OFF-LINE FIR DIGITAL FILTERS

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FIR digital filters for off-line analysis

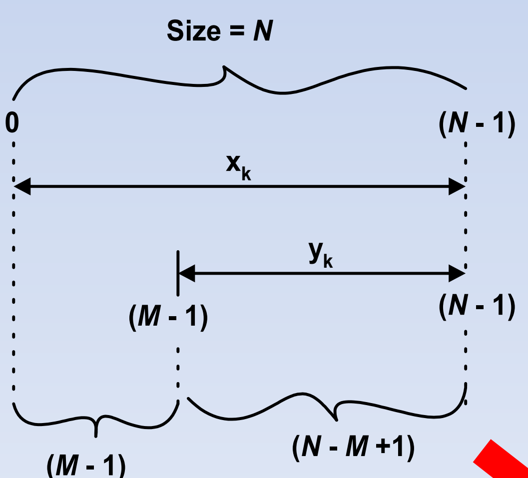
Let x_k, $k = 0,1,2,\ldots,(N\text{-}1)$ be the off-line input data having a size of *N*.

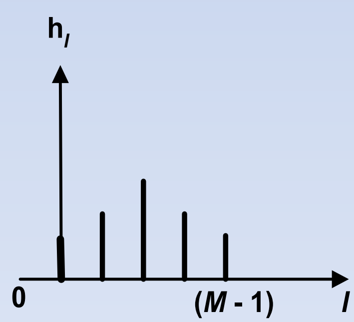
Let an *M*-tap FIR filter be employed to process input data sequence x_k.

Assuming *M* to be odd and *N* > *M*, the output filter sequence may be represented as:

$$y_k = \sum_{l=0}^{M-1} h_l x_{k-l} \quad \text{, for } k = (M\text{-}1),(M\text{-}1)+1,\ldots,(N\text{-}1) \qquad \ldots.(1)$$

where h_l, $l = 0,1,2,\ldots,(M\text{-}1)$ is the causal finite impulse sequence of the filter.





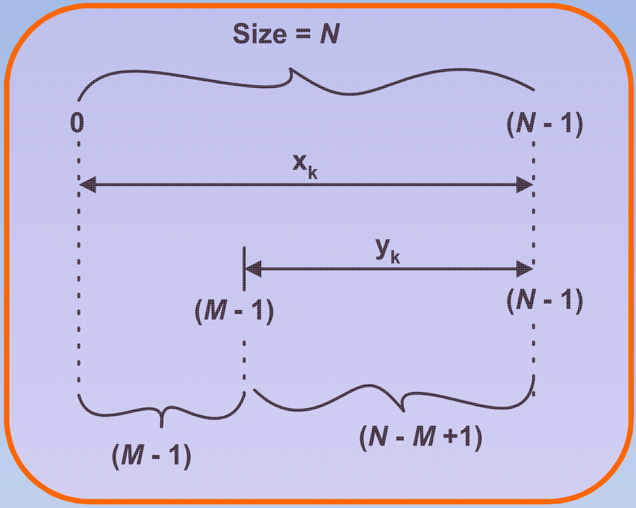
The length of output sequence is smaller than the length of input sequence.

Length of output sequence = $N - M + 1$

FIR digital filters for off-line analysis

Example:

Let $N = 10$ and $M = 3$.



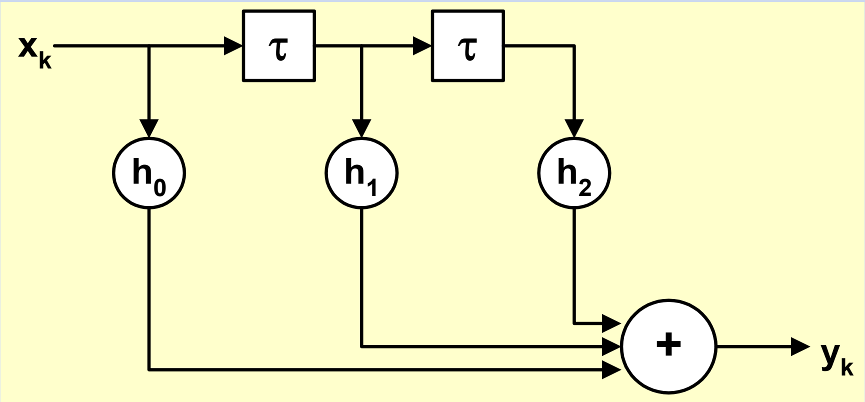
size = 10

$$x_0\, x_1\, x_2\, x_3\, x_4\, x_5\, x_6\, x_7\, x_8\, x_9$$

$$y_2\, y_3\, y_4\, y_5\, y_6\, y_7\, y_8\, y_9$$

3-1=2   10-3+1=8

Realization

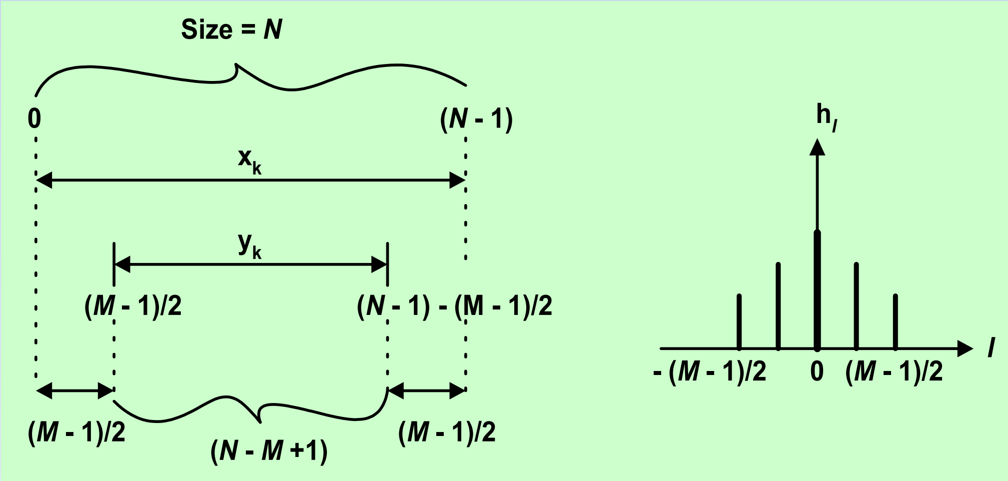


FIR digital filters for off-line analysis

To have a symmetrical separation of y_k w.r.t. x_k, relation (1) may be expressed for a non-causal FIR filters (as causality is not essential for implementation of off-line filters) as,

$$y_k = \sum_{l=-\frac{(M-1)}{2}}^{\frac{(M-1)}{2}} h_l x_{k-l}\text{, for } k = (M\text{-}1)/2, (M\text{-}1)/2+1, \ldots, (N\text{-}1)\text{-}(M\text{-}1)/2 \quad \ldots.(2)$$

where h_l, $l = -(M\text{-}1)/2,..,0,..,(M\text{-}1)/2$ is the non-causal finite impulse sequence of the filter.

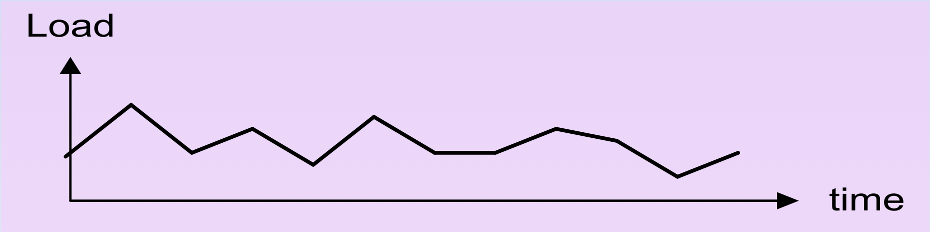


Here the spacing of y_k with respect to x_k is symmetrical.

FIR digital filters for off-line analysis

Examples of off-line data sequence

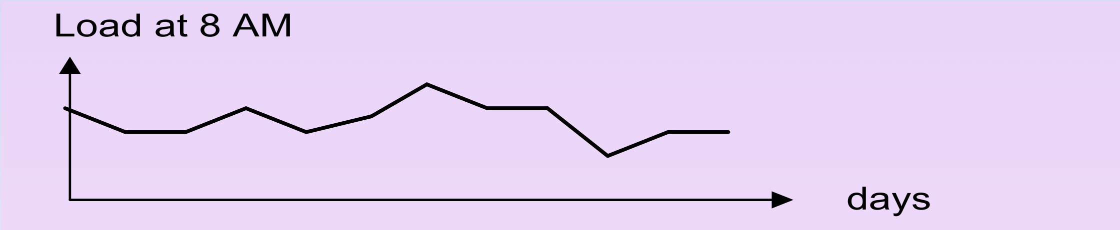
➢**Load variation of a particular place with respect to time of day.**



FIR digital filters for off-line analysis

Examples of off-line data sequence

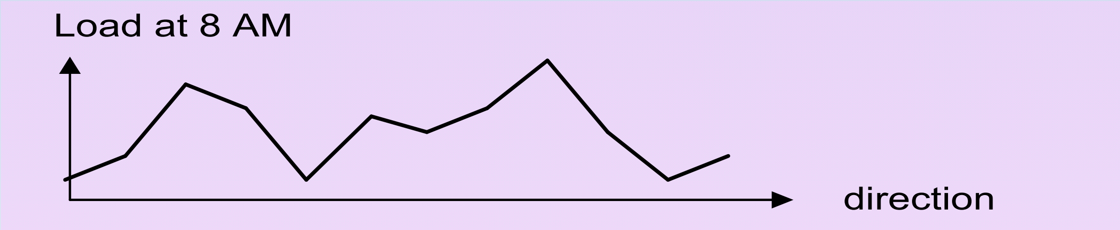
- Load variation of a particular place with respect to time of day.
- **Load variation of a particular place with respect to days at a particular time**.



FIR digital filters for off-line analysis

Examples of off-line data sequence

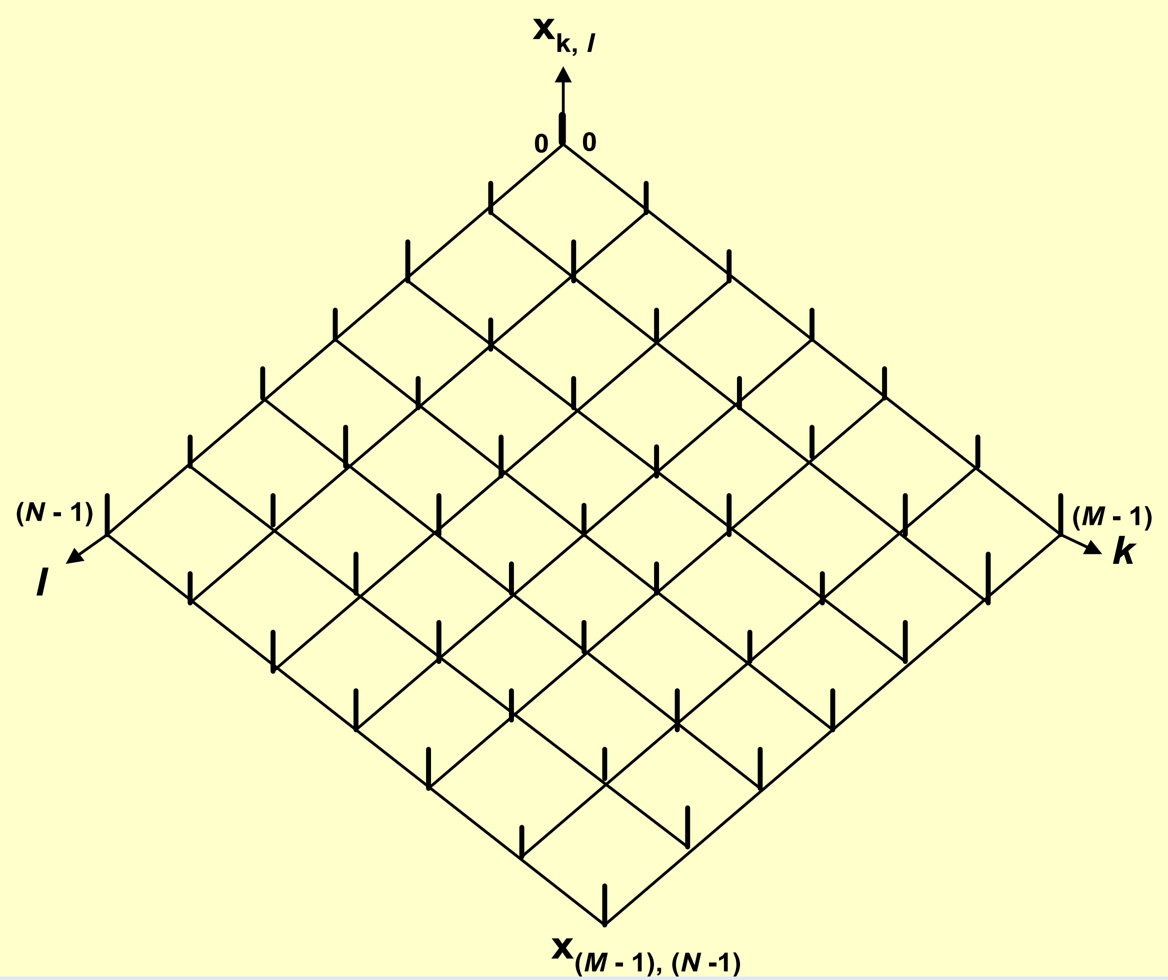
- Load variation of a particular place with respect to time of day.
- Load variation of a particular place with respect to days at a particular time.
- **Load variation along a particular direction in an area at a particular time.**



FIR digital filters for off-line analysis of 2-D data

Let the 2-dimensional data sequence of size ($M \times N$) be

$x_{k,l}$ $, k = 0,1,\ldots,(M\text{-}1),\ l = 0,1,\ldots,(N\text{-}1)$



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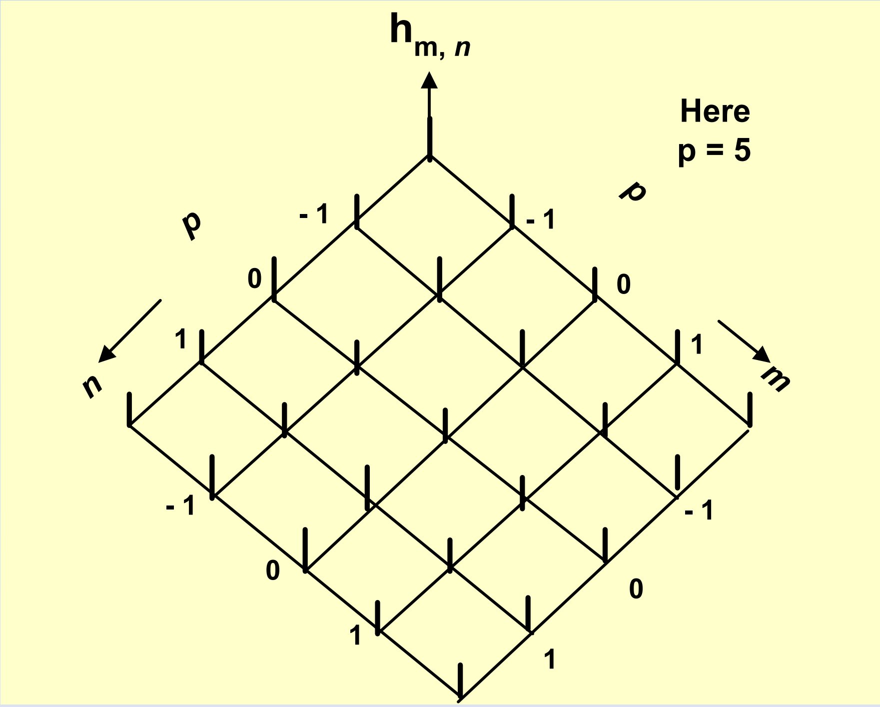
Let the 2-dimensional data sequence of size ($M \times N$) be

$$x_{k,l}, k = 0,1,\ldots,(M-1),\ l = 0,1,\ldots,(N-1)$$

Let a ($p \times p$) tap 2-dimensional FIR digital filter be employed to process $x_{k,l}$, assuming M, $N > p$ and p is odd.



2-D finite impulse sequence of FIR digital filter

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Let a ($p \times p$) tap 2-dimensional FIR digital filter be employed to process $x_{k,l}$, assuming M, $N > p$ and p is odd.

Then the filter output sequence may be expressed as (extending relation (2) for 2-dimensional)

$$y_{k,l} = \sum_{m=-(p-1)/2}^{(p-1)/2} \sum_{n=-(p-1)/2}^{(p-1)/2} h_{m,n}\, x_{k-m,l-n} \qquad \ldots.(3)$$

for $k = (p-1)/2,\ldots,(M-1)-((p-1)/2)$ and $l = (p-1)/2,\ldots,(N-1)-((p-1)/2)$

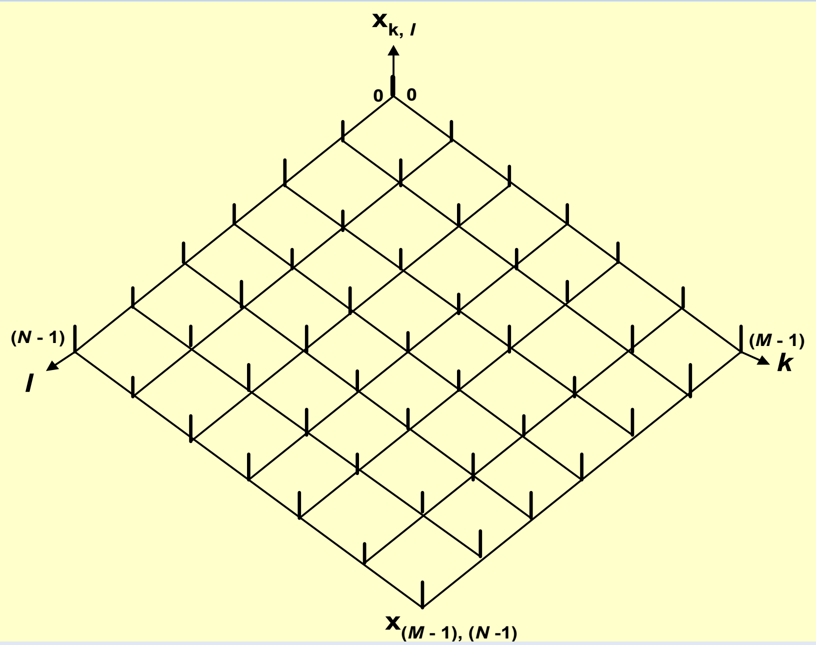
where $h_{m,n}$, $m = -(p-1)/2,\ldots,0,\ldots,(p-1)/2$ and $n = -(p-1)/2,\ldots,0,\ldots,(p-1)/2$ is the non-causal 2-dimensional finite impulse sequence of the 2-dimensional digital FIR filter. Relation (3) represents a ***2-dimensional convolution summation***.

FIR digital filters for off-line analysis of 2-D data

$$y_{k,l} = \sum_{m=-(p-1)/2}^{(p-1)/2} \sum_{n=-(p-1)/2}^{(p-1)/2} h_{m,n}\, x_{k-m,l-n} \quad \text{....(3)}$$

for $k = (p\text{-}1)/2, \ldots, (M\text{-}1)\text{-}((p\text{-}1)/2)$ and $l = (p\text{-}1)/2, \ldots, (N\text{-}1)\text{-}((p\text{-}1)/2)$

Relation (3) suggests that 2-dimensional filtering operation may be realized by performing 1-dimensional filtering operation *M* times along *k*-axis over *N*-data along *l*-axis.



FIR digital filters for off-line analysis of 2-D data

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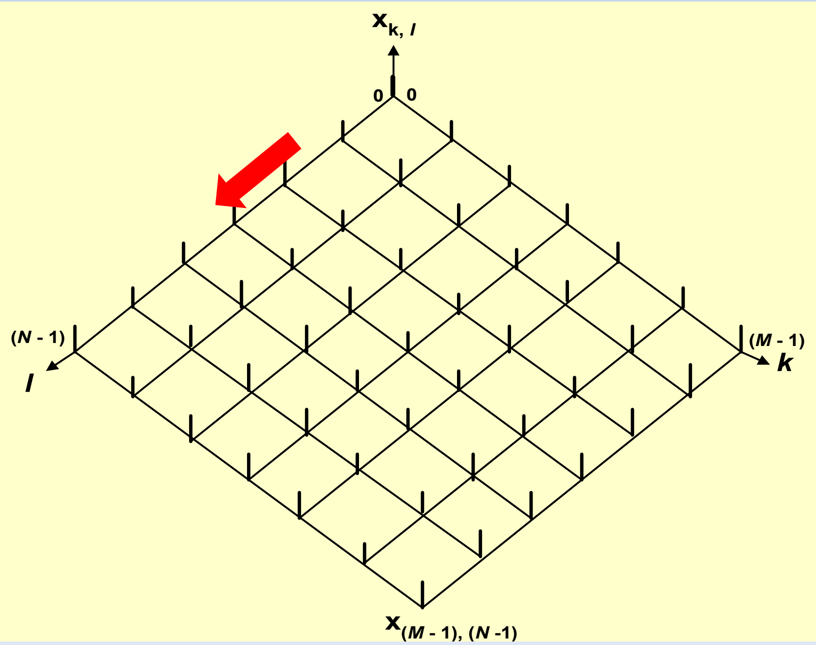

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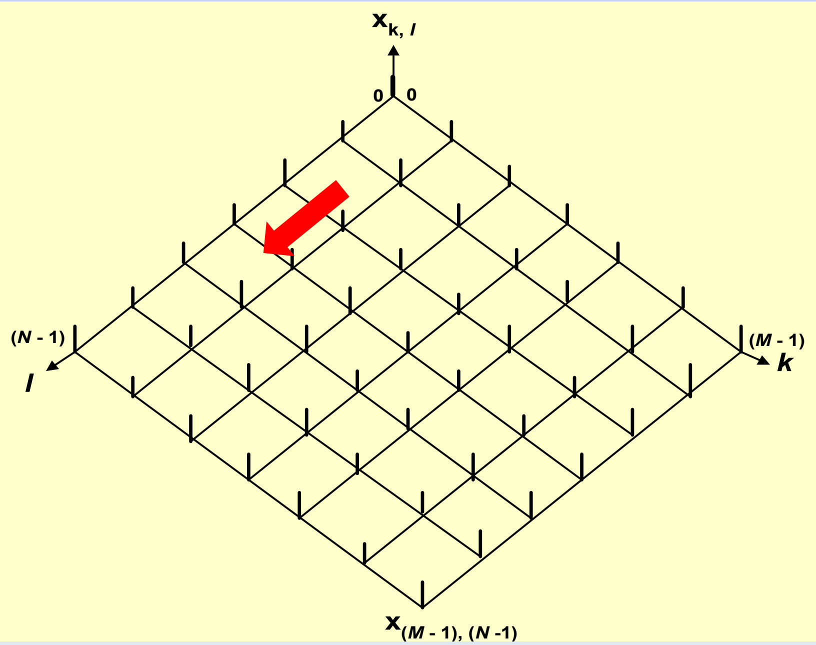

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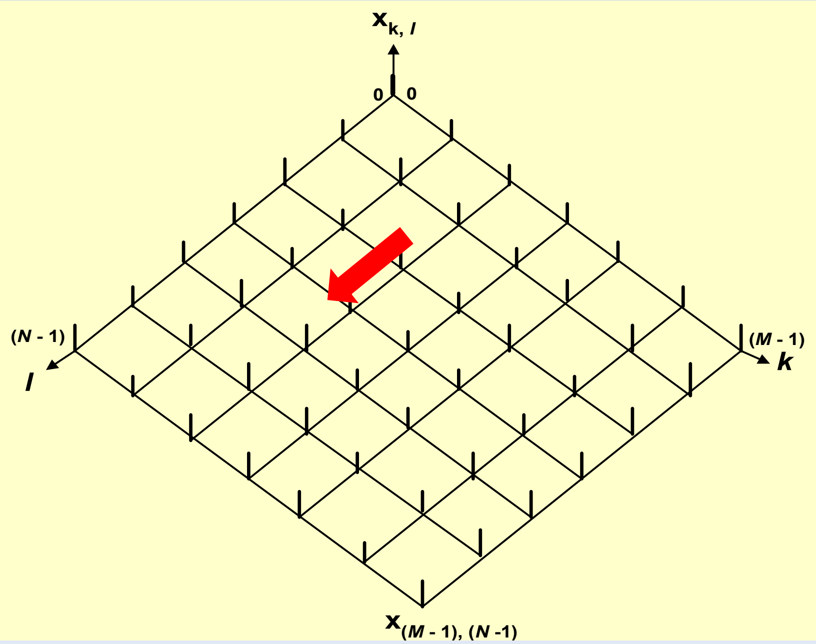

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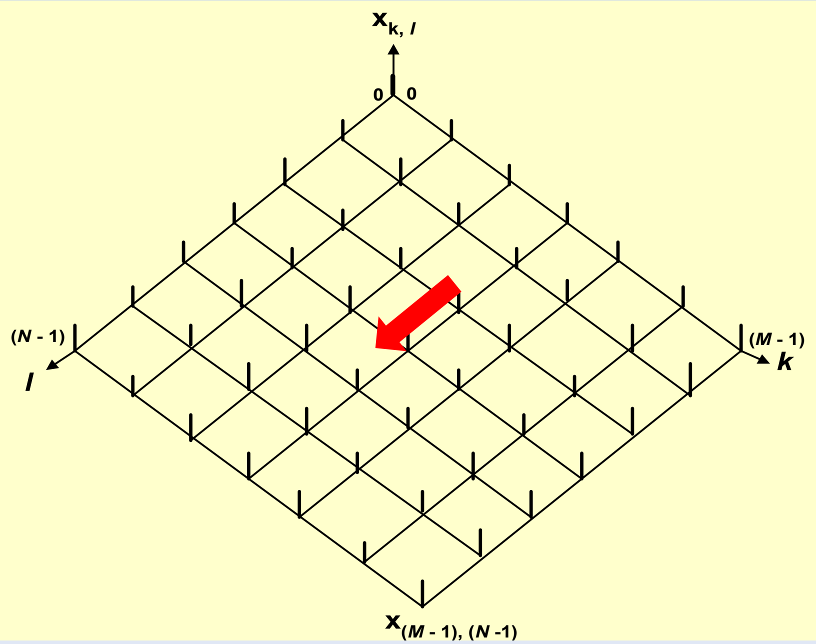

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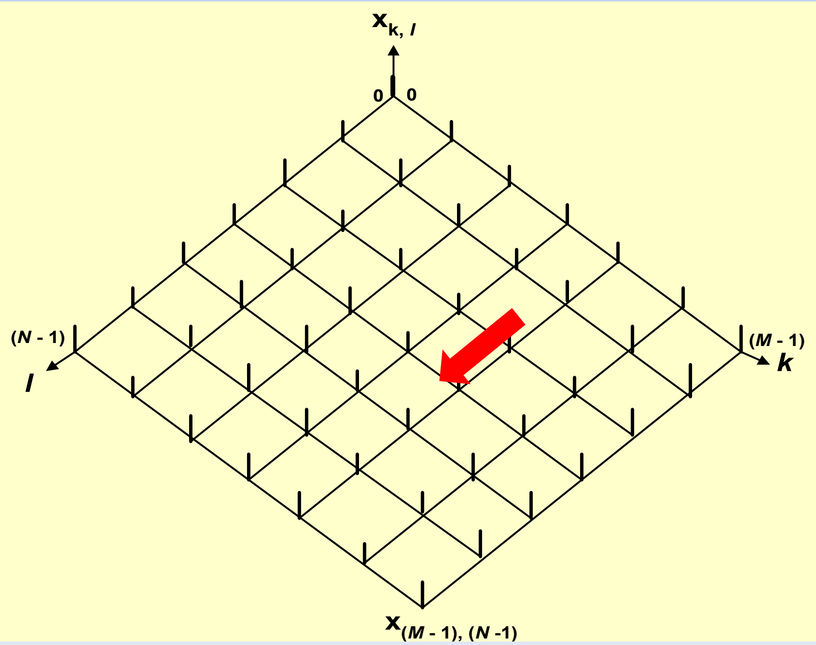

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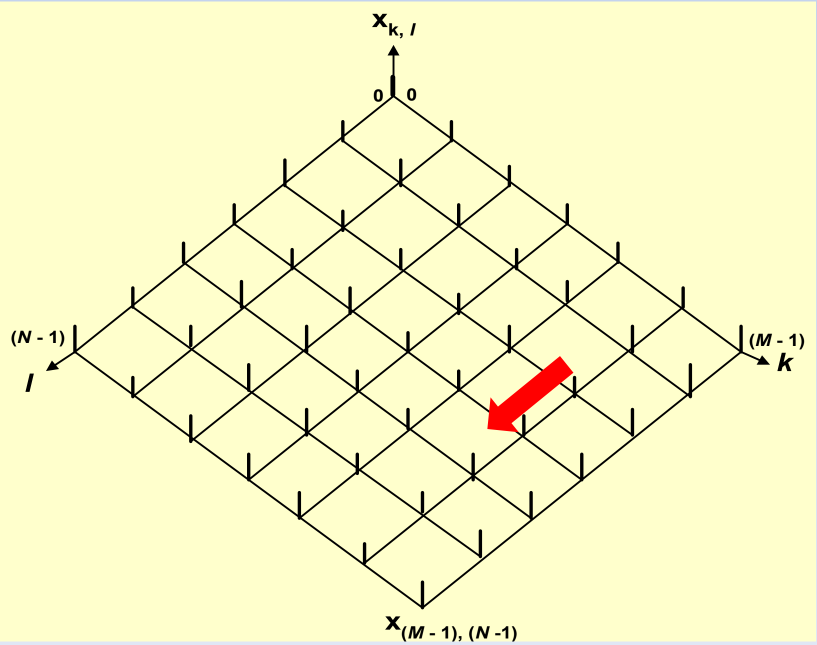

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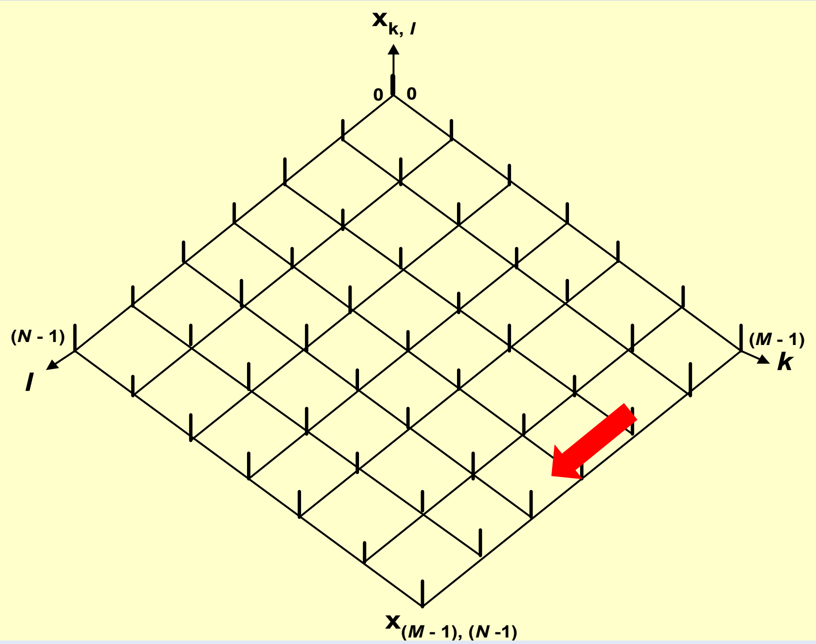
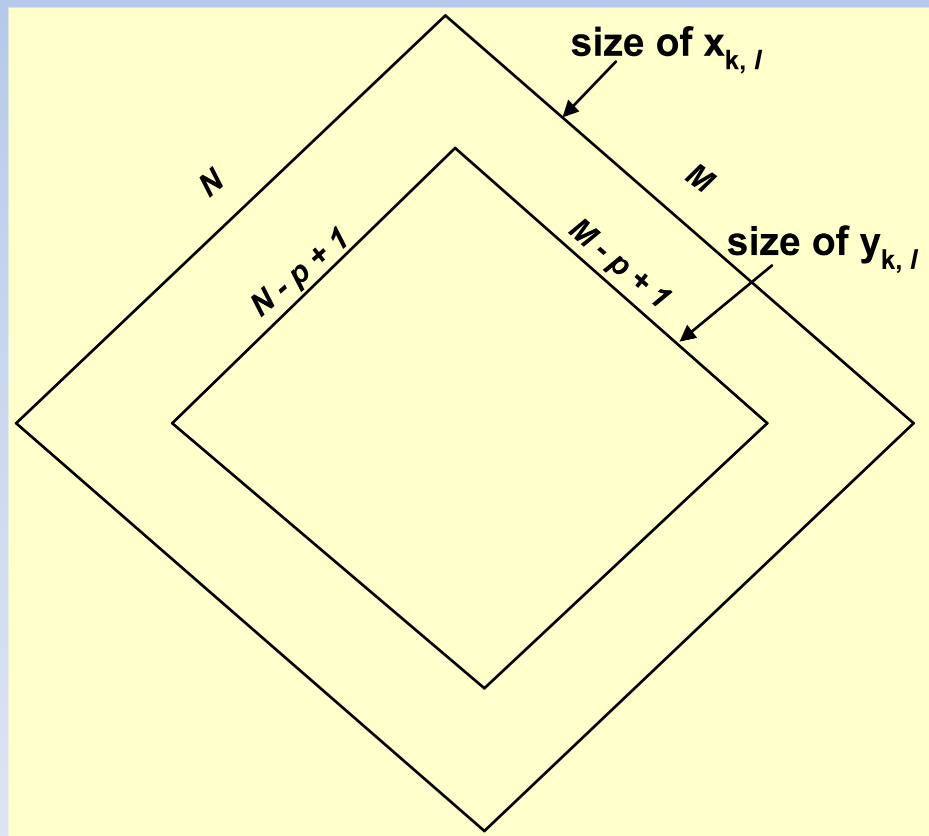


FIR digital filters for off-line analysis of 2-D data

Filter input-output space:



Thank You