Hot-Wire Anemometers

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Hot-Wire Anemometers

Constant Current Anemometer (CCA)

Wire current is maintained constant

Equilibrium wire temperature is a measure of flow velocity

Constant Temperature Anemometer (CTA)

Wire temperature is maintained constant

Equilibrium wire current is a measure of flow velocity

Hot-Wire Anemometers

Energy balance equation for the hot wire in equilibrium condition:

$$I^2 R_w = hA\left(T_w - T_f\right)$$

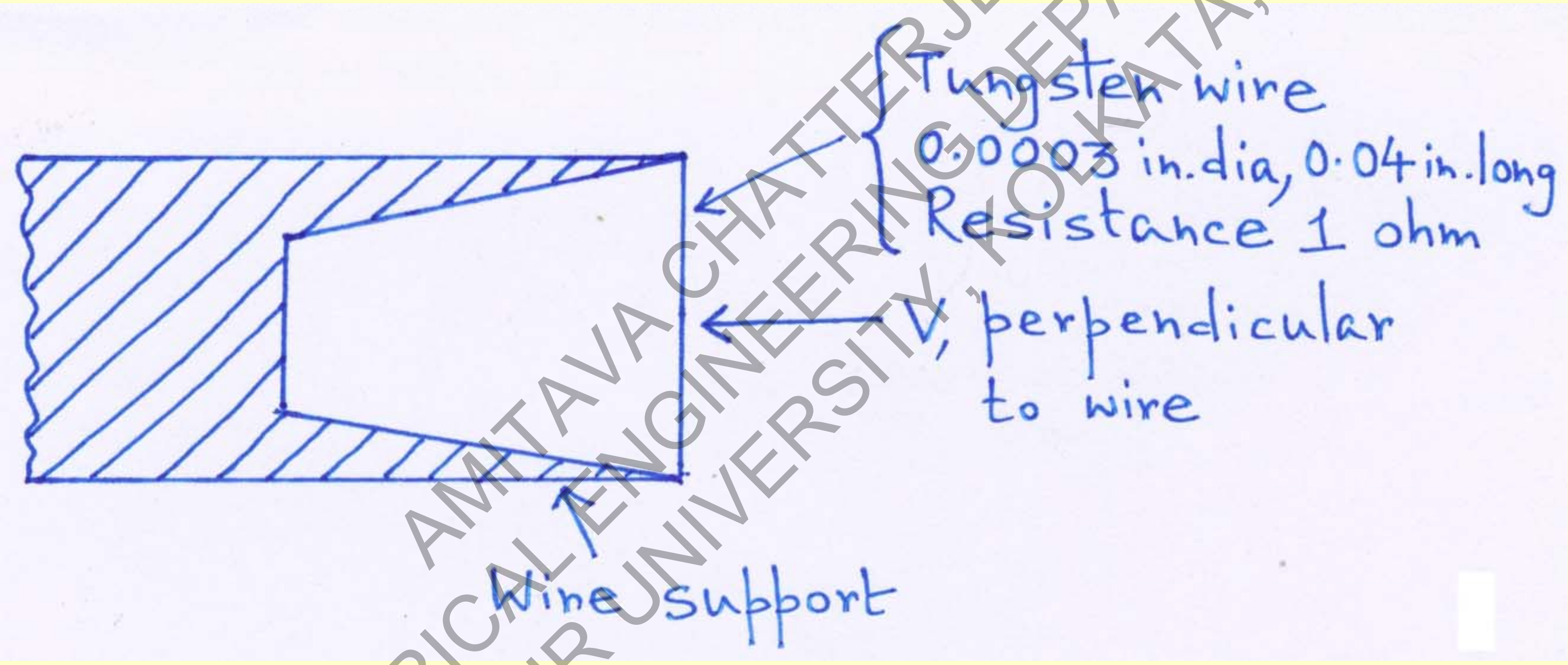
$I =$ wire current
$R_w =$ wire resistance
$T_w =$ wire temperature
$T_f =$ temperature of flowing fluid
$h =$ film coefficient of heat transfer
$A =$ heat-transfer area

✓ ***h*** **is mainly a function of flow velocity for a given fluid density.**

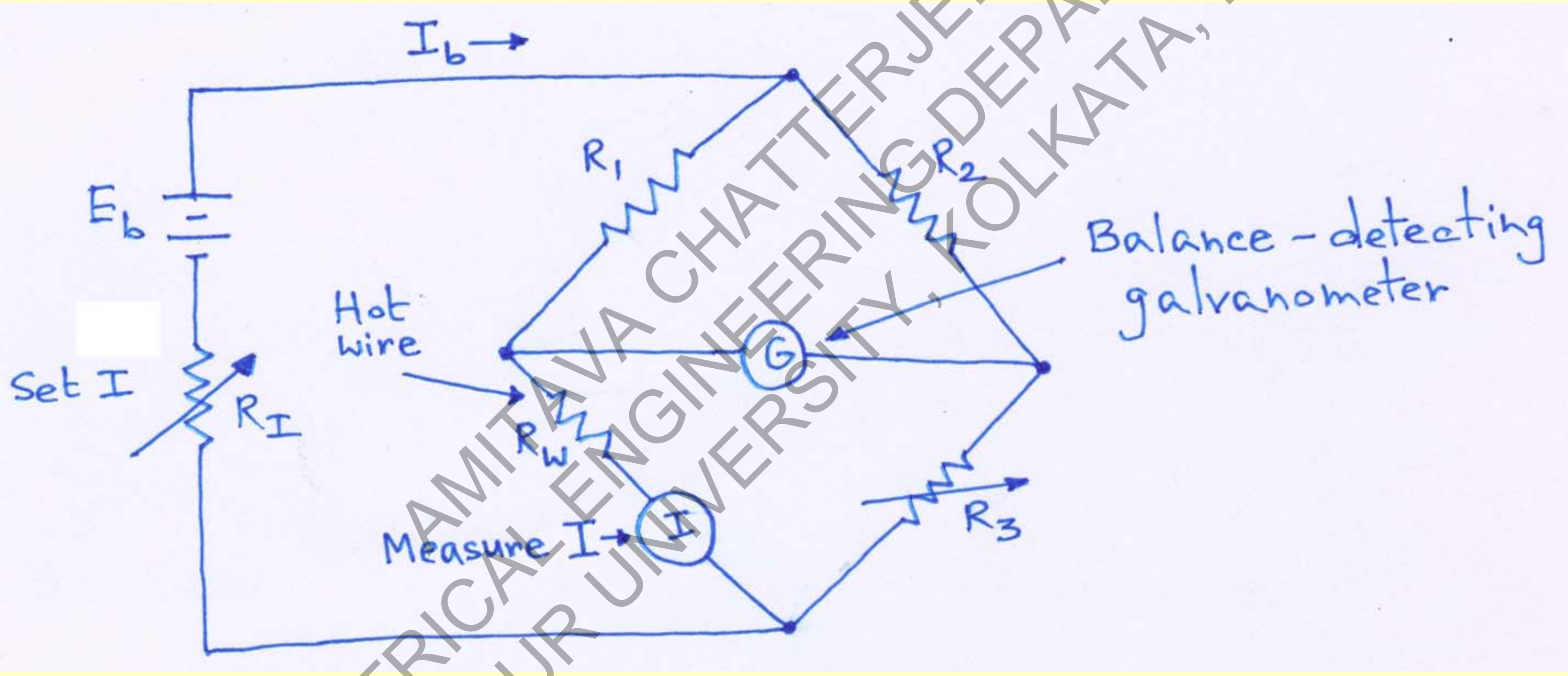
King's Law

$$h = C_0 + C_1\sqrt{V}$$

Measurement of Average velocity using CTA in “Manual Balance” Mode



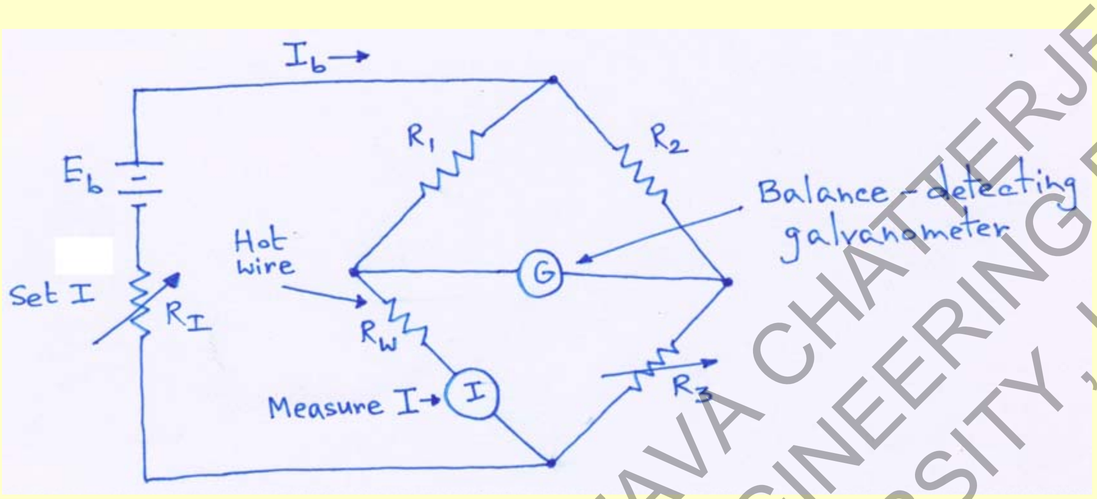
Measurement of Average velocity using CTA in "Manual Balance" Mode (contd…)

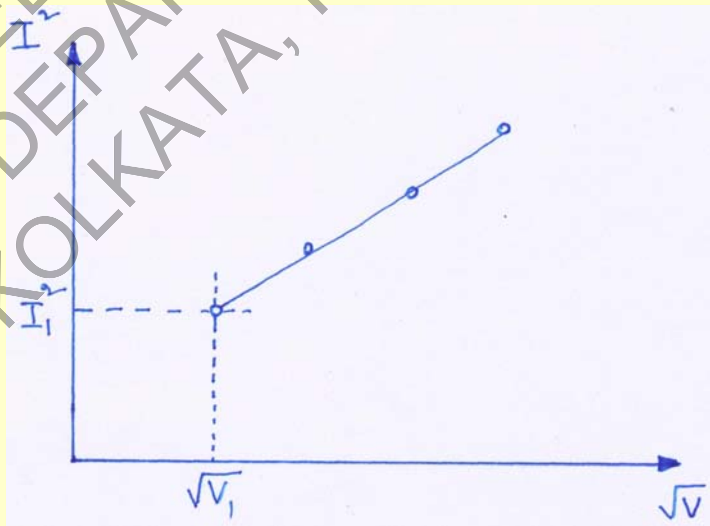


Measurement of Average velocity using CTA in "Manual Balance" Mode (contd…)

- ✓ **For accurate work, a given hot-wire probe must be calibrated in the fluid in which it is to be used.**

Measurement of Average Velocity using CTA in “Manual Balance” Mode (contd…)





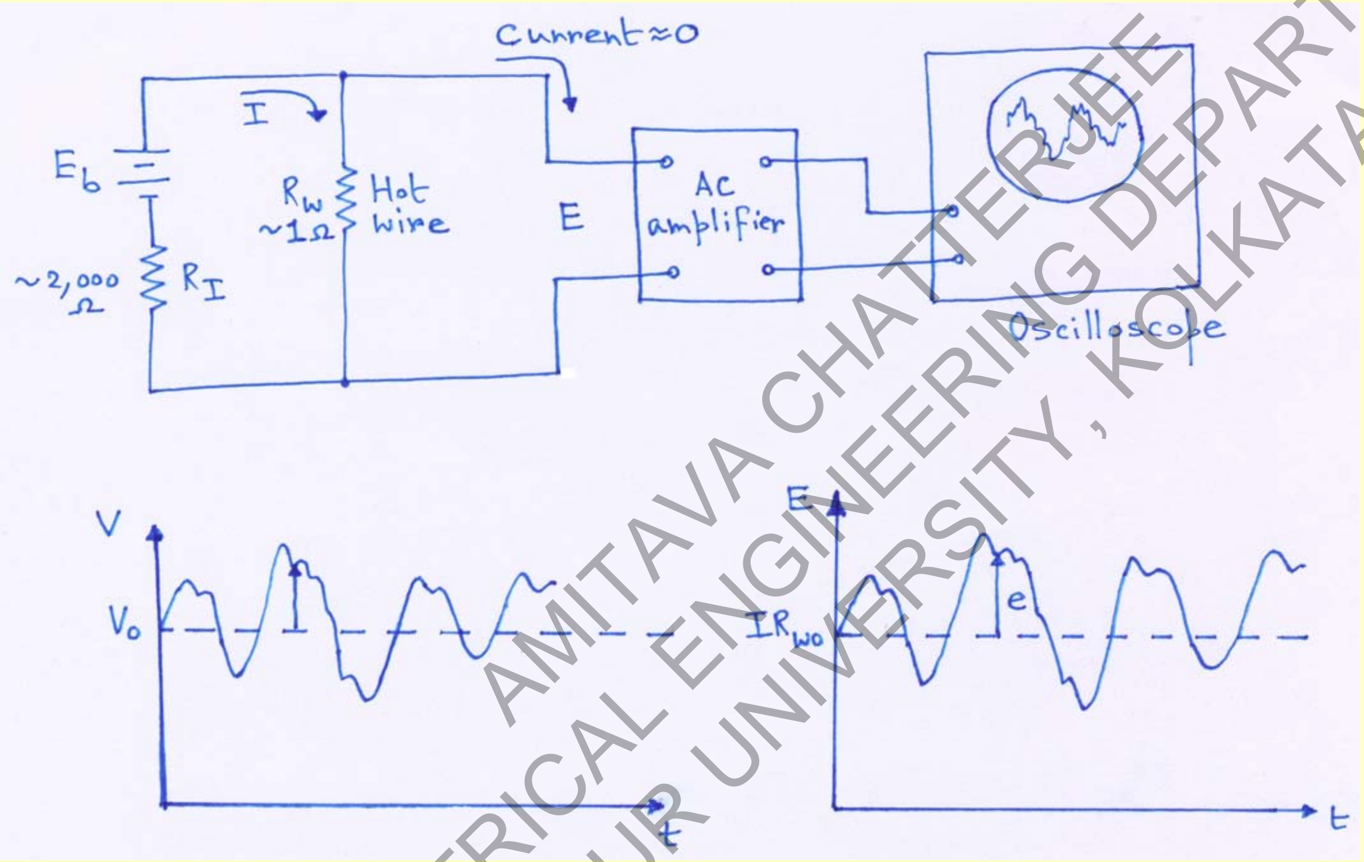
At the equilibrium condition:

$$I^2 = \frac{A(T_w - T_f)(C_0 + C_1\sqrt{V})}{R_w}$$

$$= C_2 + C_3\sqrt{V}$$

The calibration curve should be a straight line.

Measurement of Fluctuating Velocity using Constant Current Anemometer



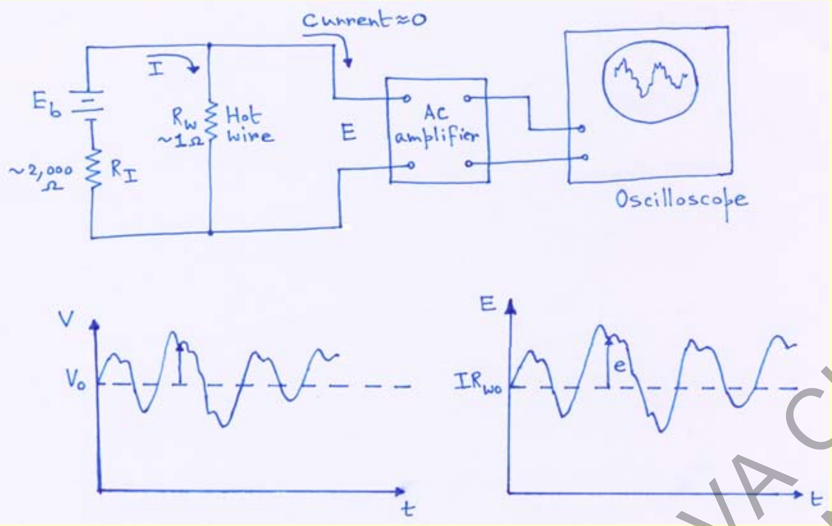
$$V = V_0 + v$$

$$R_w = R_{w0} + r_w$$

$$T_w = K_{tr}\left(R_{w0} + r_w\right)$$

- ✓ **The current can be assumed constant at *I*, even if R_w changes, since $R_I >> R_w$.**

Measurement of Fluctuating Velocity using Constant Current Anemometer



Electrical energy generated – energy lost by convection = energy stored in wire

Energy lost by convection:

$$A(T_w - T_f)\left(C_0 + C_1\sqrt{V}\right)dt$$

$$f(V) = f(V_0 + v) = C_0 + C_1\sqrt{V}$$

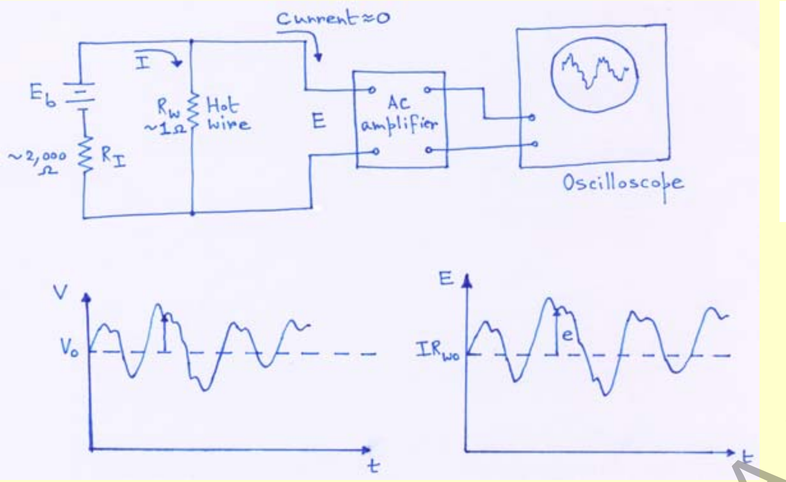
$$\approx f(V_0) + f'(V)\big|_{V=V_0}.h \approx \left(C_0 + C_1\sqrt{V_0}\right) + \frac{\partial f}{\partial V}\bigg|_{V=V_0}(V - V_0)$$

$$\Rightarrow C_0 + C_1\sqrt{V} \approx \left(C_0 + C_1\sqrt{V_0}\right) + K_v v$$

$$I^2(R_{w0} + r_w)dt - A(T_w - T_f)\left(C_0 + C_1\sqrt{V_0} + K_v v\right)dt = MCdT_w$$

M **= mass of wire**
C **= specific heat of wire**

Measurement of Fluctuating Velocity using Constant Current Anemometer



$$I^2R_{w0} + I^2 r_w - A\left[K_{tr}\left(R_{w0} + r_w\right) - T_f\right]\left(C_0 + C_1\sqrt{V_0} + K_v v\right) = MCK_{tr}\frac{dr_w}{dt}$$

Also, $I^2R_{w0} - A\left(K_{tr}R_{w0} - T_f\right)\left(C_0 + C_1\sqrt{V_0}\right) = 0$

and the term containing the product $r_w v$ of two small quantities can be neglected.

Voltage across R_w: $IR_w = I\left(R_{w0} + r_w\right) = E_0 + e$

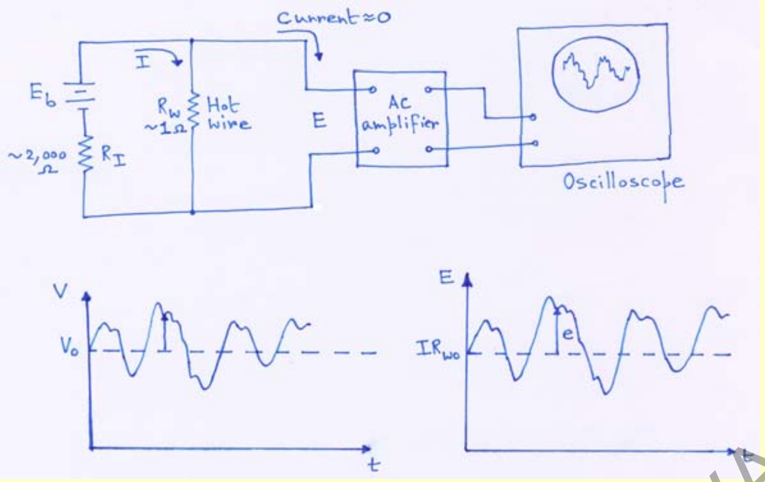
$$\frac{e(s)}{v(s)} = \frac{Ir_w(s)}{v(s)} = \frac{IAK_v\left(K_{tr}R_{w0} - T_f\right)}{I^2 - AK_{tr}\left(C_0 + C_1\sqrt{V_0}\right) - sK_{tr}MC} = \frac{K}{1 + s\tau}$$

where,

$$K = \frac{-K_v AI\left(K_{tr}R_{w0} - T_f\right)}{AK_{tr}\left(C_0 + C_1\sqrt{V_0}\right) - I^2}$$

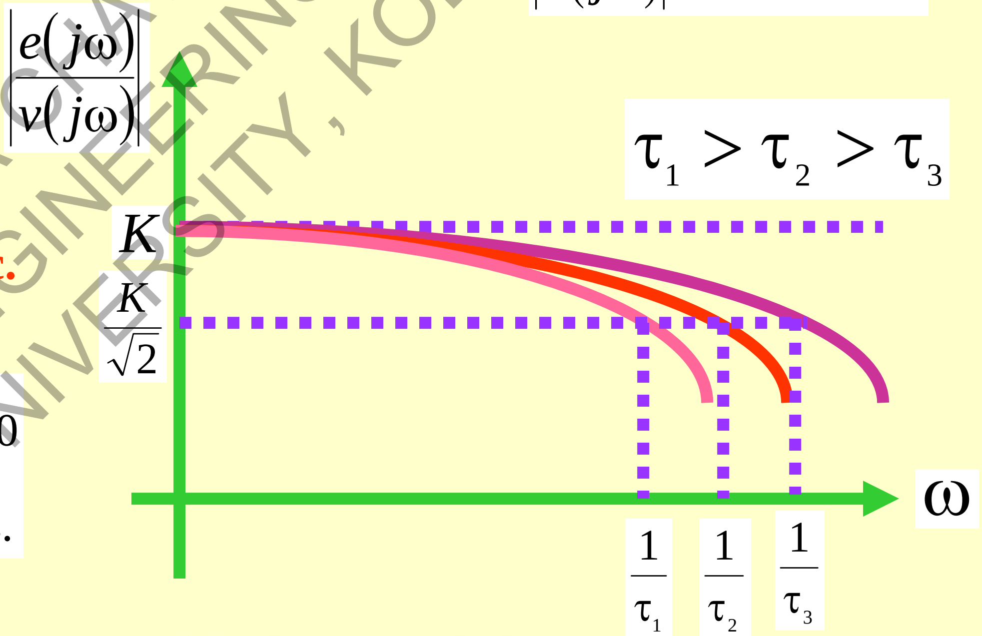
$$\tau = \frac{MCK_{tr}}{AK_{tr}\left(C_0 + C_1\sqrt{V_0}\right) - I^2}$$

Measurement of Fluctuating Velocity using Constant Current Anemometer



Frequency response: $\frac{e(j\omega)}{v(j\omega)} = \frac{K}{1 + j\omega\tau}$

Magnitude response: $\left|\frac{e(j\omega)}{v(j\omega)}\right| = \frac{K}{\sqrt{1+\omega^2\tau^2}}$



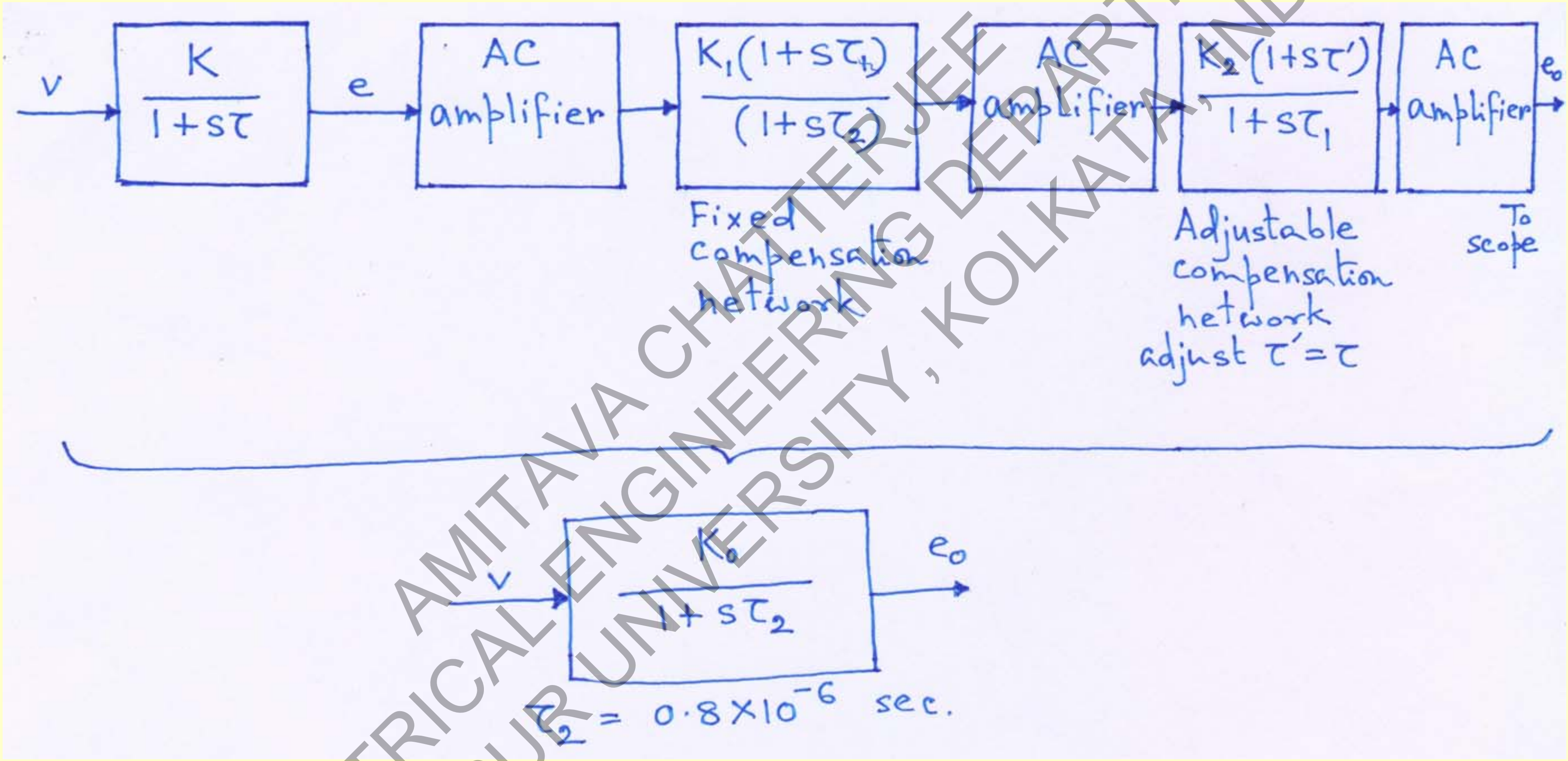
τ can not be made less than 0.001 sec.

Cut off frequency: $\omega' = \frac{1}{0.001} = 1000$

or, $f' \approx 160$ Hz.

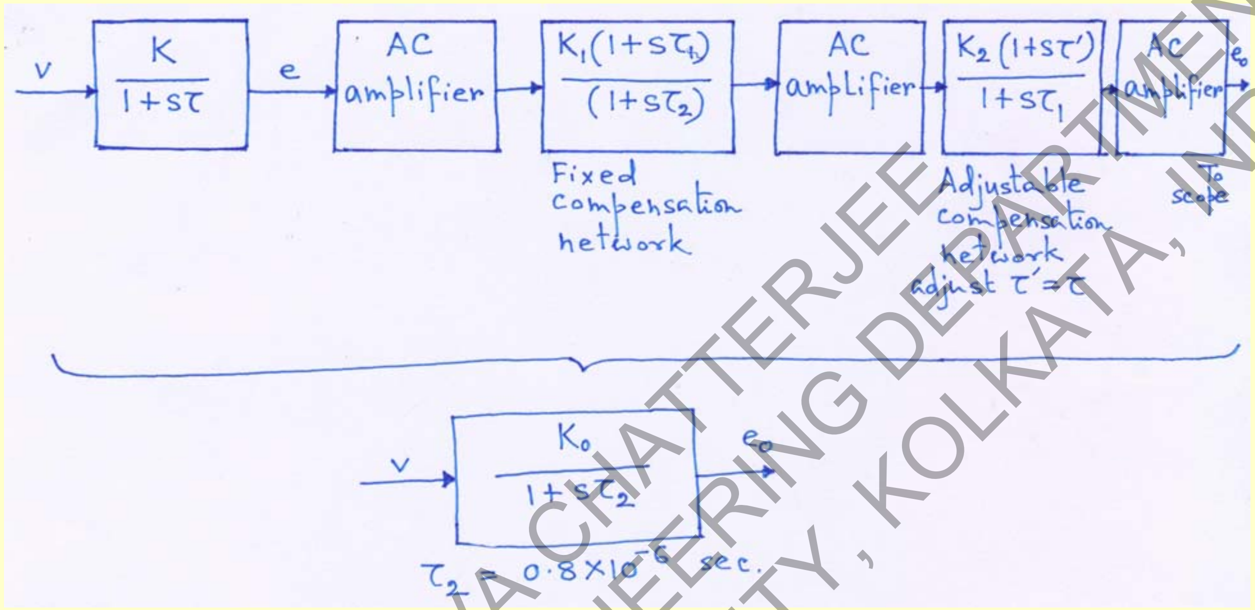
- ✓ **For turbulence studies, we have to add electrical dynamic compensation scheme.**

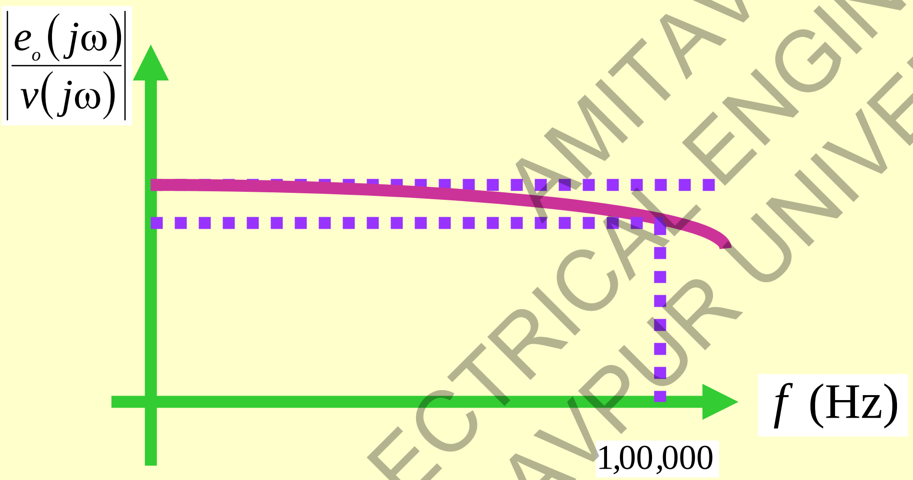
Electrical Dynamic Compensation

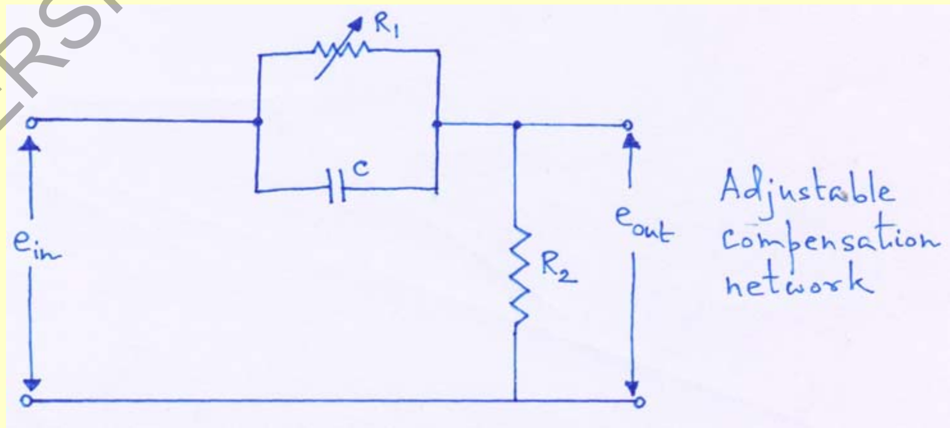


$\tau_2 = 0.8 \times 10^{-6}$ sec.

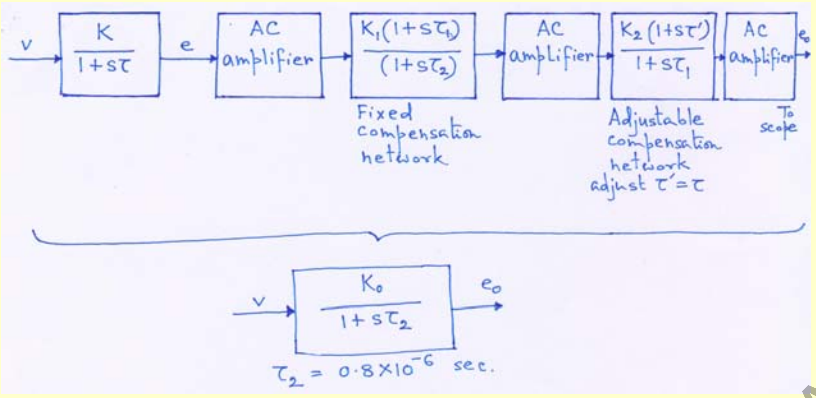
Electrical Dynamic Compensation

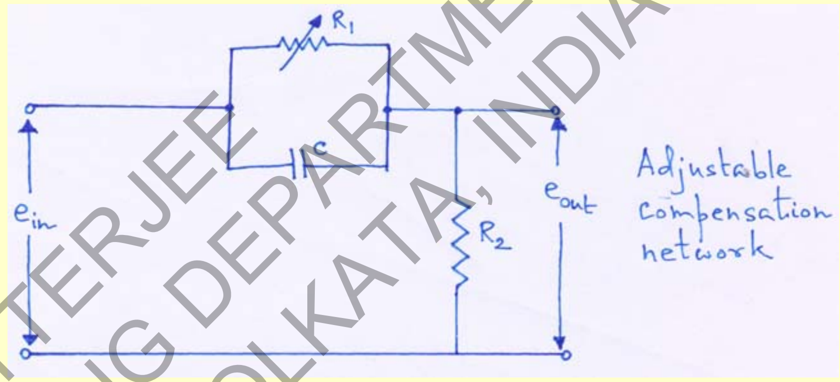






Electrical Dynamic Compensation





Transfer Function of the Compensation Network:

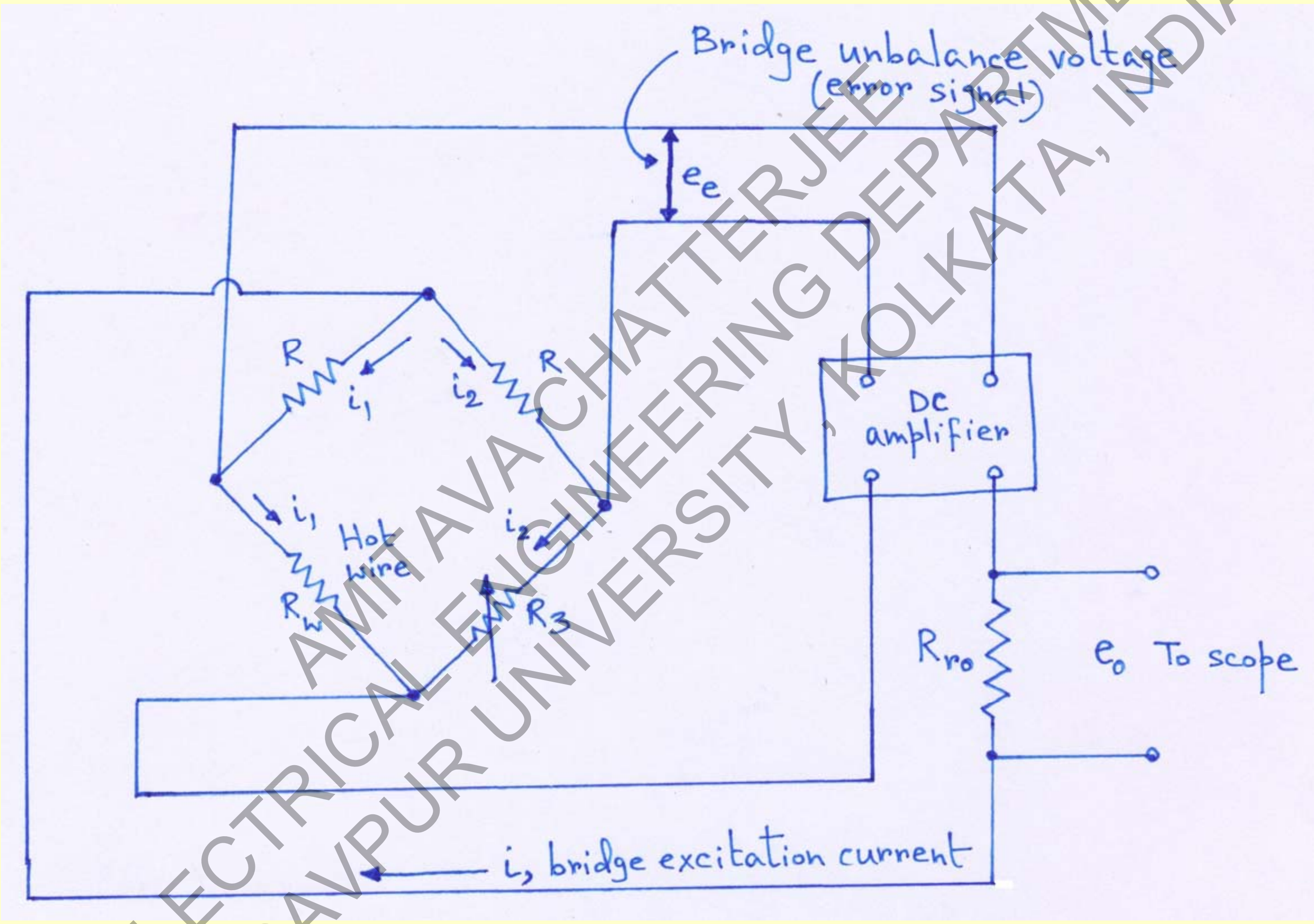
$$\frac{e_{out}(s)}{e_{in}(s)} = \frac{R_2}{R_2 + \left(\dfrac{R_1 \times \dfrac{1}{sC}}{R_1 + \dfrac{1}{sC}}\right)}$$

$$\frac{e_{out}(s)}{e_{in}(s)} = K\left(\frac{1+s\tau_{11}}{1+s\tau_{22}}\right)$$

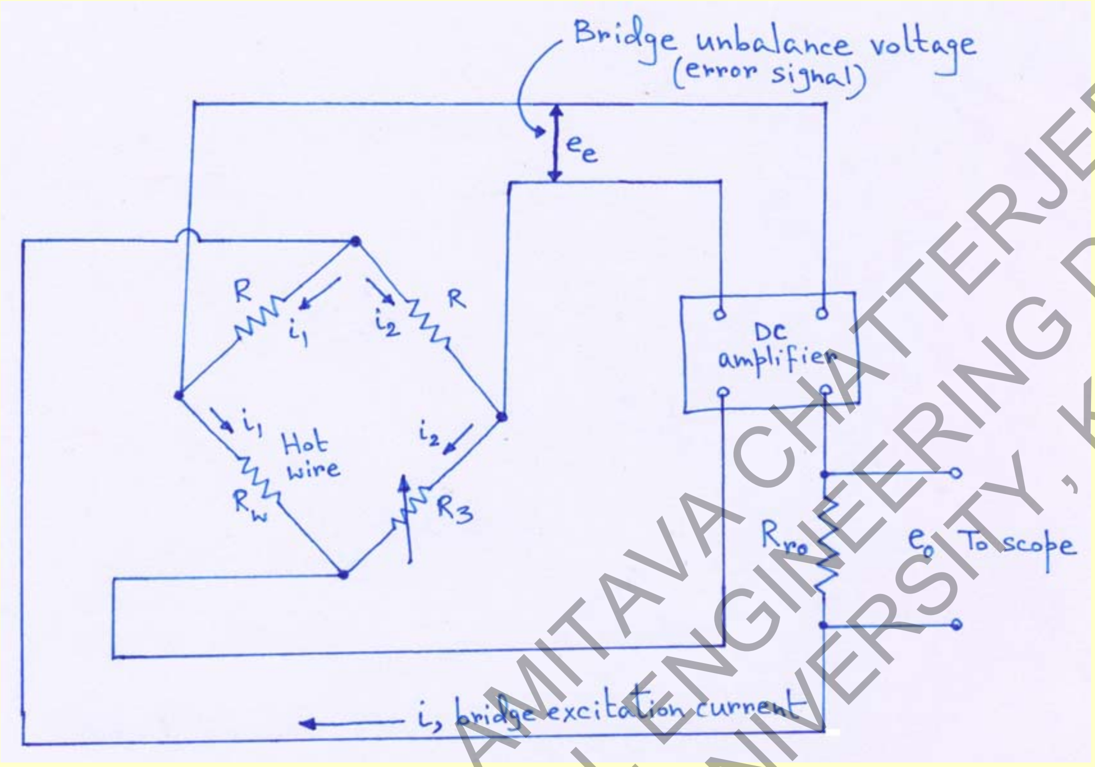
where:

$$K = \frac{R_2}{R_1 + R_2}, \tau_{11} = CR_1, \tau_{22} = C\frac{R_1 R_2}{R_1 + R_2}$$

Measurement of Fluctuating Velocity using Constant Temperature Anemometer



Measurement of Fluctuating Velocity using Constant Temperature Anemometer



- ✓ Here, the bridge balancing operation is made automatic by employing a feedback arrangement.

Fluctuating components of current:

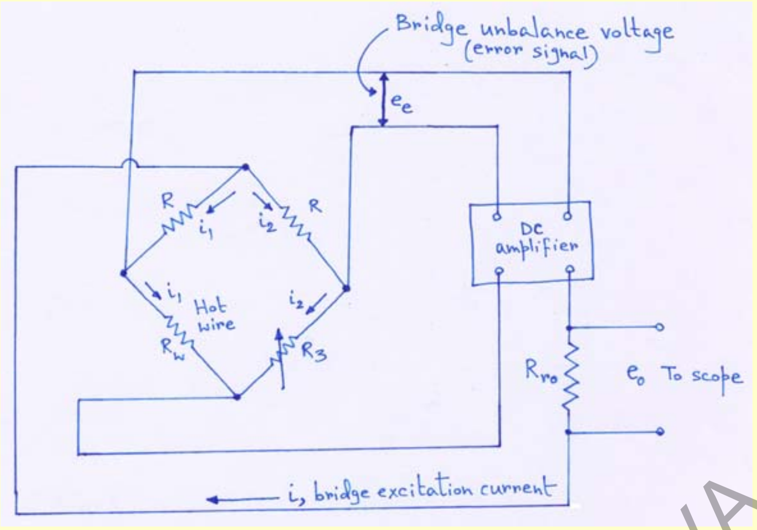
$$i_1 = \frac{R+R_3}{2R+R_w+R_3}i;$$

$$i_2 = \frac{R+R_w}{2R+R_w+R_3}i;$$

Energy balance equation: Electrical energy generated – energy lost by convection = energy stored in wire

$$(I_m+i_1)^2(R_{w0}+r_w) - A\left[K_{cr}(R_{w0}+r_w)-T_f\right]\left(C_0+C_1\sqrt{V_0}\right) = MCK_{cr}\frac{dr_w}{dt}$$ (velocity = V_0 = constant)

Measurement of Fluctuating Velocity using Constant Temperature Anemometer



Energy balance equation:

$$(I_m + i_1)^2 (R_{w0} + r_w) - A[K_{cr}(R_{w0} + r_w) - T_f](C_0 + C_1\sqrt{V_0}) = MCK_{cr}\frac{dr_w}{dt}$$

Initial steady condition:

$$I_m^2 R_{w0} = A(K_{cr} R_{w0} - T_f)(C_0 + C_1\sqrt{V_0})$$

and the terms containing product of two small quantities are neglected, i.e.:

$$2I_m i_1 r_w \quad \text{and} \quad i_1^2 (R_{w0} + r_w)$$

$$\frac{r_w(s)}{i_1(s)} = \frac{K_i}{1+s\tau} = \frac{K_e'/I_m}{1+s\tau};$$

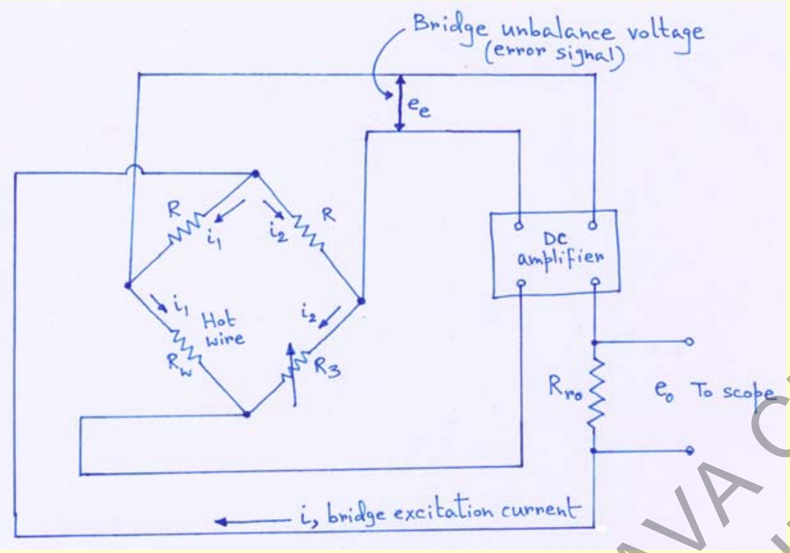
$$K_i = \frac{2I_m R_{w0}}{AK_{cr}(C_0 + C_1\sqrt{V_0}) - I_m^2};$$

$$\tau = \frac{MCK_{cr}}{AK_{cr}(C_0 + C_1\sqrt{V_0}) - I_m^2};$$

The effect of velocity on r_w:

$$\frac{r_w(s)}{v(s)} = \frac{K/I_m}{1+s\tau} \left(\because \frac{e(s)}{v(s)} = \frac{I_m r_w(s)}{v(s)} = \frac{K}{1+s\tau}\right)$$

Measurement of Fluctuating Velocity using Constant Temperature Anemometer



From theory of superposition, the total effect of i_1 and v on r_w:

$$(s\tau+1)r_w(s)=\frac{Kv(s)+K_e^{/}i_1(s)}{I_m}$$

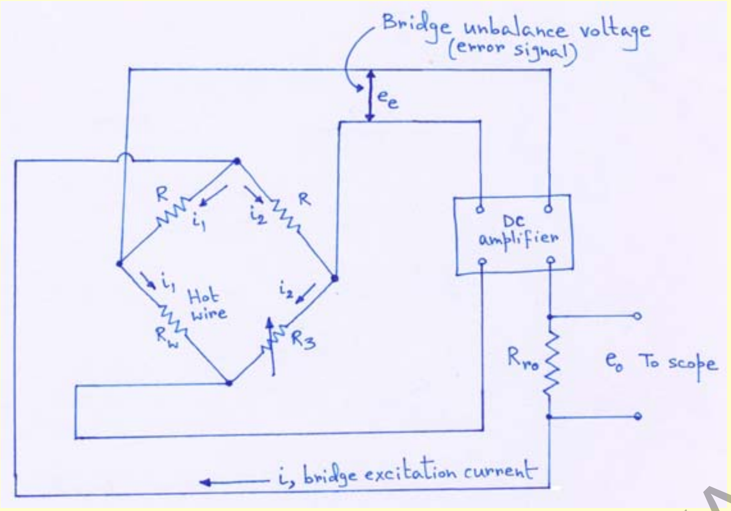
Now, a change in r_w causes a bridge unbalance voltage change as:

$$e_e(s)=\frac{I_m R}{R+R_{w0}}r_w(s)=-K_b r_w(s)$$

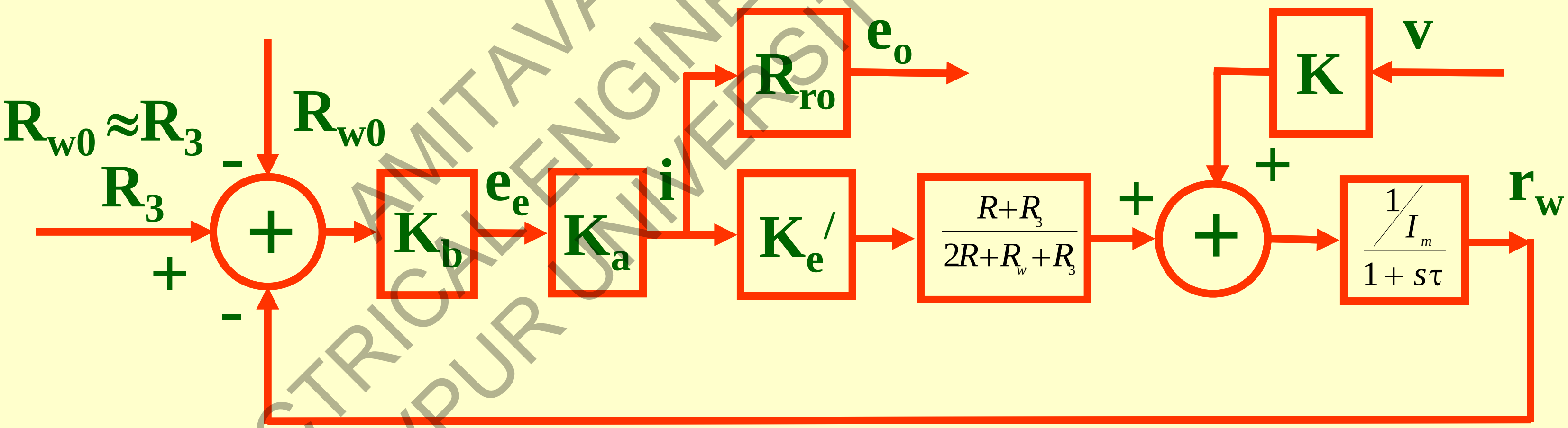
From d.c. amplifier: $i(s)=K_a e_e(s)$

✓ Using these relations, a block diagram representation can be developed.

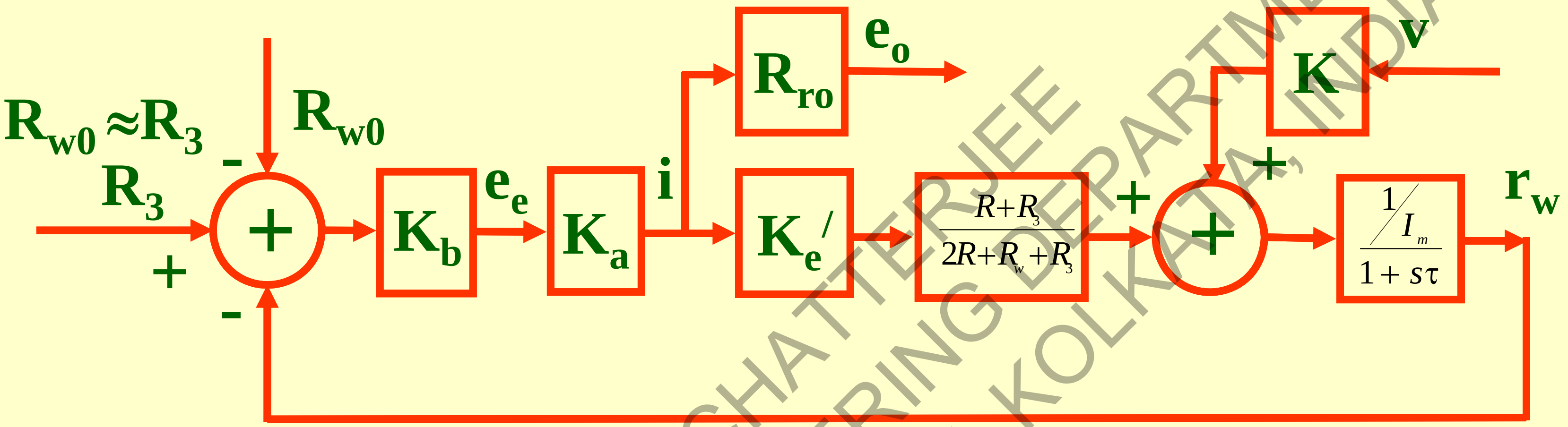
Measurement of Fluctuating Velocity using Constant Temperature Anemometer



Block Diagram Representation:



Measurement of Fluctuating Velocity using Constant Temperature Anemometer

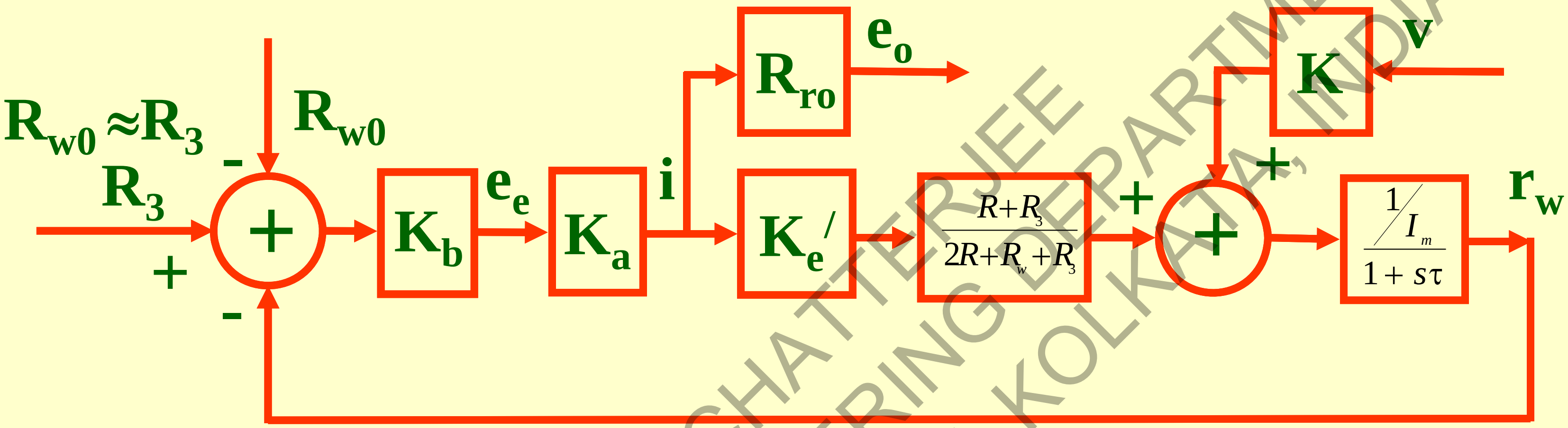


The relation between input *v* and output *e_o*:

$$\left(K_e^{/}\frac{e_o(s)}{R_{ro}}\cdot\frac{R+R_3}{2R+R_w+R_3}+Kv(s)\right)\frac{\left(1/I_m\right)\left(-K_bK_aR_{ro}\right)}{(1+s\tau)}=e_o(s)$$

$$\frac{e_o(s)}{v(s)}=\frac{K_{ct}}{1+s\tau_{ct}};\quad K_{ct}=-\frac{\left(KK_bK_aR_{ro}/I_m\right)}{\left[1+K_e^{/}\left(\frac{R+R_3}{2R+R_w+R_3}\right).\frac{K_bK_a}{I_m}\right]};\quad \tau_{ct}=\frac{\tau}{\left[1+K_e^{/}\left(\frac{R+R_3}{2R+R_w+R_3}\right).\frac{K_bK_a}{I_m}\right]}$$

Measurement of Fluctuating Velocity using Constant Temperature Anemometer



$$\frac{e_o(s)}{v(s)} = \frac{K_{ct}}{1+s\tau_{ct}};\quad K_{ct} = -\frac{\left(KK_bK_aR_{ro}/I_m\right)}{\left[1+K_e'\left(\frac{R+R_3}{2R+R_w+R_3}\right).\frac{K_bK_a}{I_m}\right]};\quad \tau_{ct} = \frac{\tau}{\left[1+K_e'\left(\frac{R+R_3}{2R+R_w+R_3}\right).\frac{K_bK_a}{I_m}\right]}$$

- ✓ K_{ct} and τ_{ct} can be taken as constants. This is because the term containing the factor $\left(\frac{R+R_3}{2R+R_w+R_3}\right)$ also contains K_a, which is very high. The fluctuations in R_w are quite small and hence the factor $\left(\frac{R+R_3}{2R+R_w+R_3}\right)$ can be taken as (1/2).

Comparison of CCAs and CTAs

- ✓ **The feedback type CTAs provide preferable solutions, compared to CCAs.**

- ✓ **In the CCAs, current must be set high enough, to heat the wire considerably above the fluid temperature.**

⬇

- ✓ **A sudden drop in flow velocity may cause the hot wire to burn out.**

- ✓ **The CTAs naturally overcome this problem, as the feedback arrangement automatically sets the wire current to maintain a safe and desired wire temperature, for every velocity.**

- ✓ **The time constant of the CTAs, τ_{ct}, is always much less than τ, since an amplifier with a high gain K_a is used.**

Thank You