

# **An Introduction to Image Enhancement Techniques**

by

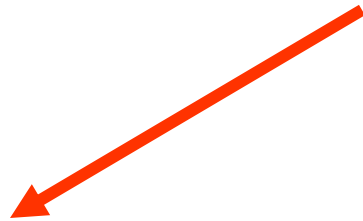
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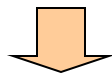
# Image Enhancement Techniques

- ✓ The principal objective of image enhancement is to process an image so that the result is more suitable than the original image for a particular application.

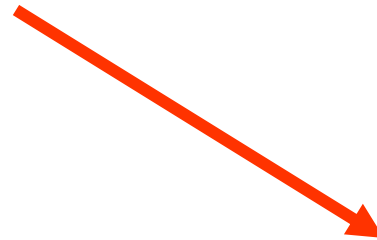
## Image Enhancement Techniques



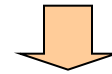
### Spatial Domain Approach



Spatial domain refers to the image plane itself and involves direct manipulation of the pixels of an image



### Frequency Domain Approach



Frequency domain processing techniques are based on modifying the Fourier Transform of an image

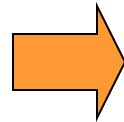
# Image Enhancement Techniques

## *Spatial Domain Approaches*

**Image Processing functions in the spatial domain:**



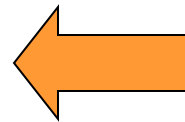
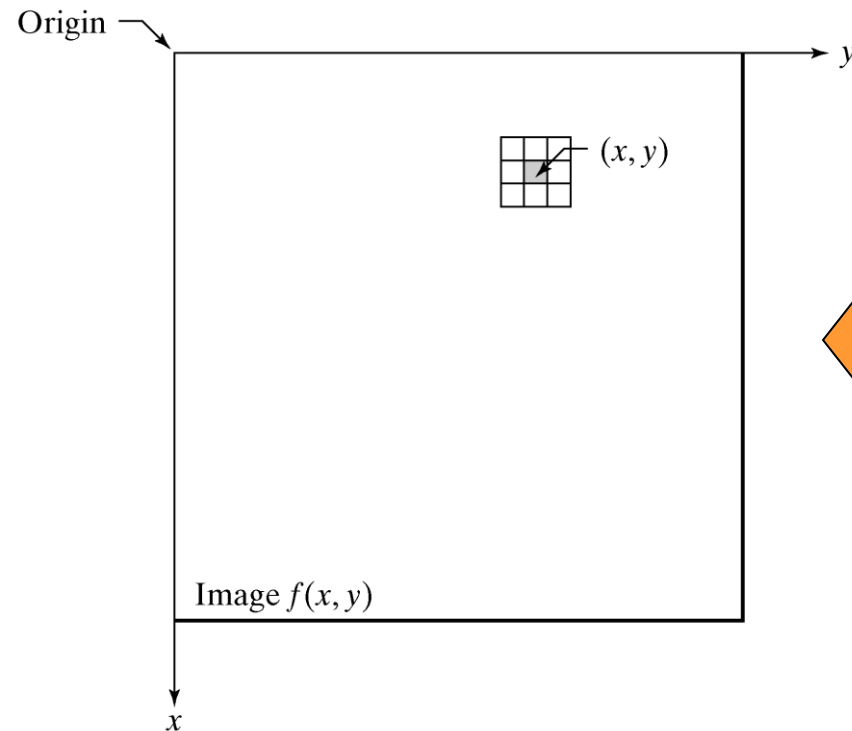
$$g(x, y) = T[f(x, y)]$$



$f(x, y)$ : The input image

$g(x, y)$ : The processed output image

$T$ : an operation on  $f$  defined over some neighborhood of  $(x, y)$



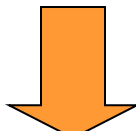
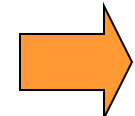
**A  $3 \times 3$  neighborhood about a point  $(x, y)$  in an image.**

✓ **Note:** The smallest neighborhood size is  $1 \times 1$ .

# Image Enhancement Techniques

## *Spatial Domain Approaches*

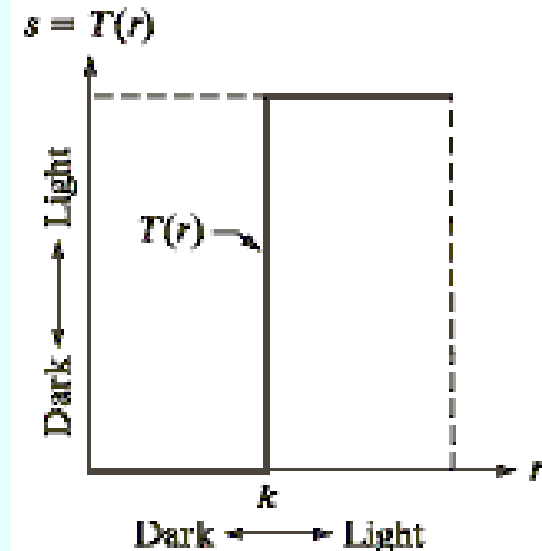
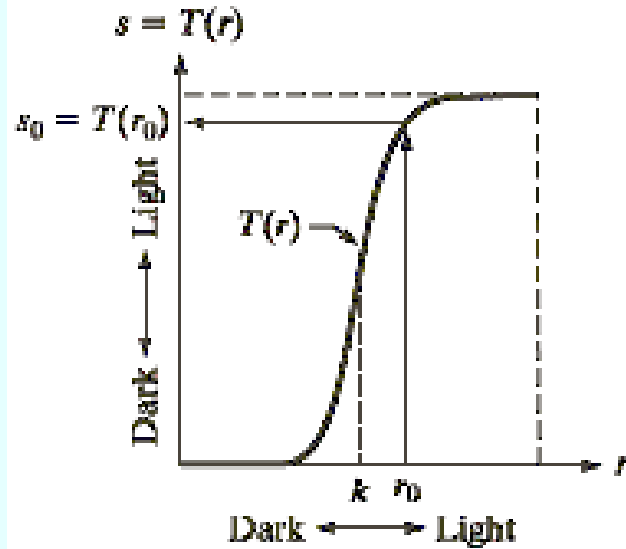
For a neighborhood size of  $1 \times 1$  :

  
$$s = T(r)$$


$r$ : gray level of  $f(x,y)$  at  $(x,y)$

$s$ : gray level of  $g(x,y)$  at  $(x,y)$

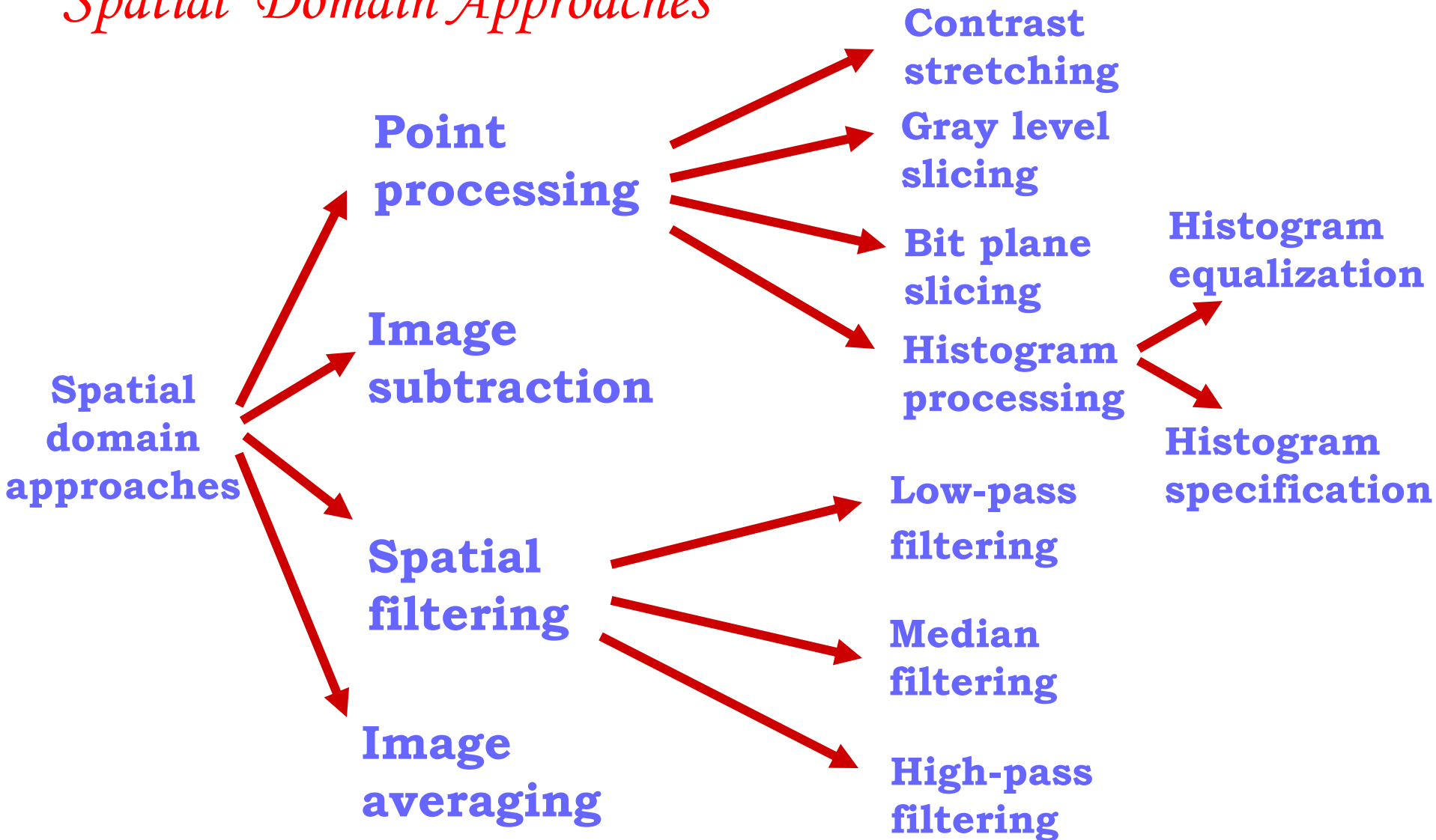
$T$ : a gray level transformation function



**Gray level transformation functions  
for Contrast Enhancement.**

# Image Enhancement Techniques

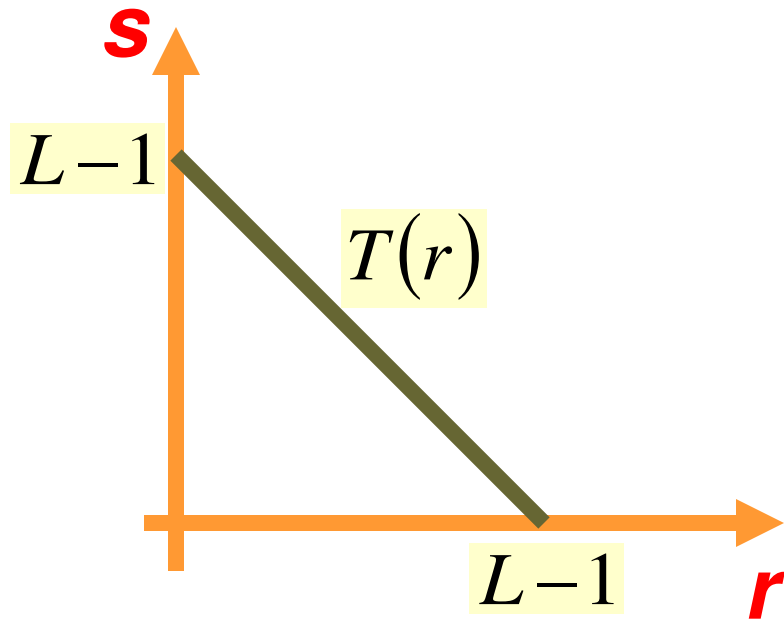
## *Spatial Domain Approaches*



# Spatial Domain Techniques

## *Point Processing Techniques*

### Negative of an Image



$s = L - 1 - r$ , for gray levels  
in the range  $[0, L - 1]$



Original **Pepper** Image



Negative of the **Pepper** Image

# Spatial Domain Techniques

## *Point Processing Techniques*

### Contrast Stretching

- ✓ **The possible causes of a low contrast image are:**
  - **Poor illumination**
  - **Lack of dynamic range in the imaging sensor**
  - **Wrong setting of the lens aperture during image acquisition**

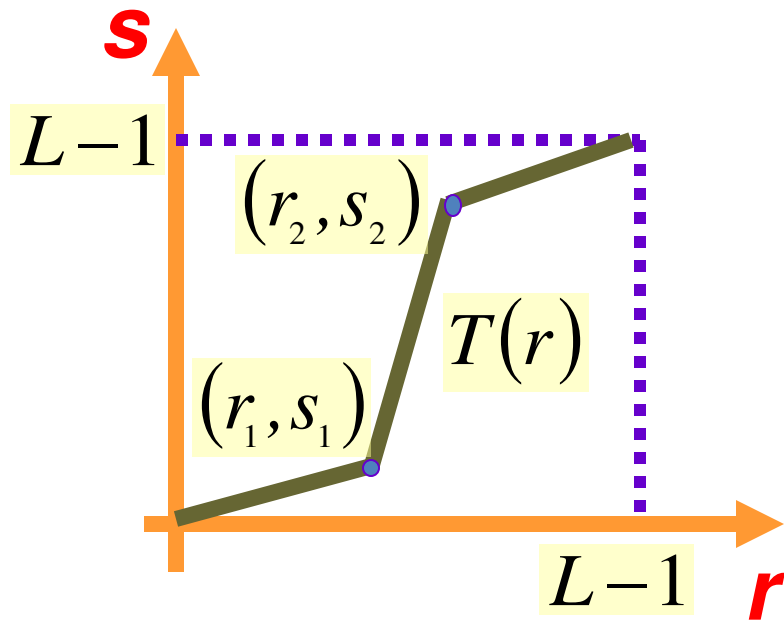
### *Solution ??*

- ✓ **Contrast stretching attempts to **increase the dynamic range** of the gray levels of the image being processed.**

# Spatial Domain Techniques

## *Point Processing Techniques*

### Contrast Stretching



- ✓ The locations of the points  $(r_1, s_1)$  and  $(r_2, s_2)$  control the shape of the transformation function.



**A low Contrast Image**



**Enhanced Image after Contrast Stretching**

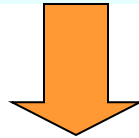


# Spatial Domain Techniques

## *Point Processing Techniques*

### Dynamic Range Compression

- ✓ Sometimes dynamic range of a processed image far exceeds the capability of the display device.



- ✓ This results in only the brightest parts of the image being visible on the display screen.

### *Solution ??*

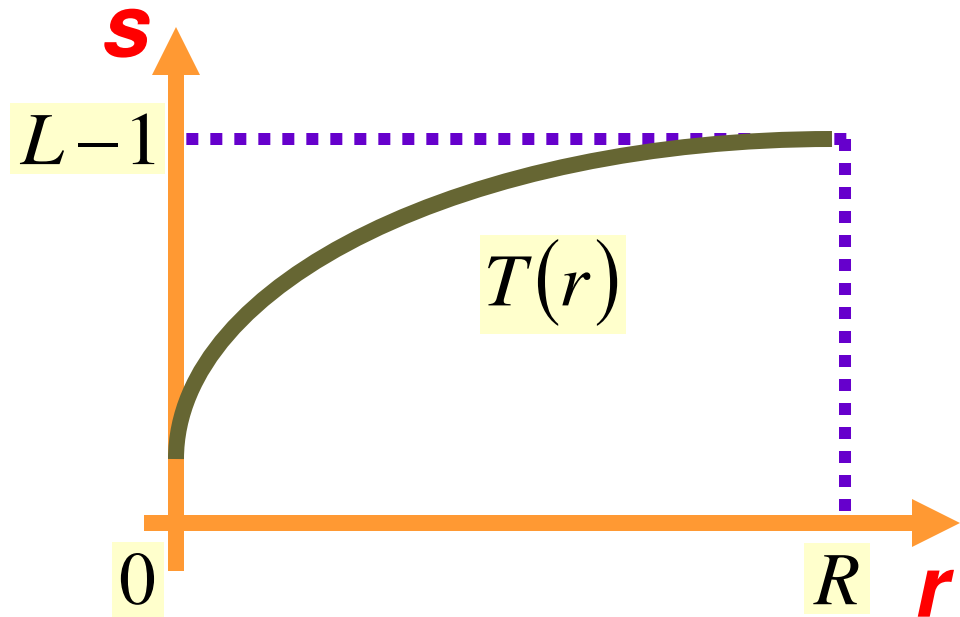
- ✓ **Compress the dynamic range** of the pixel values by using an intensity transformation function:  $s = c \log_{10}(1 + |r|)$ .

Here  $c$  = a scaling constant.

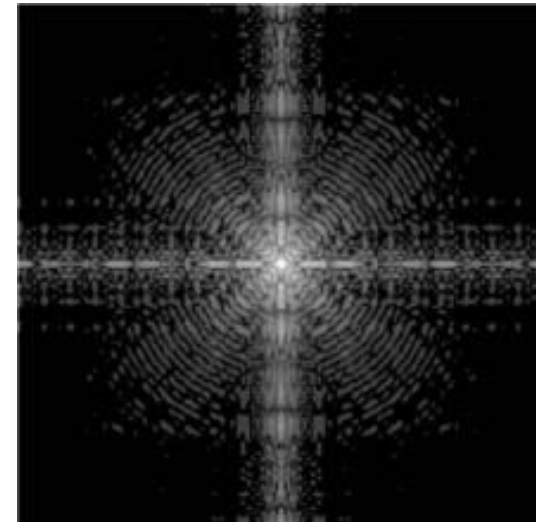
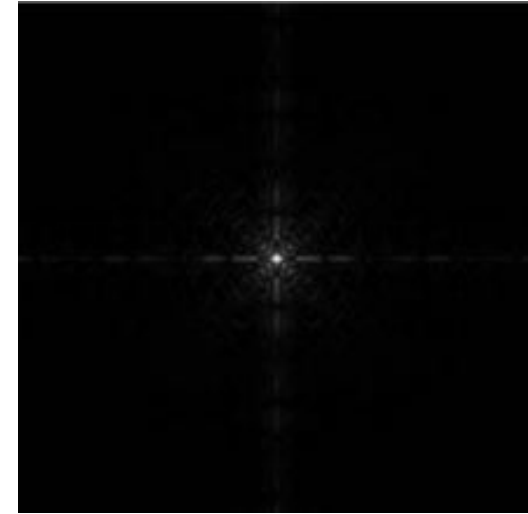
# Spatial Domain Techniques

## *Point Processing Techniques*

### Dynamic Range Compression



A Fourier spectrum,  $[0, R] = [0, 1.5 \times 10^6]$ , scaled linearly for display in 8-bit system.



Log transformed image.

✓ *Example:* For  $[0, R] = [0, 2.5 \times 10^6]$ ,

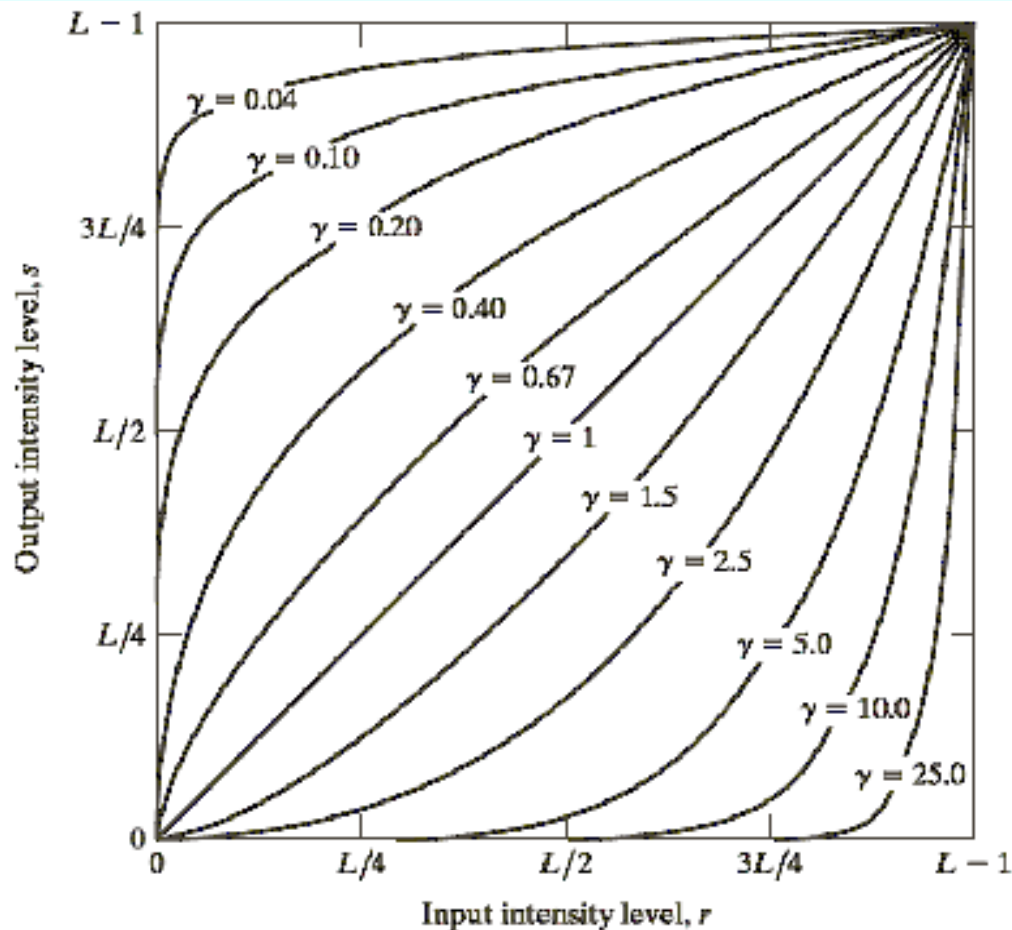
$$\log_{10}(1 + |r|) \in [0, 6.4].$$

To scale this range up to  $[0, L-1] = [0, 255]$ , scaling factor  $c = (255/6.4)$ .

# Spatial Domain Techniques

## *Point Processing Techniques*

### Power-Law (Gamma) Transformations

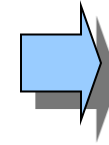
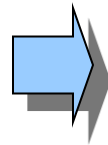


✓ **Note:** This method is popularly known as *gamma correction method*.

$s = cr^\gamma$ , with  $c$  and  $\gamma$  positive constants.

# Spatial Domain Techniques

## Contrast Enhancement using Power-Law Transformations



**MRI of a  
fractured  
human spine.**

**Power-Law  
transformed  
image with  $c = 1$   
and  $\gamma = 0.6$ .**

**Power-Law  
transformed  
image with  $c = 1$   
and  $\gamma = 0.3$ .**

# Spatial Domain Techniques

## *Point Processing Techniques*

### Gray-level Slicing

- ✓ Sometimes we need to **highlight a specific range of gray levels in an image.**
- ✓ Possible **application areas are masses of water in satellite imagery, enhancement of flaws in x-ray images etc.**

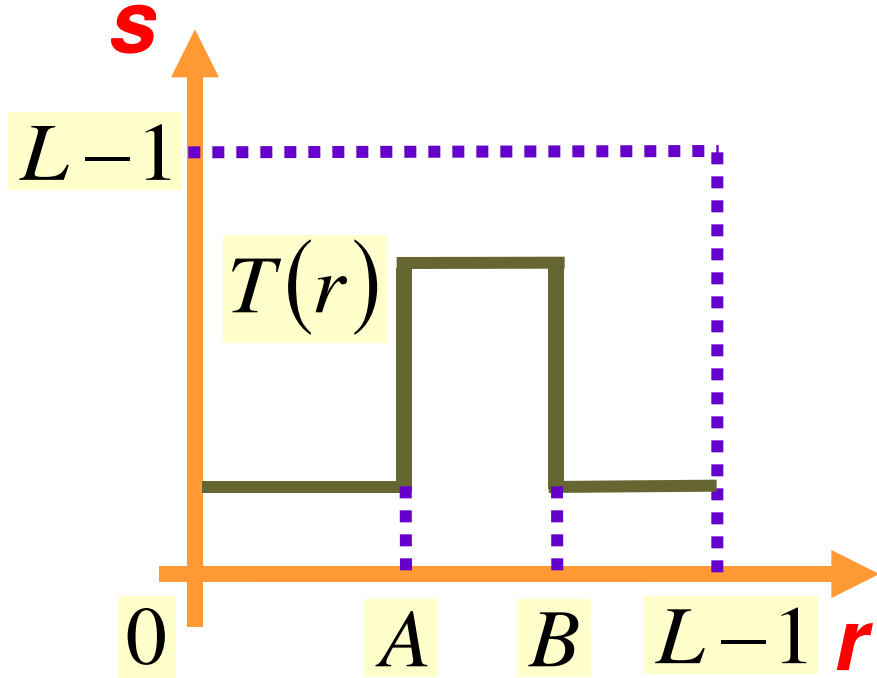
### *Solution ??*

- ✓ Use one of the two basic approaches of gray-level slicing:
  - *Approach 1* – All gray levels in the range of interest are displayed using a high value and the rest using a low value.
  - *Approach 2* – Brightens the desired range of gray levels but preserves the background and gray-level tonalities in an image.

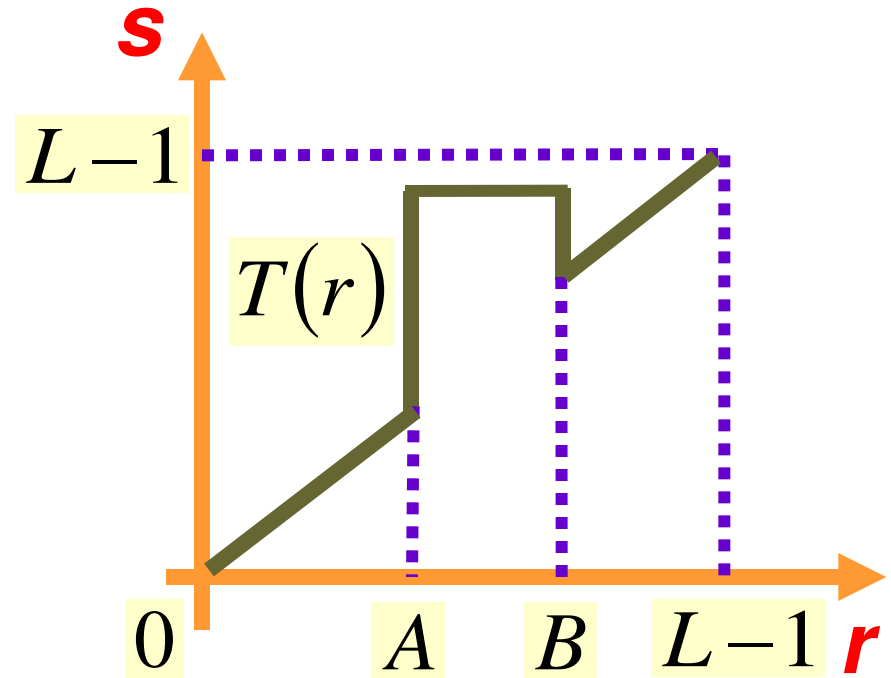
# Spatial Domain Techniques

## *Point Processing Techniques*

### Gray-level Slicing - Approach 1



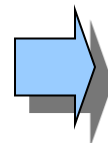
### Gray-level Slicing - Approach 2



# Spatial Domain Techniques

## *Point Processing Techniques*

### Comparison of Gray-level Slicing Approaches



**An Aortic  
Angiogram.**

**Transformed  
image obtained  
with *slicing*  
approach - 1.**

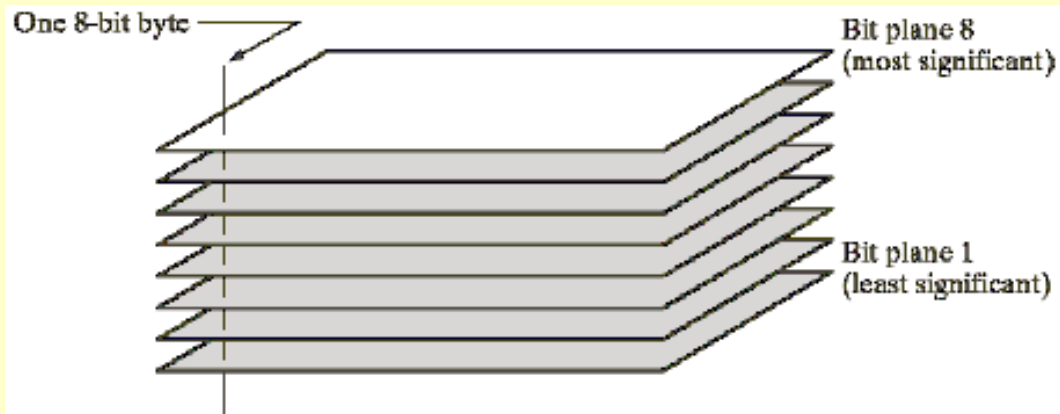
**Transformed  
image obtained  
with *slicing*  
approach - 2.**

# Spatial Domain Techniques

## *Point Processing Techniques*

### Bit-Plane Slicing

- ✓ Sometimes it is desirable to highlight the **contribution made by specific bits** to the total image appearance.
- ✓ The image can be imagined to be composed of **eight 1-bit planes**, plane 0 for the LSB to plane 7 for the MSB.



**Bit-plane representation of an 8-bit digital image.**

**Highest order bits contain visually significant data.**

**Lowest order bits contain more subtle details.**

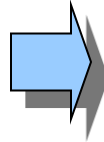


# Spatial Domain Techniques

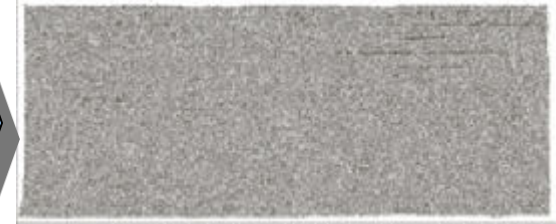
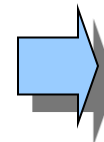
## Example of Bit-Plane Slicing



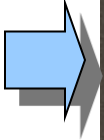
(a)



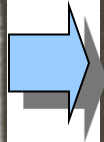
(b)



(c)



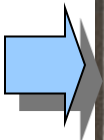
(d)



(e)



(f)



(g)



(h)



(i)

**(a) An 8-bit gray scale image. (b) through (i) Bit planes 1 through 8 with bit plane 1 for LSB. Each bit plane is a binary image.**

# Spatial Domain Techniques

## *Point Processing Techniques*

- ✓ In point-processing techniques, choice of different intensity transformation functions gives rise to different types of image enhancement.
- ✓ However, choice of a suitable function for a given input image is rather cumbersome and a trial-and-error based approach can be rather time consuming.

## *Solution ??*

- ✓ An improved approach utilizes a systematic, automated solution for a suitable transformation mapping by **employing information from image histogram**. This can be achieved by utilizing:

- Histogram Equalization Technique, or
- Histogram Specification Technique.

# Spatial Domain Techniques

## *Spatial Filtering*

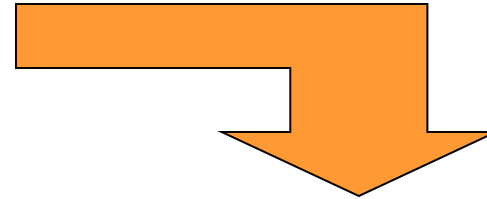
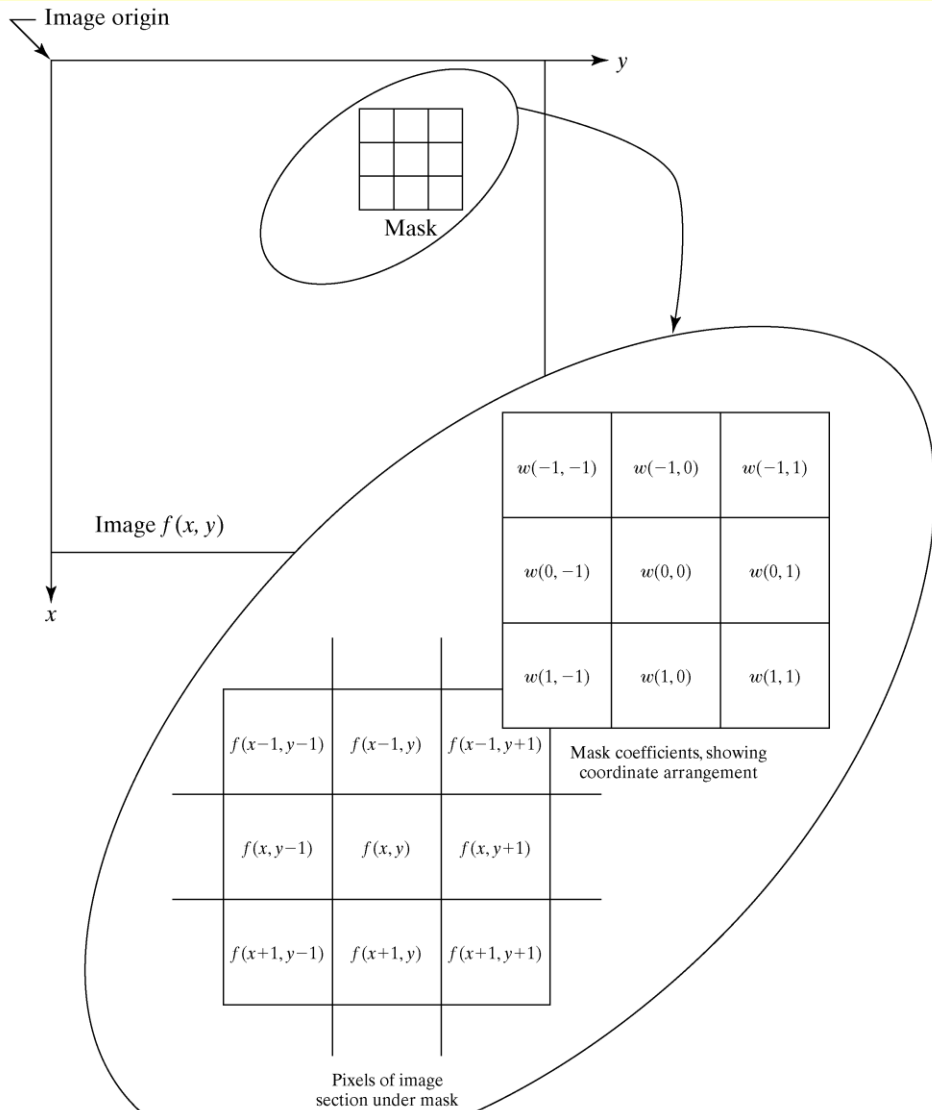
- ✓ *Low pass filters* attenuate or eliminate high frequency components in the Fourier domain. Low pass filtering gives rise to **image blurring**.
- ✓ *High pass filters* attenuate or eliminate low frequency components in the Fourier domain. High pass filtering gives rise to **sharpening of edges and other sharp details**.

## *How to Implement ??*

- ✓ Utilize suitable *two-dimensional masks* of suitable size e.g.  $3 \times 3$ , or  $5 \times 5$ , or  $7 \times 7$ , with **appropriate mask co-efficients**.

# Spatial Domain Techniques

## *Spatial Filtering*

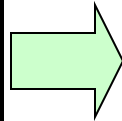


**A  $3 \times 3$  filter mask and the image neighborhood or sub-image under its influence.**

# Spatial Domain Techniques

## *Spatial Filtering*

$z_1$	$z_2$	$z_3$
$z_4$	$z_5$	$z_6$
$z_7$	$z_8$	$z_9$



$w_1$	$w_2$	$w_3$
$w_4$	$w_5$	$w_6$
$w_7$	$w_8$	$w_9$

=

$z_1$	$z_2$	$z_3$
$z_4$	$z_{5new}$	$z_6$
$z_7$	$z_8$	$z_9$

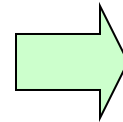
**A 3 × 3 image section under consideration**

**A 3 × 3 filter mask employed for this image section**

**Output of the filter for this image section**

$$z_{5new} = f(R)$$

$$R = w_1 z_1 + w_2 z_2 + \dots + w_9 z_9 = \sum_{i=1}^9 w_i z_i$$



**R = Response of the mask**

The mask is centered on the image pixel whose new intensity value is to be calculated. This calculation is performed for each pixel separately by moving the mask to center it on the pixel under consideration.

# Smoothing Spatial Filters

## *Low pass Spatial Filtering*

$$\frac{1}{9} \times$$

1	1	1
1	1	1
1	1	1

(a)

$$\frac{1}{16} \times$$

1	2	1
2	4	2
1	2	1

(b)

Two typical  $3 \times 3$  low pass filter masks

- ✓ A *Low pass spatial filter* must have all **positive coefficients**.
- ✓ For a *Low pass spatial filter mask* shown above in (a), the operation is also popularly termed as **neighborhood averaging**. This averaging causes blurring and loss of sharpness.
- ✓ For a *filter mask* shown in (b), it is called **weighted averaging**.

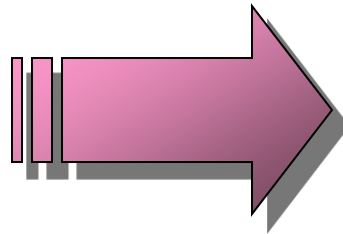
# Smoothing Spatial Filters

## Median Filtering

- ✓ **Median filters** are nonlinear filters employed with an objective of **noise reduction, without blurring.**

$z_1$	$z_2$	$z_3$
$z_4$	$z_5$	$z_6$
$z_7$	$z_8$	$z_9$

**A  $3 \times 3$  image section under consideration**



**Median filtering**

$z_1$	$z_2$	$z_3$
$z_4$	$z_{5new}$	$z_6$
$z_7$	$z_8$	$z_9$

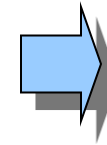
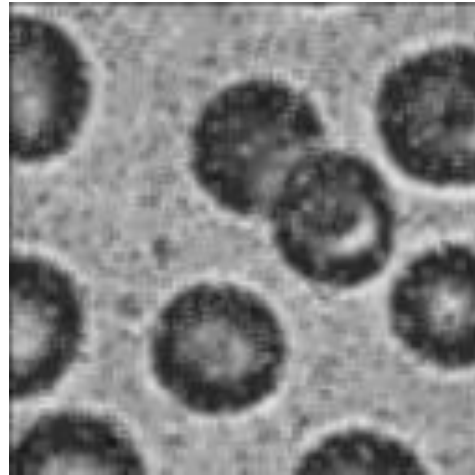
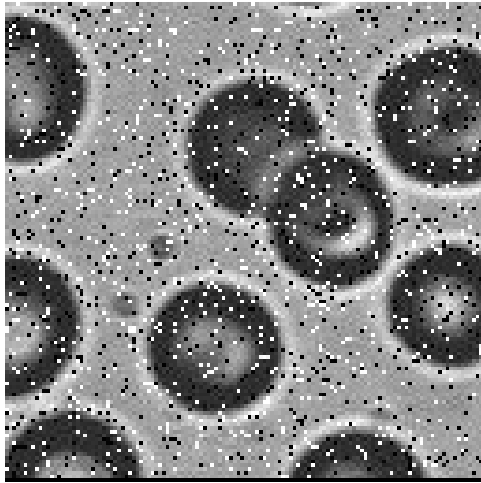
**Output of the filter for this image section**

$$z_{5new} = med(z_1, z_2, z_4, z_6, z_8, z_9)$$

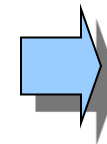
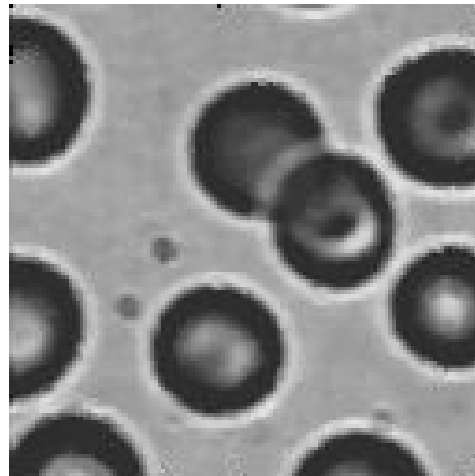
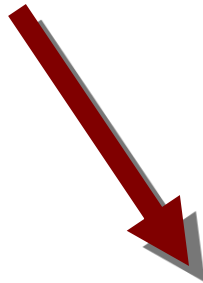
The filter is most effective when the noise pattern consists of spike-like components and it is of utmost importance to preserve edge sharpness.

# Smoothing Spatial Filters

## *A Comparison of Average and Median Filtering - I*



**Processed image  
employing a  $3 \times 3$   
averaging mask.**



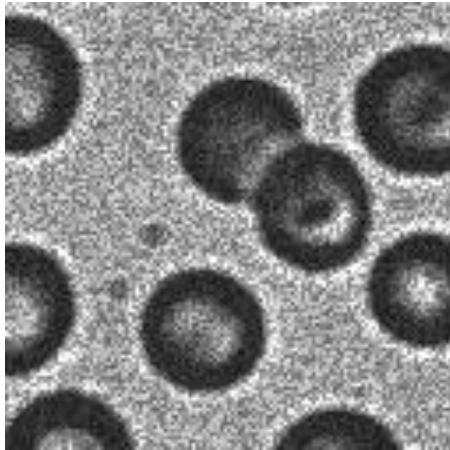
**Processed image  
employing a  $3 \times 3$   
median filter.**

**A Blood cell image  
corrupted by salt-and-  
pepper noise.**

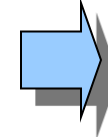
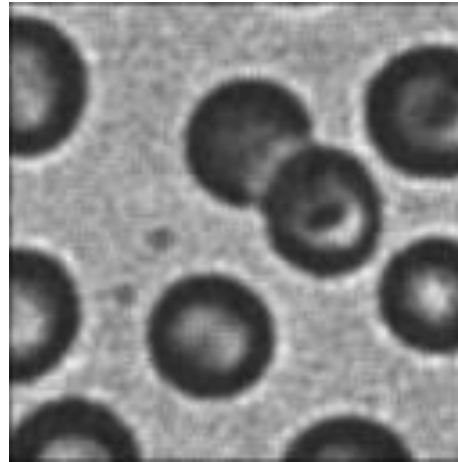
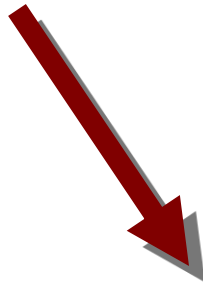


# Smoothing Spatial Filters

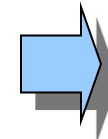
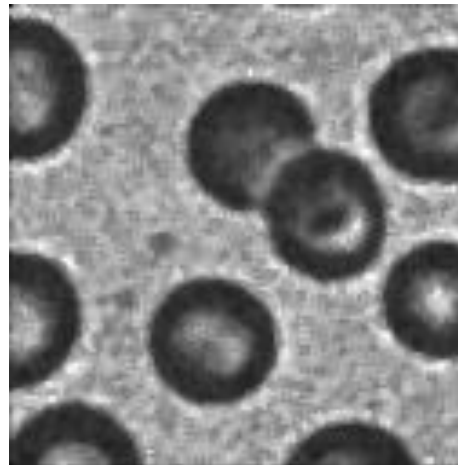
## *A Comparison of Average and Median Filtering - II*



Same Blood cell image corrupted by speckle noise.



Processed image employing a  $3 \times 3$  averaging mask.



Processed image employing a  $3 \times 3$  median filter.

# Sharpening Spatial Filters

## *Derivative Filters*

- ✓ **The differentiation operation is expected to sharpen an image.**
- ✓ **One can use either first derivative or second derivative information.**

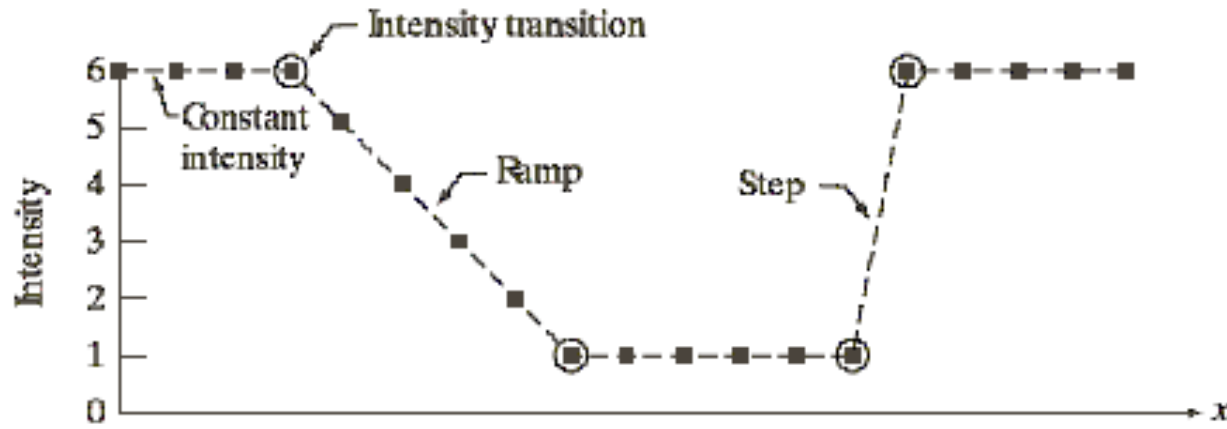
*Digital Approximation of First Derivative:*

$$\frac{\partial f(x, y)}{\partial x} = f(x + 1, y) - f(x, y)$$

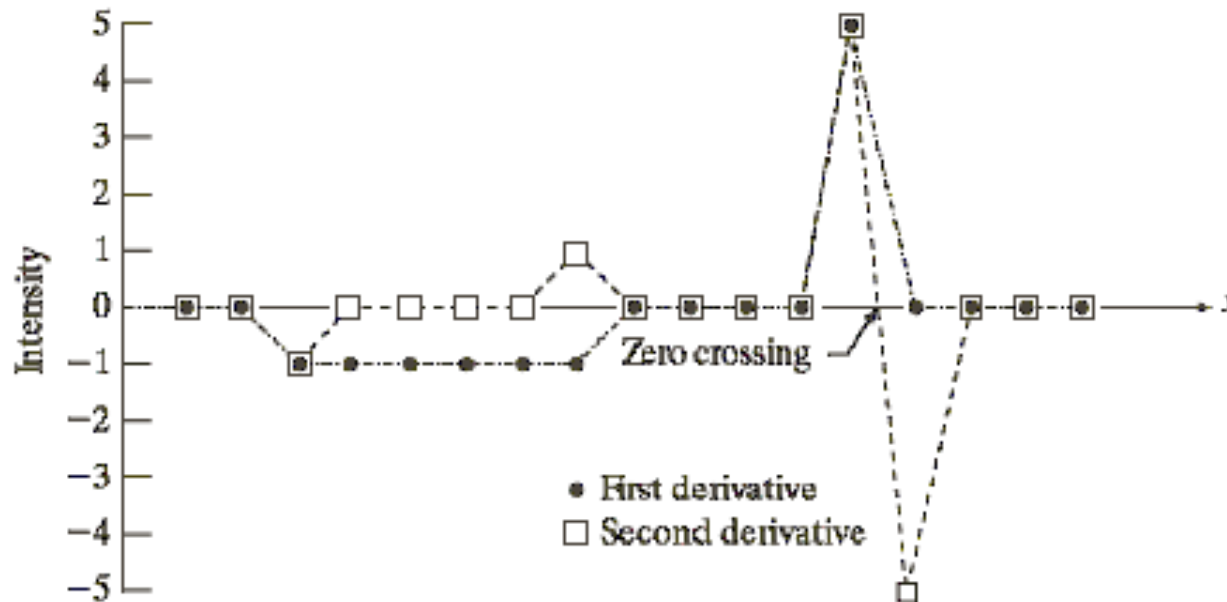
*Digital Approximation of Second Derivative:*

$$\frac{\partial^2 f(x, y)}{\partial x^2} = f(x + 1, y) + f(x - 1, y) - 2f(x, y)$$

# Sharpening Spatial Filters



Scan line	6	6	6	6	5	4	3	2	1	1	1	1	1	1	6	6	6	6	6
1st derivative	0	0	-1	-1	-1	-1	-1	0	0	0	0	0	0	5	0	0	0	0	0
2nd derivative	0	0	-1	0	0	0	0	0	1	0	0	0	0	5	-5	0	0	0	0



**First and second derivatives of a horizontal, single-row intensity profile of an image.**

# Sharpening Spatial Filters

## *Constraints of using a Definition for First Derivative Filters*

- ✓ **Must be zero** in areas of *constant intensity*.
- ✓ **Must be non-zero** at the *onset of an intensity step or ramp*.
- ✓ **Must be non-zero** along *ramps*.

## *Constraints of using a Definition for Second Derivative Filters*

- ✓ **Must be zero** in areas of *constant intensity*.
- ✓ **Must be non-zero** at the *onset and end of an intensity step or ramp*.
- ✓ **Must be zero** along *ramps of constant slope*.

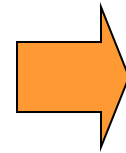
# Sharpening Spatial Filters

## Implementing a First Derivative Filter for Image Sharpening

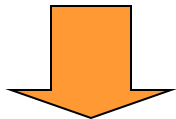


By applying the *Gradient*.

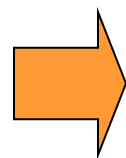
The gradient of a function  $f(x, y)$  at coordinates  $(x, y)$  is defined as the two-dimensional column vector:



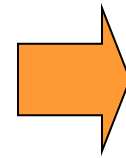
$$\nabla f \equiv \text{grad}(f) \equiv \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$



The magnitude (length) of vector  $\nabla f$ :



$$M(x, y) = \text{mag}(\nabla f) \\ = \sqrt{g_x^2 + g_y^2}$$



$$M(x, y) \approx |g_x| + |g_y|$$

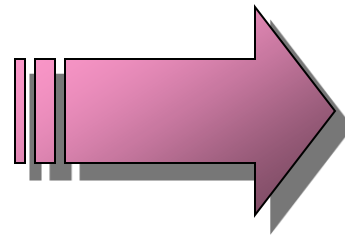
$M(x, y)$  is an image of same size as the original and called the **gradient image**. The **computation of this gradient** is the basis for various approaches to develop first derivative filter.

# Sharpening Spatial Filters

## First Derivative Filters

$z_1$	$z_2$	$z_3$
$z_4$	$z_5$	$z_6$
$z_7$	$z_8$	$z_9$

A  $3 \times 3$  image section under consideration



Derivative filtering

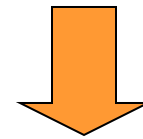
$$M(x, y) = \left[ (z_8 - z_5)^2 + (z_6 - z_5)^2 \right]^{1/2}$$

Approximated form:

$$M(x, y) \approx |z_8 - z_5| + |z_6 - z_5|$$

Another approach using cross-differences:

$$M(x, y) \approx \left[ (z_9 - z_5)^2 + (z_8 - z_6)^2 \right]^{1/2}$$



$$\nabla f \approx |z_9 - z_5| + |z_8 - z_6|$$

These equations can be implemented using masks of size  $2 \times 2$ .

- ✓ **Constraint:** Masks of even sizes are awkward to implement. Hence an approximation with  $3 \times 3$  neighborhood is preferred.

# Sharpening Spatial Filters

## *First Derivative Filters*

The masks for 1<sup>st</sup> derivative operators of size 3 × 3:

-1	-2	-1
0	0	0
1	2	1

-1	0	1
-2	0	2
-1	0	1

**Sobel operators**

$$g_x = \frac{\partial f}{\partial x} = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)$$

$$g_y = \frac{\partial f}{\partial y} = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)$$

$$M(x, y) \approx \left| (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3) \right| + \left| (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7) \right|$$

# Sharpening Spatial Filters

## *First Derivative Filters*

The masks for 1<sup>st</sup> derivative operators of size 3 × 3:

-1	-1	-1
0	0	0
1	1	1

-1	0	1
-1	0	1
-1	0	1

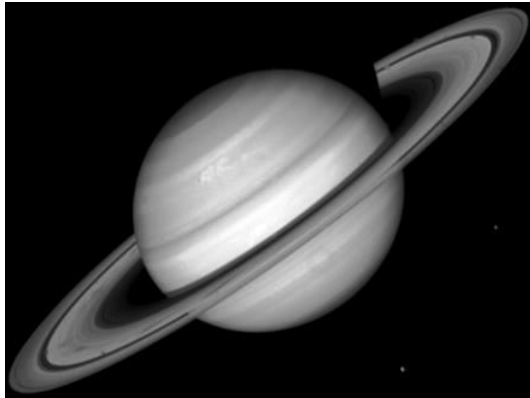
## **Prewitt operators**

$$M(x, y) \approx \left| (z_7 + z_8 + z_9) - (z_1 + z_2 + z_3) \right| \\ + \left| (z_3 + z_6 + z_9) - (z_1 + z_4 + z_7) \right|$$

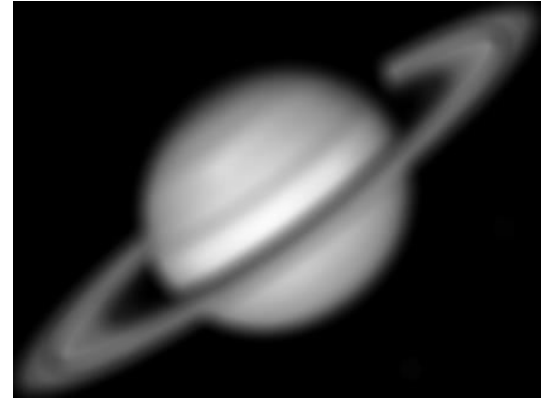


# Sharpening Spatial Filters

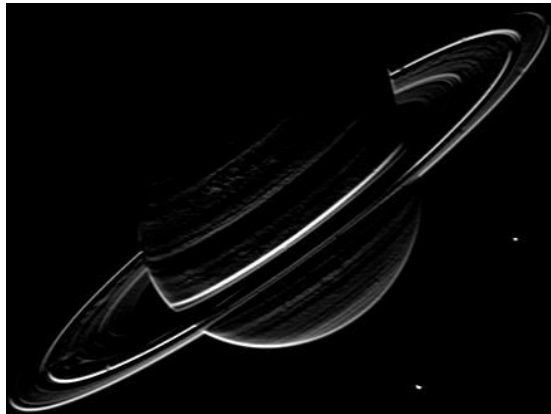
## *Example of Sobel Masks*



**Original 'Saturn' image.**



**Blurred 'Saturn' image.**



**Output of *Sobel Vertical Mask*.**



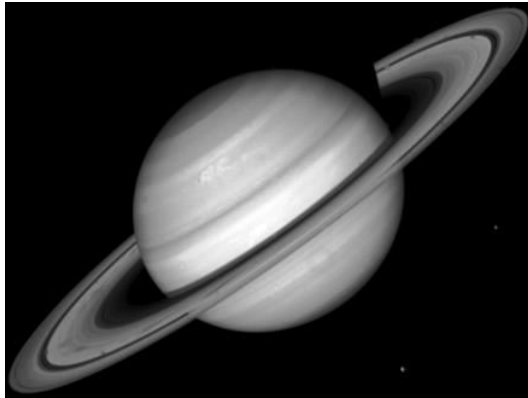
**Output of *Sobel Horizontal Mask*.**



**Output of *Sobel Gradient Mask*.**

# Sharpening Spatial Filters

*Example of Sobel Masks – contd ...*



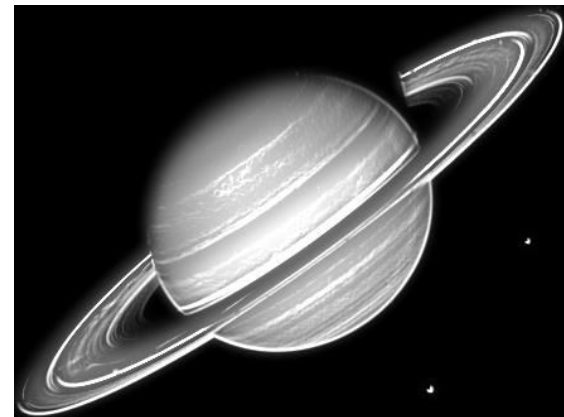
**Original 'Saturn' image.**



**Blurred 'Saturn' image.**



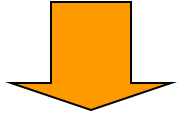
**Output of Sobel Gradient Mask.**



**Output of Sobel Gradient Mask added to the Blurred image.**

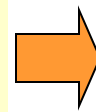
# Sharpening Spatial Filters

## Implementing a Second Derivative Filter for Image Sharpening



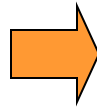
By applying the *Laplacian*.

The Laplacian of an image function  $f(x, y)$  of two variables:



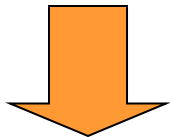
$$\nabla^2 f(x, y) = \frac{\partial^2 f(x, y)}{\partial x^2} + \frac{\partial^2 f(x, y)}{\partial y^2}$$

Common digital approximations of the second derivatives:



$$\frac{\partial^2 f(x, y)}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f(x, y)}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$



$$\nabla^2 f(x, y) = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

# Sharpening Spatial Filters

## *Second Derivative Filters*

The masks for 2<sup>nd</sup> derivative operators of size 3 × 3:

0	1	0
1	-4	1
0	1	0

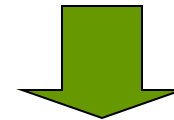
Mask 1



Mask employing **Laplacian** considering conventional **horizontal** and **vertical** directions

1	1	1
1	-8	1
1	1	1

Mask 2



Mask employing **Laplacian** considering four directions: (a) **horizontal**, (b) **vertical**, (c) **+45°** and (c) **-45°** directions

# Sharpening Spatial Filters

## *Second Derivative Filters*

The masks for 2<sup>nd</sup> derivative operators of size  $3 \times 3$ :

0	-1	0
-1	4	-1
0	-1	0

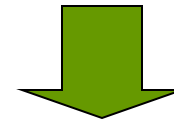
Mask 3



A Laplacian Mask employing **negative** of the Laplacian Mask 1

-1	-1	-1
-1	8	-1
-1	-1	-1

Mask 4



A Laplacian Mask employing **negative** of the Laplacian Mask 2

# Sharpening Spatial Filters

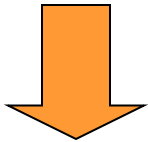
## *Second Derivative Filters*

*There is a Problem ...*

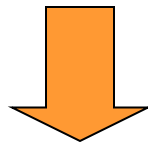
The Laplacian is a derivative operator that highlights intensity discontinuities in an image and, in the process, de-emphasizes image regions having slow variations in intensity profile.

*How to preserve the original background features and yet perform sharpening operation ??*

Utilize the Laplacian in the following manner:



$$g(x, y) = f(x, y) + c[\nabla^2 f(x, y)]$$



$f(x, y)$ : The input image

$g(x, y)$ : The sharpened output image

$c$ : a constant

$c = -1$ , for Masks 1 and 2.  $c = +1$ , for Masks 3 and 4.

# Sharpening Spatial Filters

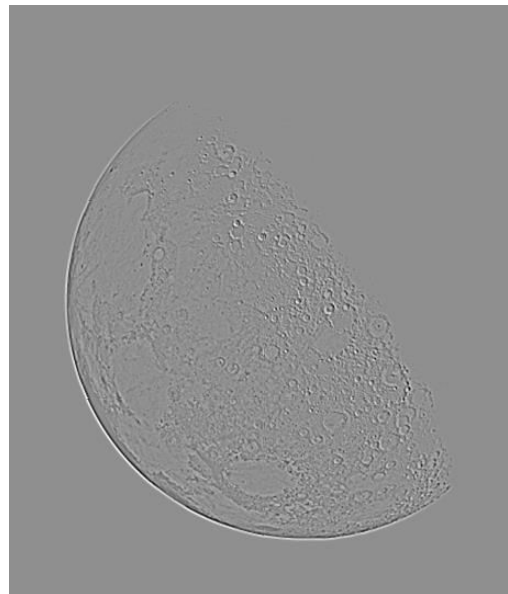
*Example of  
Laplacian Filters*



**'Blurry Moon'**  
image.



**Output of  
Laplacian Mask 1  
(without scaling).**



**Output of  
Laplacian Mask 1  
(with scaling).**



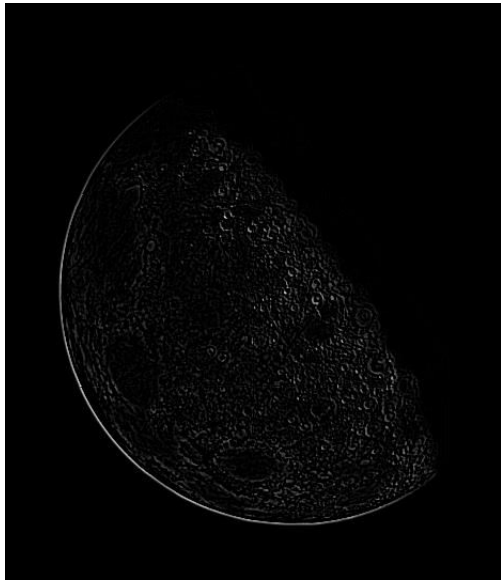
**Output of *non-scaled*  
Mask 1 subtracted from  
the *Blurry* image.**

# Sharpening Spatial Filters

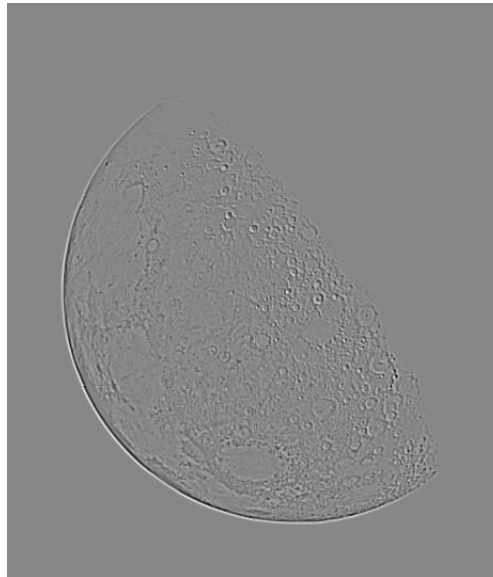
*Example of  
Laplacian Filters  
– contd...*



**'Blurry Moon'**  
**image.**



**Output of  
Laplacian Mask 2  
(without scaling).**



**Output of  
Laplacian Mask 2  
(with scaling).**



**Output of *non-scaled*  
Mask 2 subtracted from  
the *Blurry* image.**



# Sharpening Spatial Filters

## *Performance Comparison of Laplacian Filters*



**'Blurry Moon'**  
image.



**Performance of  
the *Laplacian*  
*Mask 1*.**



**Performance of  
the *Laplacian*  
*Mask 2*.**

# Sharpening Spatial Filters

*High pass Spatial Filtering using  
First and Second Derivatives*

An Important point ...

- ✓ For a *high pass spatial filter mask*, whether utilizing first derivative or second derivative, the **sum of the mask coefficients is always zero.**

# Sharpening Spatial Filters

## *Unsharp Masking and High-boost Filtering*

Create a blurred version of the original image

$$\rightarrow f(x, y) \Rightarrow \bar{f}(x, y)$$

Subtract the blurred version from the original

$$\rightarrow g_{mask}(x, y) = f(x, y) - \bar{f}(x, y)$$

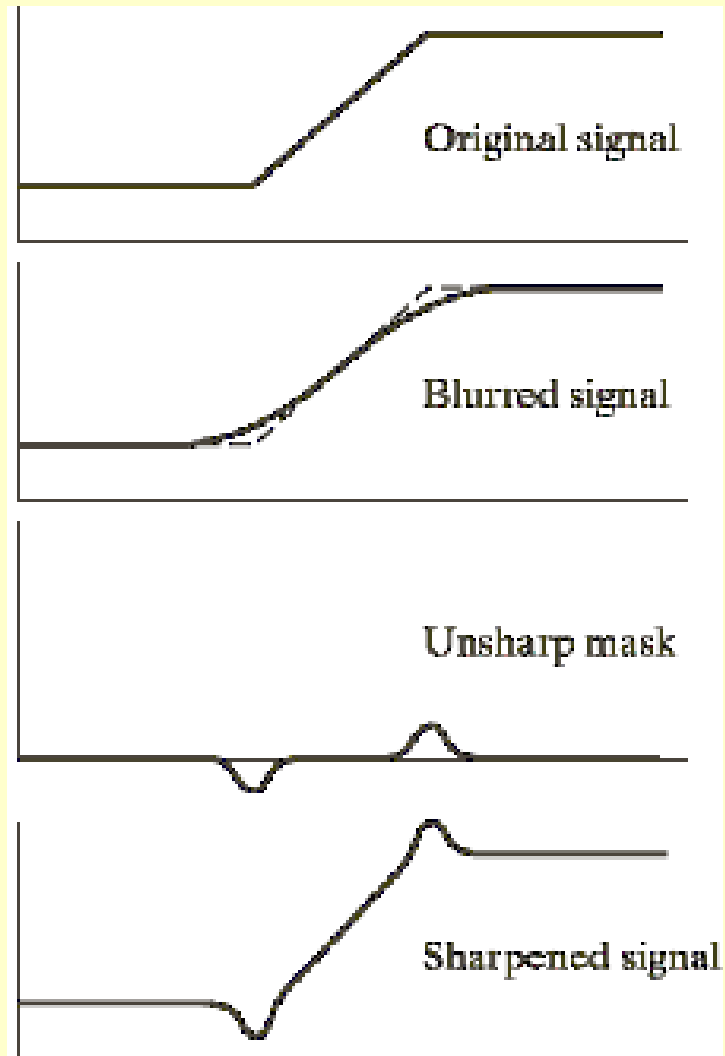
Add this mask to the original image

$$\rightarrow g(x, y) = f(x, y) + k * g_{mask}(x, y)$$

- ✓ When  $k = 1$ , we have *unsharp masking*.
- ✓ When  $k > 1$ , we have *high-boost filtering*.

# Sharpening Spatial Filters

## *Illustration of Unsharp Masking*



# Frequency Domain Methods

- ✓ Frequency domain techniques for image enhancement utilize *convolution theorem in two-dimensions*.
- ✓ The *two-dimensional Discrete Fourier Transform (DFT) pair* is given as:

$$F(u, v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) \exp[-j2\pi(ux/M + vy/N)]$$

$$u = 0, 1, 2, \dots, M-1 \quad \text{and} \quad v = 0, 1, 2, \dots, N-1$$

and

$$f(x, y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v) \exp[j2\pi(ux/M + vy/N)]$$

$$x = 0, 1, 2, \dots, M-1 \quad \text{and} \quad y = 0, 1, 2, \dots, N-1$$

# Frequency Domain Methods

*Convolution Theorem in Two-Dimensions:*

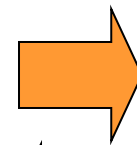
$$f(x, y) * g(x, y) \Leftrightarrow F(u, v)G(u, v)$$

and

$$f(x, y)g(x, y) \Leftrightarrow F(u, v) * G(u, v)$$

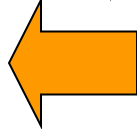
*In Image Enhancement Problems:*

$$g(x, y) = h(x, y) * f(x, y)$$

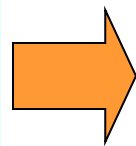


$h(x, y)$ : input image  
 $f(x, y)$ : a linear, position  
invariant operator  
 $g(x, y)$ : output image

$$G(u, v) = H(u, v)F(u, v)$$



$G$ ,  $H$  and  $F$  are FTs of  $g$ ,  
 $h$ , and  $f$ .  $H(u, v)$  is the  
process transfer function.

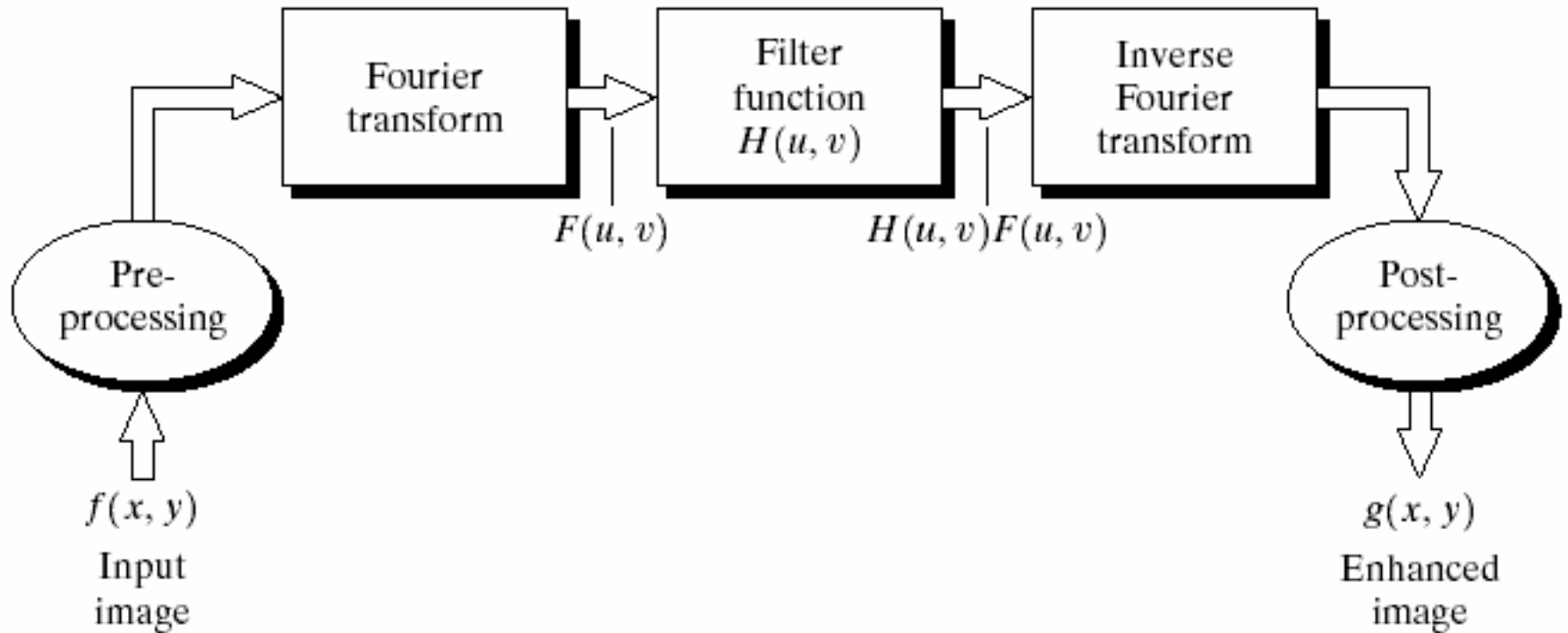


*Our Problem:*

$f(x, y)$  is given. Select  $H(u, v)$   
such that  $g(x, y)$  exhibit some  
highlighted feature of  $f(x, y)$ .

# Frequency Domain Filtering

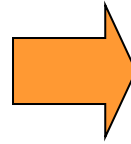
Frequency domain filtering operation



# Ideal Lowpass Filter (ILPF)

*Transfer function of a 2-D ILPF:*

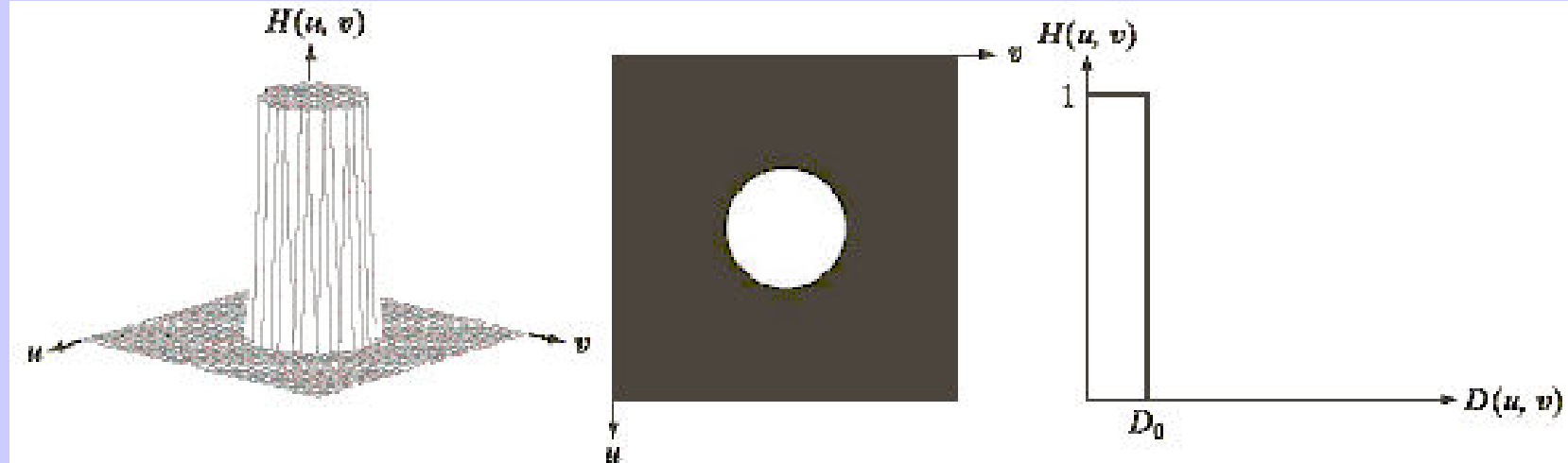
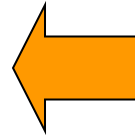
$$H(u, v) = \begin{cases} 1 & \text{if } D(u, v) \leq D_0 \\ 0 & \text{if } D(u, v) > D_0 \end{cases}$$



$D_0$ : a specified non-negative quantity;

$D(u, v)$ : Distance of the point  $(u, v)$  from the origin of the frequency plane

$$D(u, v) = \sqrt{u^2 + v^2}$$



(a)

(b)

(c)

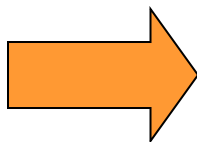
**(a) A 3-D perspective plot of an ILPF T.F., (b) Filter displayed as an image, (c) Filter radial cross section.**



# Butterworth Lowpass Filter (BLPF)

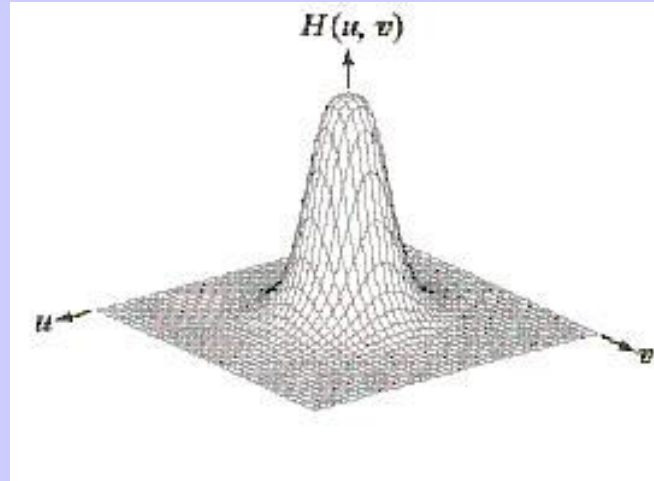
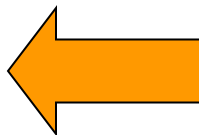
*Transfer function of a 2-D BLPF:*

$$H(u, v) = \frac{1}{1 + \left[ \frac{D(u, v)}{D_0} \right]^{2n}}$$

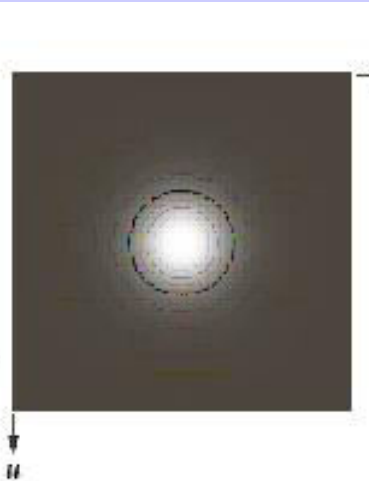


$n$  is the order of the filter.  
The cut-off frequency locus  
is at a distance  $D_0$  from the  
origin.

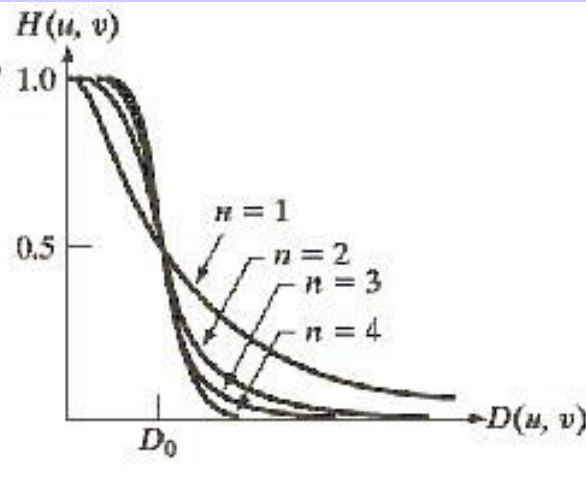
$$D(u, v) = \sqrt{u^2 + v^2}$$



(a)



(b)



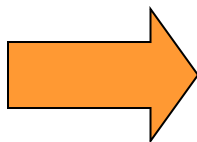
(c)

**(a) A 3-D perspective plot of a BLPF T.F., (b) Filter displayed as an image, (c) Filter radial cross section of orders 1-4.**

# Gaussian Lowpass Filter (GLPF)

*Transfer function of a 2-D GLPF:*

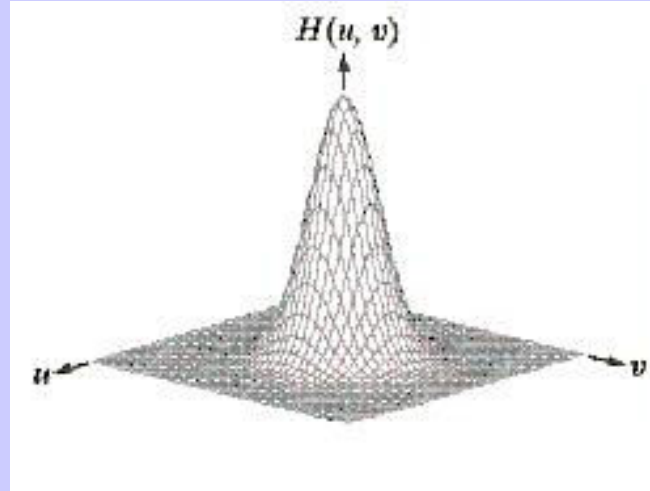
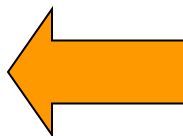
$$H(u, v) = e^{-\frac{D^2(u, v)}{2\sigma^2}}$$



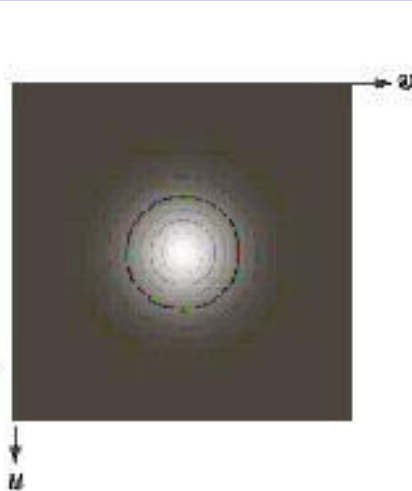
$\sigma$  is a measure of the spread of the Gaussian curve.

The cut-off frequency locus is at a distance  $D_0$  from the origin.

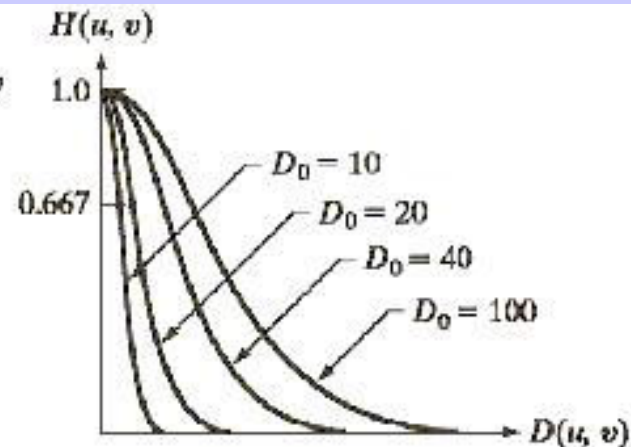
$$D(u, v) = \sqrt{u^2 + v^2}$$



(a)



(b)



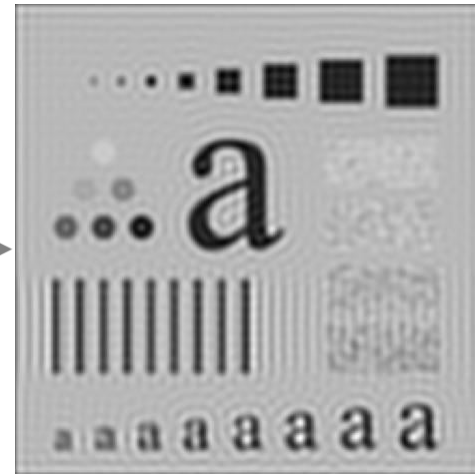
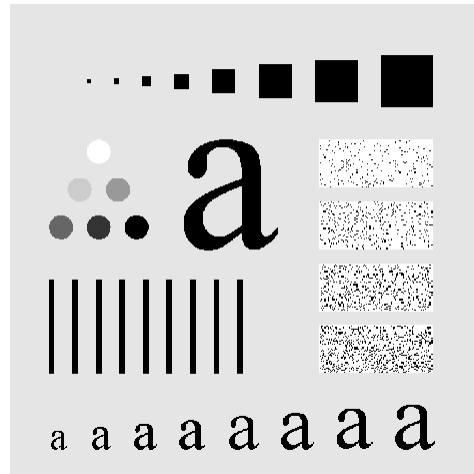
(c)

(a) A 3-D perspective plot of a GLPF T.F., (b) Filter displayed as an image, (c) Filter radial cross section for various  $D_0$ .

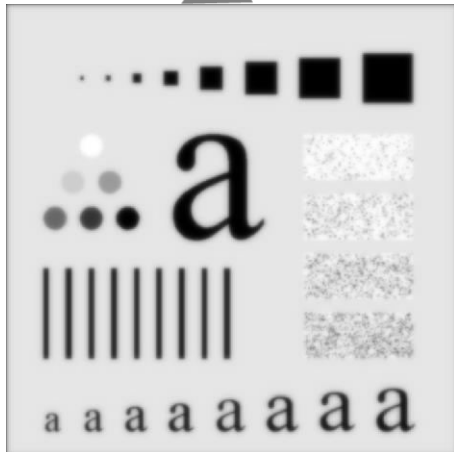
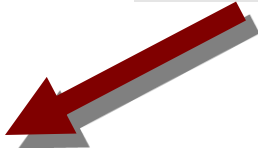
# Lowpass Filters

## Comparison of Ideal, Butterworth and Gaussian Filtering - I

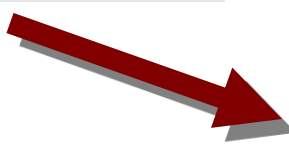
Original image of a test pattern.



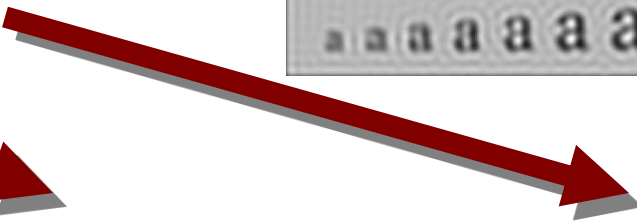
ILPF with  $D_0 = 80$



BLPF with  $n = 1, D_0 = 80$



BLPF with  $n = 2, D_0 = 80$

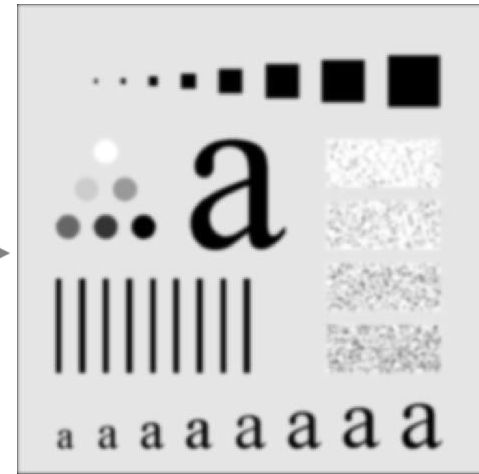
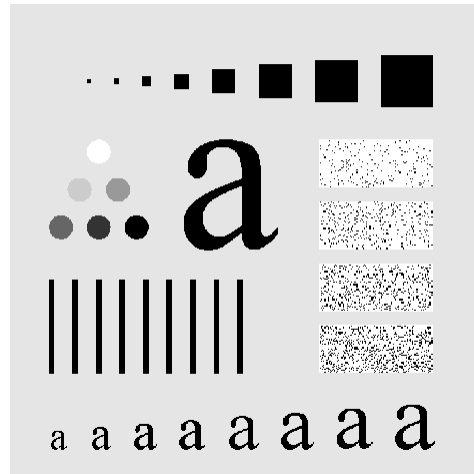


GLPF with  $D_0 = 80$

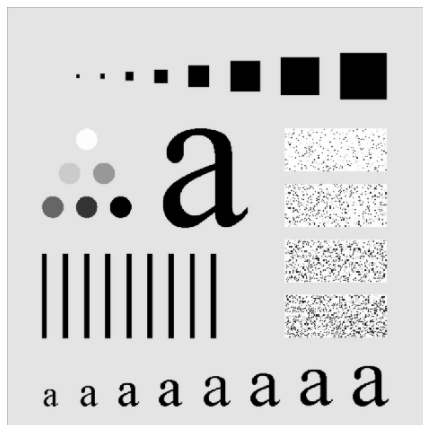
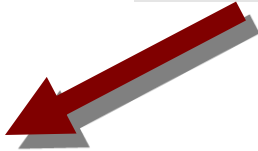
# Lowpass Filters

## Comparison of Ideal, Butterworth and Gaussian Filtering - II

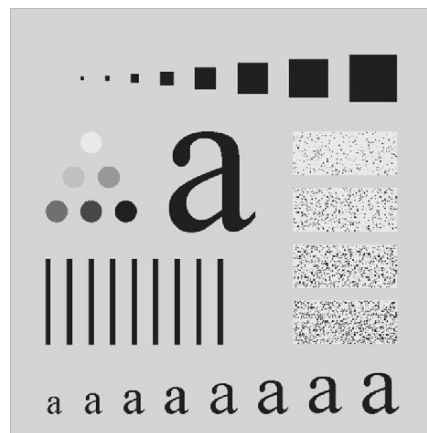
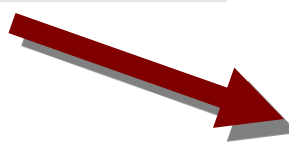
Original  
image of a  
test pattern.



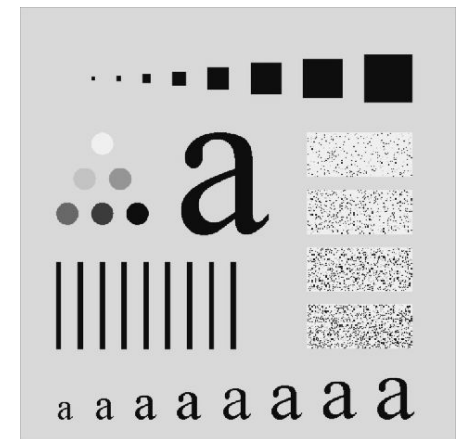
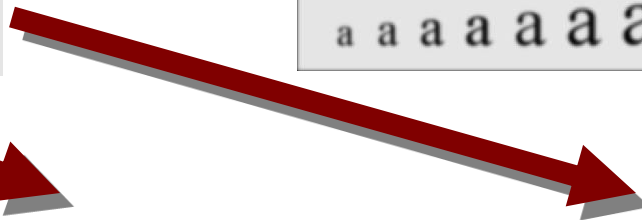
ILPF with  
 $D_0 = 480$



BLPF with  
 $n = 1, D_0 = 480$



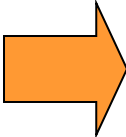
BLPF with  
 $n = 2, D_0 = 480$

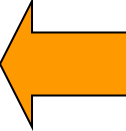


GLPF with  
 $D_0 = 480$

# Ideal Highpass Filter (IHPF)

*Transfer function of a 2-D IHPF:*

$$H(u, v) = \begin{cases} 0 & \text{if } D(u, v) \leq D_0 \\ 1 & \text{if } D(u, v) > D_0 \end{cases}$$


$$H_{hp}(u, v) = 1 - H_{lp}(u, v)$$


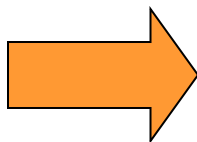
$D_0$ : specified cut-off distance from the origin of the frequency plane;  
 $D(u, v)$ : Distance of the point  $(u, v)$  from the origin of the frequency plane

# Butterworth Highpass Filter (BHPF)

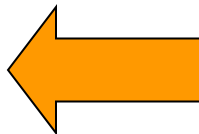
*Transfer function of a 2-D BHPF:*

$$H(u, v) = \frac{1}{1 + \left[ \frac{D_0}{D(u, v)} \right]^{2n}}$$

$$D(u, v) = \sqrt{u^2 + v^2}$$



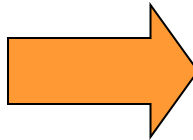
**$n$  is the order of the filter.  
The cut-off frequency locus  
is at a distance  $D_0$  from the  
origin.**



# Gaussian Highpass Filter (GHPF)

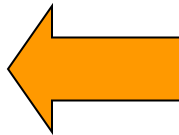
*Transfer function of a 2-D GHPF:*

$$H(u, v) = 1 - e^{-\frac{D^2(u, v)}{2D_0^2}}$$



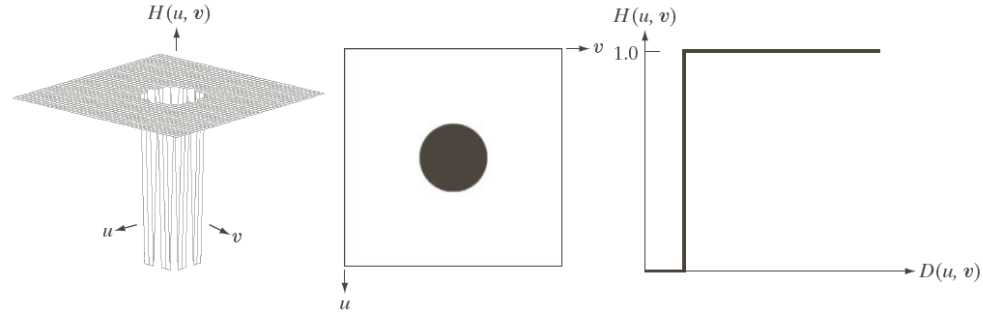
The cut-off frequency locus is at a distance  $D_0$  from the origin.

$$D(u, v) = \sqrt{u^2 + v^2}$$

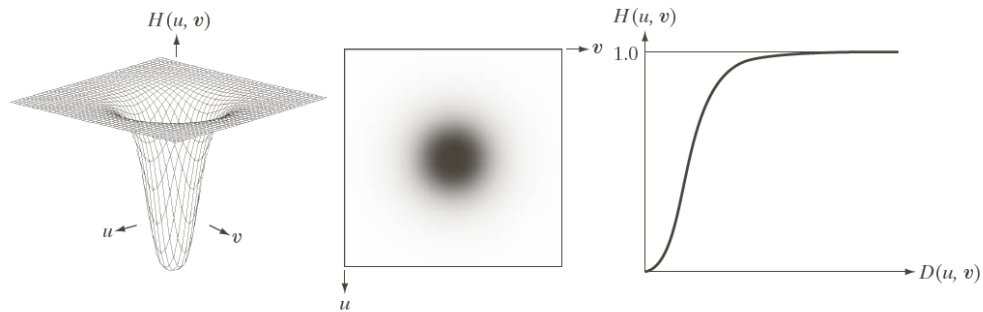


# Highpass Filters

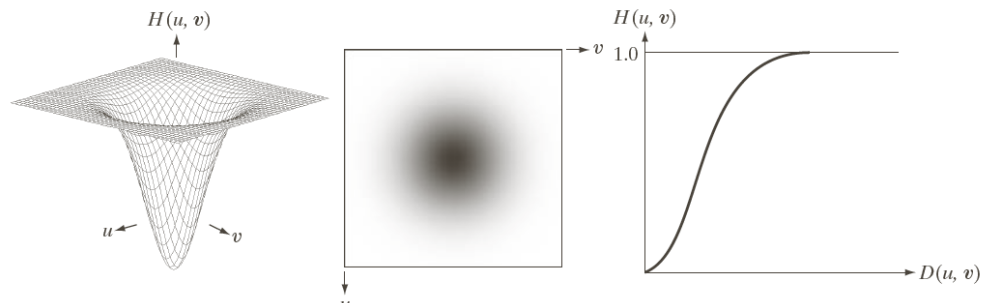
(a)



(b)



(c)



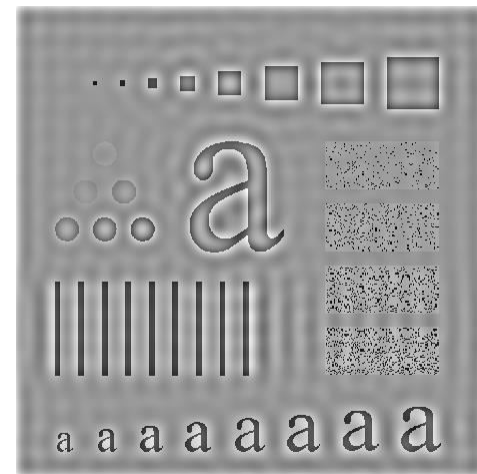
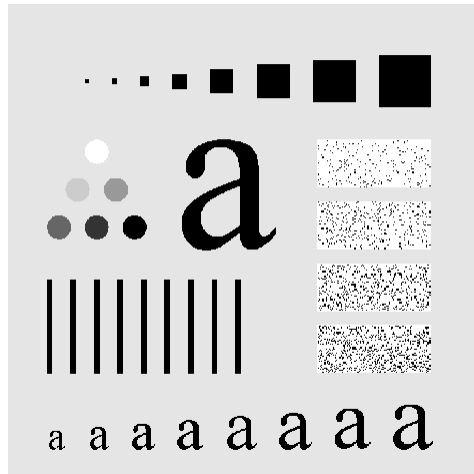
**3-D perspective plot, image representation, and radial cross section for (a) IHPF, (b) BHPF, and (c) GHPF.**



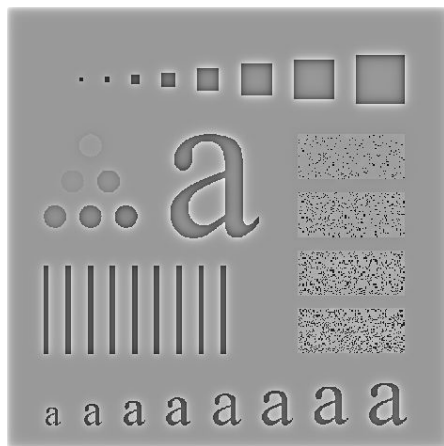
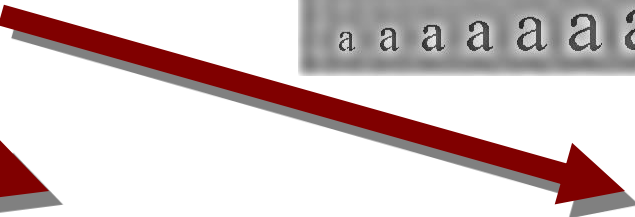
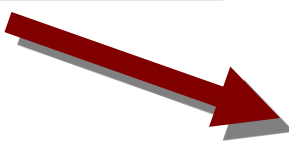
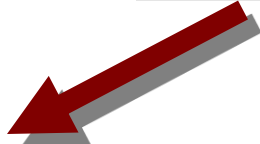
# Highpass or Sharpening Filters

*Comparison of Ideal, Butterworth and Gaussian Filtering - I*

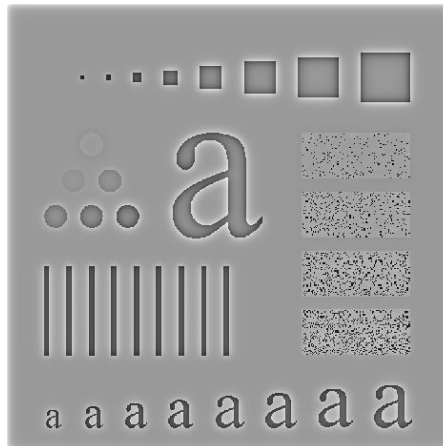
Original  
image of a  
test pattern.



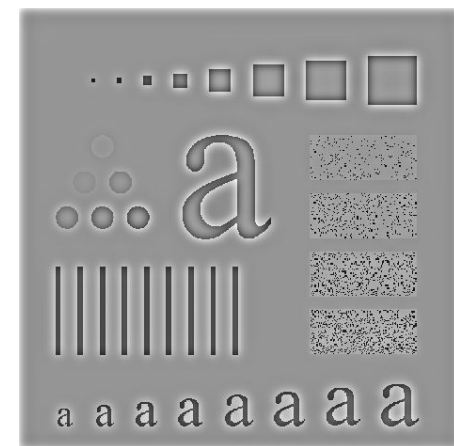
**IHPF**  
with  
 $D_0 = 40$



**BHPF with**  
 $n = 1, D_0 = 40$



**BHPF with**  
 $n = 2, D_0 = 40$

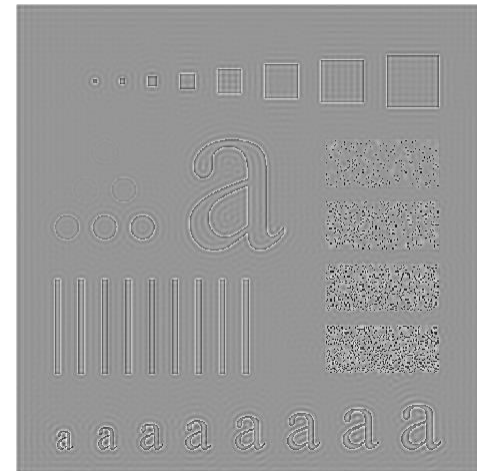
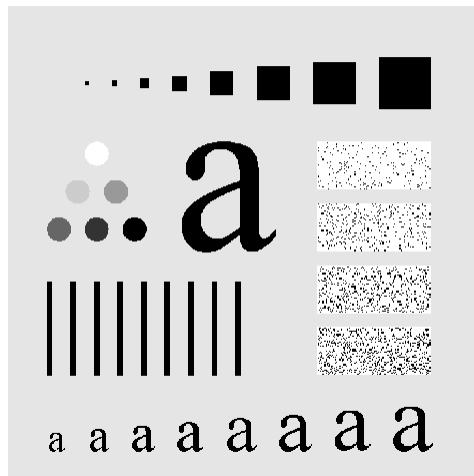


**GHPF with**  
 $D_0 = 40$

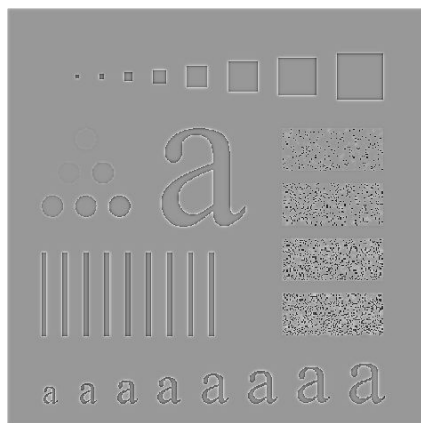
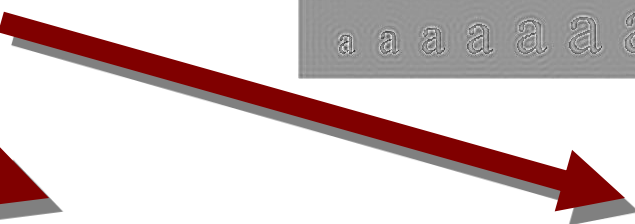
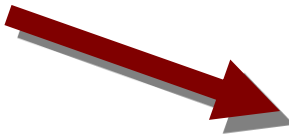
# Highpass or Sharpening Filters

## Comparison of Ideal, Butterworth and Gaussian Filtering - II

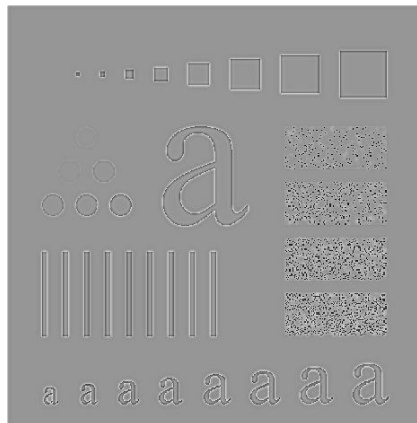
Original image of a test pattern.



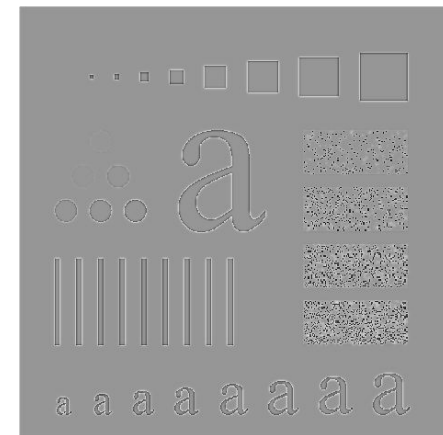
IHPF with  $D_0 = 150$



BHPF with  $n = 1, D_0 = 150$



BHPF with  $n = 2, D_0 = 150$

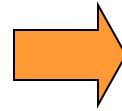


GHPF with  $D_0 = 150$

# High-Frequency Emphasis Filtering (HFEF)

*Transfer function of a HFEEF:*

$$H_{hfe}(u, v) = a + b * H_{hp}(u, v)$$



**$a$ : offset;  $b$ : multiplier;**

**$H_{hp}(u, v)$ : High pass filter T.F.**

The offset term is used to preserve the low frequency components.

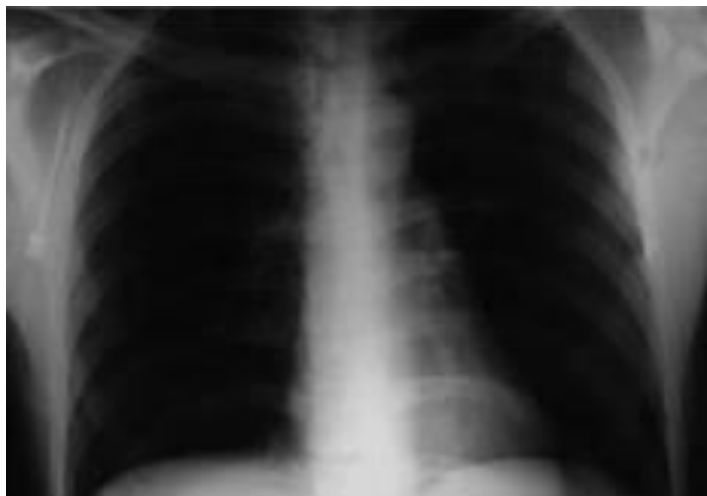
The multiplier term is used to highlight the high frequency components.

- ✓ **Note:** This technique requires a post-filtering processing to redistribute the gray levels.

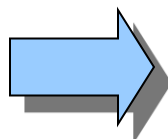
*Candidate tool for post-filtering ??*

- ✓ **Histogram equalization** is a popular option because it can provide contrast enhancement.

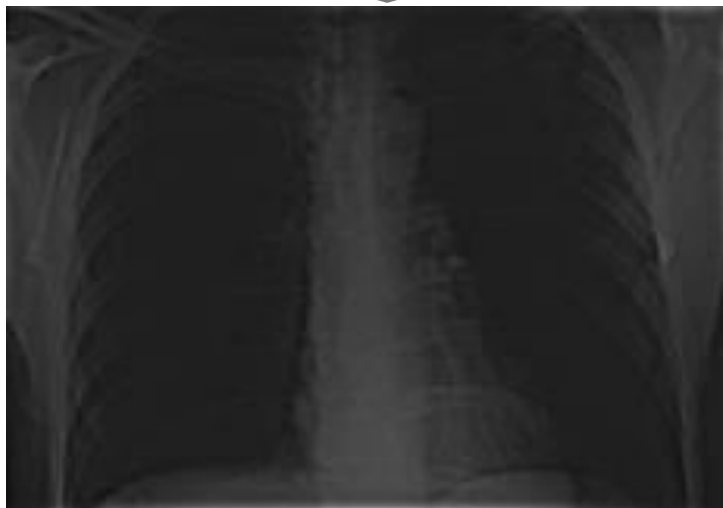
# High-Frequency Emphasis Filtering (HFEF)



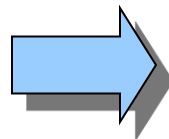
A chest X-ray image.



Processed image employing BHPF ( $n = 2$ ,  $D_0 = 5\%$  of image vertical dimension).



Processed image after high-frequency emphasis filtering ( $a = 0.5$ ,  $b = 2.0$ ).



Processed image after HFEF, followed by histogram equalization.

# Homomorphic Filtering

- ✓ This method utilizes the illumination-reflectance model.

$$f(x, y) = i(x, y)r(x, y) \quad \Rightarrow \quad \begin{array}{l} i(x, y): \text{illumination component;} \\ r(x, y): \text{reflectance component.} \end{array}$$

*There is a problem ...*

- ✓ This equation can not be used directly to operate separately on the frequency components of illumination and reflectance.

*Why ??*

- ✓ Because the Fourier Transform of the product of the two functions is not separable.

$$\mathcal{F}\{f(x, y)\} \neq \mathcal{F}\{i(x, y)\}\mathcal{F}\{r(x, y)\}$$

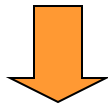
# Homomorphic Filtering (contd...)

Let us define:

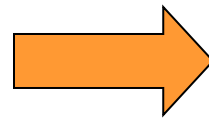
$$z(x, y) = \ln f(x, y) = \ln i(x, y) + \ln r(x, y)$$



$$\mathcal{F}\{z(x, y)\} = \mathcal{F}\{\ln f(x, y)\} = \mathcal{F}\{\ln i(x, y) + \ln r(x, y)\}$$



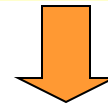
$$Z(u, v) = I(u, v) + R(u, v)$$



$I(u, v)$ : F.T. of  $\ln i(x, y)$ ;  
 $R(u, v)$ : F.T. of  $\ln r(x, y)$ .

Processing  $Z(u, v)$  with a Homomorphic Filter T.F.  $H(u, v)$ :

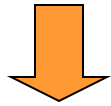
$$S(u, v) = H(u, v)Z(u, v) = H(u, v)I(u, v) + H(u, v)R(u, v)$$



$$\begin{aligned} s(x, y) &= \mathcal{F}^{-1}\{S(u, v)\} = \mathcal{F}^{-1}\{H(u, v)I(u, v)\} + \mathcal{F}^{-1}\{H(u, v)R(u, v)\} \\ &= i'(x, y) + r'(x, y) \end{aligned}$$

# Homomorphic Filtering (contd...)

- ✓ As  $z(x, y)$  was formed by taking the logarithm of the original image  $f(x, y)$ , the inverse operation on  $s(x, y)$  should give us the desired enhanced image  $g(x, y)$ .



$$\begin{aligned}g(x, y) &= \mathbf{exp}[s(x, y)] \\ &= \mathbf{exp}[i'(x, y)]\mathbf{exp}[r'(x, y)] \\ &= i_0(x, y)r_0(x, y)\end{aligned}$$

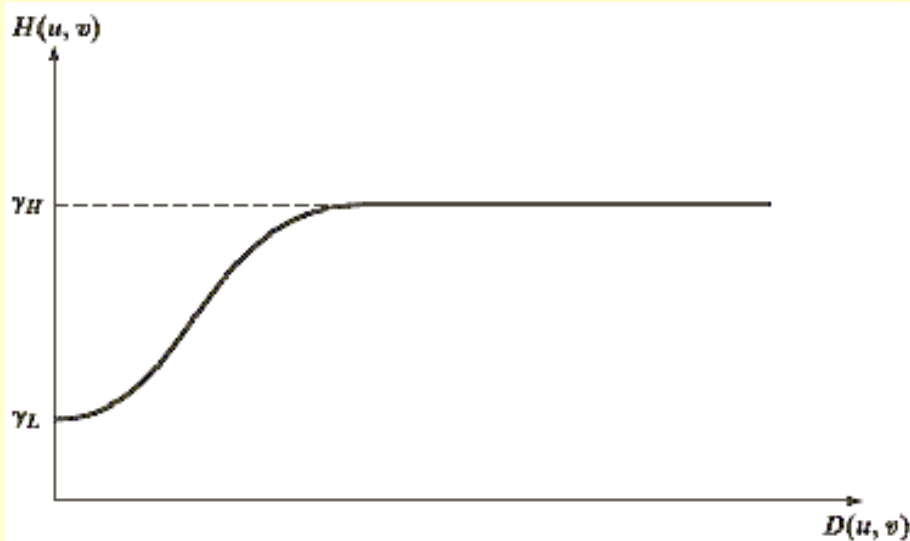
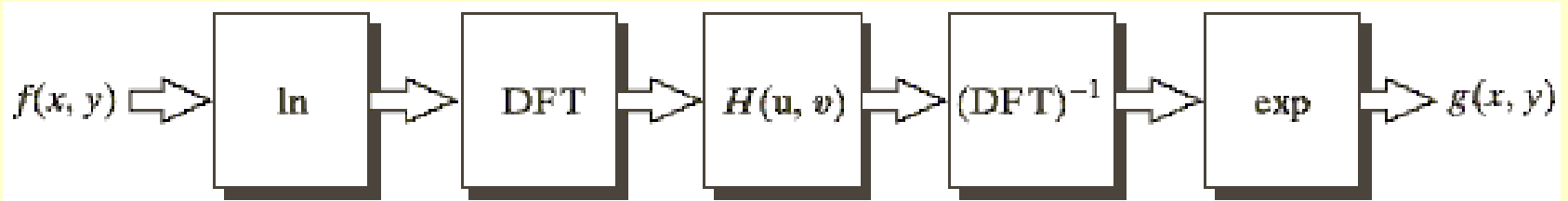
$i_0(x, y)$ : illumination component of the output image;

$r_0(x, y)$ : reflectance component of the output image;

*What is the salient feature of this approach ??*

This approach works on separating the illumination and reflectance components of an image so that the Homomorphic Filter T.F. can work on these components separately.

# Homomorphic Filtering (contd...)



Illumination Component is characterized by slow spatial variations.

Reflectance Component is characterized by abrupt variations.

**Cross section of a circularly symmetric Homomorphic Filter function.**



# Homomorphic Filtering (contd...)



**A full body PET  
(Positron Emission  
Tomography) scan.**



**Homomorphic Filtered  
enhanced image.**

# Image Enhancement Techniques

## *References:*

- ❑ **R. C. Gonzalez and R. E. Woods. Digital Image Processing. Pearson Education Inc., 2008.**
- ❑ **R. C. Gonzalez, R. E. Woods, and S. L. Eddins. Digital Image Processing using MATLAB<sup>®</sup>. Pearson Education, Inc. 2005.**
- ❑ **S. Annadurai and R. Shanmugalakshmi. Fundamentals of Digital Image Processing. Pearson Education, Inc. 2007.**

**Thank You.**