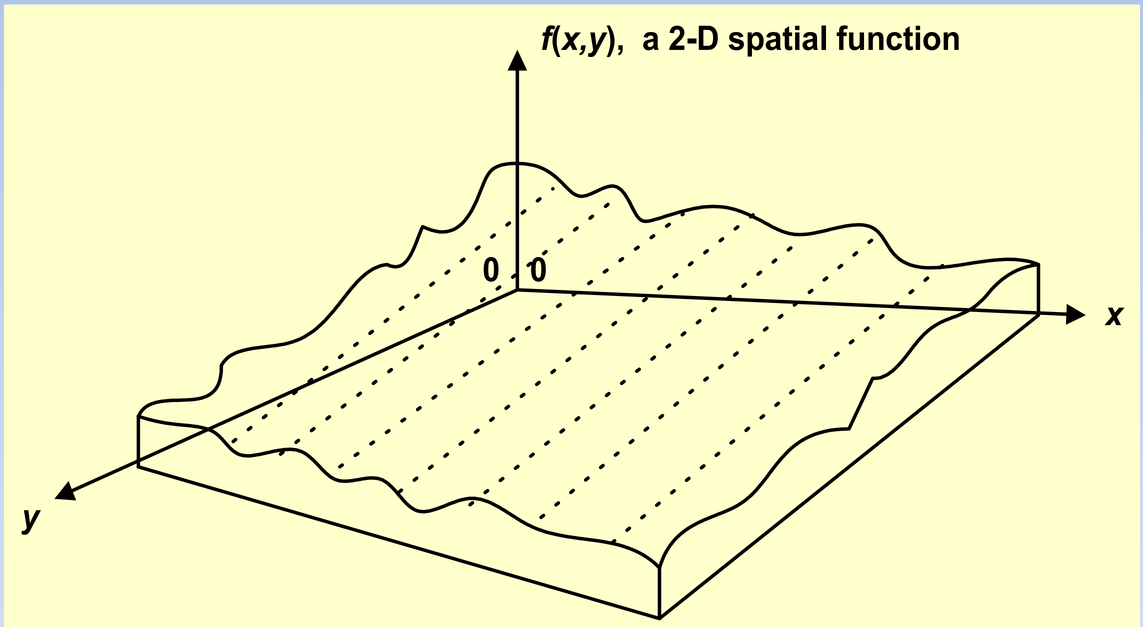
INTRODUCTION TO IMAGE PROCESSING

Prof. Anjan Rakshit and Prof. Amitava Chatterjee
Electrical Measurement and Instrumentation Laboratory,
Electrical Engineering Department,
Jadavpur University, Kolkata, India.

Black and White Image as a 2-D Continuous Function of Space

Let the gray level at a particular position [*x*,*y*] of an image be $f(x,y)$, a continuous function of position as shown below:



Black and White Image as a 2-D Continuous Function of Space

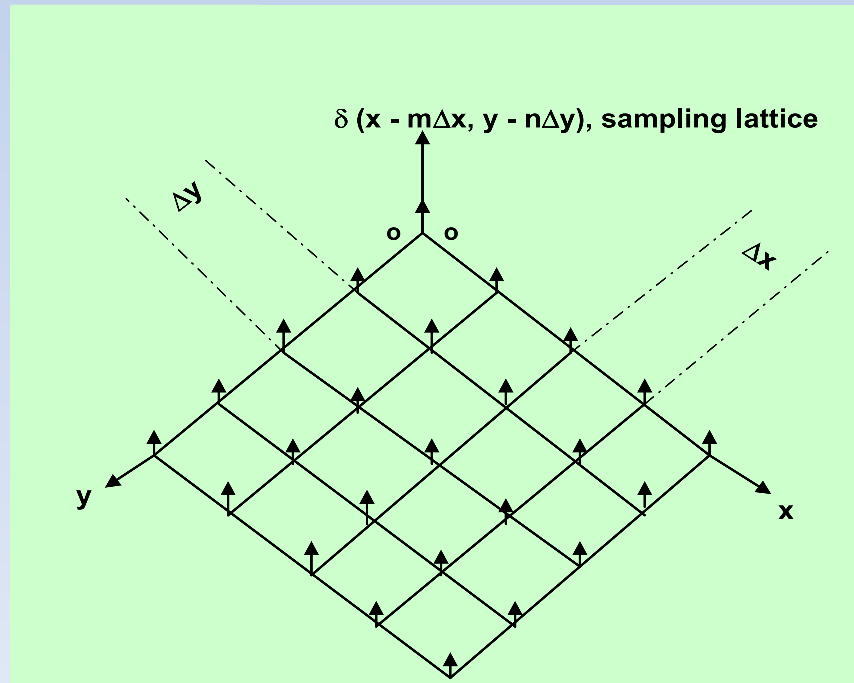
The digital or discrete version of the image $f(x,y)$ may be obtained as:

$$f^*(x,y) = f(x,y).s(x,y) \qquad \text{....(1)}$$

where $s(x,y)$ is a 2-dimensional sampling function (sampling lattice) defined as,

$$s(x,y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta(x - m\Delta x, y - n\Delta y) \qquad \text{....(2)}$$

where Δx and Δy are the spacing of the samples along x-axis and y-axis respectively.



Black and White Image as a 2-D Continuous Function of Space

$$f^*(x,y) = f(x,y).s(x,y)$$

$$s(x,y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta(x - m\Delta x, y - n\Delta y)$$

Thus, $f^*(x,y) = f(x,y) \sum_{m=-\alpha}^{\alpha} \sum_{n=-\alpha}^{\alpha} \delta(x - m\Delta x, y - n\Delta y)$

$$= \sum_{m=-\alpha}^{\alpha} \sum_{n=-\alpha}^{\alpha} f(x,y)\delta(x - m\Delta x, y - n\Delta y)$$

$$= \sum_{m=-\alpha}^{\alpha} \sum_{n=-\alpha}^{\alpha} f_{m,n}\delta(x - m\Delta x, y - n\Delta y) \qquad \text{....(3)}$$

where $f_{m,n}$ is a 2-D sampled sequence m = 0,±1, ±2,.., n = 0, ±1, ±2,..

Black and White Image as a 2-D Continuous Function of Space

For a finite size image let the size be $M \times N$ pixels (pixel stands for picture elements or samples) then $f_{m,n}$ becomes a 2-dimensional finite sequence with m = 0,1,2,..,(M-1) and n = 0,1,2,..,(N-1).

The image may be expressed as x_{max} x y_{max} where $x_{max} = (M-1)\Delta x$ and $y_{max} = (N-1)\Delta y$.

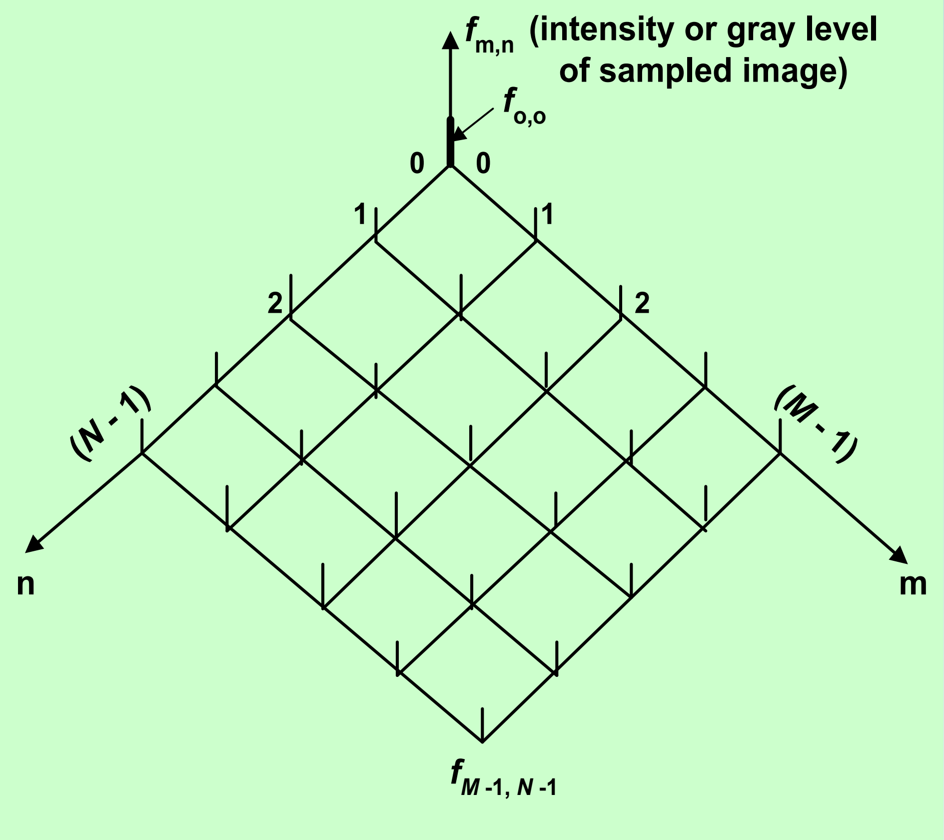


Image Filtering – A Two-Dimensional Filtering Problem

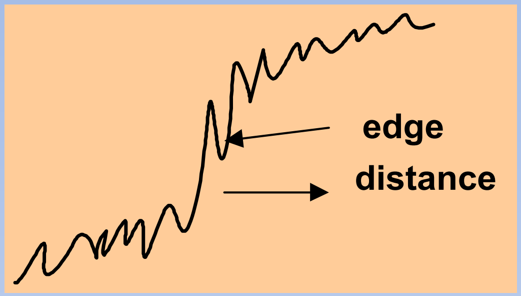
Image filtering may be done in spatial domain or in frequency domain (considering a 2-D Fourier transform of the image).

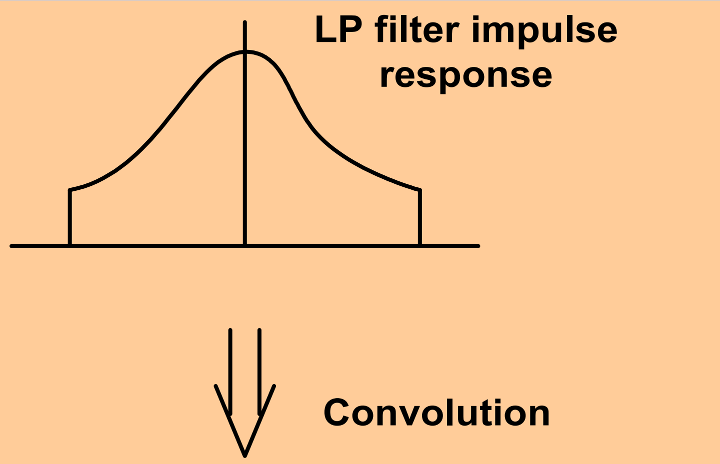
In spatial domain filtering, 2-dimensional convolution operation may be performed between the 2-dimensional image sequence and a finite tap 2-dimensional FIR filter impulse sequence.

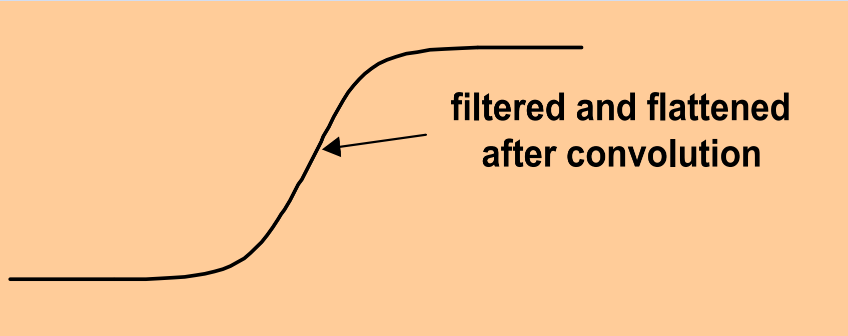
Type of Filters:

- **Low-pass**, to remove high frequency noise.
- **Band-pass**, to enhance some spatial frequency range.
- **High-pass**, to sharpen the image by enhancing high spatial frequencies.

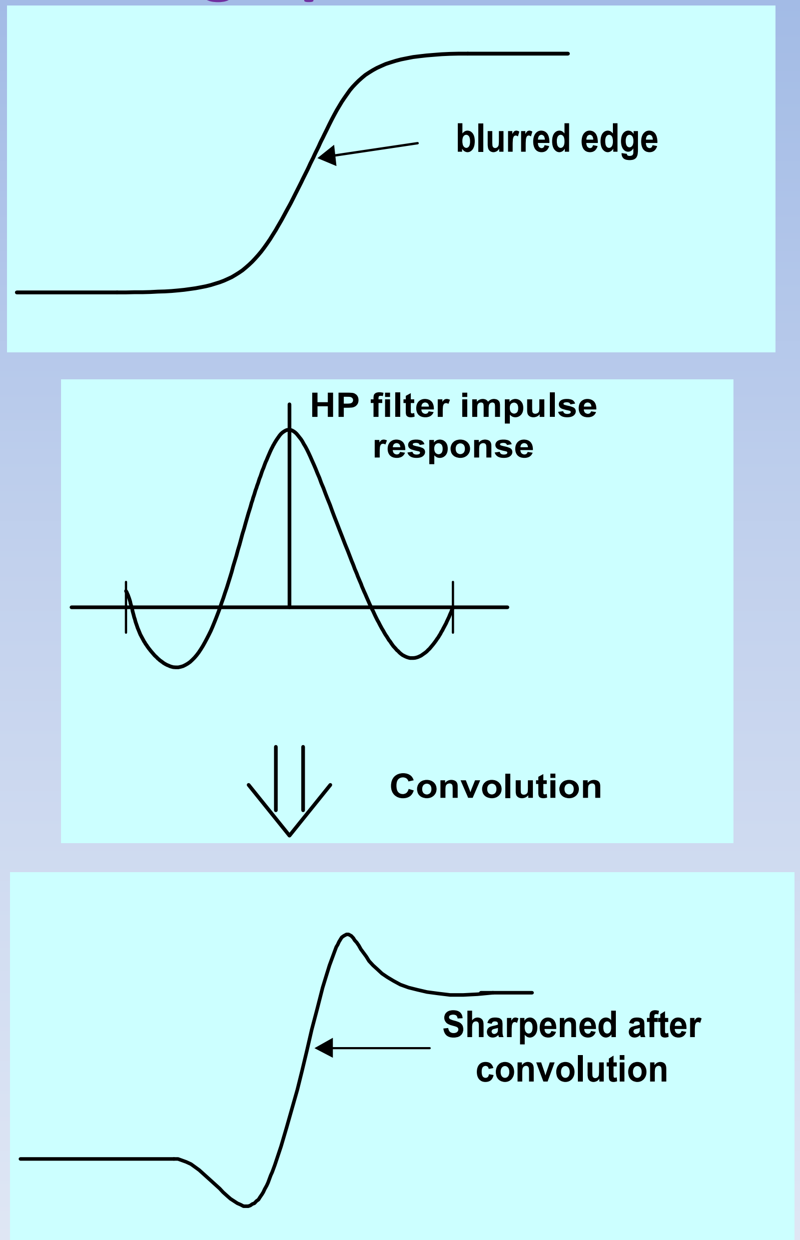
Low-pass Filter







High-pass Filter



FIR Image Filters

Let a ($p \times p$) tap FIR filter be employed to filter the 2-dimensional pixel sequence $f_{m,n}$, $m = 0,1,2,..,(M\text{-}1)$, $n = 0,1,2,..,(N\text{-}1)$.

If $h_{k,l}$, $k,l = -(p\text{-}1)/2,..,0,..,(p\text{-}1)/2$, be the 2-dimensional impulse sequence of the filter then the filtered output pixel sequence may be represented as (local or neighborhood operation):

$$f'_{m,n} = \sum_{k=-\frac{p-1}{2}}^{\frac{p-1}{2}} \sum_{l=-\frac{p-1}{2}}^{\frac{p-1}{2}} h_{k,l} \; f_{m-k,n-l} \qquad ....(4)$$

for $m = (p\text{-}1)/2,\ldots,(M\text{-}1)\text{-}((p\text{-}1)/2)$ and $n = (p\text{-}1)/2,\ldots,(N\text{-}1)\text{-}((p\text{-}1)/2)$

The finite 2-dimensional sequence $h_{k,l}$ is called the **convolution mask**.

FIR Image Filters

Low-pass (3×3) masks

$$\frac{1}{6}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad \text{averaging mask}$$

$$\frac{1}{10}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

FIR Image Filters

Low-pass (3×3) masks

Scale factor →

$$\frac{1}{6}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad \text{averaging mask}$$

$$\frac{1}{10}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

FIR Image Filters

High-pass (3×3) masks

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 9 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ -2 & 5 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

Low-pass FIR Image Filter

Original image

$$\frac{1}{6}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Convolution mask

Filtered image

Low-pass FIR Image Filter

Noisy image

$$\frac{1}{6}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Convolution mask

Filtered image

High-pass FIR Image Filter

Original image

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

Convolution mask

Filtered image

Contrast Enhancement by Histogram Equalization

Let the variable '*a*' represent the gray level of the pixels in the image to be enhanced.

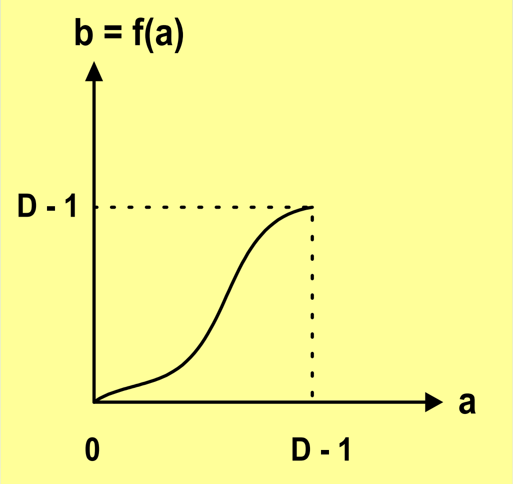
Let the gray level be quantized from 0,1,2,…,D – 1, where D is the maximum number of gray levels (e.g. for 8-bit quantization, D = 256).

Thus the range of *a* is $0 \leq a \leq D-1$.

Let each pixel of the input image be modified by the following point operation:

$$b(x,y) = f[a(x,y)] \quad \text{….(5)}$$

where $b(x,y)$ is the output pixel level at (x,y),
$a(x,y)$ is the input pixel level at (x,y) and
$f[a(x,y)]$ is the gray scale transformation function.



Relation (5) may be expressed as

$$b = f(a) \quad \text{….(6)}$$

Contrast Enhancement by Histogram Equalization

The histogram of an image is

H_u = number of pixels with gray level *'u'* for $0 \leq u \leq (D-1)$ ....(7)

The area of the image (in term of pixels) may be expressed as

$$A = \sum_{u=0}^{D-1} H_u \quad ....(8)$$

The cumulative histogram is defined as

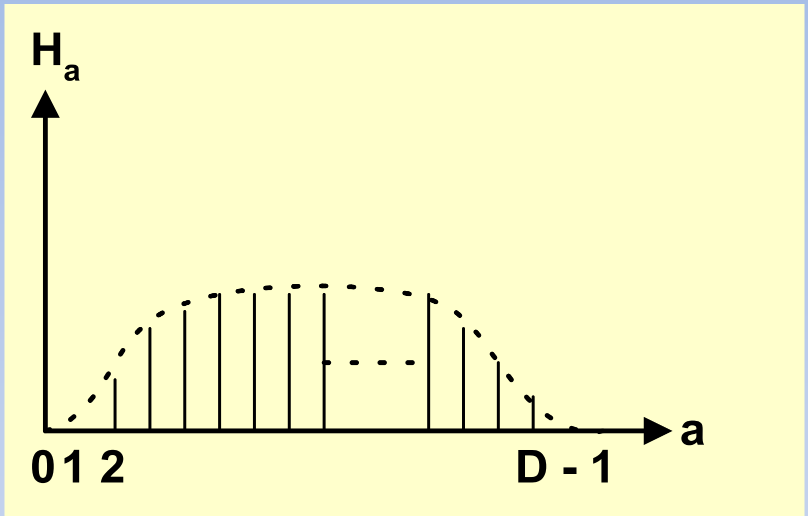
$$A_p = \sum_{u=0}^{p} H_u$$, the area enclosing gray levels from 0 to '*p*' ....(9)

Now, the histogram of the input image is H_a for $0 \leq a \leq (D-1)$.

And the area of the input image is

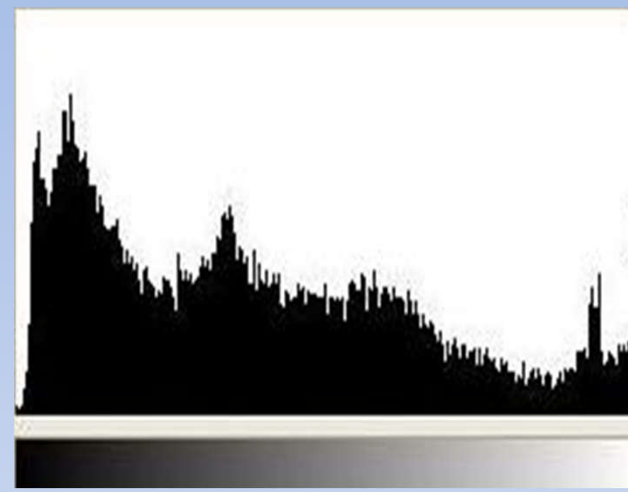
$$A = \sum_{a=0}^{D-1} H_a$$

Histogram of the Input Image



Histogram of normal input image

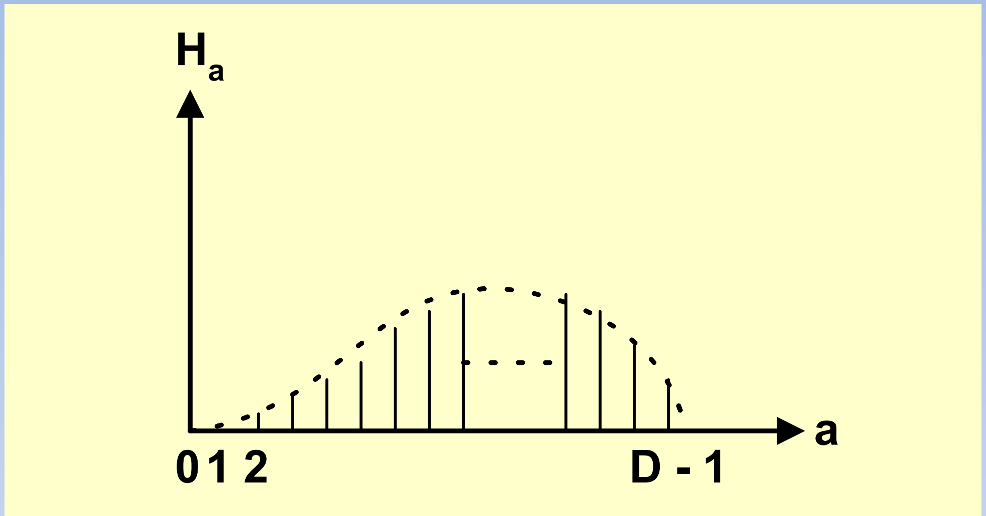
Histogram of the Input Image: An Example



Normal input image

Histogram

Histogram of the Input Image



Histogram of over exposed input image

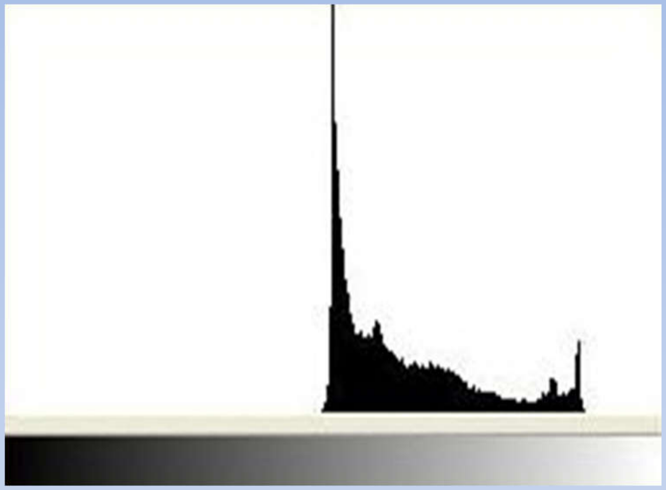
Histogram of the Input Image: An Example

Low contrast

Over exposed input image

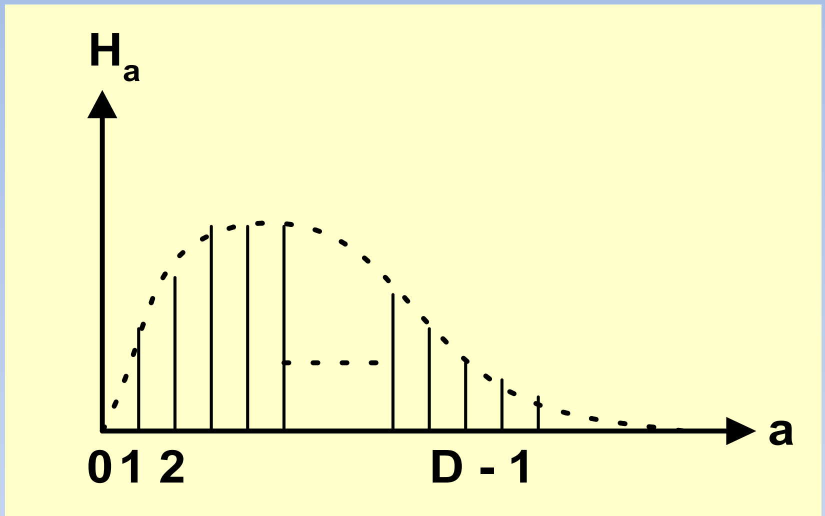
Histogram of the Input Image: An Example

Over exposed input image



Histogram

Histogram of the Input Image



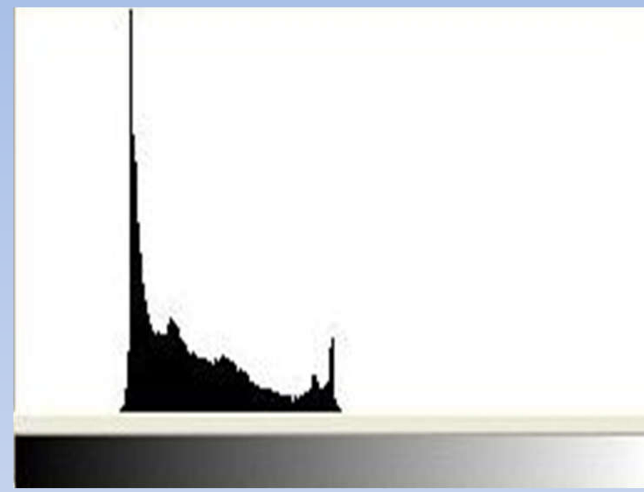
Histogram of under exposed input image

Histogram of the Input Image: An Example

Low contrast

Under exposed input image

Histogram of the Input Image: An Example

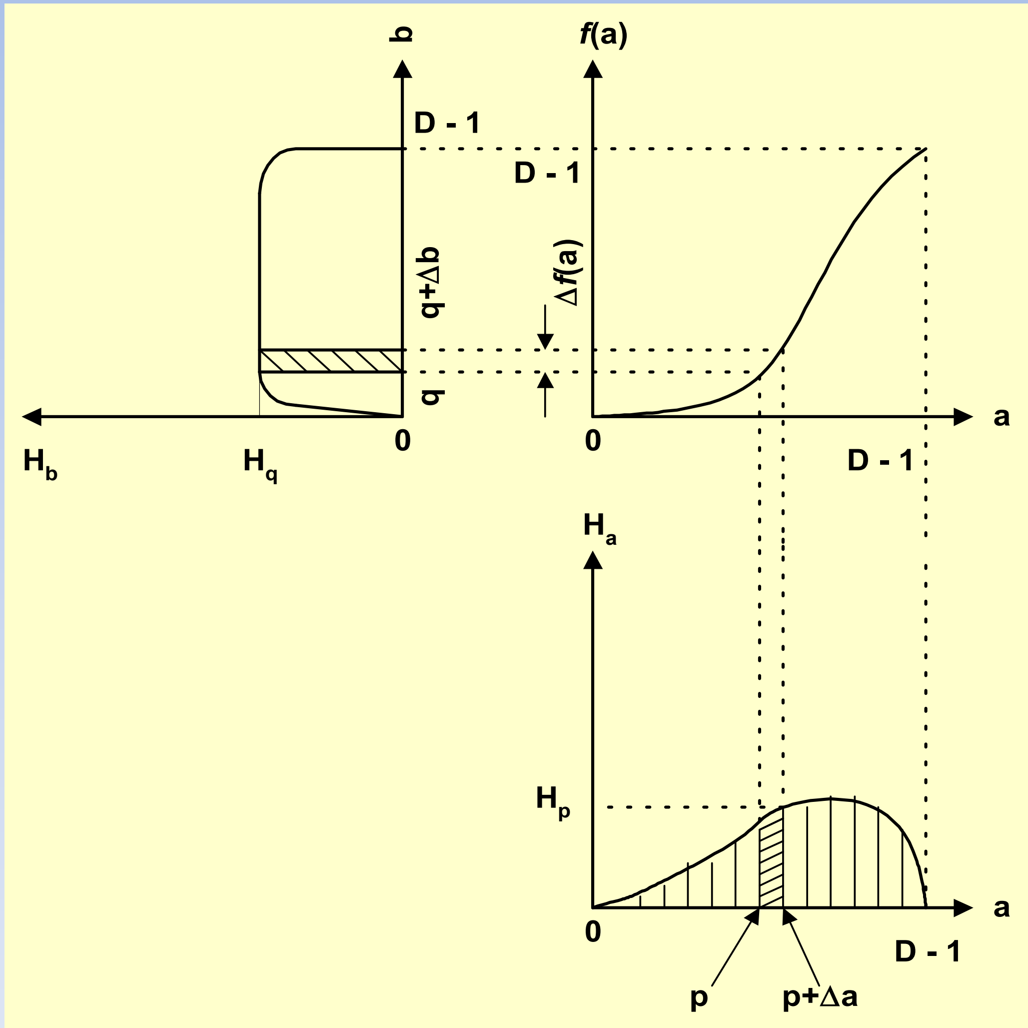


Under exposed input image

Histogram

Contrast Enhancement by Histogram Equalization

The input and output histograms may be related as



The number of output pixels having gray level between *q* and *q*+Δ*b* equals the number of input pixels with gray level between *p* and *p* + Δ*a* as transformation is performed on point basis.

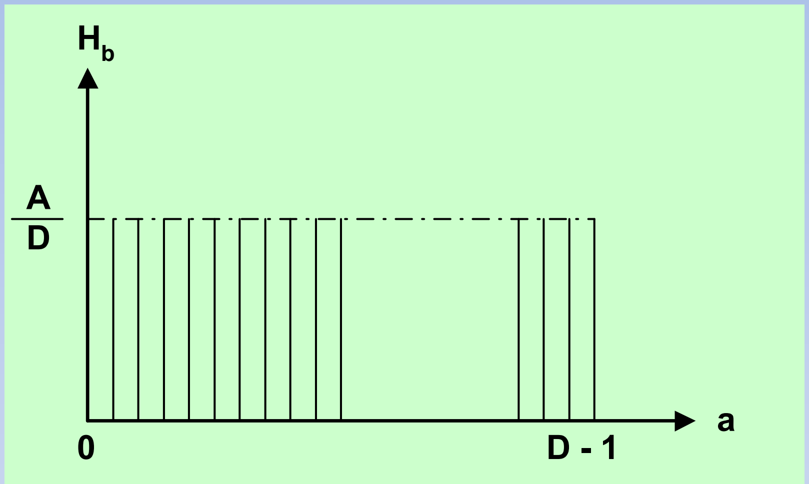
Thus,

$$\Delta a H p = \Delta b H q \quad \text{....(10)}$$

if Δ*a* and Δ*b* are small.

Contrast Enhancement by Histogram Equalization

To improve contrast, the histogram of the output image may be assigned a flat shape as



Here,

$$\sum_{b=0}^{D-1} H_b = \sum_{b=0}^{D-1} A/D = \frac{A}{D}.D = A,$$ the area of the image.

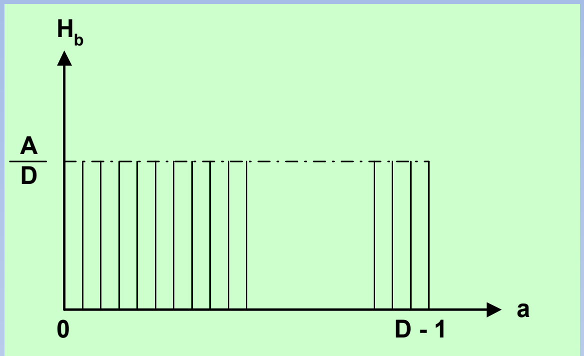
Now from relation (10)

$$\Delta a Hp = \Delta b Hq \quad \text{....(10)}$$

$$\frac{\Delta b}{\Delta a} = \frac{H_p}{H_q}$$

Contrast Enhancement by Histogram Equalization

$$\frac{\Delta b}{\Delta a} = \frac{H_p}{H_q}$$



Now, $\Delta b = \Delta f(a)$

Thus,

$$\frac{\Delta f(a)}{\Delta(a)} = \frac{H_p}{\left(\frac{A}{D}\right)},$$ as $H_q = A/D$, for flat histogram

Contrast Enhancement by Histogram Equalization

$$\frac{\Delta b}{\Delta a} = \frac{H_p}{H_q}$$

Now, $\Delta b = \Delta f(a)$

Thus,

$$\frac{\Delta f(a)}{\Delta(a)} = \frac{H_p}{\left(\frac{A}{D}\right)},$$ as $H_q = A/D$, for flat histogram

or, $$\Delta f(a) = \left(\frac{D}{A}\right) H_p . \Delta(a) \qquad \text{....(11)}$$

Contrast Enhancement by Histogram Equalization

$$\Delta f\left(a\right)=\left(\frac{D}{A}\right)H_p.\Delta\left(a\right) \qquad \text{….(11)}$$

Now, $\Delta a = a - (a-1)$, considering quantization of gray level.

$= 1$

And, $\Delta f(a) = f(a) - f(a-1)$

Then, $f(a) - f(a-1) = (D/A)H_p$

Now, summing from 0 to a,

$$\sum_{u=0}^{a} f\left(u\right)-f\left(u-1\right)=\frac{D}{A}\sum_{u=0}^{a} H_u$$

or, $$f\left(a\right)-f\left(-1\right)=\frac{D}{A}A_a$$

Contrast Enhancement by Histogram Equalization

$$f\left(a\right)-f\left(-1\right)=\frac{D}{A}A_a$$

Now, $f(-1) = 0$ and A_a is the cumulative histogram.

Thus,

$$f\left(a\right)=\frac{D}{A}A_a \qquad \text{….(12)}$$

Thus from relation (12), the output pixel gray level of a point (x,y) $b = f(a)$, may be obtained by calculating A_a of the input histogram for the input gray level '*a*' of the point (x,y) of the input image.

Contrast Enhancement by Histogram Equalization

Original image

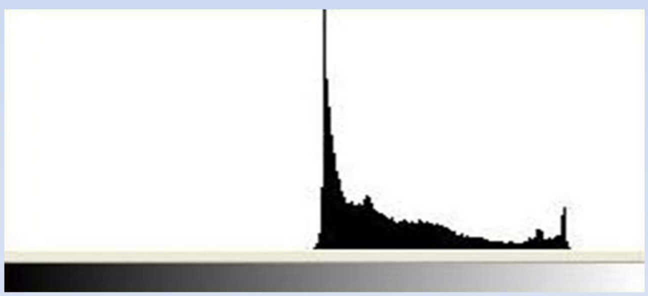
Histogram equalized

Histogram

Histogram

Contrast Enhancement by Histogram Equalization

Original image



Histogram

Histogram

References

Reference: **Digital Image Processing** by Gonzalez & Woods.

Thank You