Piezoelectric Transducers

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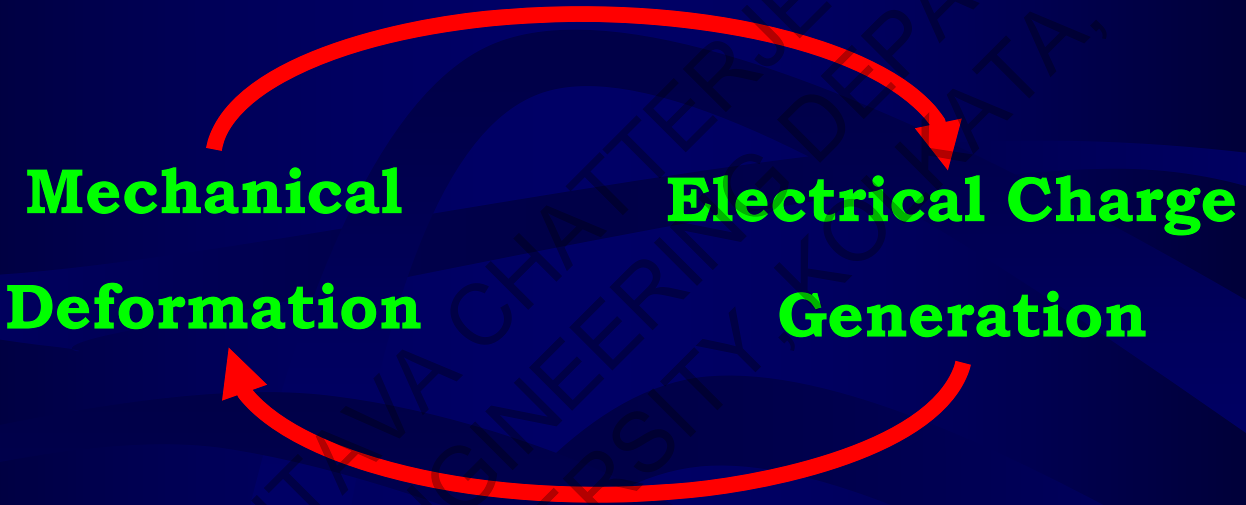
Piezoelectric Transducers

Piezoelectric Effect:



Mechanical Deformation

Electrical Charge Generation

- ✓ **Piezoelectric Transducers employ the principle of electromechanical energy conversion, in both directions.**
- ✓ **The mechanical input/electrical output direction is the basis of many instruments used for measuring acceleration, force, and pressure.**

Piezoelectric Transducers

Piezoelectric Materials:

- ❖**Natural crystals (quartz, rochelle salt, etc.)**
- ❖**Synthetic crystals (lithium sulphate, ammonium dihydrogen phosphate, etc.)**
- ❖**Polarized ferroelectric ceramics (barium titanate etc.)**

✓ **Because of their natural asymmetric structure, the crystal materials exhibit this effect without further processing.**

✓ **The ferroelectric ceramics must be artificially polarized by applying a strong electric field to the material (while it is heated to a temperature above the Curie point of that material) and then slowly cooling with the field still applied.**

Piezoelectric Transducers

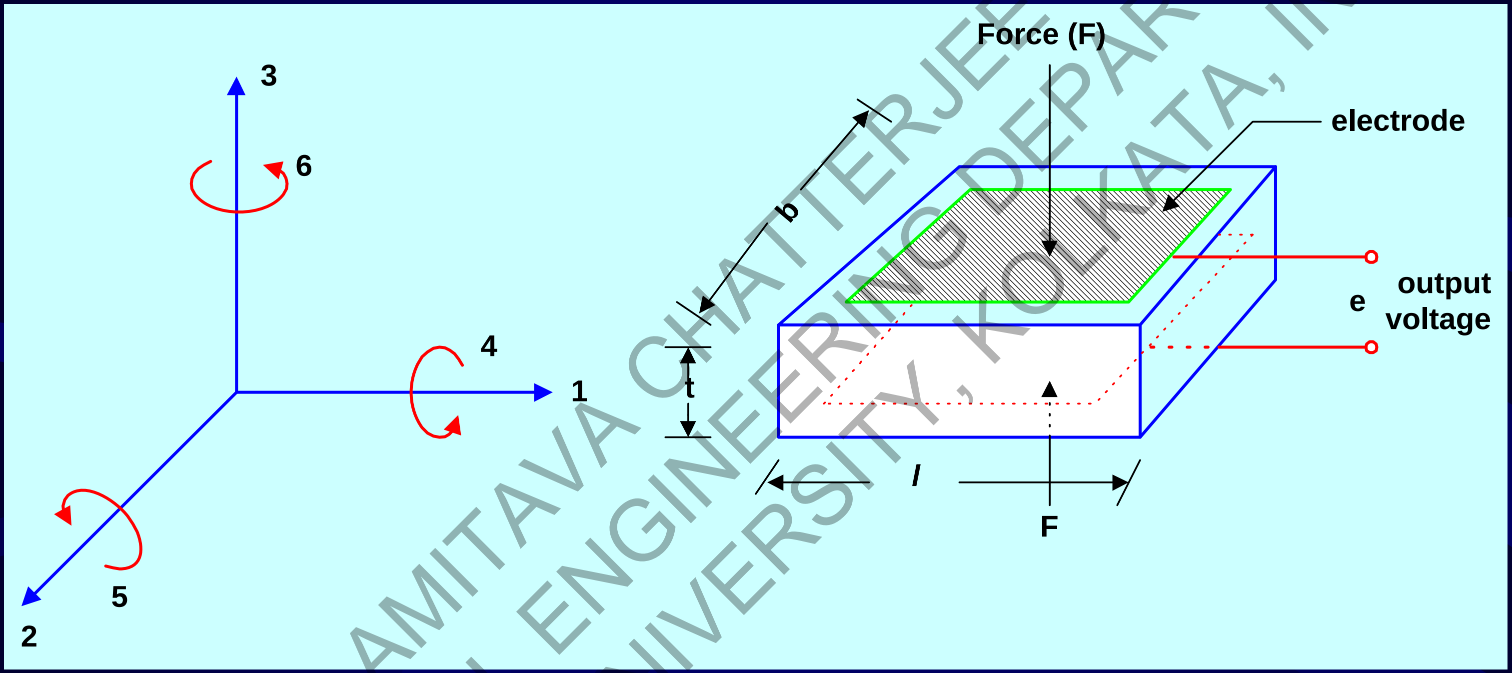
Different Modes of Mechanical Deformations:

- **Thickness-expansion**
- **Transverse-expansion**
- **Thickness-shear**
- **Face-shear**

✓ **The piezoelectric effect is direction sensitive. Here tension produces a definite voltage polarity while compression produces the opposite.**

Piezoelectric Transducers

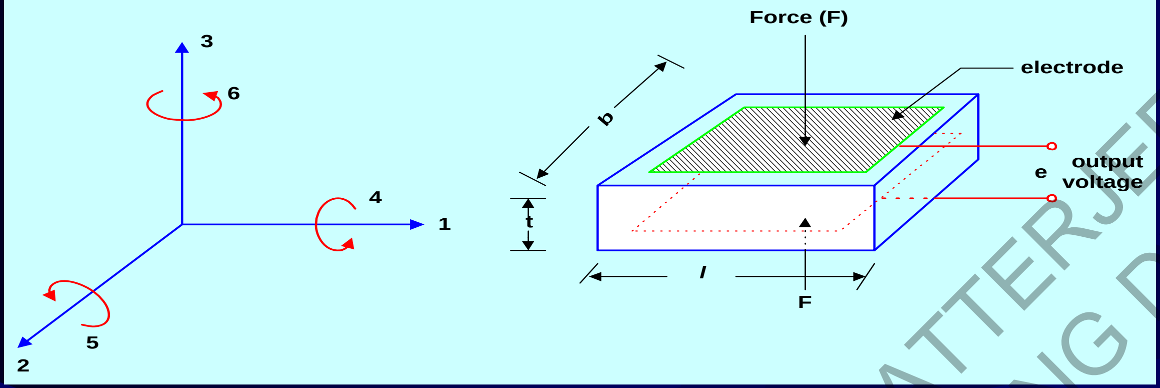
Piezoelectric Effect under Thickness-Expansion Mode (e.g. Barium Titanate):



Axis System **Piezoelectric Transducer**

- ✓ **Mechanical deformation generates a charge and this charge appears as a voltage across the electrodes** $\left(E = \frac{Q}{C}\right)$.

Piezoelectric Effect under Thickness-Expansion Mode (e.g. Barium Titanate):



'g' constant:

$$g_{33} \overset{\Delta}{=} \frac{\text{electric field generated in direction 3}}{\text{mechanical stress applied in direction 3}} = \frac{e/t}{F/lb}$$

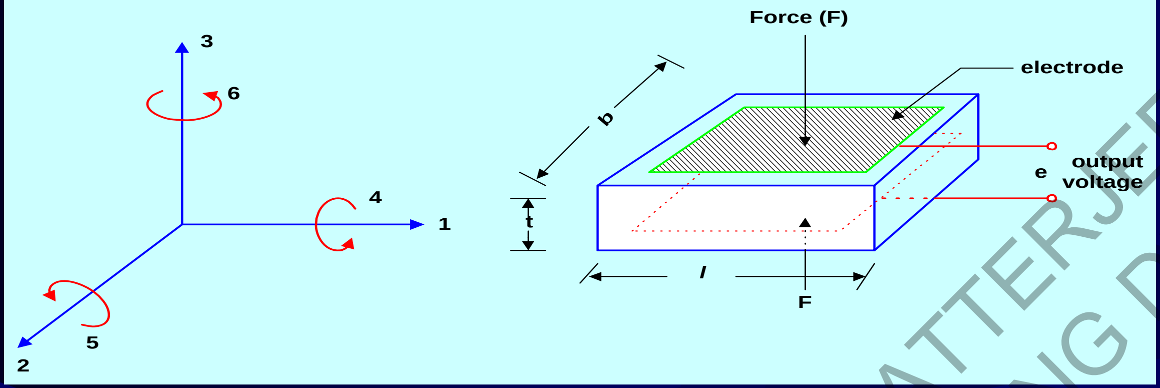
Voltage Output = $e = g_{33} \times \frac{F}{lb} \times t = g_{33} \times \text{stress} \times \text{thickness}$

Typical Values

12×10^{-3} (V/m) / (N/m^2) for barium titanate

50×10^{-3} (V/m) / (N/m^2) for quartz

Piezoelectric Effect under Thickness-Expansion Mode (e.g. Barium Titanate):



'd' constant:

$$d_{33} \overset{\Delta}{=} \frac{\textbf{charge generated in direction 3}}{\textbf{Force applied in direction 3}} = \frac{Q}{F} = \frac{Q}{(lb)} \frac{1}{F/(lb)}$$

$$d_{33} = \varepsilon g_{33},$$ **ε = permittivity of the material**

Typical Values

Permittivity of barium titanate: 12.5×10^{-9} F/m.

Permittivity of quartz : 4.06×10^{-11} F/m.

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Piezoelectric Displacement Transducers

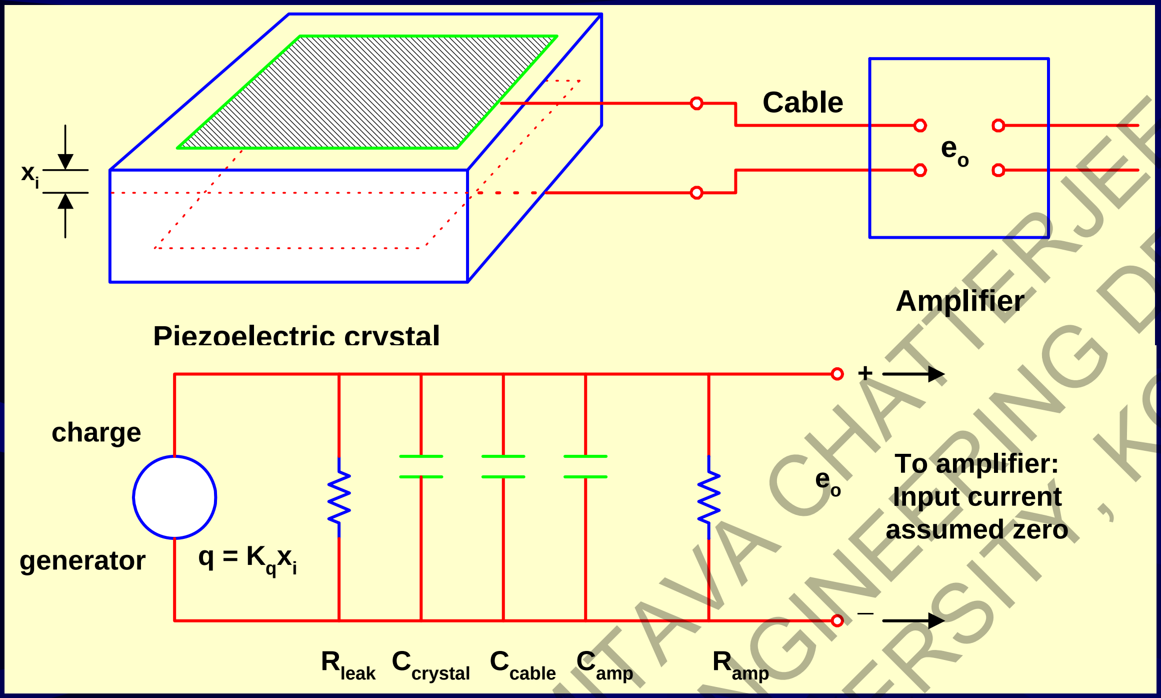
- ✓ For **displacement transducers**, the pertinent quantity is **output voltage (or charge) per unit deflection** (instead of stress or force). This is because it is really the “deflection” that causes the charge generation. Hence a **knowledge of modulus of elasticity** is also needed.

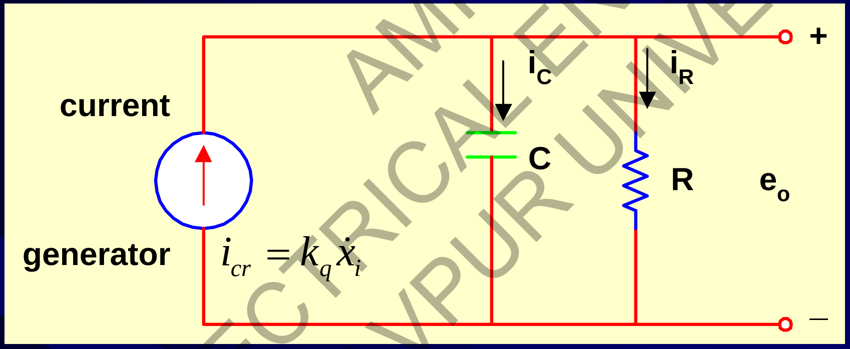
Typical Values

Modulus of elasticity of quartz = 8.6×10^{10} N/m^2

Modulus of elasticity of barium titanate = 12×10^{10} N/m^2

Piezoelectric Displacement Transducers





$$R \triangleq \frac{R_{amp} R_{leak}}{R_{amp} + R_{leak}} \approx R_{amp}$$

$$C \triangleq C_{crystal} + C_{cable} + C_{amp}$$

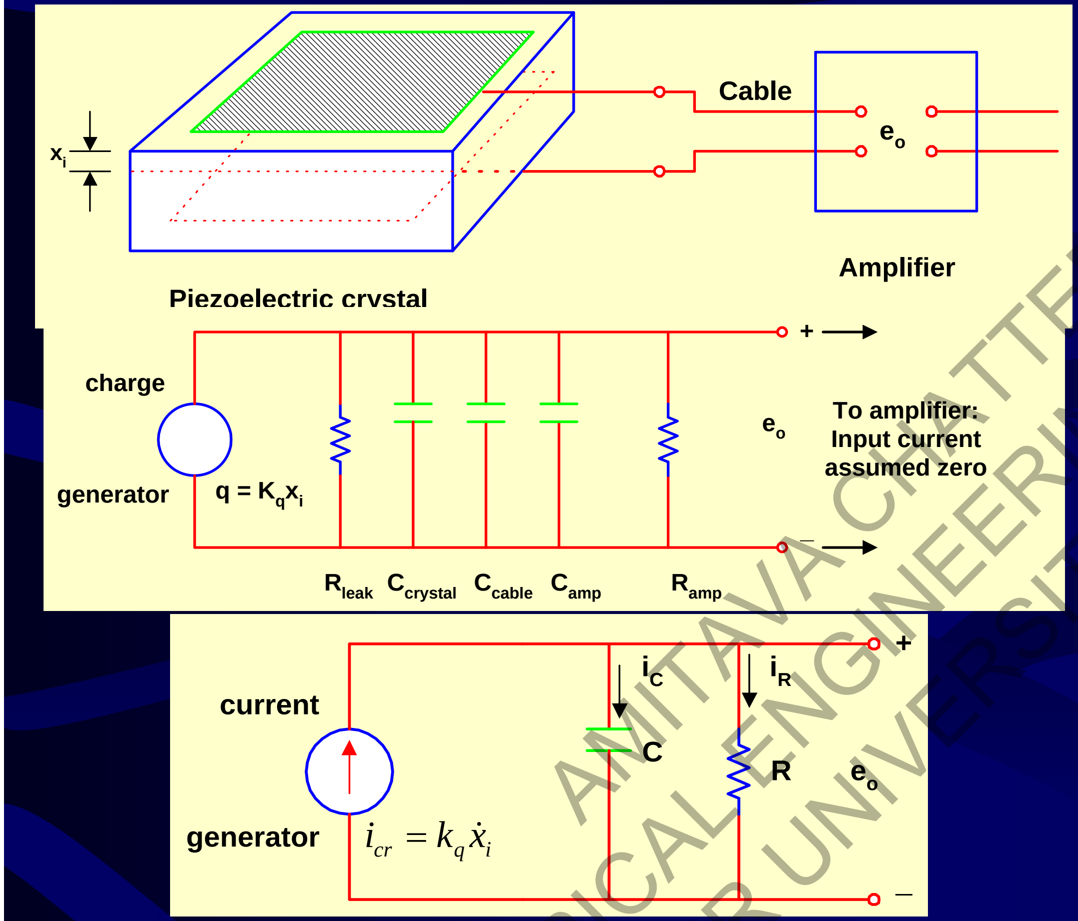
$$q = K_q x_i$$

$$\begin{cases} K_q = \textbf{charge sensitivity (c/m)} \\ x_i = \textbf{displacement (m)} \end{cases}$$

$$i_{cr} = \frac{dq}{dt} = K_q \frac{dx_i}{dt}$$

$$i_{cr} = i_c + i_R$$

Piezoelectric Displacement Transducers



$$e_o = e_c = \frac{\int i_c\,dt}{C} = \frac{\int (i_{cr} - i_R)\,dt}{C}$$

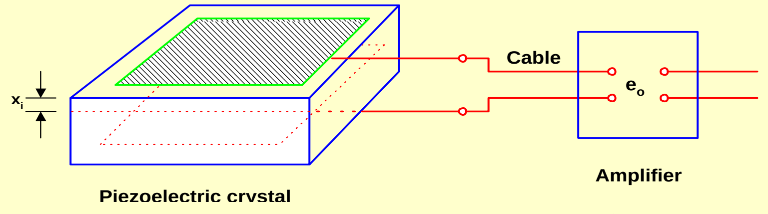
$$\frac{E_o(s)}{X_i(s)} = \frac{K_q sR}{1 + sRC} = \frac{\frac{K_q}{C}s}{s + \frac{1}{RC}} = \frac{Ks}{s + \frac{1}{T}}$$

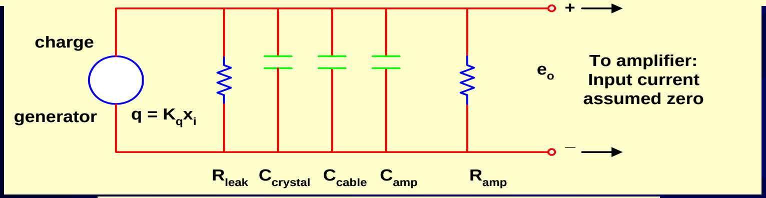
$$\begin{cases} K \overset{\Delta}{=} \textbf{voltage sensitivity} = \dfrac{K_q}{C}\ \textbf{(V/m)} \\ T \overset{\Delta}{=} \textbf{time constant} = RC\ \textbf{(s)} \end{cases}$$

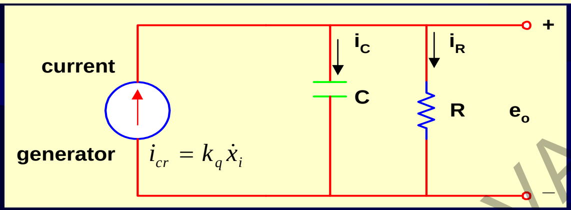
The steady state response to a constant x_i is zero.
Thus we cannot measure static displacements.

Piezoelectric Displacement Transducers

Frequency Response

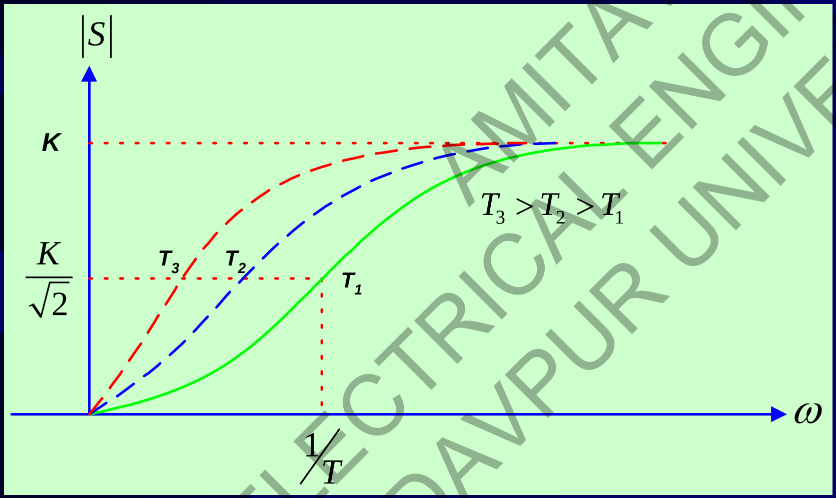




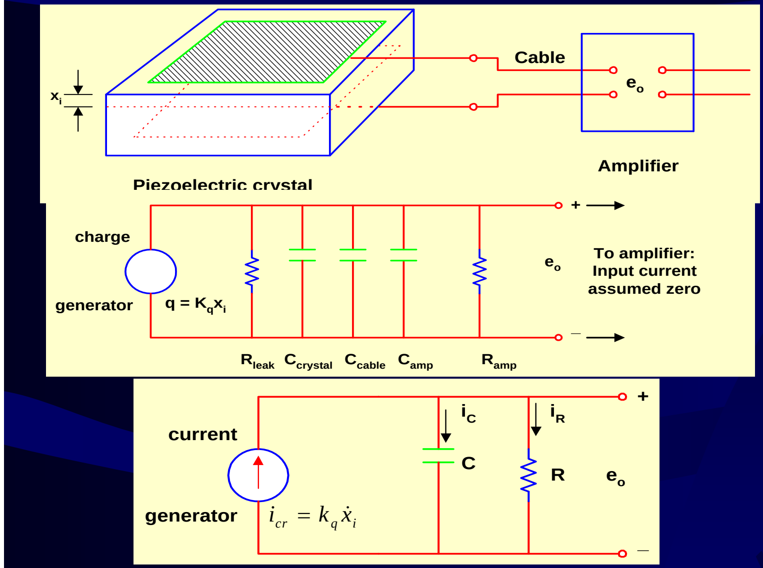


$$\frac{E_o(j\omega)}{X_i(j\omega)} = \frac{j\omega K}{j\omega + \frac{1}{T}} = \frac{j\omega KT}{j\omega T + 1}$$

$$\left|\frac{E_o(j\omega)}{X_i(j\omega)}\right| = \frac{K\omega T}{\sqrt{1+\omega^2 T^2}} = |S|$$



Piezoelectric Displacement Transducers



Static Response

Steady state response for constant or static $x_i = 0$.

Dynamic Response

For vibratory x_i (sinusoidal), flat amplitude response, within a specified percentage, restricts the lowest frequency possible.

For 5% basis, the frequency must exceed ω_1 given by:

$$\omega_1 = \frac{3.04}{T} \Leftarrow (1-0.05)^2 = \frac{(\omega_1 T)^2}{(\omega_1 T)^2 + 1}$$

The low frequency limit is set by the time constant T – higher the T, lower the ω_1.

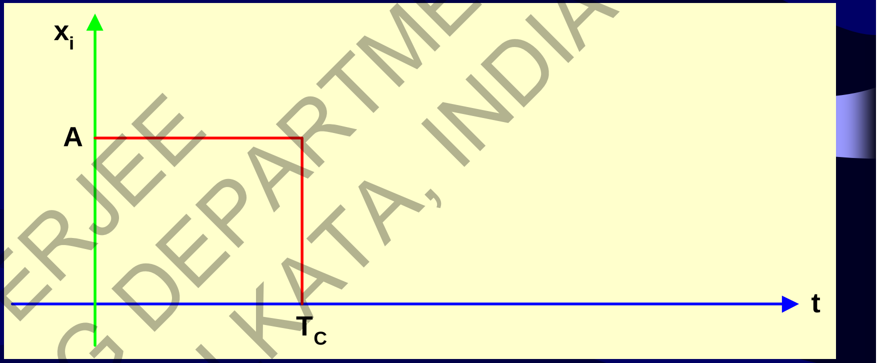
Piezoelectric Displacement Transducers

Pulse Response

Let the input be a pulse excitation:

$$x_i = Au(t) - Au(t - T_c)$$

$$X_i(s) = \frac{A}{s} - e^{-sT_c} \cdot \frac{A}{s}$$



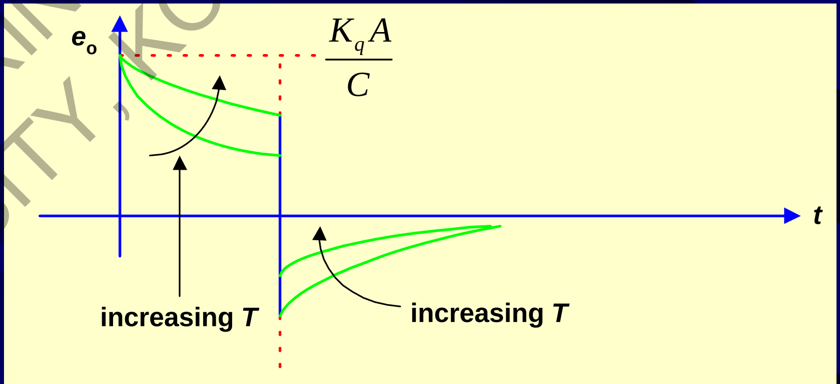
The differential equation describing the response:

$$C\frac{de_o}{dt} + \frac{e_o}{R} = K_q\left(\frac{dx_i}{dt}\right)$$

$$T\frac{de_o}{dt} + e_o = KT\left(\frac{dx_i}{dt}\right)\left(K = \frac{K_q}{C} \textbf{ and } T = RC\right)$$

$$E_o(s) = AKT\frac{1 - e^{-sT_c}}{sT + 1} = AK\frac{1 - e^{-sT_c}}{s + \frac{1}{T}}$$

$$e_o(t) = AKe^{-\frac{t}{T}}u(t) - AKe^{-\frac{(t - T_c)}{T}}u(t - T_c)$$

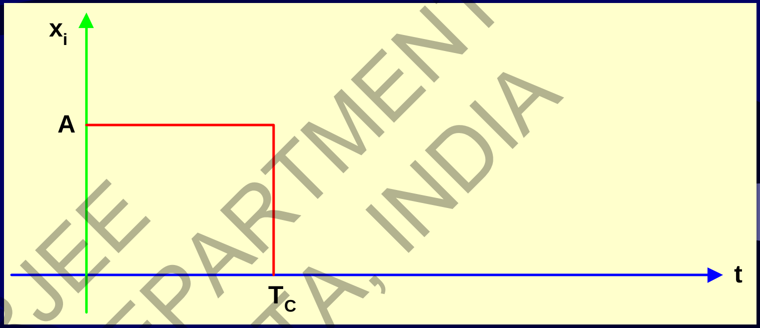


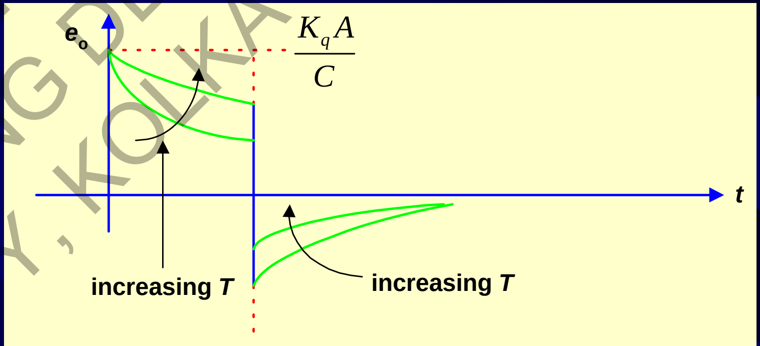
Piezoelectric Displacement Transducers

Pulse Response

Conclusion:

- ✓ **A large *T* is desirable for faithful reproduction of x_i.**
- ✓ **If the *decay* and *undershoot* at $t = T_c$ are to be kept within 5% of the true value (i.e. the pulse height), *T* must be at least $20T_c$.**
- ✓ ***T* can be increased by increasing either or both *R* and *C*.**





Increasing C:

By connecting an external shunt capacitor across the transducer terminals .

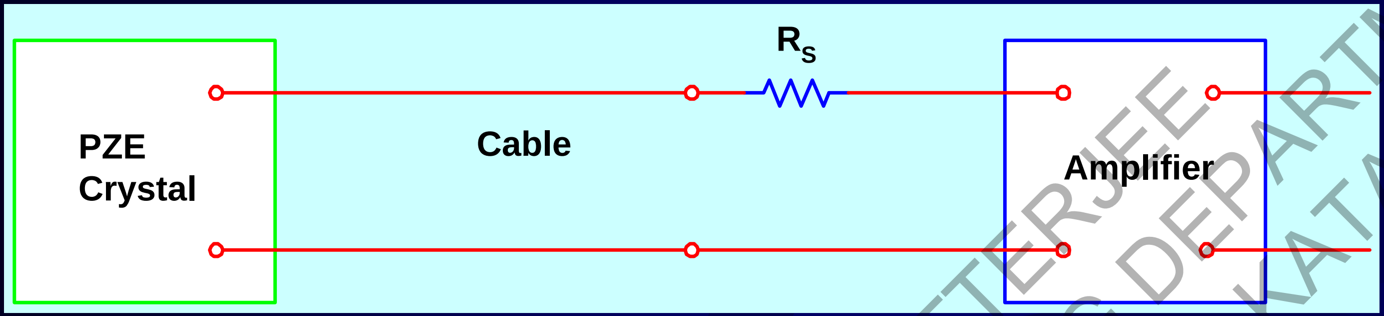
Increasing R:

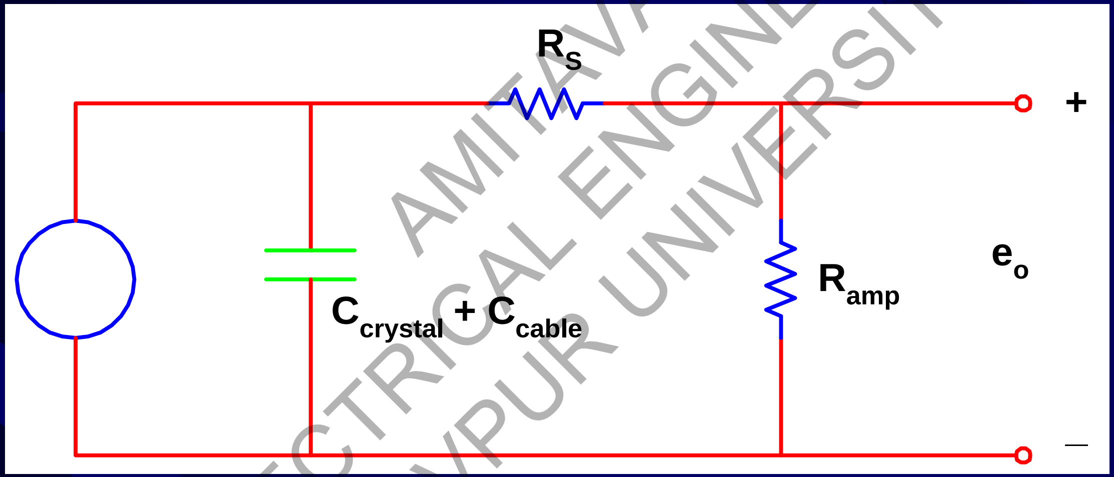
By connecting a series resistance R_s external to the amplifier.

Price Paid ??

A loss in sensitivity as $K = \frac{K_q}{C}$.

Connecting Series Resistance External to the Amplifier





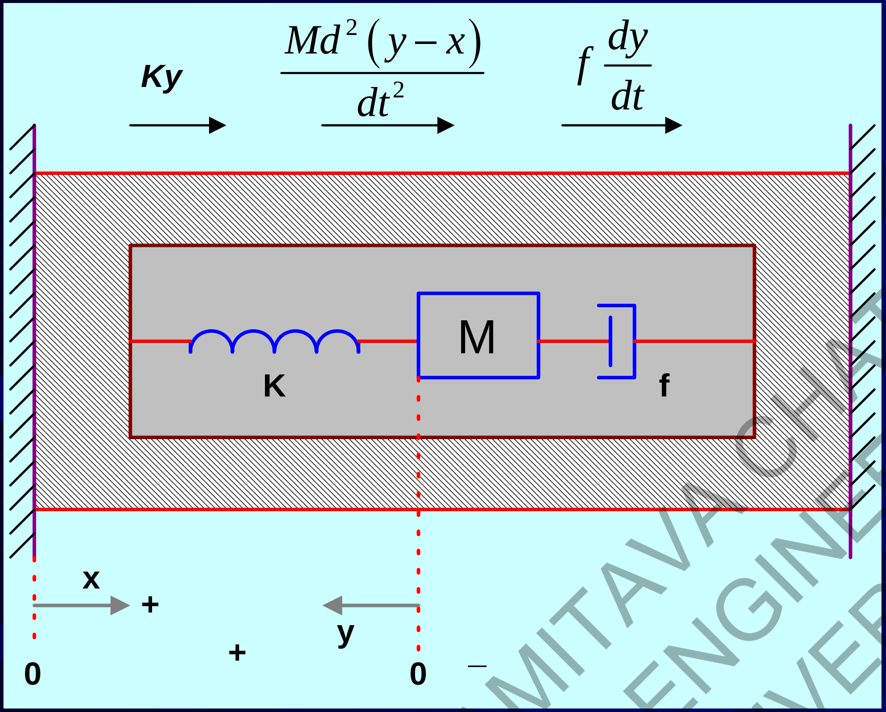
$$K \triangleq \frac{K_q}{C}\left(\frac{R_{amp}}{R_{amp}+R_s}\right)$$

$$T \triangleq \left(R_{amp}+R_s\right)C$$

$$C \triangleq \left(C_{crystal}+C_{cable}\right)$$

Piezoelectric Accelerometers

General Description of Accelerometers



Simplified Diagram of an Accelerometer (a spring-mass-dashpot system)

Force equation of the system:

$$M\frac{d^2(y-x)}{dt^2}+f\frac{dy}{dt}+Ky=0$$

$$M\frac{d^2y}{dt^2}+f\frac{dy}{dt}+Ky=M\frac{d^2x}{dt^2}=Ma$$

x = displacement of the moving object (or frame) with respect to a fixed reference frame,

y = displacement of the mass M with respect to accelerometer frame,

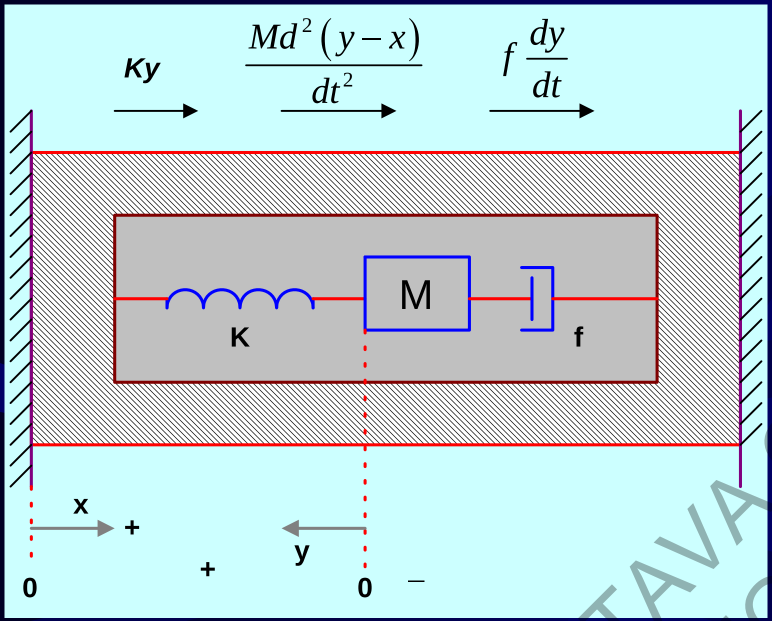
a = input acceleration.

For our system: $K_s x_o + B\dot{x}_o = M\ddot{x}_M = M\left(\ddot{x}_i - \ddot{x}_o\right)$

x_i and x_M are absolute displacements, and x_o is chosen zero when gravity force is acting along the x-axis statically.

Piezoelectric Accelerometers

General Description of Accelerometers



$$K_s x_o + B\dot{x}_o = M\ddot{x}_M = M\left(\ddot{x}_i - \ddot{x}_o\right)$$

$$\Downarrow$$

$$K_s X_o(s) + BsX_o(s) = s^2 M\left[X_i(s) - X_o(s)\right]$$

$$\Downarrow$$

$$\frac{X_o(s)}{s^2 X_i(s)} = \frac{K}{\frac{s^2}{\omega_n^2} + \frac{2\varsigma}{\omega_n}s + 1}$$

$$\omega_n \triangleq \sqrt{\frac{K_s}{M}},\ \varsigma \triangleq \frac{B}{2\sqrt{K_s M}} \text{ and } K \triangleq \frac{1}{\omega_n^2} \text{ cm/(cm/s}^2\text{)}$$

Output: $e_o = K_e x_o$ $\Rightarrow$ **Acceleration to Voltage T.F. :** $\frac{E_o(s)}{s^2 X_i(s)} = \frac{KK_e}{\frac{s^2}{\omega_n^2} + \frac{2\varsigma}{\omega_n}s + 1}$

Piezoelectric Accelerometers

The T.F. of our system: $$\frac{E_o(s)}{s^2 X_i(s)} = \left(\frac{\frac{K_q}{C}\cdot \tau s}{\tau s + 1}\right)\left(\frac{\frac{1}{\omega_n^2}}{\frac{s^2}{\omega_n^2} + \frac{2\zeta}{\omega_n}s + 1}\right)$$

$$\left(\frac{\frac{K_q}{C}\cdot \tau s}{\tau s + 1}\right)$$ ➡ appears due to cable, buffer amplifier etc.

✓ The low frequency response of this accelerometer is limited by the piezoelectric characteristic $\frac{\tau s}{\tau s + 1}$.

✓ The high frequency response is limited by mechanical resonance.

Piezoelectric Accelerometers

- ✓ The accurate frequency range of such accelerometers, for flat amplitude response within a specified percentage, is $\frac{3}{\tau} < \omega < \frac{\omega_n}{5}$.

(5% high at high frequency end and 5% low at low frequency end)

Using magnitude response of the piezoelectric transducer:

$$(1-0.05)^2 = \frac{(\omega_1\tau)^2}{(\omega_1\tau)^2+1}$$

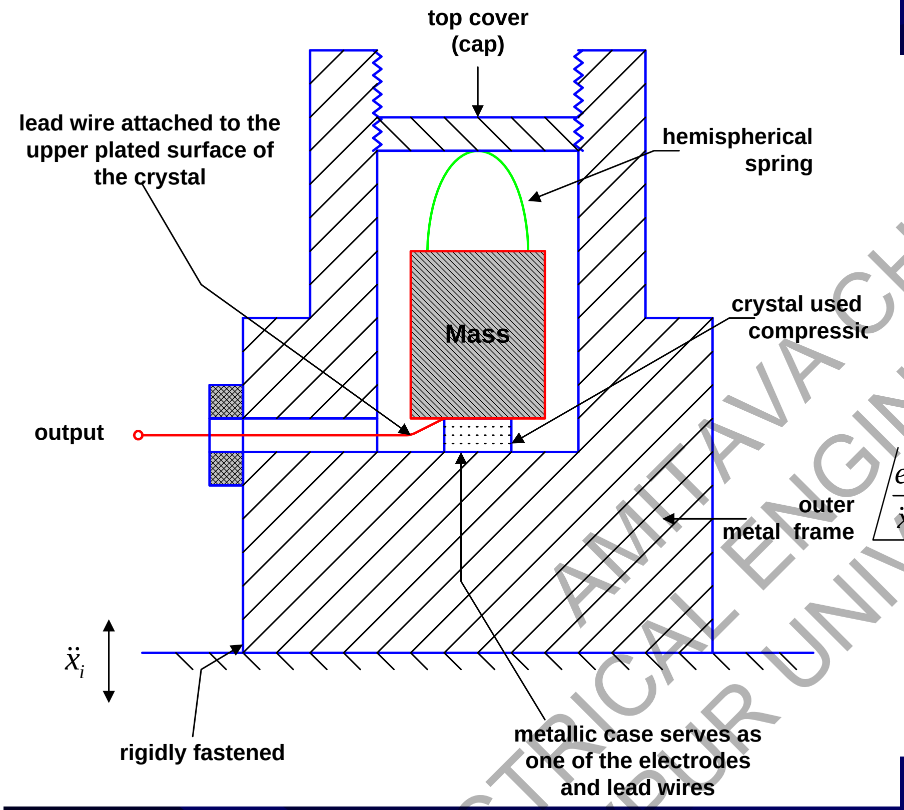
or $$\omega_1 = \frac{3.04}{\tau} \approx \frac{3}{\tau}$$

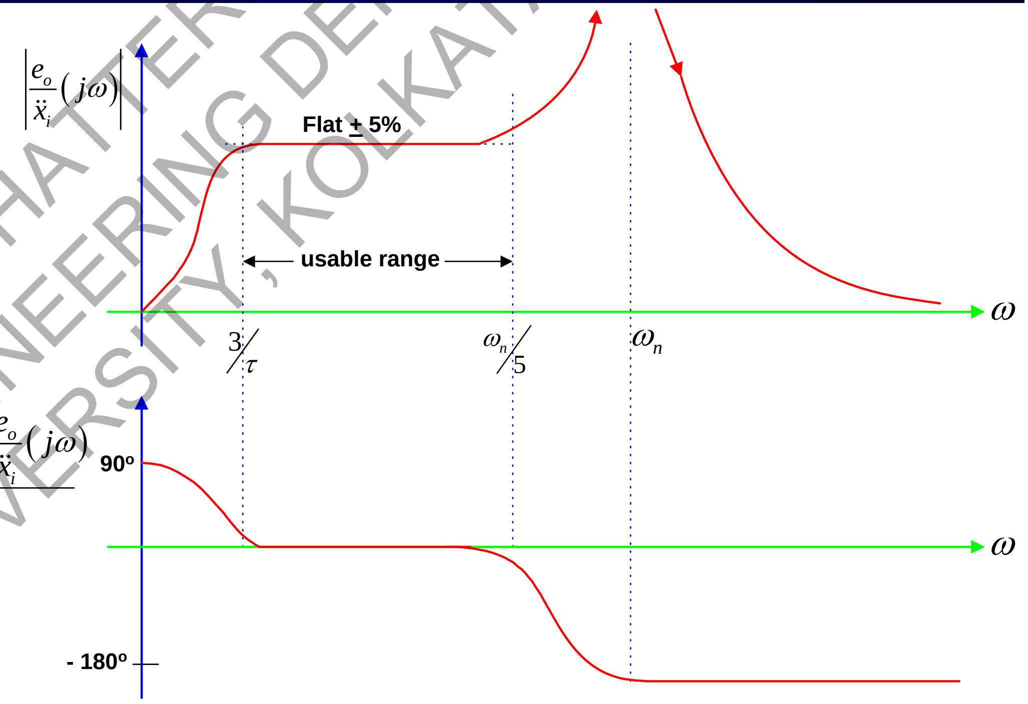
- ✓ The damping ratio ζ of piezoelectric accelerometers is extremely low (almost zero for most practical purposes).

Piezoelectric Accelerometers

The Construction:

Frequency Response:

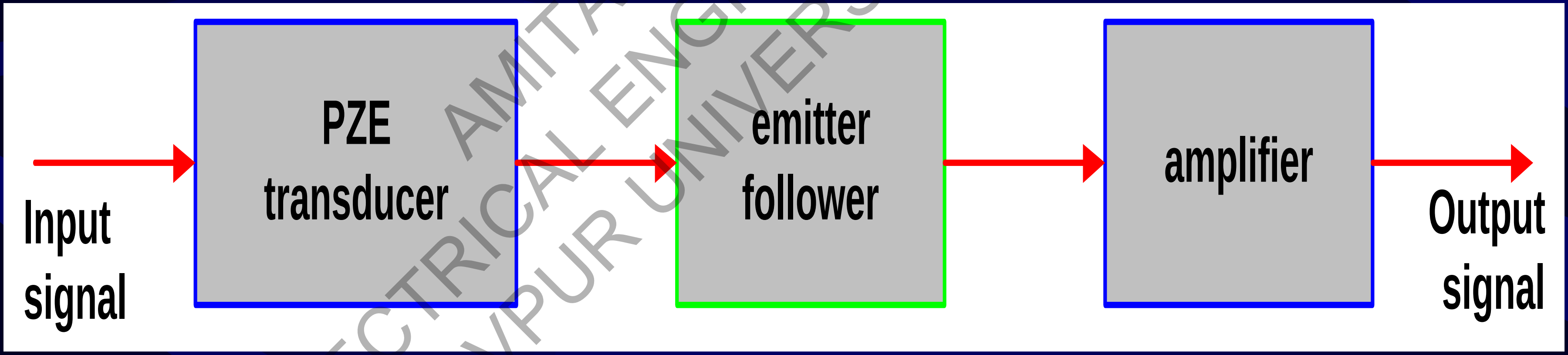




Piezoelectric Accelerometers

✓ Accurate low frequency response requires large τ, which is usually achieved by use of high impedance amplifiers (e.g. instrumentation amplifiers or charge amplifiers).

Use of a Buffer Amplifier Stage:



Input signal

PZE transducer

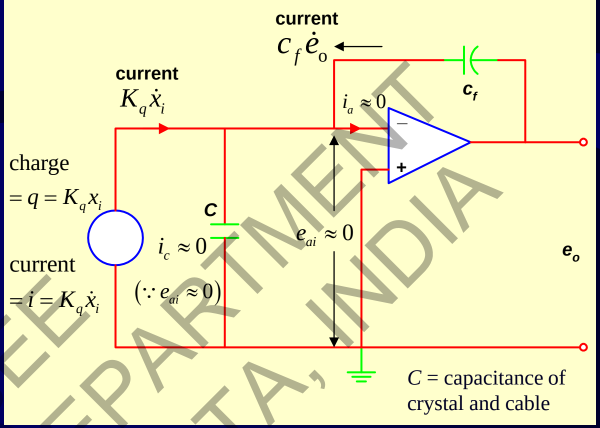
emitter follower

amplifier

Output signal

Charge Amplifier

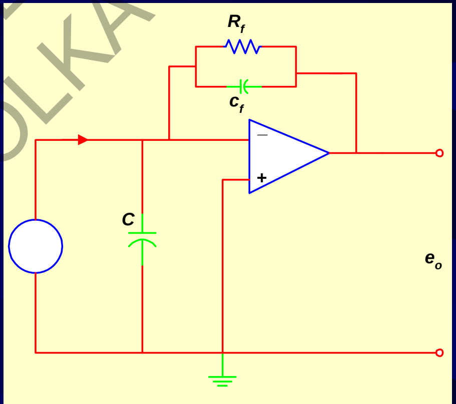
Fig (a) gives the ideal form:



Assumption: **input voltage e_{ai} and current i_a of the op-amp ≈ 0.**

$$K_q \frac{dx_i}{dt} = -C_f \frac{de_o}{dt} \Rightarrow e_o = -\frac{K_q x_i}{C_f}$$

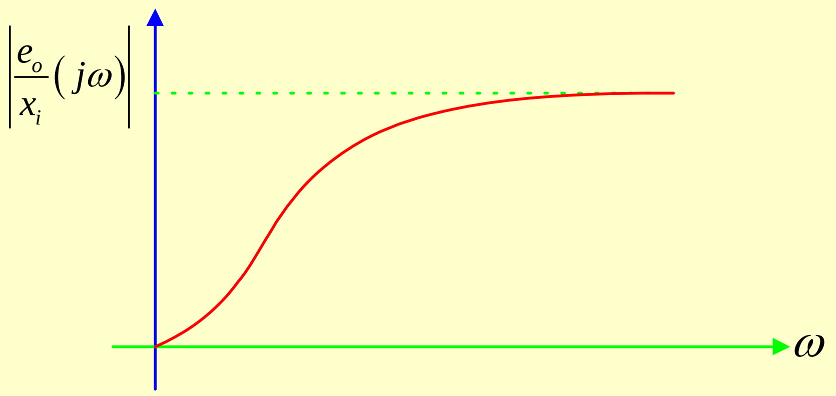
Fig (b) gives the practical ckt.:



$$\frac{E_o(s)}{X_i(s)} = -\frac{sK\tau}{1+s\tau} = -\frac{sK}{s+\frac{1}{\tau}},$$

$$K \triangleq \frac{K_q}{C_f} \text{ (V/m)}, \quad \tau \triangleq R_f C_f \text{ (sec)}$$

Amplitude response:



Charge Amplifier

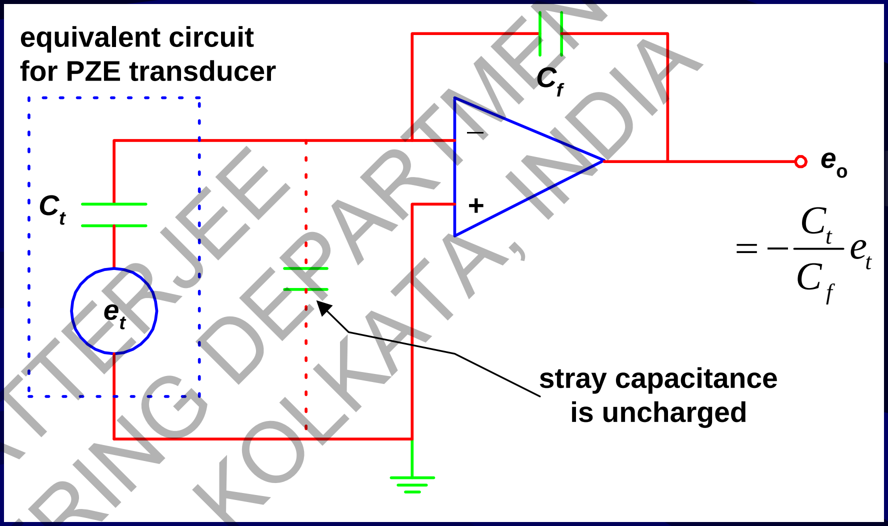
An Alternate Representation:

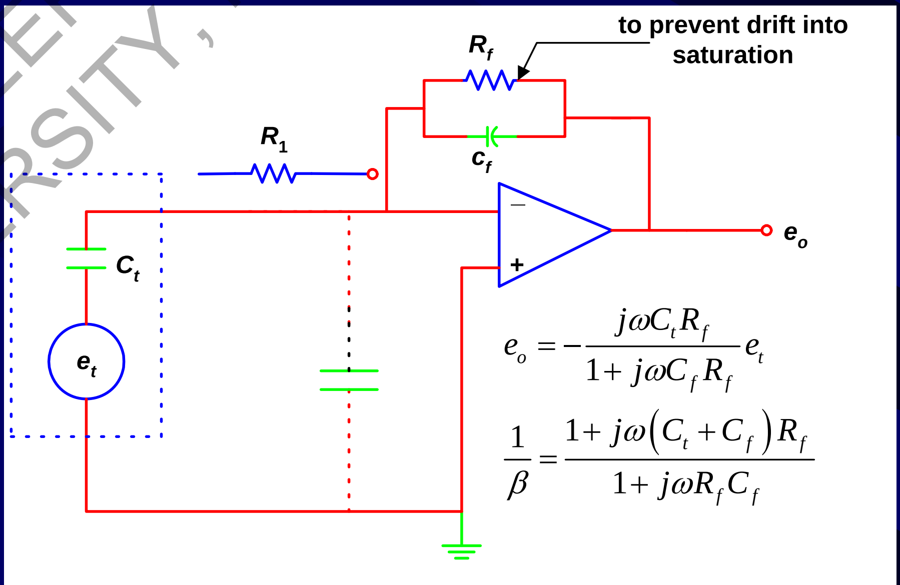
- ✓ Here, the output of the capacitive transducer is represented by an equivalent circuit, consisting of a voltage source e_t in series with a capacitance C_t.

The amplifier output: $e_o = -\left(\frac{C_t}{C_f}\right)e_t$

- ✓ The presence of R_f limits the lower bandwidth limit of the charge amplifier to the frequency:

$$f_L = \frac{1}{2\pi C_f R_f}$$





Thank You