional Controller



$$-(x)=\left(\frac{a}{a+b}\right)e-\left(\frac{b}{a+b}\right)K_b m$$

Now m = $-K_n.x$, where K_n is the nozzle gain.

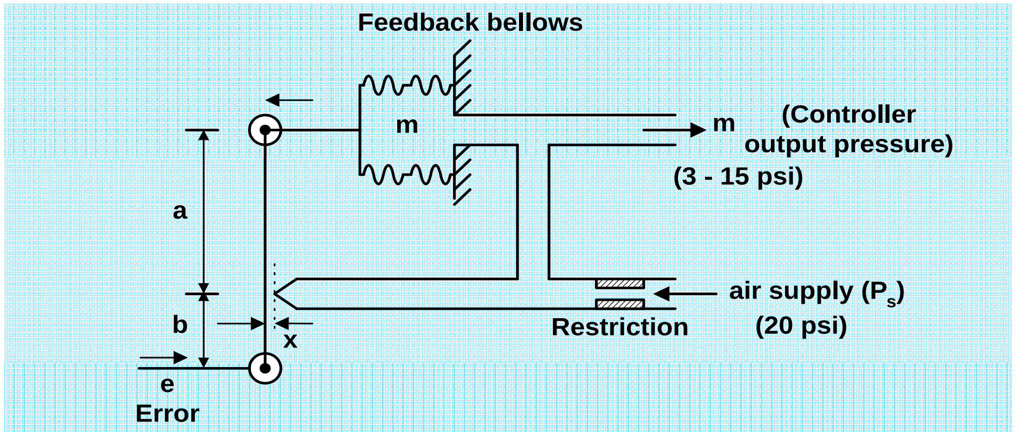
Thus,

$$m/K_n=\left(\frac{a}{a+b}\right)e-\left(\frac{b}{a+b}\right)K_b m$$

or

$$m\left[\frac{1}{K_n}+\left(\frac{b}{a+b}\right)K_b\right]=\left(\frac{a}{a+b}\right)e$$

Pneumatic Proportional Controller



$$-(x) = \left(\frac{a}{a+b}\right)e - \left(\frac{b}{a+b}\right)K_b m$$

Now m = $-K_n .x$, where K_n is the nozzle gain.

Thus,

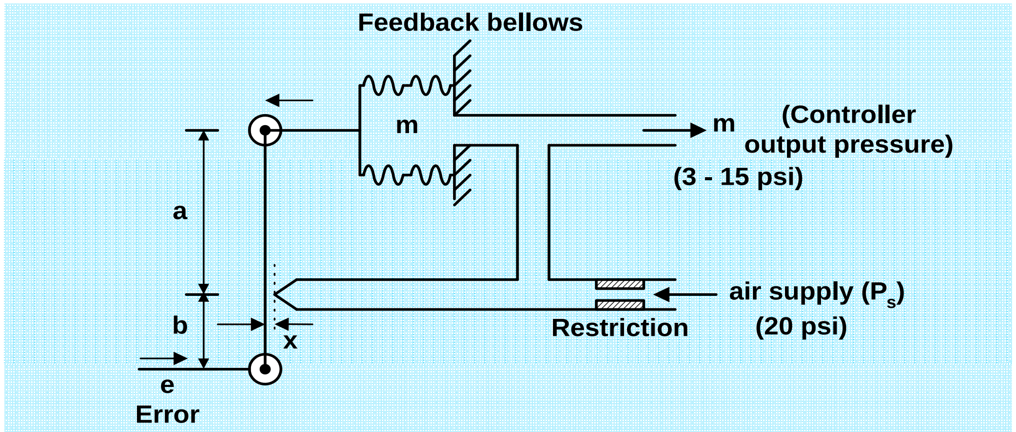
$$m/K_n = \left(\frac{a}{a+b}\right)e - \left(\frac{b}{a+b}\right)K_b m$$

or

$$m\left[\frac{1}{K_n} + \left(\frac{b}{a+b}\right)K_b\right] = \left(\frac{a}{a+b}\right)e$$

if K_n is very high, then $\frac{1}{K_n} \approx 0,$

Pneumatic Proportional Controller

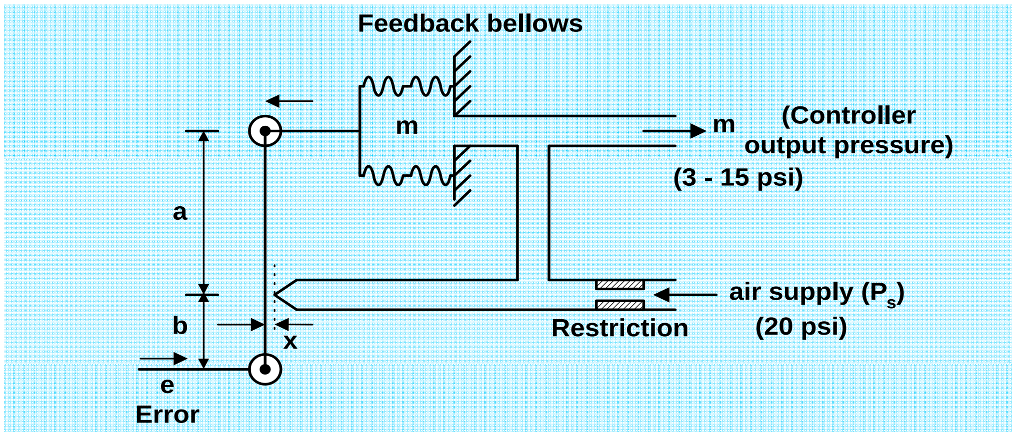


This gives

$$m = \frac{\left(\dfrac{a}{a+b}\right)}{\left(\dfrac{b}{a+b}\right)K_b} e = \left(\frac{a}{bK_b}\right) e = K_p . e$$

where K_p = proportional gain = $\dfrac{a}{b K_b}$

Pneumatic Proportional Controller



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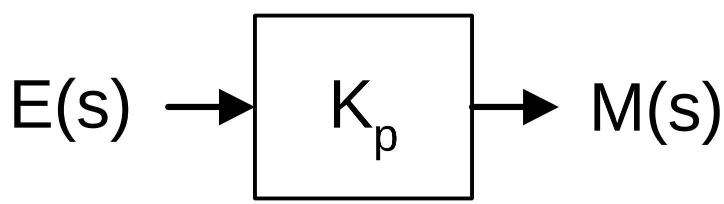
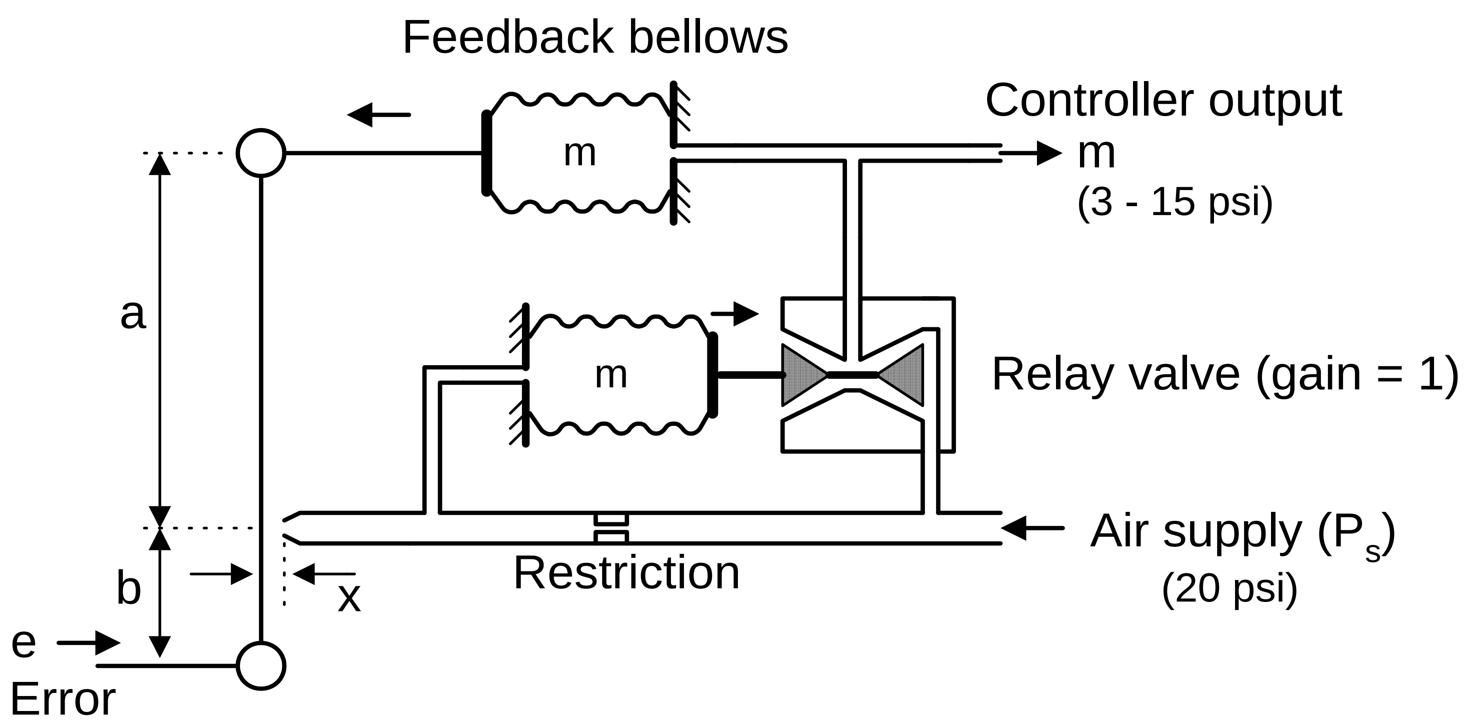
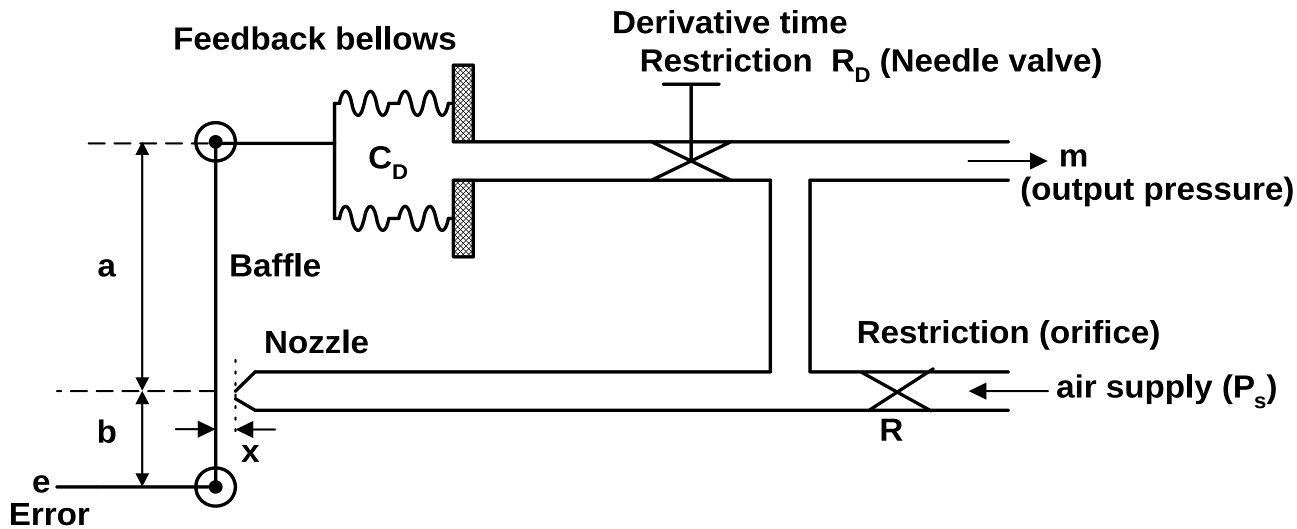
Simplified block diagram of the controller



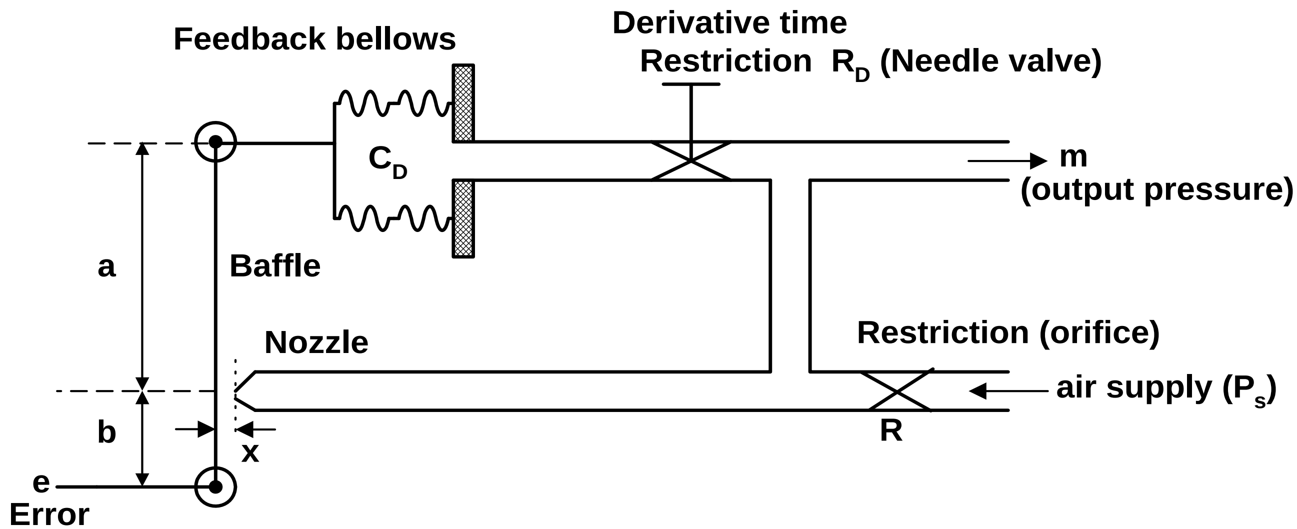
Pneumatic proportional controller with a direct-acting relay



Pneumatic Proportional-Derivative Controller



Pneumatic Proportional-Derivative Controller



The Baffle-Nozzle separation may be expressed as:

$$-X(s)=\left(\frac{a}{a+b}\right)E(s)-\left(\frac{b}{a+b}\right)\left(\frac{K_b}{1+sT_D}\right)M(s) \qquad \text{(assuming } R_D >> R\text{)}$$

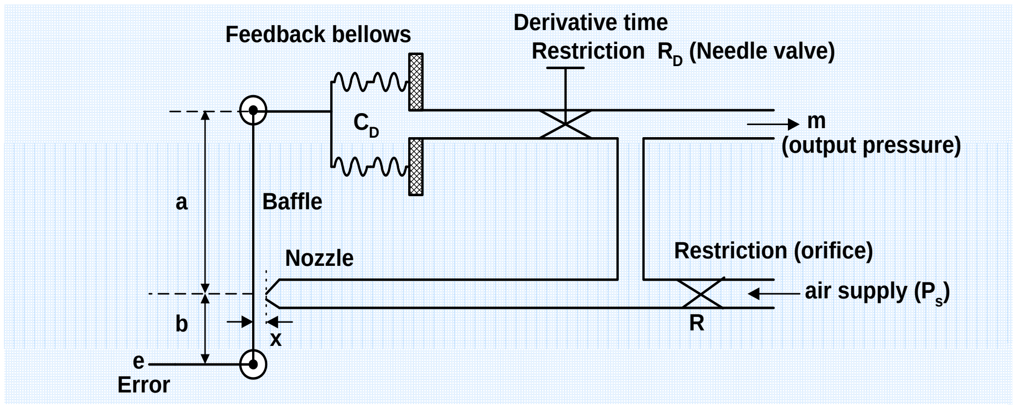
where K_b = Bellows stiffness factor,

T_D = Derivative time

= $R_D C_D$, (assuming $R_D >> R$),

C_D = Capacity of the bellows.

Pneumatic Proportional-Derivative Controller

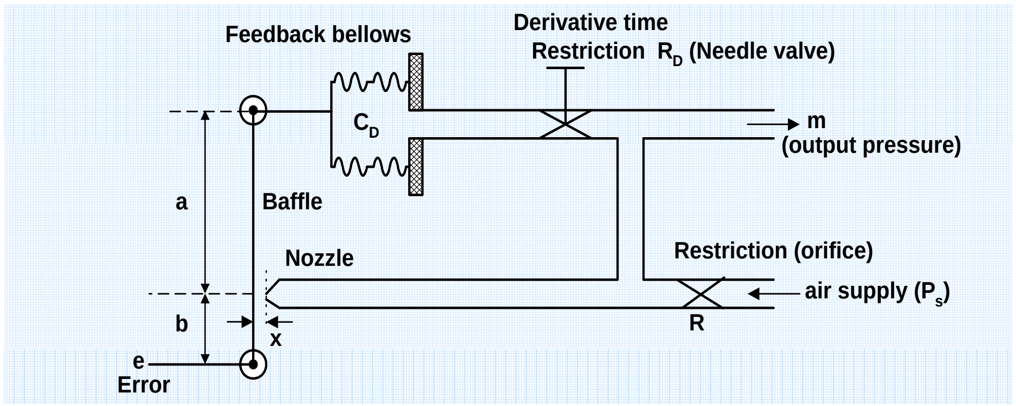


$$-X(s)=\left(\frac{a}{a+b}\right)E(s)-\left(\frac{b}{a+b}\right)\left(\frac{K_b}{1+sT_D}\right)M(s)$$

Now $M(s)=-K_n\cdot X(s)$

where K_n is the nozzle gain.

Pneumatic Proportional-Derivative Controller



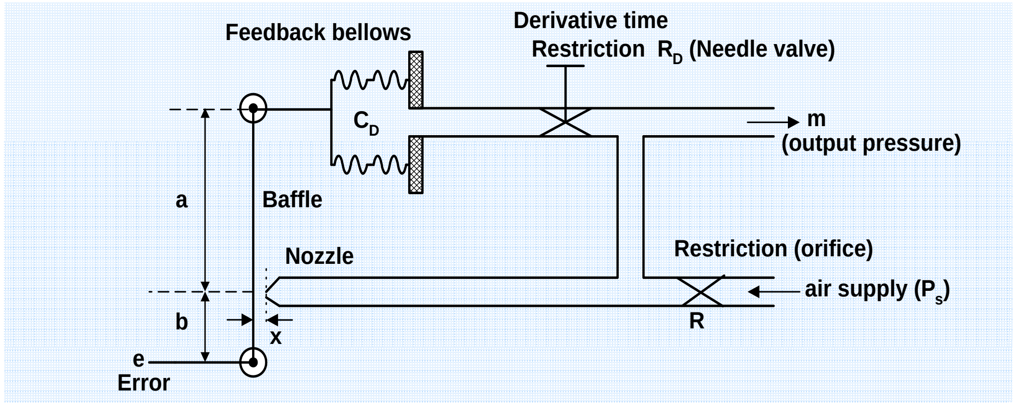
$$-X(s)=\left(\frac{a}{a+b}\right)E(s)-\left(\frac{b}{a+b}\right)\left(\frac{K_b}{1+sT_D}\right)M(s)$$

Now $M(s)=-K_n \cdot X(s)$

where K_n is the nozzle gain.

Thus $$\frac{M(s)}{K_n}=\left(\frac{a}{a+b}\right)E(s)-\left(\frac{b}{a+b}\right)\left(\frac{K_b}{1+sT_D}\right)M(s)$$

Pneumatic Proportional-Derivative Controller



$$-X(s)=\left(\frac{a}{a+b}\right)E(s)-\left(\frac{b}{a+b}\right)\left(\frac{K_b}{1+sT_D}\right)M(s)$$

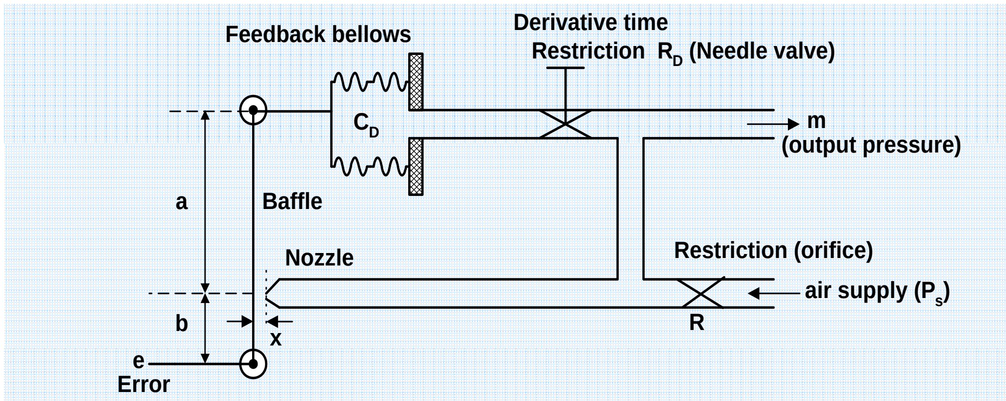
Now $M(s)=-K_n\cdot X(s)$

where K_n is the nozzle gain.

Thus $$\frac{M(s)}{K_n}=\left(\frac{a}{a+b}\right)E(s)-\left(\frac{b}{a+b}\right)\left(\frac{K_b}{1+sT_D}\right)M(s)$$

or, $$M(s)\left[\frac{1}{K_n}+\left(\frac{b}{a+b}\right)\left(\frac{K_b}{1+sT_D}\right)\right]=\left(\frac{a}{a+b}\right)E(s)$$

Pneumatic Proportional-Derivative Controller



$$-X(s)=\left(\frac{a}{a+b}\right)E(s)-\left(\frac{b}{a+b}\right)\left(\frac{K_b}{1+sT_D}\right)M(s)$$

Now $M(s)=-K_n\cdot X(s)$

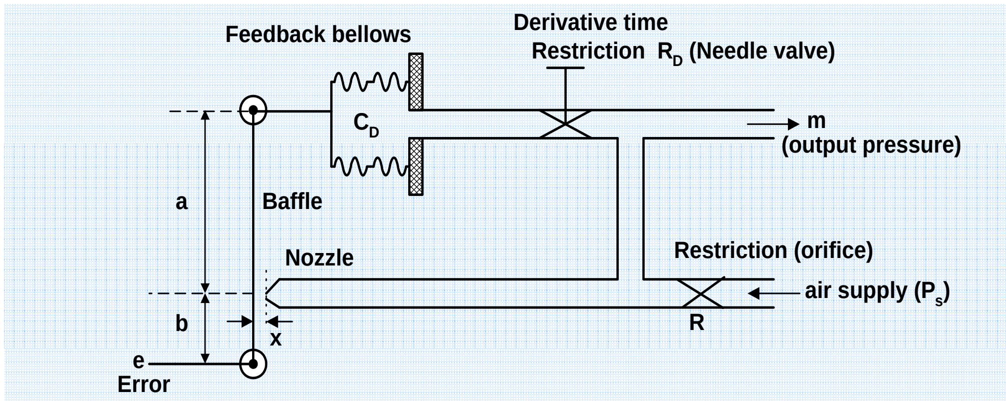
where K_n is the nozzle gain.

Thus $$\frac{M(s)}{K_n}=\left(\frac{a}{a+b}\right)E(s)-\left(\frac{b}{a+b}\right)\left(\frac{K_b}{1+sT_D}\right)M(s)$$

or, $$M(s)\left[\frac{1}{K_n}+\left(\frac{b}{a+b}\right)\left(\frac{K_b}{1+sT_D}\right)\right]=\left(\frac{a}{a+b}\right)E(s)$$

Now, $K_n >> 1, \frac{1}{K_n}\approx 0,$

Pneumatic Proportional-Derivative Controller



$$-X(s)=\left(\frac{a}{a+b}\right)E(s)-\left(\frac{b}{a+b}\right)\left(\frac{K_b}{1+sT_D}\right)M(s)$$

Now $M(s)=-K_n\cdot X(s)$

where K_n is the nozzle gain.

Thus $$\frac{M(s)}{K_n}=\left(\frac{a}{a+b}\right)E(s)-\left(\frac{b}{a+b}\right)\left(\frac{K_b}{1+sT_D}\right)M(s)$$

or, $$M(s)\left[\frac{1}{K_n}+\left(\frac{b}{a+b}\right)\left(\frac{K_b}{1+sT_D}\right)\right]=\left(\frac{a}{a+b}\right)E(s)$$

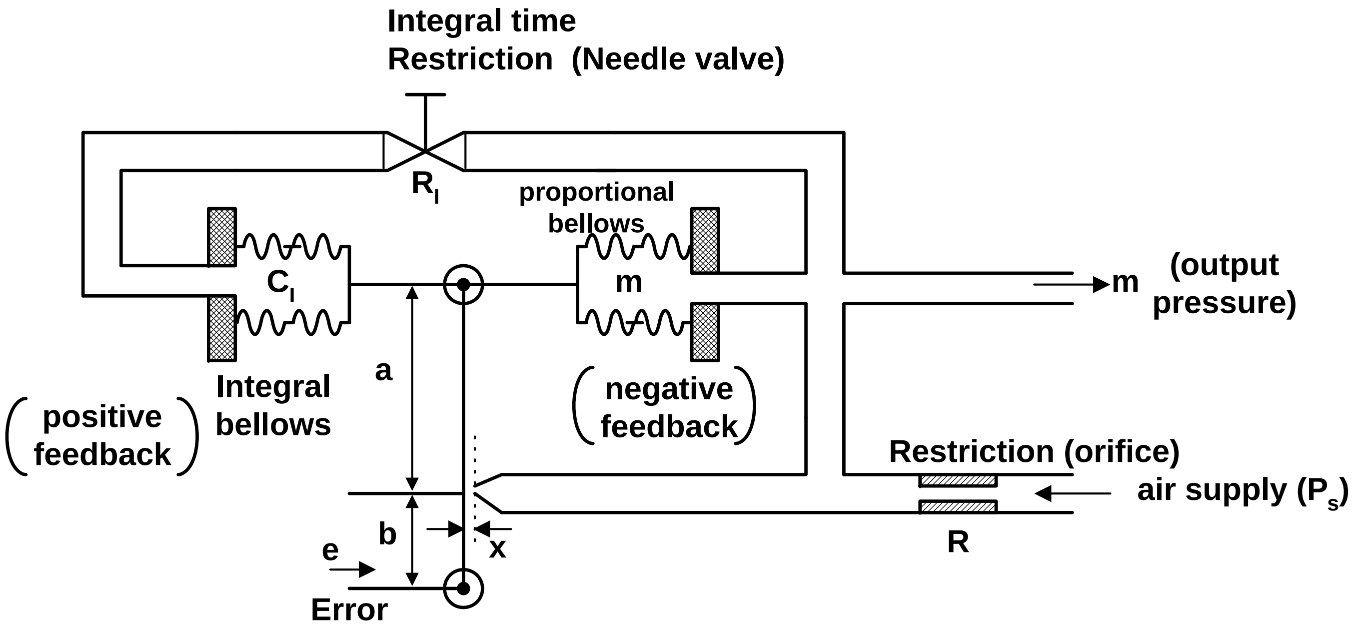
Now, $K_n >> 1, \frac{1}{K_n}\approx 0,$

$$\therefore \frac{M(s)}{E(s)}\approx\left(\frac{a}{bK_b}\right)(1+sT_D)$$

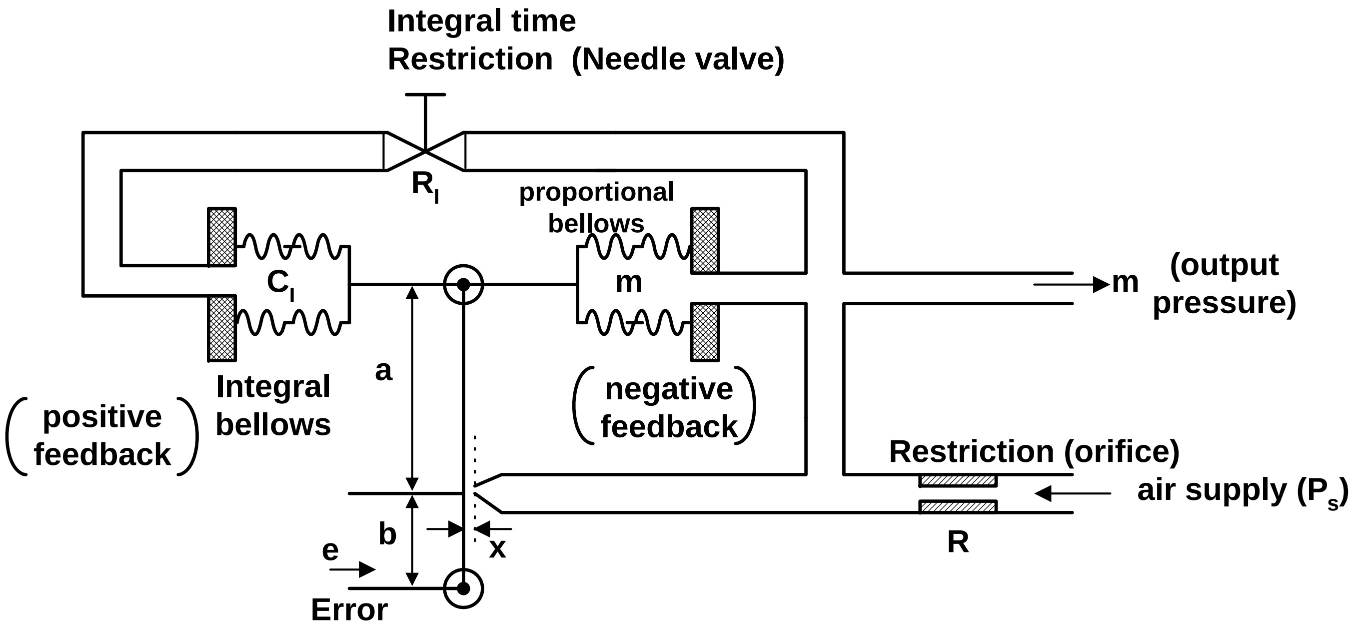
$$=K_p(1+sT_D)$$

where K_p = proportional gain = $\frac{a}{bK_b}$

Pneumatic Proportional-Integral Controller



Pneumatic Proportional-Integral Controller



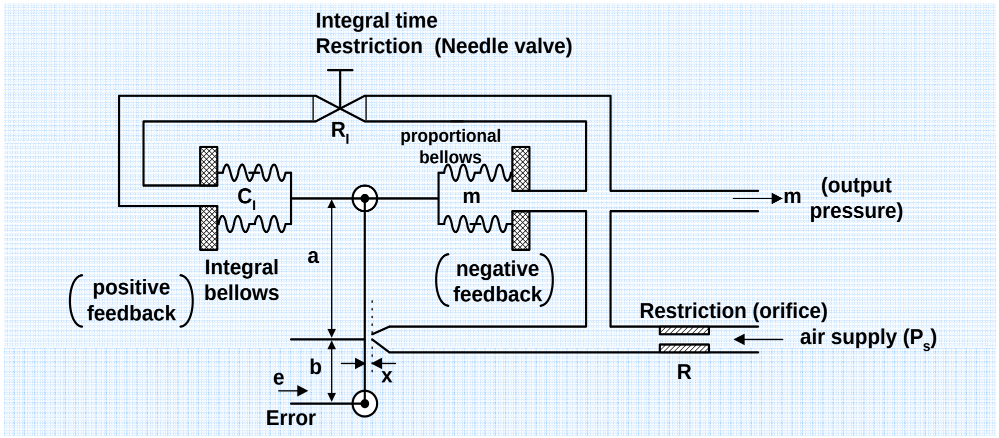
The Baffle-Nozzle separation may be expressed as: (assuming $R_I >> R$ and same stiffness for both bellows)

$$-X(s) = \left(\frac{a}{a+b}\right)E(s) - \left(\frac{b}{a+b}\right)K_b M(s) + \left(\frac{b}{a+b}\right)\left(\frac{K_b}{1+sT_i}\right)M(s)$$

where T_i = Integral time
= $R_I C_I$

and C_I = Capacity of integral bellows
K_b = Bellows stiffness factor.

Pneumatic Proportional-Integral Controller



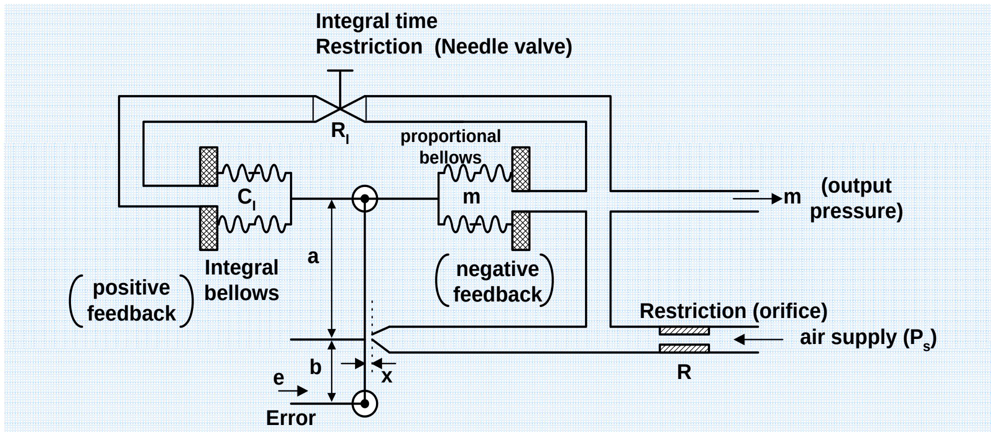
$$-X(s)=\left(\frac{a}{a+b}\right)E(s)-\left(\frac{b}{a+b}\right)K_bM(s)+\left(\frac{b}{a+b}\right)\left(\frac{K_b}{1+sT_i}\right)M(s)$$

Hence,

$$-X(s)=\left(\frac{a}{a+b}\right)E(s)-\left(1-\frac{1}{1+sT_i}\right)\left(\frac{b}{a+b}\right)K_bM(s)$$

$$=\left(\frac{a}{a+b}\right)E(s)-\left(\frac{1}{1+\frac{1}{sT_i}}\right)\left(\frac{b}{a+b}\right)K_bM(s)$$

Pneumatic Proportional-Integral Controller



$$-X(s) = \left(\frac{a}{a+b}\right)E(s) - \left(\frac{b}{a+b}\right)K_b M(s) + \left(\frac{b}{a+b}\right)\left(\frac{K_b}{1+sT_i}\right)M(s)$$

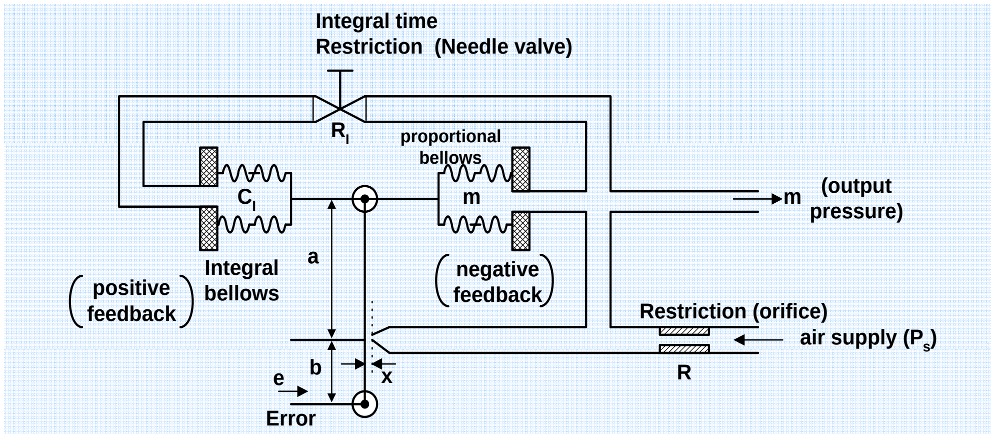
Hence,

$$-X(s) = \left(\frac{a}{a+b}\right)E(s) - \left(1 - \frac{1}{1+sT_i}\right)\left(\frac{b}{a+b}\right)K_b M(s)$$

$$= \left(\frac{a}{a+b}\right)E(s) - \left(\frac{1}{1+\frac{1}{sT_i}}\right)\left(\frac{b}{a+b}\right)K_b M(s)$$

Now, $M(s) = -K_n . X(s)$

Pneumatic Proportional-Integral Controller



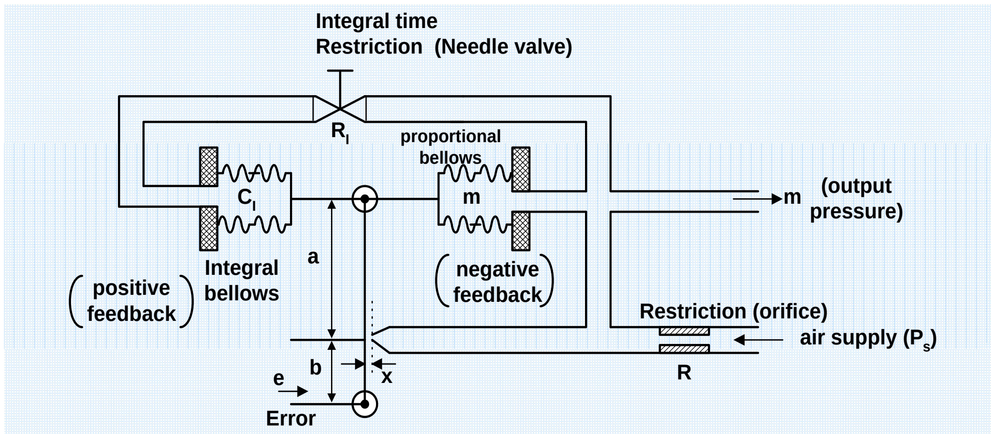
$$-X(s)=\left(\frac{a}{a+b}\right)E(s)-\left(1-\frac{1}{1+sT_i}\right)\left(\frac{b}{a+b}\right)K_bM(s)$$

$$=\left(\frac{a}{a+b}\right)E(s)-\left(\frac{1}{1+\frac{1}{sT_i}}\right)\left(\frac{b}{a+b}\right)K_bM(s)$$

and $\quad M(s)=-K_n.X(s)$

Thus, assuming $K_n \gg 1$, i.e. $\frac{1}{K_n}\approx 0,$

Pneumatic Proportional-Integral Controller



$$-X(s)=\left(\frac{a}{a+b}\right)E(s)-\left(1-\frac{1}{1+sT_i}\right)\left(\frac{b}{a+b}\right)K_bM(s)$$

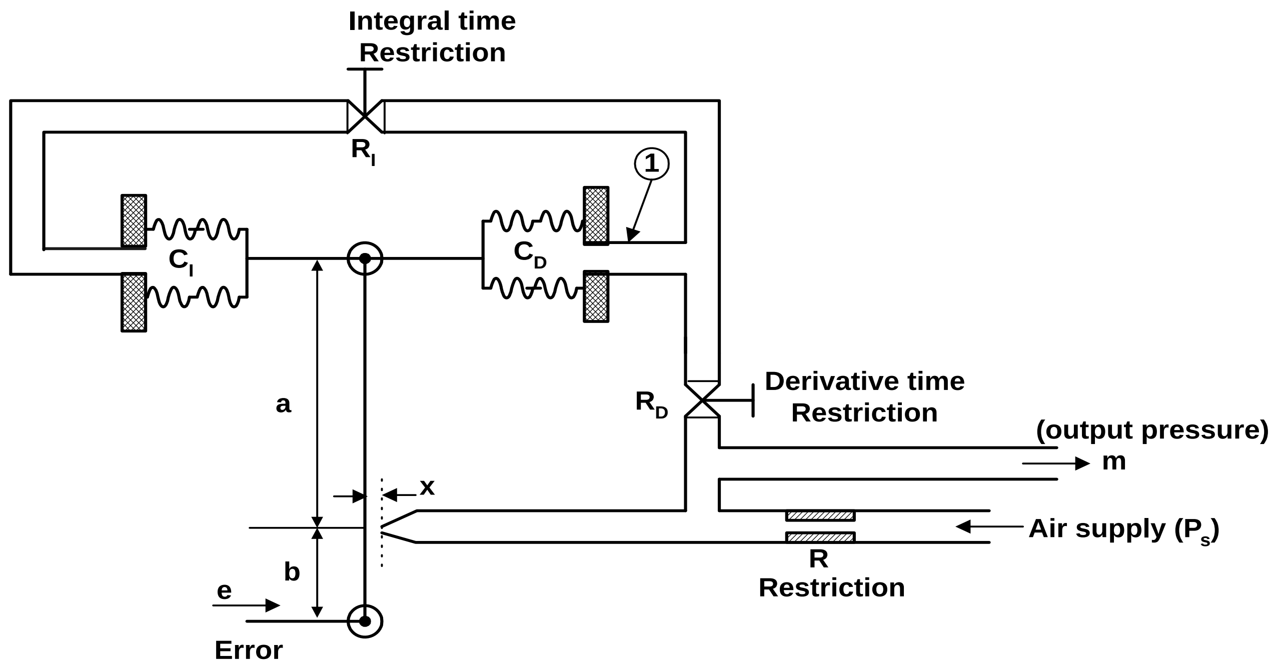
$$=\left(\frac{a}{a+b}\right)E(s)-\left(\frac{1}{1+\frac{1}{sT_i}}\right)\left(\frac{b}{a+b}\right)K_bM(s)$$

and $M(s)=-K_n.X(s)$

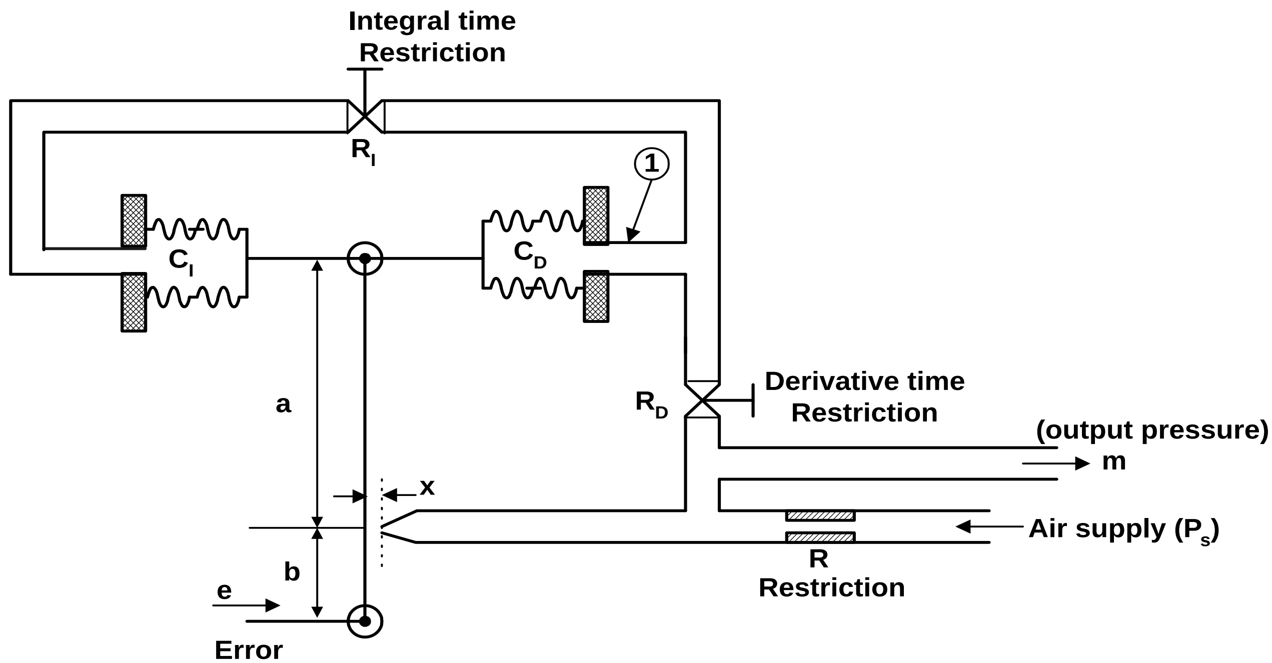
Thus, assuming $K_n >> 1$, i.e. $\frac{1}{K_n}\approx 0,$

$$\frac{M(s)}{E(s)}=\frac{a}{bK_b}\left(1+\frac{1}{sT_i}\right)=K_p\left(1+\frac{1}{sT_i}\right),$$ where $$K_p=\frac{a}{bK_b}$$

Pneumatic Proportional-Integral-Derivative Controller



Pneumatic Proportional-Integral-Derivative Controller



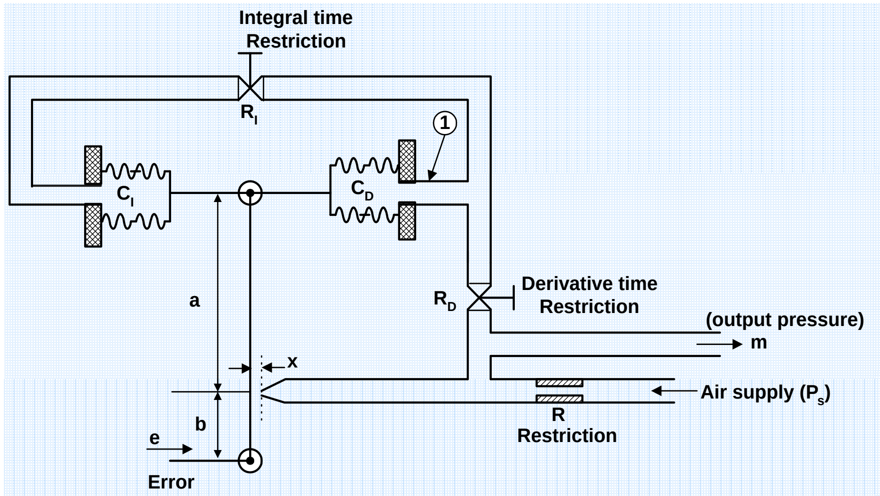
The Baffle-Nozzle separation may be expressed as: (assuming $R_I \gg R_D \gg R$)

$$-X(s) = \left(\frac{a}{a+b}\right)E(s) - \left(\frac{b}{a+b}\right)\left(\frac{K_b}{1+sT_D}\right)M(s)$$

$$+\left(\frac{b}{a+b}\right)\left(\frac{K_b}{(1+sT_i)(1+sT_D)}\right)M(s)$$

$$= \left(\frac{a}{a+b}\right)E(s) - \left(\frac{bK_b}{a+b}\right)\left(\frac{1}{(1+sT_D)} - \frac{1}{(1+sT_i)(1+sT_D)}\right)M(s)$$

Pneumatic Proportional-Integral-Derivative Controller



$$-X(s)=\left(\frac{a}{a+b}\right)E(s)-\left(\frac{b}{a+b}\right)\left(\frac{K_b}{1+sT_D}\right)M(s)$$

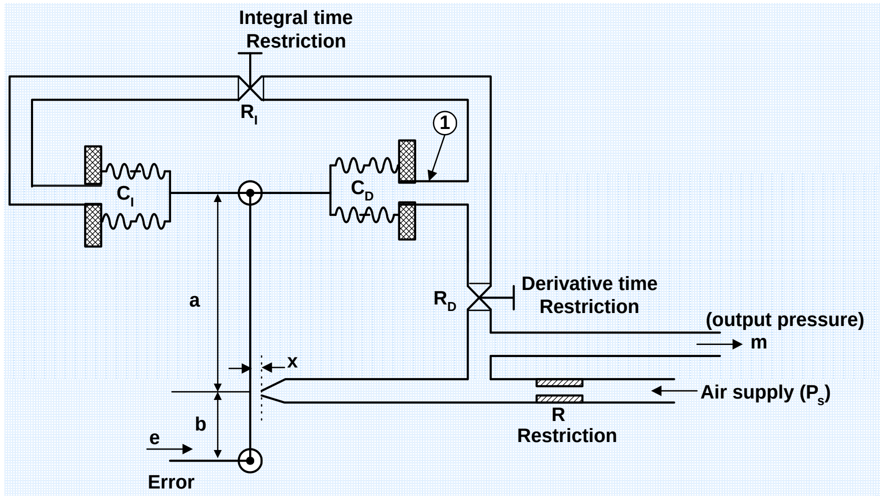
$$+\left(\frac{b}{a+b}\right)\left(\frac{K_b}{(1+sT_i)(1+sT_D)}\right)M(s)$$

$$=\left(\frac{a}{a+b}\right)E(s)-\left(\frac{bK_b}{a+b}\right)\left(\frac{1}{(1+sT_D)}-\frac{1}{(1+sT_i)(1+sT_D)}\right)M(s)$$

Now,

$$\frac{1}{1+sT_D}-\frac{1}{(1+sT_i)(1+sT_D)}=\frac{1+sT_i-1}{(1+sT_i)(1+sT_D)}$$

Pneumatic Proportional-Integral-Derivative Controller



$$-X(s) = \left(\frac{a}{a+b}\right)E(s) - \left(\frac{b}{a+b}\right)\left(\frac{K_b}{1+sT_D}\right)M(s)$$

$$+\left(\frac{b}{a+b}\right)\left(\frac{K_b}{(1+sT_i)(1+sT_D)}\right)M(s)$$

$$= \left(\frac{a}{a+b}\right)E(s) - \left(\frac{bK_b}{a+b}\right)\left(\frac{1}{(1+sT_D)} - \frac{1}{(1+sT_i)(1+sT_D)}\right)M(s)$$

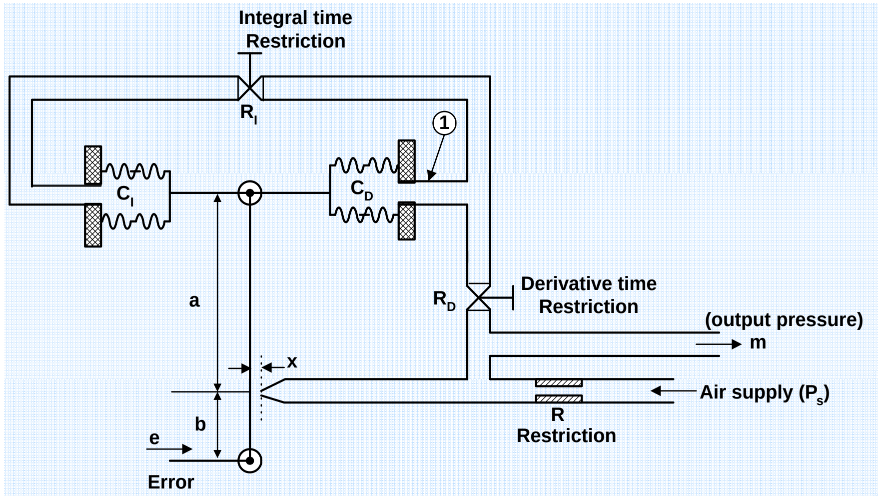
Now,

$$\frac{1}{1+sT_D} - \frac{1}{(1+sT_i)(1+sT_D)} = \frac{1+sT_i-1}{(1+sT_i)(1+sT_D)}$$

$$= \frac{sT_i}{1+sT_i+sT_D+s^2T_iT_D}$$

$$= \frac{1}{1+\frac{1}{sT_i}+\frac{T_D}{T_i}+sT_D}$$

Pneumatic Proportional-Integral-Derivative Controller



$$-X(s)=\left(\frac{a}{a+b}\right)E(s)-\left(\frac{b}{a+b}\right)\left(\frac{K_b}{1+sT_D}\right)M(s)$$

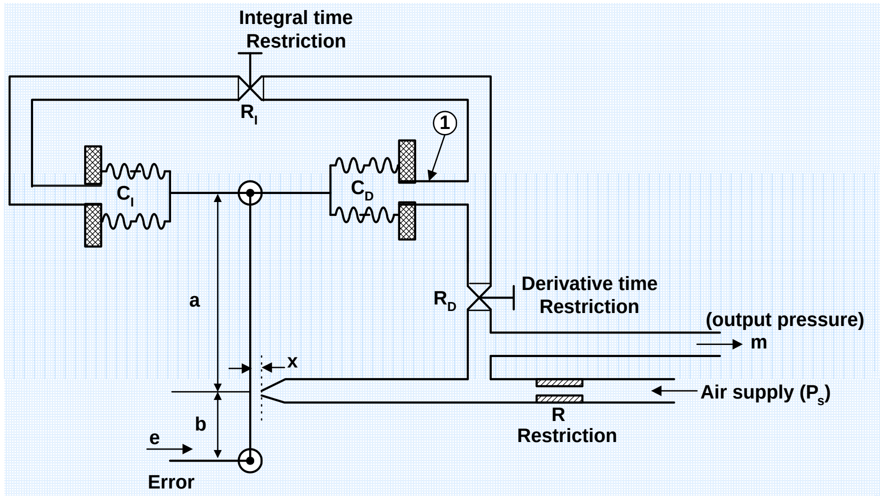
$$+\left(\frac{b}{a+b}\right)\left(\frac{K_b}{(1+sT_i)(1+sT_D)}\right)M(s)$$

$$=\left(\frac{a}{a+b}\right)E(s)-\left(\frac{bK_b}{a+b}\right)\left(\frac{1}{(1+sT_D)}-\frac{1}{(1+sT_i)(1+sT_D)}\right)M(s)$$

Therefore,

$$-X(s)=\left(\frac{a}{a+b}\right)E(s)-\left(\frac{bK_b}{a+b}\right)\left(\frac{1}{1+\frac{1}{sT_i}+\frac{T_D}{T_i}+sT_D}\right)M(s)$$

Pneumatic Proportional-Integral-Derivative Controller



$$-X(s)=\left(\frac{a}{a+b}\right)E(s)-\left(\frac{b}{a+b}\right)\left(\frac{K_b}{1+sT_D}\right)M(s)$$

$$+\left(\frac{b}{a+b}\right)\left(\frac{K_b}{(1+sT_i)(1+sT_D)}\right)M(s)$$

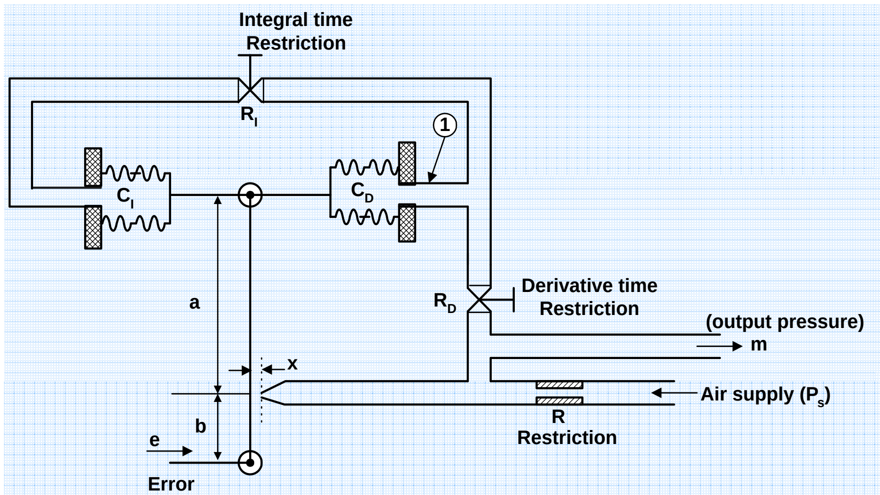
$$=\left(\frac{a}{a+b}\right)E(s)-\left(\frac{bK_b}{a+b}\right)\left(\frac{1}{(1+sT_D)}-\frac{1}{(1+sT_i)(1+sT_D)}\right)M(s)$$

Therefore,

$$-X(s)=\left(\frac{a}{a+b}\right)E(s)-\left(\frac{bK_b}{a+b}\right)\left(\frac{1}{1+\frac{1}{sT_i}+\frac{T_D}{T_i}+sT_D}\right)M(s)$$

Now, $M(s)=-K_n\cdot X(s)$

Pneumatic Proportional-Integral-Derivative Controller



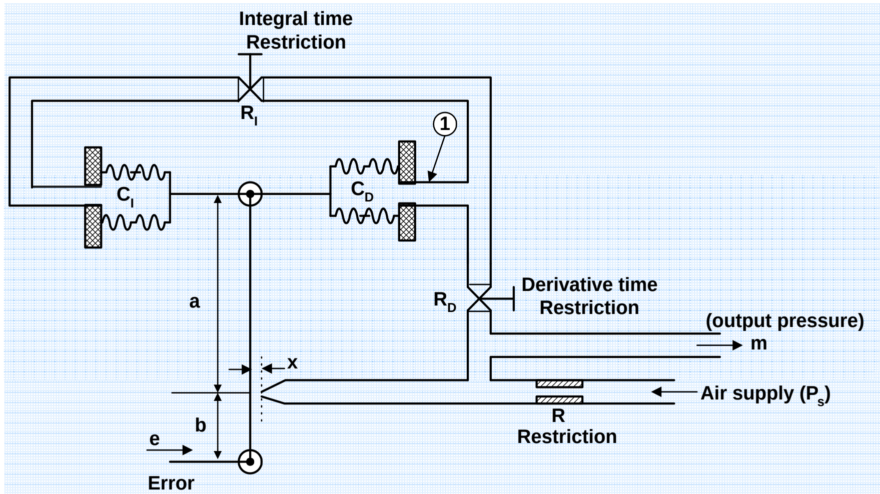
$$-X(s)=\left(\frac{a}{a+b}\right)E(s)-\left(\frac{bK_b}{a+b}\right)\left(\frac{1}{1+\frac{1}{sT_i}+\frac{T_D}{T_i}+sT_D}\right)M(s)$$

and $$M(s)=-K_n\cdot X(s)$$

Therefore,

$$\frac{M(s)}{K_n}=\left(\frac{a}{a+b}\right)E(s)-\left(\frac{bK_b}{a+b}\right)\left(\frac{1}{1+\frac{T_D}{T_i}+\frac{1}{sT_i}+sT_D}\right)M(s)$$

Pneumatic Proportional-Integral-Derivative Controller



$$-X(s)=\left(\frac{a}{a+b}\right)E(s)-\left(\frac{bK_b}{a+b}\right)\left(\frac{1}{1+\frac{1}{sT_i}+\frac{T_D}{T_i}+sT_D}\right)M(s)$$

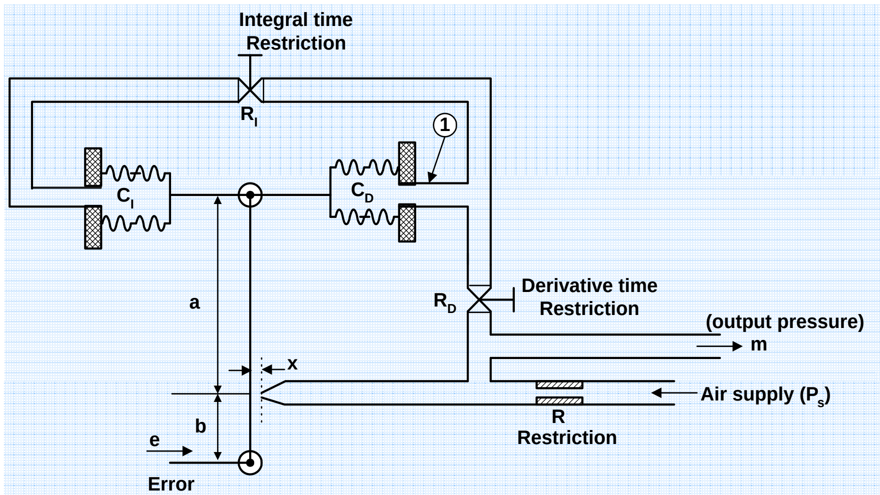
and $$M(s)=-K_n\cdot X(s)$$

Therefore,

$$\frac{M(s)}{K_n}=\left(\frac{a}{a+b}\right)E(s)-\left(\frac{bK_b}{a+b}\right)\left(\frac{1}{1+\frac{T_D}{T_i}+\frac{1}{sT_i}+sT_D}\right)M(s)$$

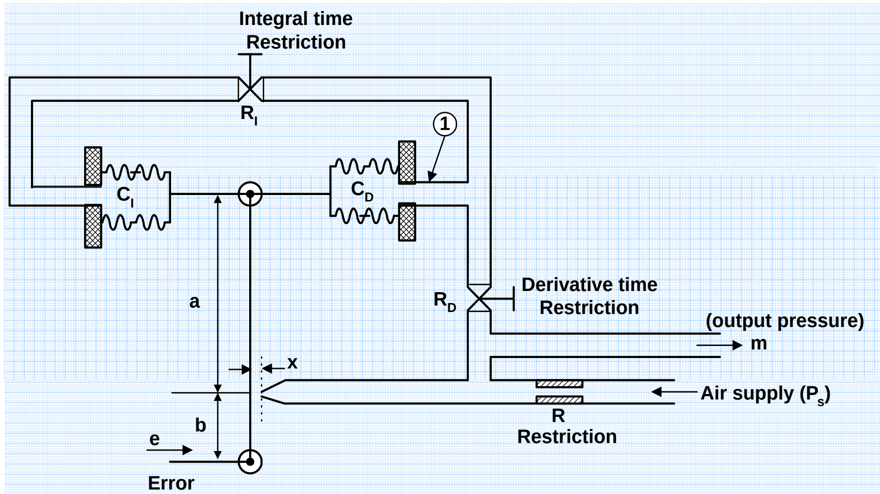
or, $$M(s)\left[\frac{1}{K_n}+\left(\frac{bK_b}{a+b}\right)\left(\frac{1}{1+\frac{T_D}{T_i}+\frac{1}{sT_i}+sT_D}\right)\right]=\left(\frac{a}{a+b}\right)E(s)$$

Pneumatic Proportional-Integral-Derivative Controller



$$M(s)\left[\frac{1}{K_n}+\left(\frac{bK_b}{a+b}\right)\left(\frac{1}{1+\frac{T_D}{T_i}+\frac{1}{sT_i}+sT_D}\right)\right]=\left(\frac{a}{a+b}\right)E(s)$$

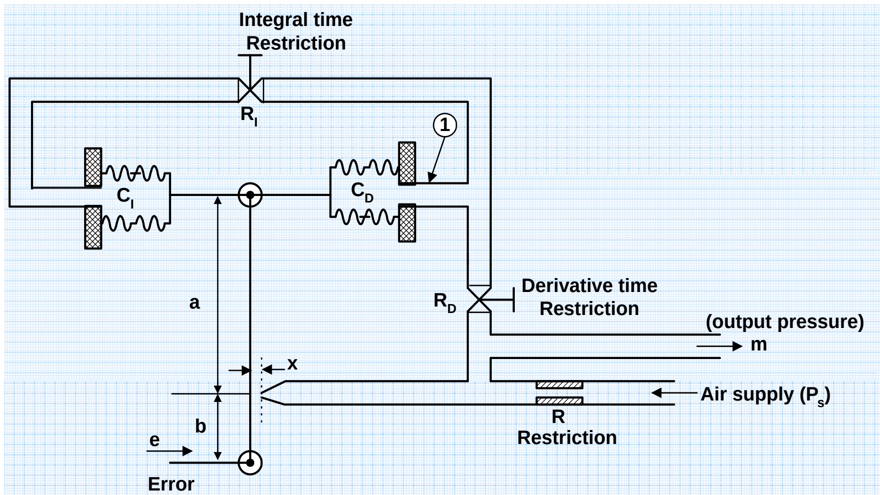
Pneumatic Proportional-Integral-Derivative Controller



$$M(s)\left[\frac{1}{K_n}+\left(\frac{bK_b}{a+b}\right)\left(\frac{1}{1+\frac{T_D}{T_i}+\frac{1}{sT_i}+sT_D}\right)\right]=\left(\frac{a}{a+b}\right)E(s)$$

as $K_n >> 1, \quad \frac{1}{K_n} \approx 0$

Pneumatic Proportional-Integral-Derivative Controller



$$M(s)\left[\frac{1}{K_n}+\left(\frac{bK_b}{a+b}\right)\left(\frac{1}{1+\frac{T_D}{T_i}+\frac{1}{sT_i}+sT_D}\right)\right]=\left(\frac{a}{a+b}\right)E(s)$$

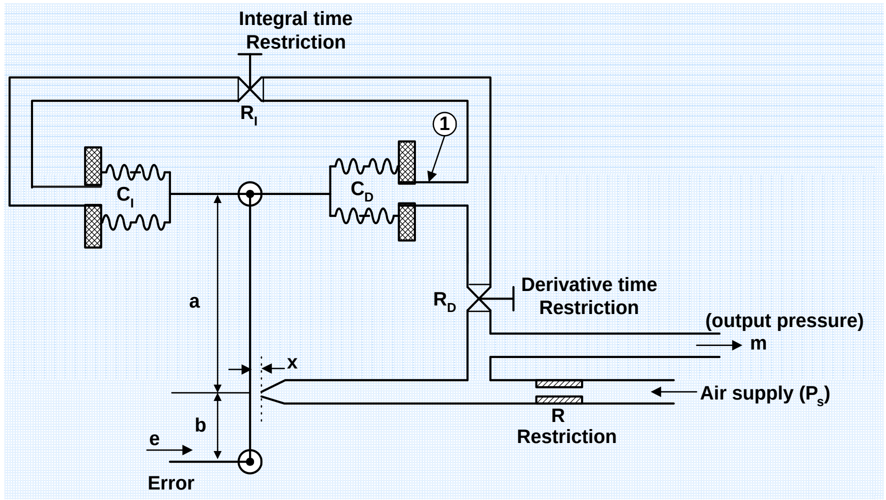
as $K_n >> 1, \quad \frac{1}{K_n} \approx 0$

Therefore,

$$\frac{M(s)}{E(s)}=\left(\frac{a}{bK_b}\right)\left(1+\frac{T_D}{T_i}+\frac{1}{sT_i}+sT_D\right)=K_p\left(1+\frac{T_D}{T_i}+\frac{1}{sT_i}+sT_D\right)$$

where $K_p=\frac{a}{bK_b}$

Pneumatic Proportional-Integral-Derivative Controller

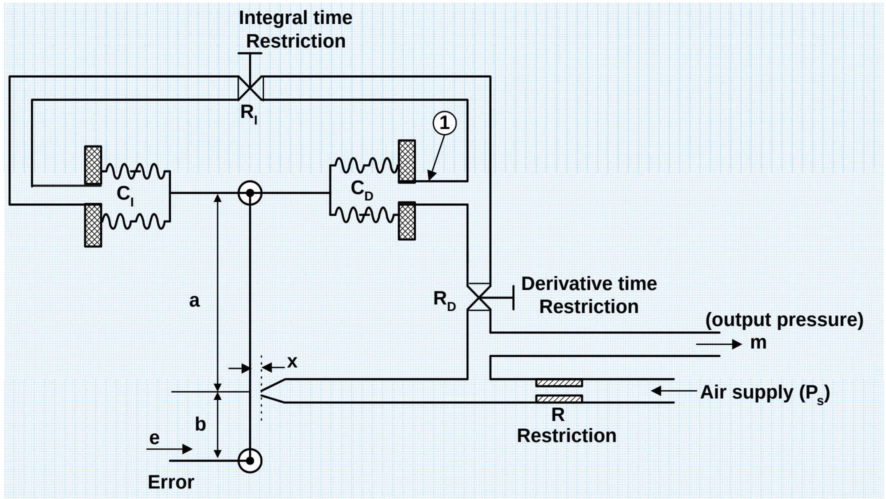


$$\frac{M(s)}{E(s)} = \left(\frac{a}{bK_b}\right)\left(1 + \frac{T_D}{T_i} + \frac{1}{sT_i} + sT_D\right) = K_p\left(1 + \frac{T_D}{T_i} + \frac{1}{sT_i} + sT_D\right)$$

where

$$K_p = \frac{a}{bK_b}$$

Pneumatic Proportional-Integral-Derivative Controller



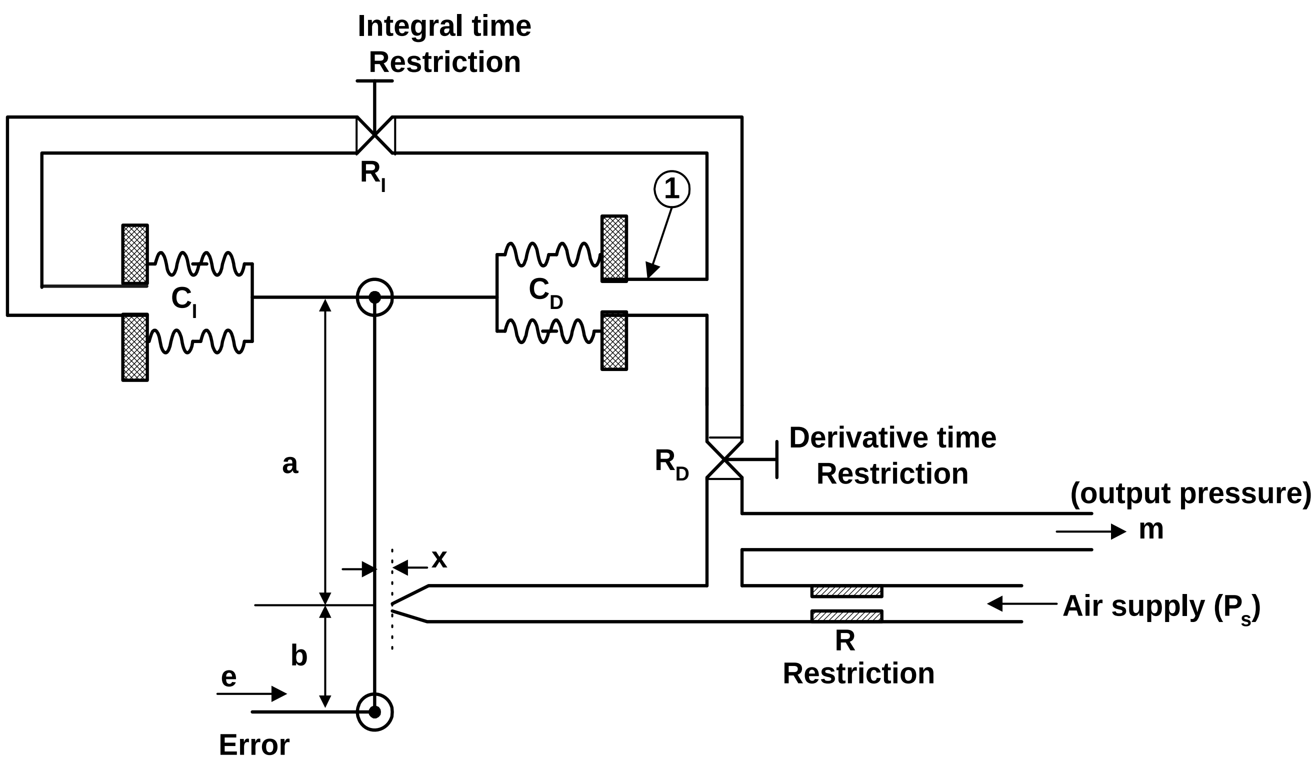
$$\frac{M(s)}{E(s)} = \left(\frac{a}{bK_b}\right)\left(1 + \frac{T_D}{T_i} + \frac{1}{sT_i} + sT_D\right) = K_p\left(1 + \frac{T_D}{T_i} + \frac{1}{sT_i} + sT_D\right)$$ where $$K_p = \frac{a}{bK_b}$$

Here,

$\left(\frac{T_D}{T_i}\right)$ imposes an interaction between integral and derivative operations of the controller. If we choose $T_i >> T_D$, the interaction reduces and the transfer function becomes

$$\frac{M(s)}{E(s)} \approx K_p\left(1 + \frac{1}{sT_i} + sT_D\right)$$ the ideal relation of a PID controller.

Pneumatic Proportional-Integral-Derivative Controller



The controller gain becomes infinite when $T_i = T_D$, if derivative time restriction is placed in position (1).

References

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Thank You