Fuzzy Systems and Fuzzy Control: An Introduction

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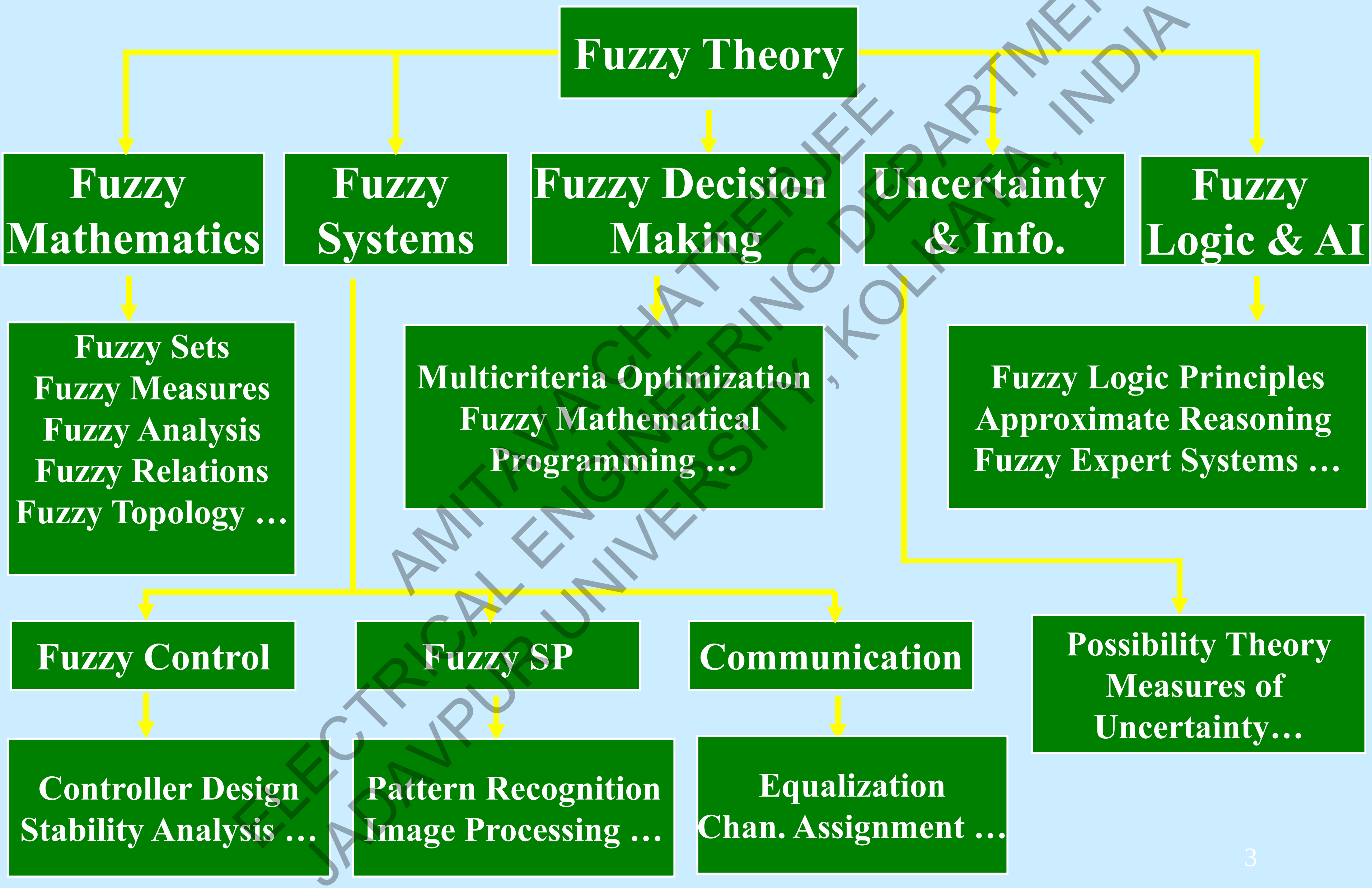
FUZZY LOGIC AND FUZZY SYSTEMS

Why do we need it?

- The theory of fuzzy logic is based on intuition and judgment.
- The requirement of a system model can be dispensed with.
- Fuzzy sets provide a smooth transition between members and non members.
- Relatively simple, fast and adaptive.
- Fuzzy systems can implement those design objectives which are difficult to be expressed mathematically and hence can be more conveniently expressed by linguistic or qualitative rules.

CLASSIFICATION OF FUZZY THEORY



CLASSICAL SETS vs. FUZZY SETS

Classical or Crisp Sets

A Crisp Set *C* is a set with a crisp boundary. For example, a crisp set *C* can be expressed as:

$C = \{x|\ x > 6\}$

In a crisp set *C* a member *x* either belongs to it or does not belong to it. Hence the membership value of a member *x* in C is either 0 or 1.

Fuzzy Sets

A Fuzzy Set *A* is a set without a crisp boundary. The transition from *belonging to a set* to *not belonging to a set* is gradual. Hence a fuzzy set *A* contains elements having varying degrees of membership in the set, ranging from 0 to 1.

FUZZY SETS

Fuzzy set *A* in *X*, can be defined as a set of ordered pairs:

$A = \{(x, \mu_A(x)) \mid x \in X\}$

X **= a collection of objects, denoted generically by *x*,**

$\mu_A(x)$ **= Membership Function (MF) of *x* in *A*,** $\mu_A(x) \in [0, 1]$.

X **is called the *Universe of Discourse* or *Universe* and the MF maps each element of *X* to a continuous membership value between 0 and 1.**

An alternative representation of Fuzzy set *A*:

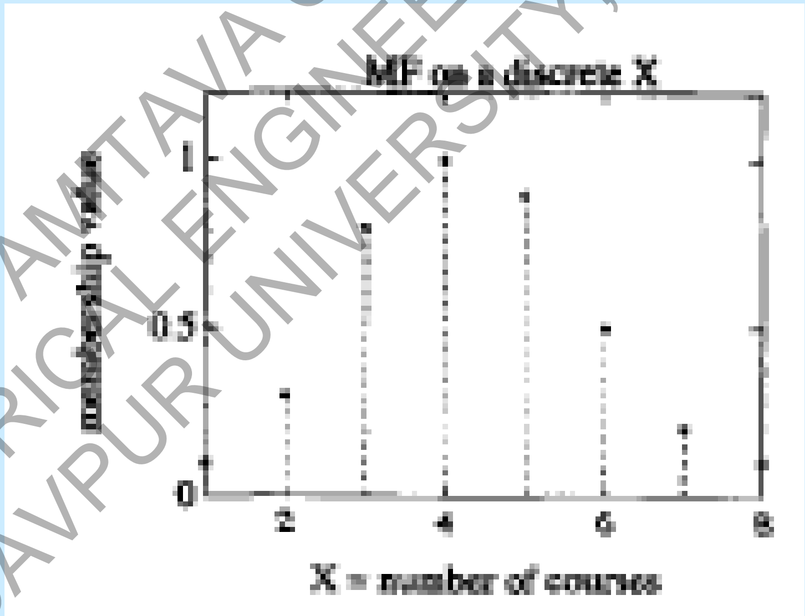
$$A = \sum_{x_i \in X} \mu_A(x_i)/x_i, \text{ if } X \text{ is discrete.}$$

$$A = \int \mu_A(x)/x, \text{ if } X \text{ is continous.}$$

FUZZY SETS WITH DISCRETE X

Let *X* = {1, 2, 3, 4, 5, 6, 7, 8} be the set of numbers of courses a student may take in a semester. Let fuzzy set *A* denote *the appropriate number of courses taken*. *A* can be given as:

A = {(1, 0.1), (2, 0.3), (3, 0.8), (4, 1), (5, 0.9), (6, 0.5), (7, 0.2), (8,0.1)}

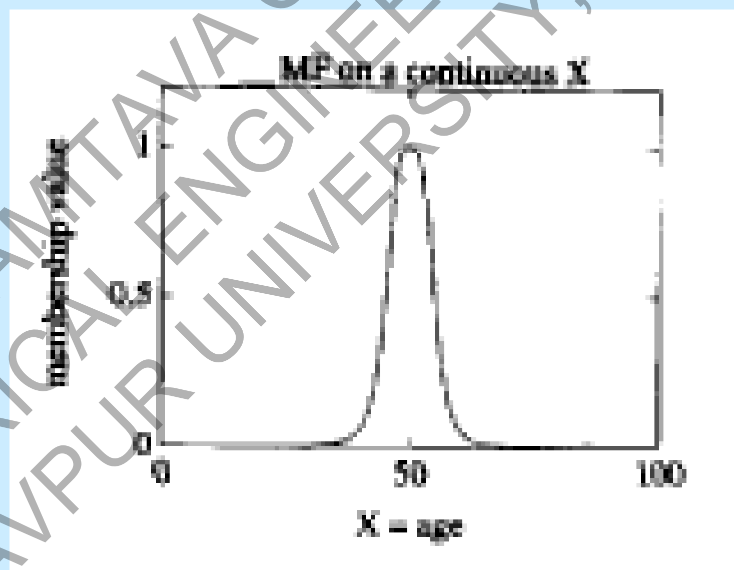


The Fuzzy Set

FUZZY SETS WITH CONTINUOUS X

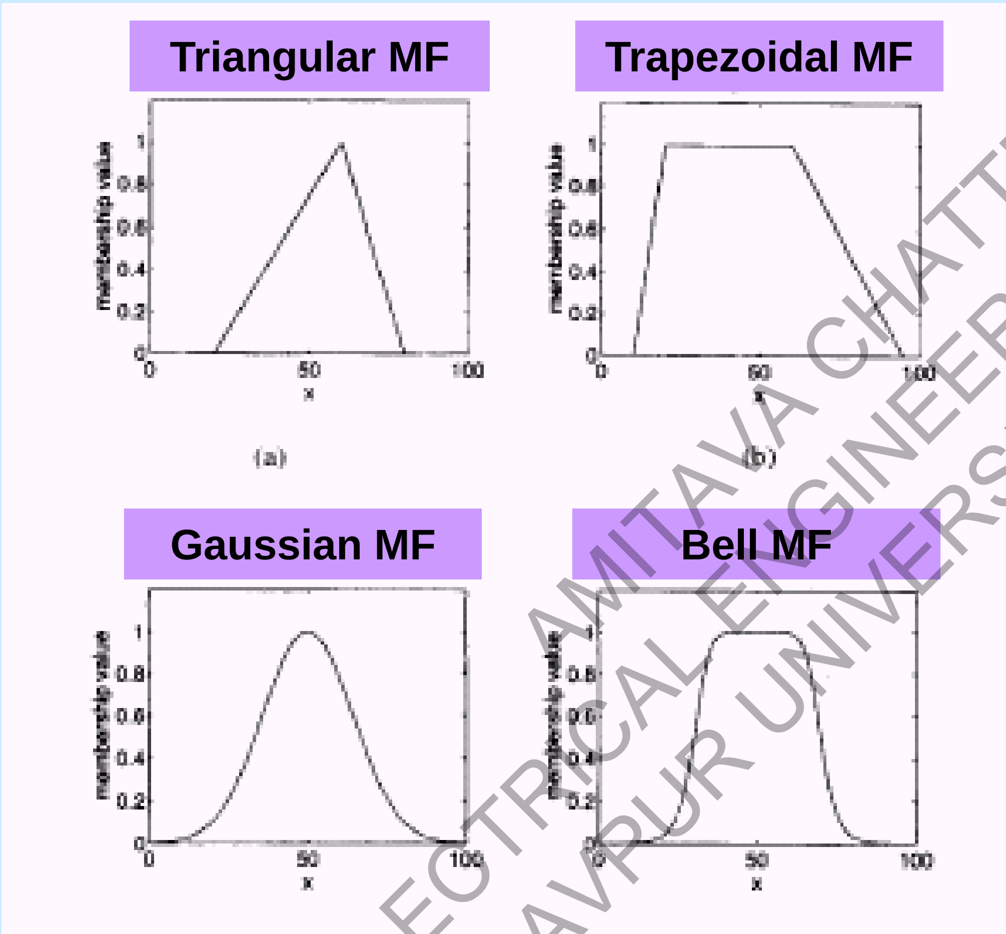
Let $X = R^+$ be the set of possible ages for human beings. Let fuzzy set B = *about 50 years old*. B can be expressed as:

$B = \{(x, \mu_B(x)) \mid x \in X\}$ where $\mu_B(x) = \dfrac{1}{1 + \left(\dfrac{x-50}{5}\right)^4}$



The Fuzzy Set

TYPICAL SHAPES OF MEMBERSHIP FUNCTIONS



$$triangle\,(x;a,b,c) = \max\left(\min\left(\frac{x-a}{b-a}, \frac{c-x}{c-b}\right), 0\right)$$

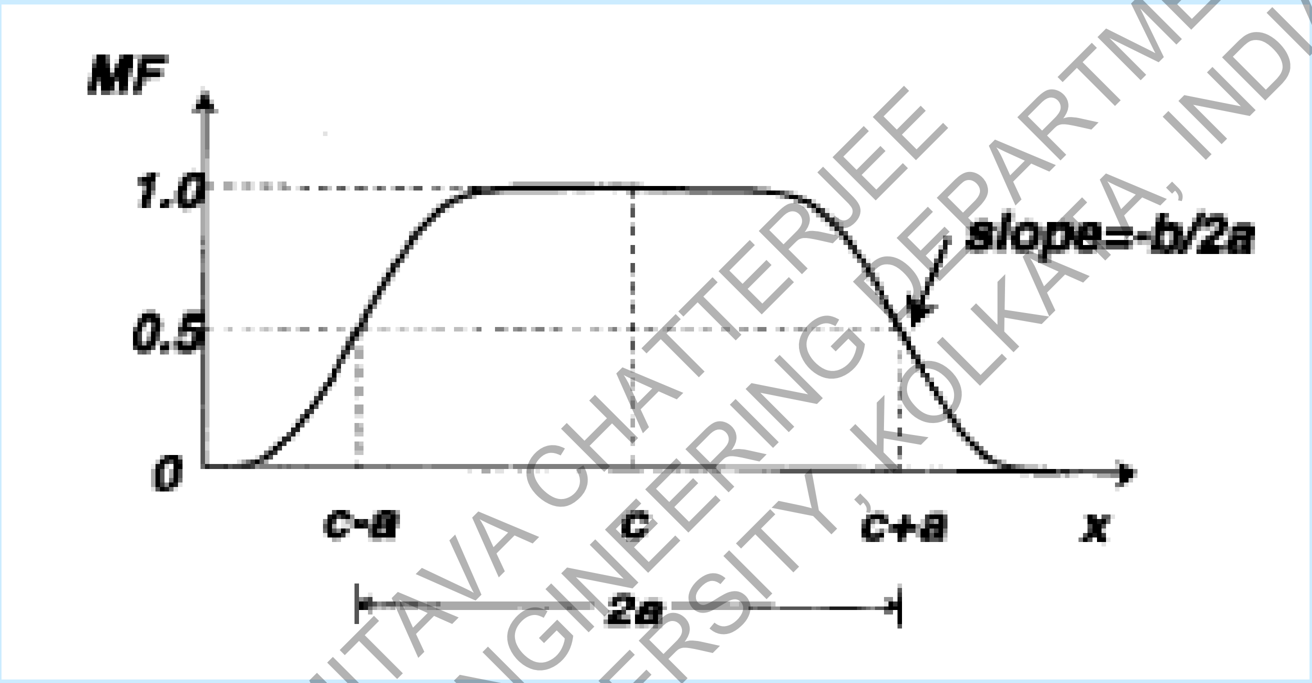
$$trapezoid\,(x;a,b,c,d) = \max\left(\min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right)$$

$$gaussian\,(x;\sigma,c) = e^{\{-[(x-c)/\sigma]^2\}}$$

$$bell\,(x;a,b,c) = \frac{1}{1+\left|\frac{x-c}{a}\right|^{2b}}$$

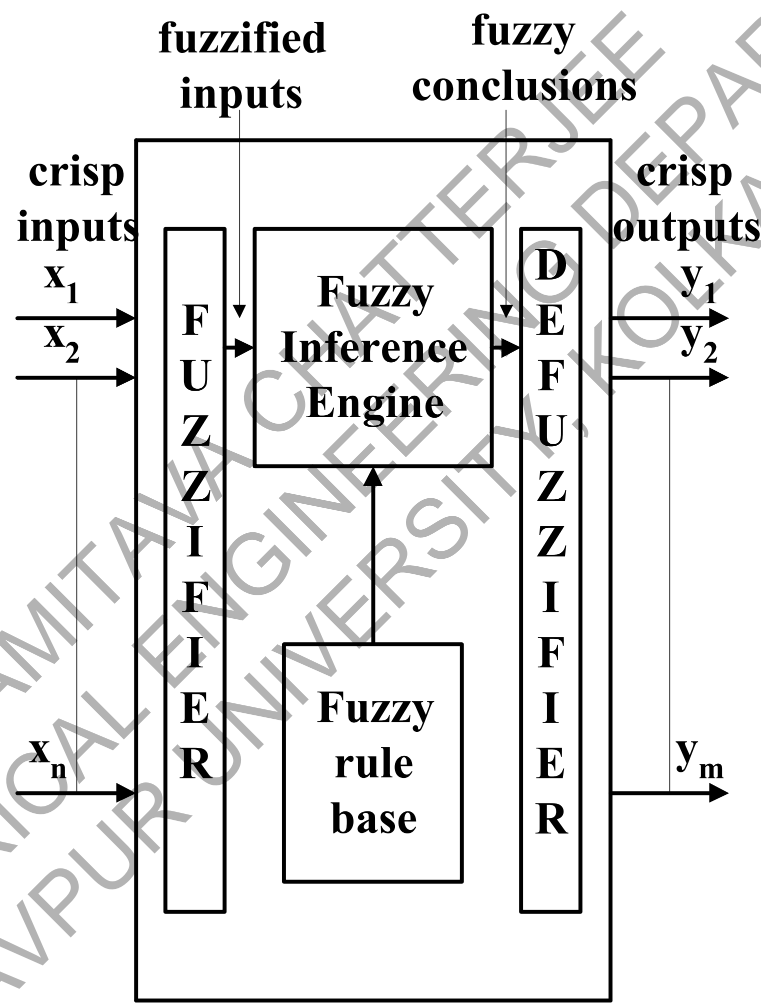
Triangular MFs **and *Trapezoidal MFs* are overwhelmingly popular for real-time implementations because of their simple formulae and computational efficiency.**

GENERALIZED BELL MEMBERSHIP FUNCTION



$$bell(x;a,b,c) = \frac{1}{1+\left|\frac{x-c}{a}\right|^{2b}}$$

A GENERIC FUZZY INFERENCE SYSTEM



FUZZY RULE-BASE

If-Then Rules:

MAMDANI-TYPE INFERENCING

Rule **: If x_1 is A_1 and x_2 is B_1, Then y is C_1,**

Rule **: If x_1 is A_2 and x_2 is B_2, Then y is C_2.**

SUGENO-TYPE INFERENCING

Rule **: If x_1 is A_1 and x_2 is B_1, Then $f_1 = r_1$,**

Rule **: If x_1 is A_2 and x_2 is B_2, Then $f_2 = r_2$.**

DEFUZZIFICATION STRATEGIES

MAMDANI-TYPE INFERENCING

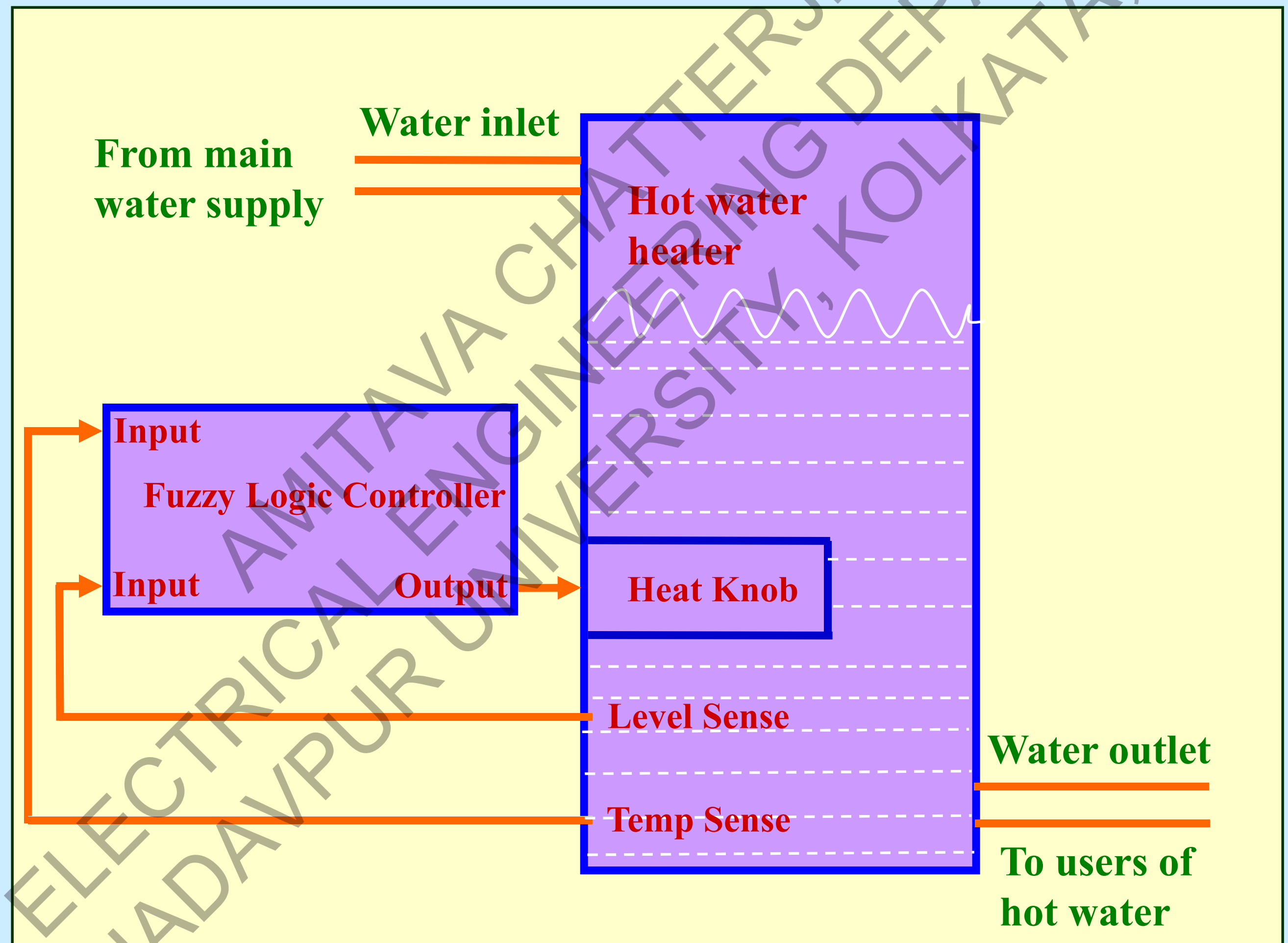
- Center-of-gravity / Center-of-area method
- Center-of-sums method
- Height method
- Center-of-largest-area method
- First-of-maxima method
- Middle-of-maxima method

SUGENO-TYPE INFERENCING

- Weighted average method

AN EXAMPLE

Fuzzy Control of a Water Heater with Mamdani-type inferencing



Fuzzy Control of a Water Heater (contd.)

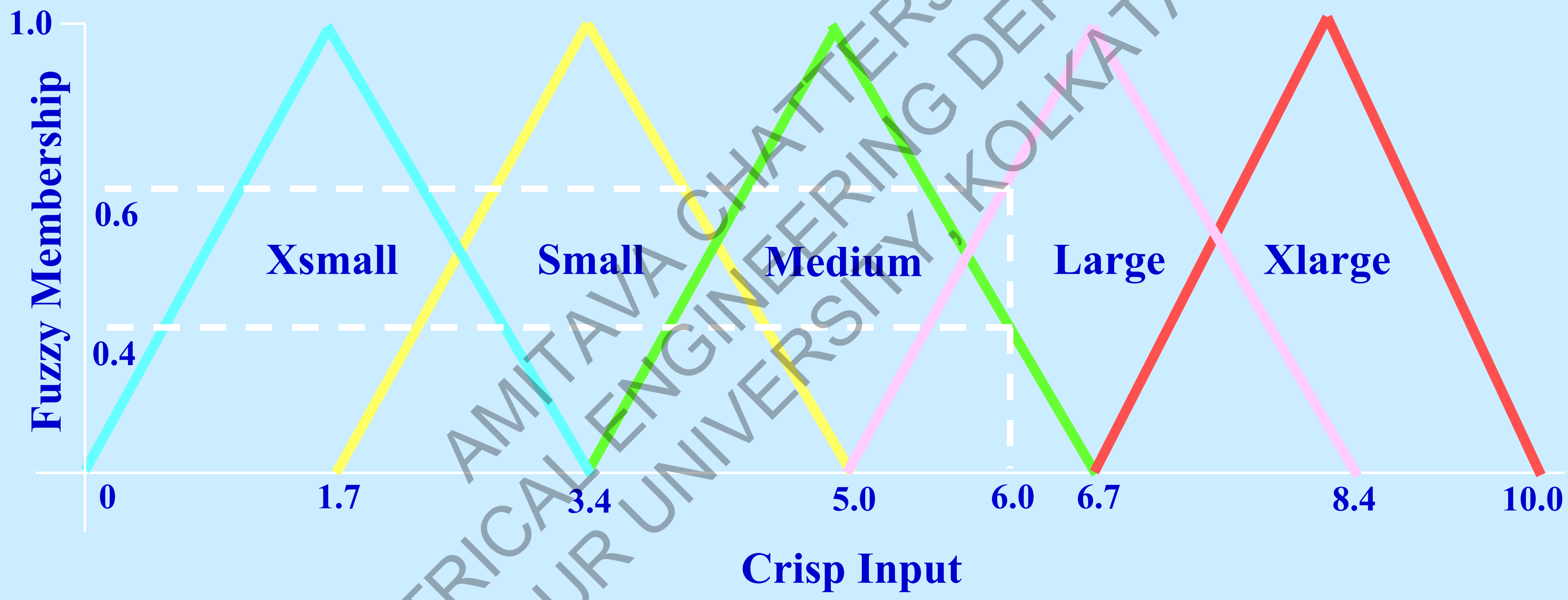
STEP 1: DEFINE INPUTS AND OUTPUTS

Variable	Minimum Value	Maximum Value
LevelSense	0	10
TempSense	0	125
HeatKnob	0	10

Universe of discourse for inputs and output

Fuzzy Control of a Water Heater (contd.)

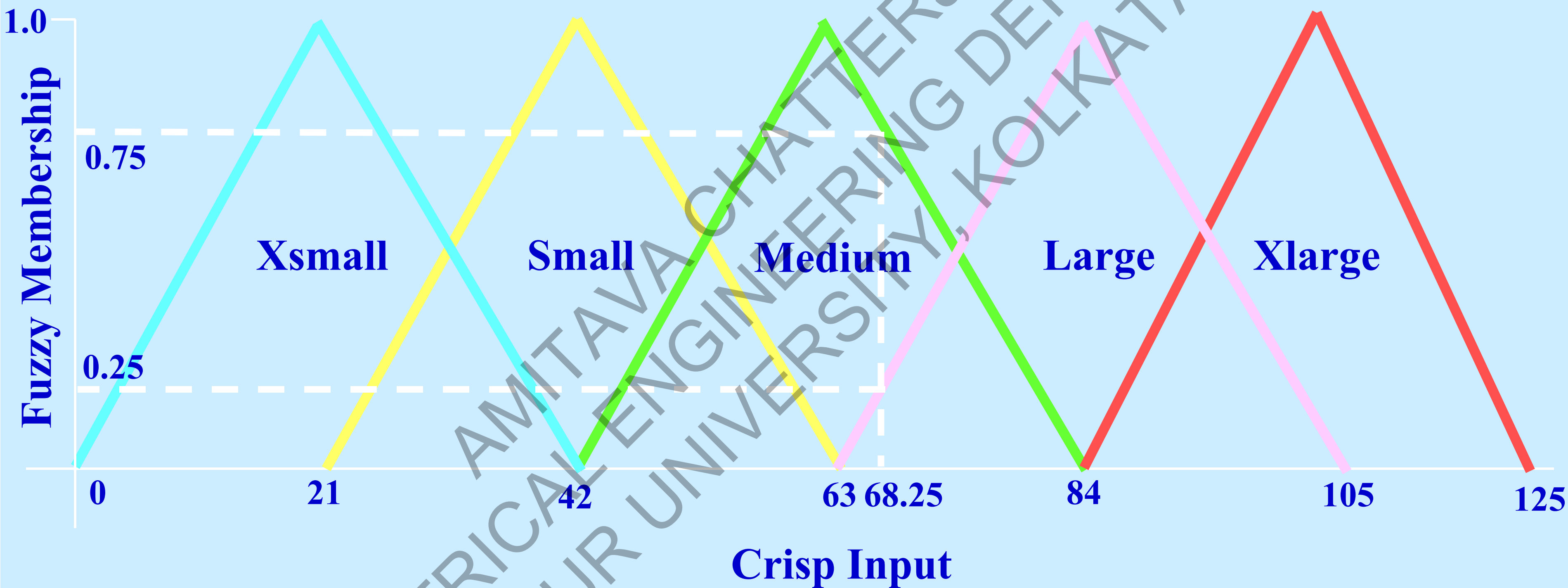
STEP 2: FUZZIFY THE INPUTS



Fuzzy membership functions for LevelSense

Fuzzy Control of a Water Heater (contd.)

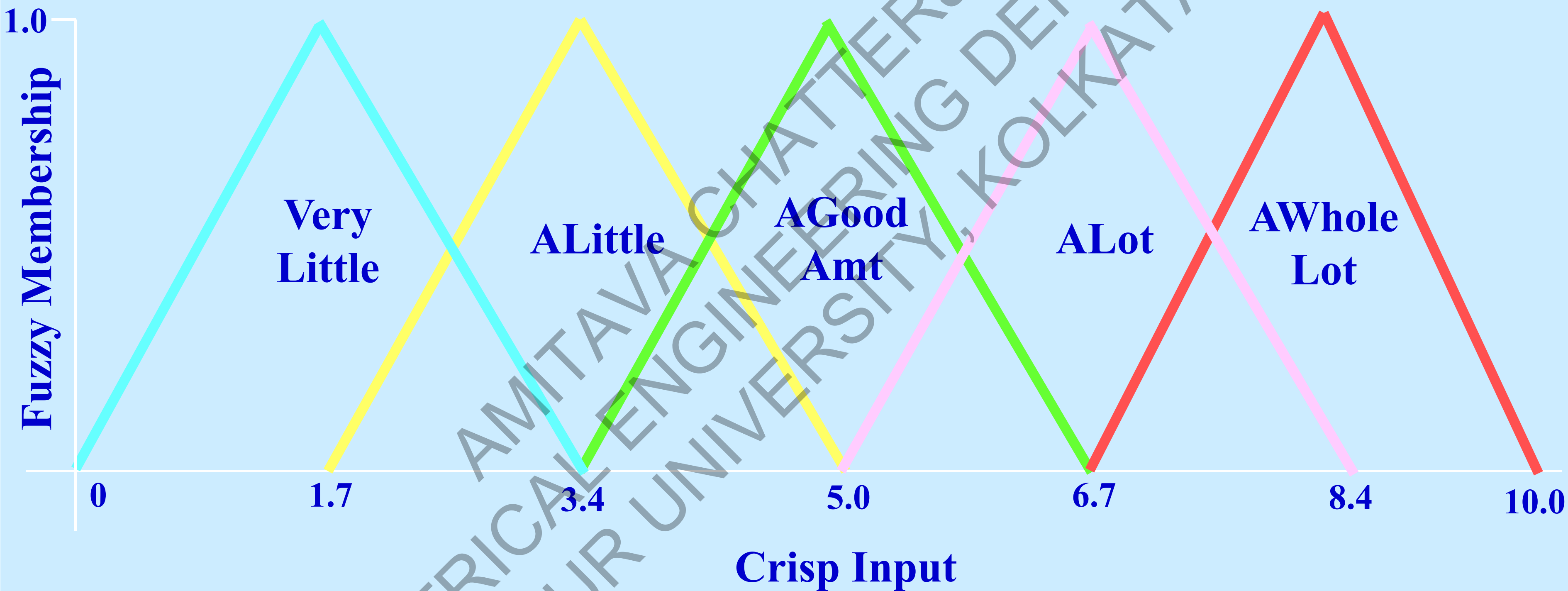
STEP 2: FUZZIFY THE INPUTS (contd.)



Fuzzy membership functions for TempSense

Fuzzy Control of a Water Heater (contd.)

STEP 3: SETUP FUZZY MEMBERSHIP FUNCTIONS FOR THE OUTPUT



Fuzzy membership functions for HeatKnob

Fuzzy Control of a Water Heater (contd.)

STEP 4: CREATE A FUZZY IF-THEN RULE BASE

TempSense → LevelSense ↓	Xsmall	Small	Medium	Large	Xlarge
Xsmall	AGoodAmt	ALittle	VeryLittle		
Small	ALot	AGoodAmt	VeryLittle	VeryLittle	
Medium	AWholeLot	ALot	AGoodAmt	VeryLittle	
Large	AWholeLot	ALot	ALot	ALittle	
Xlarge	AWholeLot	ALot	ALot	AGoodAmt	

Fuzzy rule base for output HeatKnob

Fuzzy Control of a Water Heater (contd.)

STEP 5: DEFUZZIFY THE OUTPUTS

Four rules are activated:

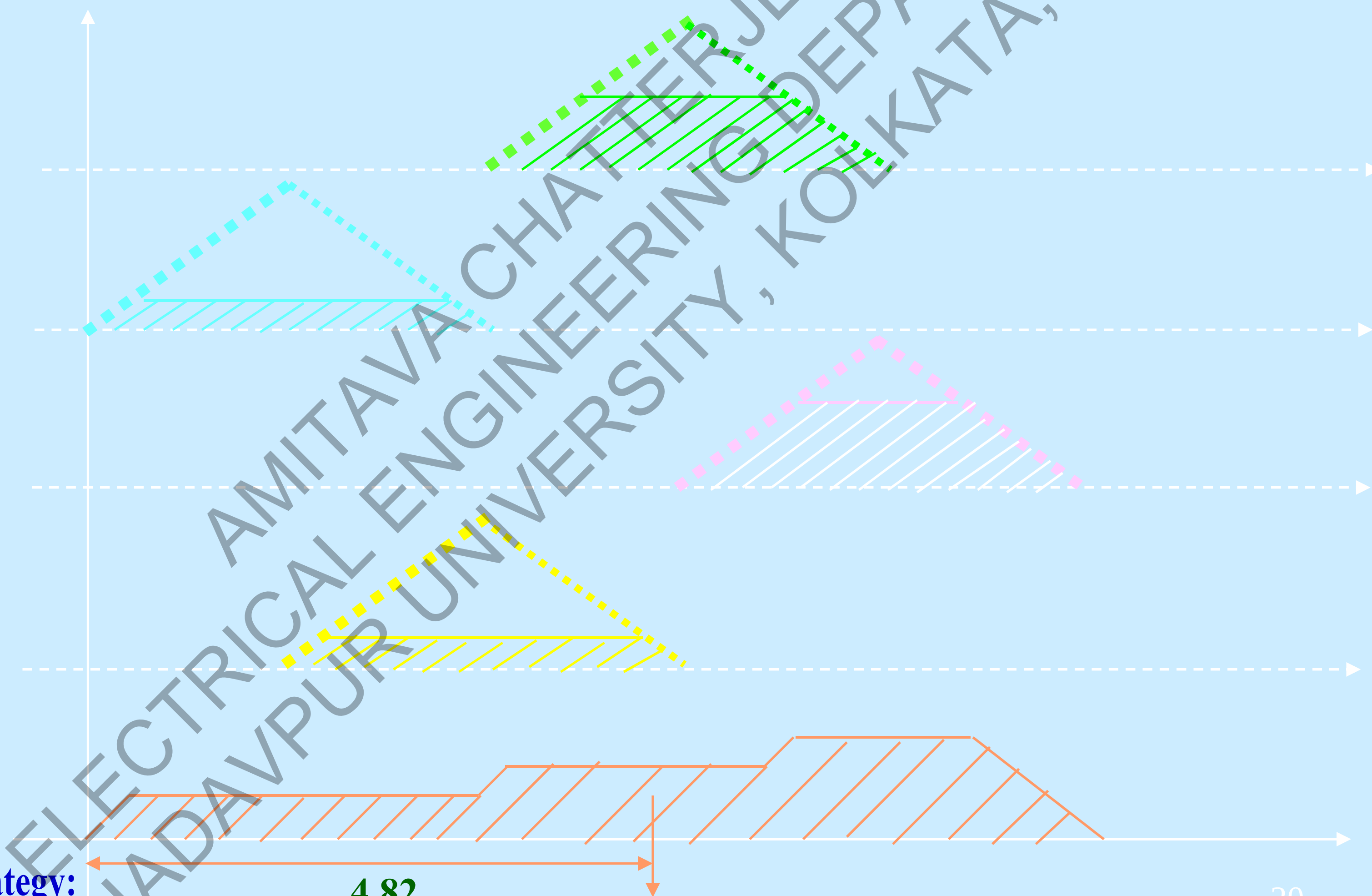
- **TempSense = Medium (0.75) AND LevelSense = Medium (0.4)**
- **TempSense = Large (0.25) AND LevelSense = Medium (0.4)**
- **TempSense = Medium (0.75) AND LevelSense = Large (0.6)**
- **TempSense = Large (0.25) AND LevelSense = Large (0.6)**

Activated fuzzified consequence with firing strength:

- **HeatKnob = AGoodAmt with firing strength = (0.75) ∧ (0.4) = (0.4)**
- **HeatKnob = VeryLittle with firing strength = (0.25) ∧ (0.4) = (0.25)**
- **HeatKnob = ALot with firing strength = (0.75) ∧ (0.6) = (0.6)**
- **HeatKnob = ALittle with firing strength = (0.25) ∧ (0.6) = (0.25)**

Fuzzy Control of a Water Heater (contd.)

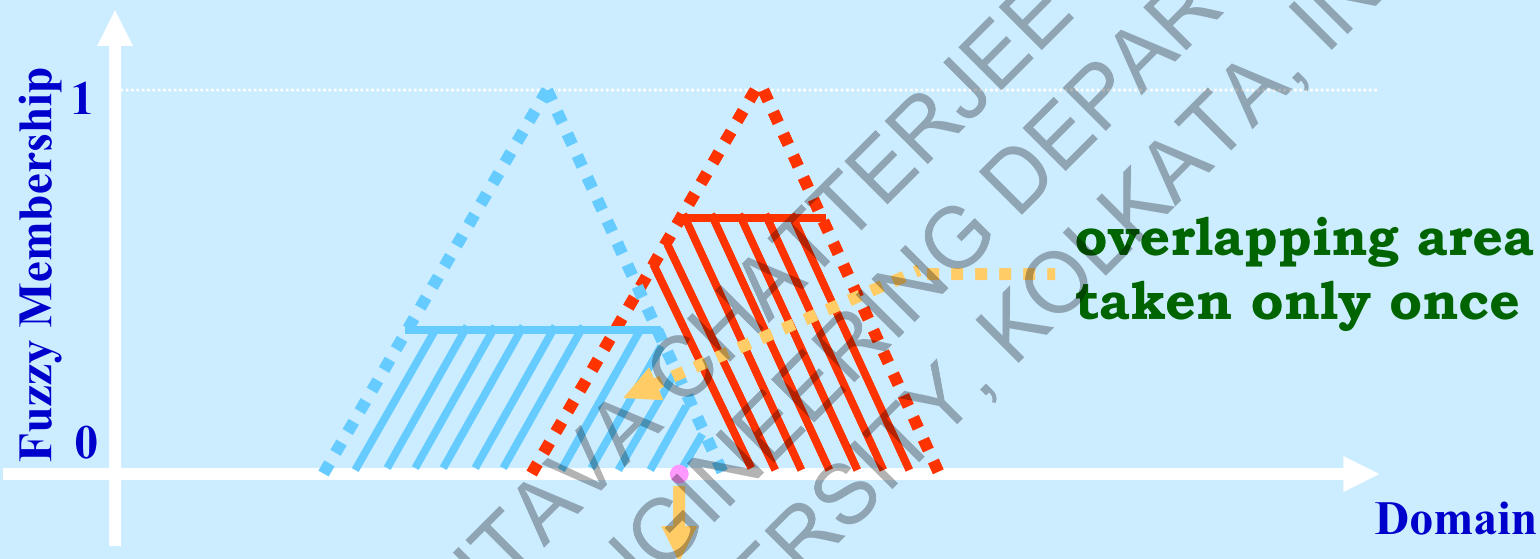
STEP 6: DETERMINE THE CRISP OUTPUT



Defuzzification Strategy: Center of Gravity

DEFUZZIFICATION STRATEGIES

CENTER-OF-GRAVITY/AREA METHOD



Graphical Representation of COG/COA Method.

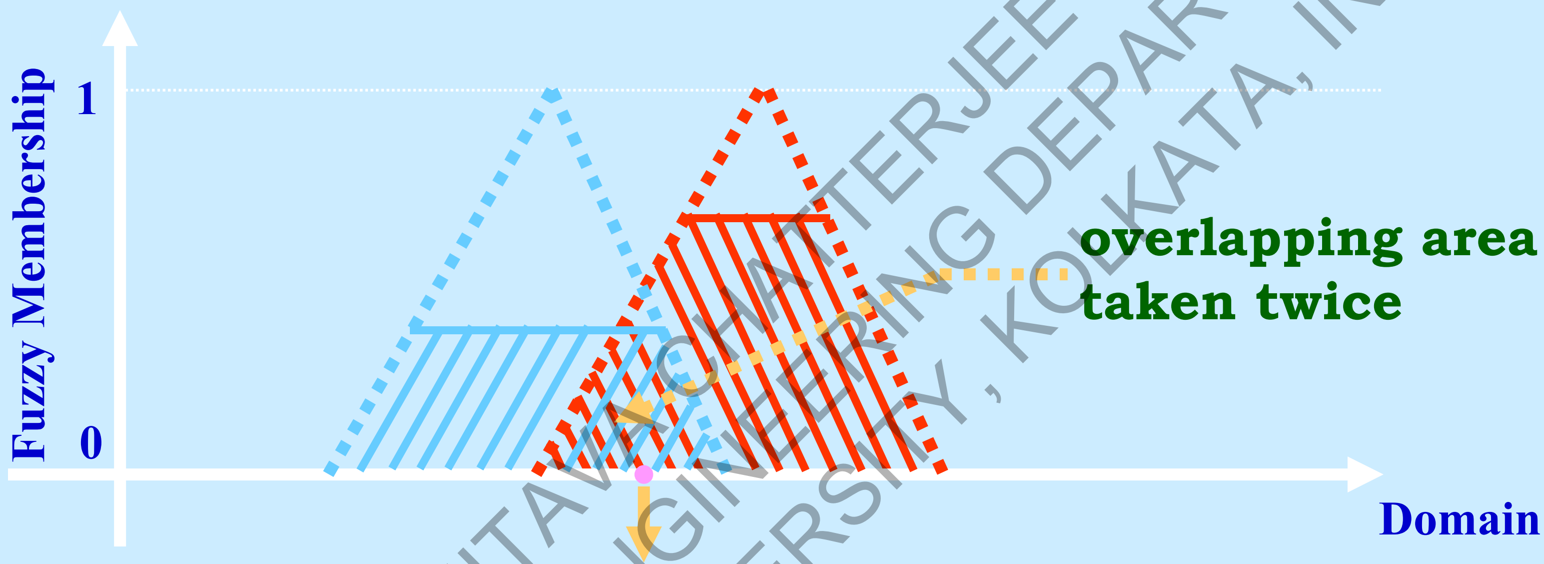
Crisp value of output y^{crisp}:

$$y^{crisp} = \frac{\int_{Y_{domain}} y \cdot \mu_Y(y)\,dy}{\int_{Y_{domain}} \mu_Y(y)\,dy} = \frac{\int_{Y_{domain}} y \cdot \max_k \mu_{CLY^{(k)}}(y)\,dy}{\int_{Y_{domain}} \max_k \mu_{CLY^{(k)}}(y)\,dy}$$

$\mu_{CLY^{(k)}}(y)$: **membership value of y in the clipped kth fuzzy set $CLY^{(k)}$**

DEFUZZIFICATION STRATEGIES

CENTER-OF-SUMS METHOD



Graphical Representation of COS Method.

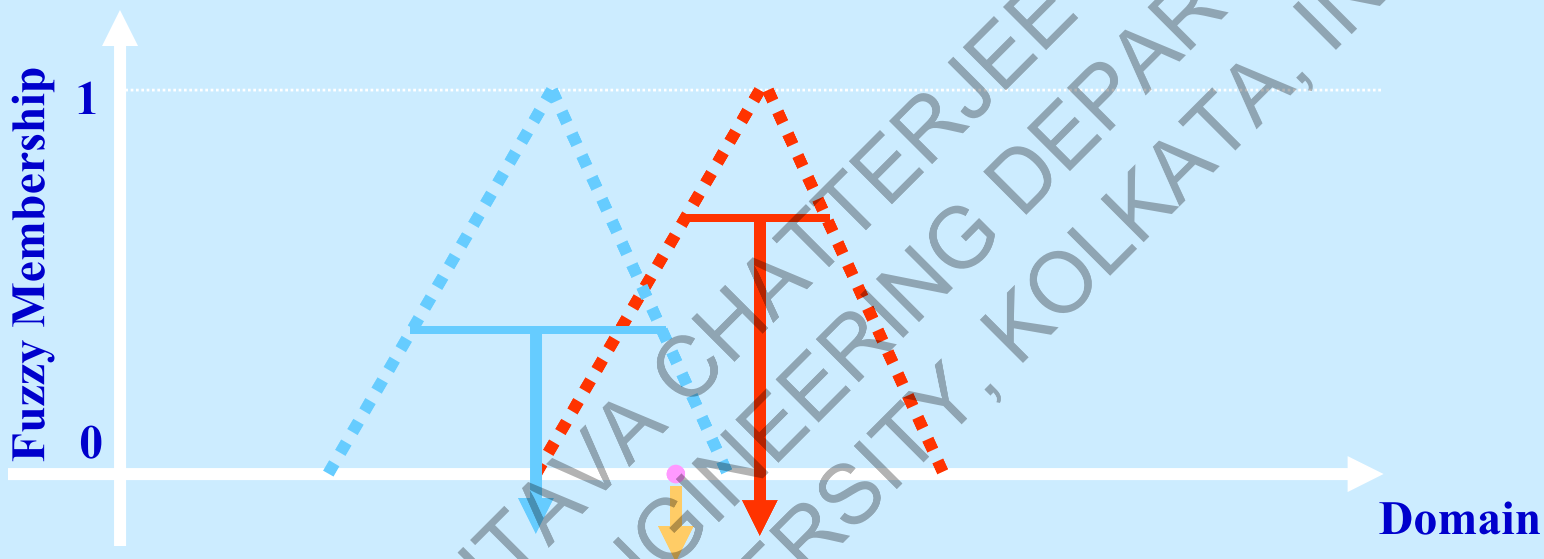
Crisp value of output y^{crisp}:

$$y^{crisp} = \frac{\int_{Y_{domain}} y \cdot \sum_{k=1}^{n} \mu_{CLY^{(k)}}(y)dy}{\int_{Y_{domain}} \sum_{k=1}^{n} \mu_{CLY^{(k)}}(y)dy}$$

$\mu_{CLY^{(k)}}(y)$: membership value of y in the clipped kth fuzzy set $CLY^{(k)}$

DEFUZZIFICATION STRATEGIES

HEIGHT METHOD



Graphical Representation of Height Method.

Crisp value of output y^{crisp}: $$y^{crisp} = \frac{\sum_{k=1}^{m} c^{(k)} f_k}{\sum_{k=1}^{m} f_k}$$

f_k: **height of the clipped** ***k*****th output fuzzy set** $CLY^{(k)}$

$c^{(k)}$: **crisp value of the output** y **corresponding to the peak value of the** ***k*****th output fuzzy set** $LY^{(k)}$

FUZZY SYSTEMS WITH SUGENO-TYPE INFERENCING

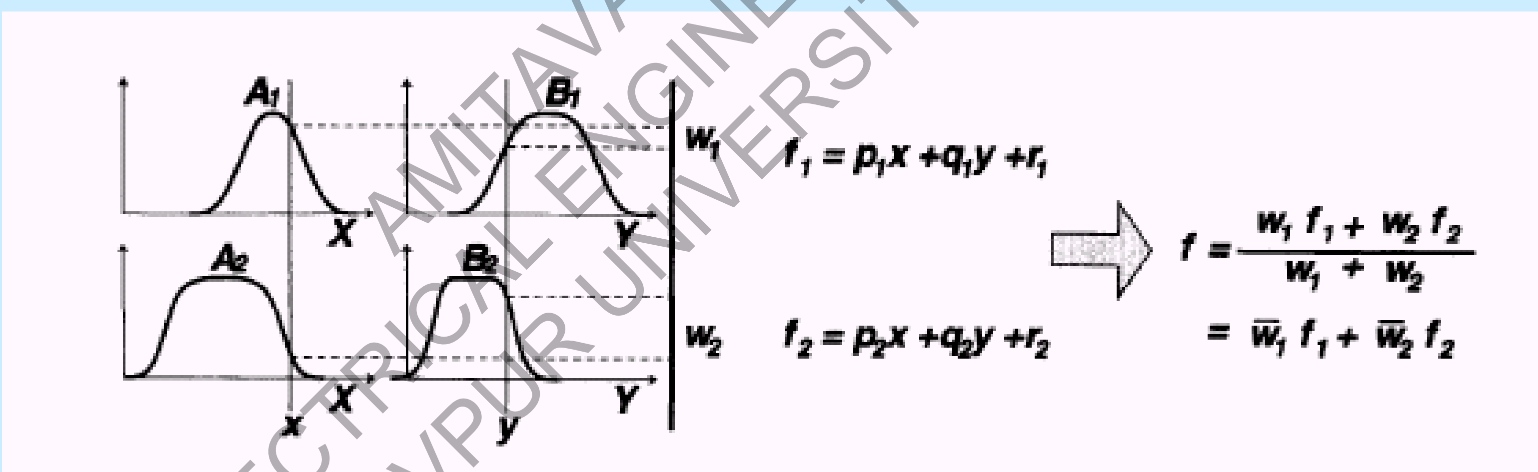
First-order Sugeno Model

Rule **1 :** **If** x **is** A_1 **and** y **is** B_1,

Then $f_1 = p_1 x + q_1 y + r_1$,

Rule **2 :** **If** x **is** A_2 **and** y **is** B_2,

Then $f_2 = p_2 x + q_2 y + r_2$.

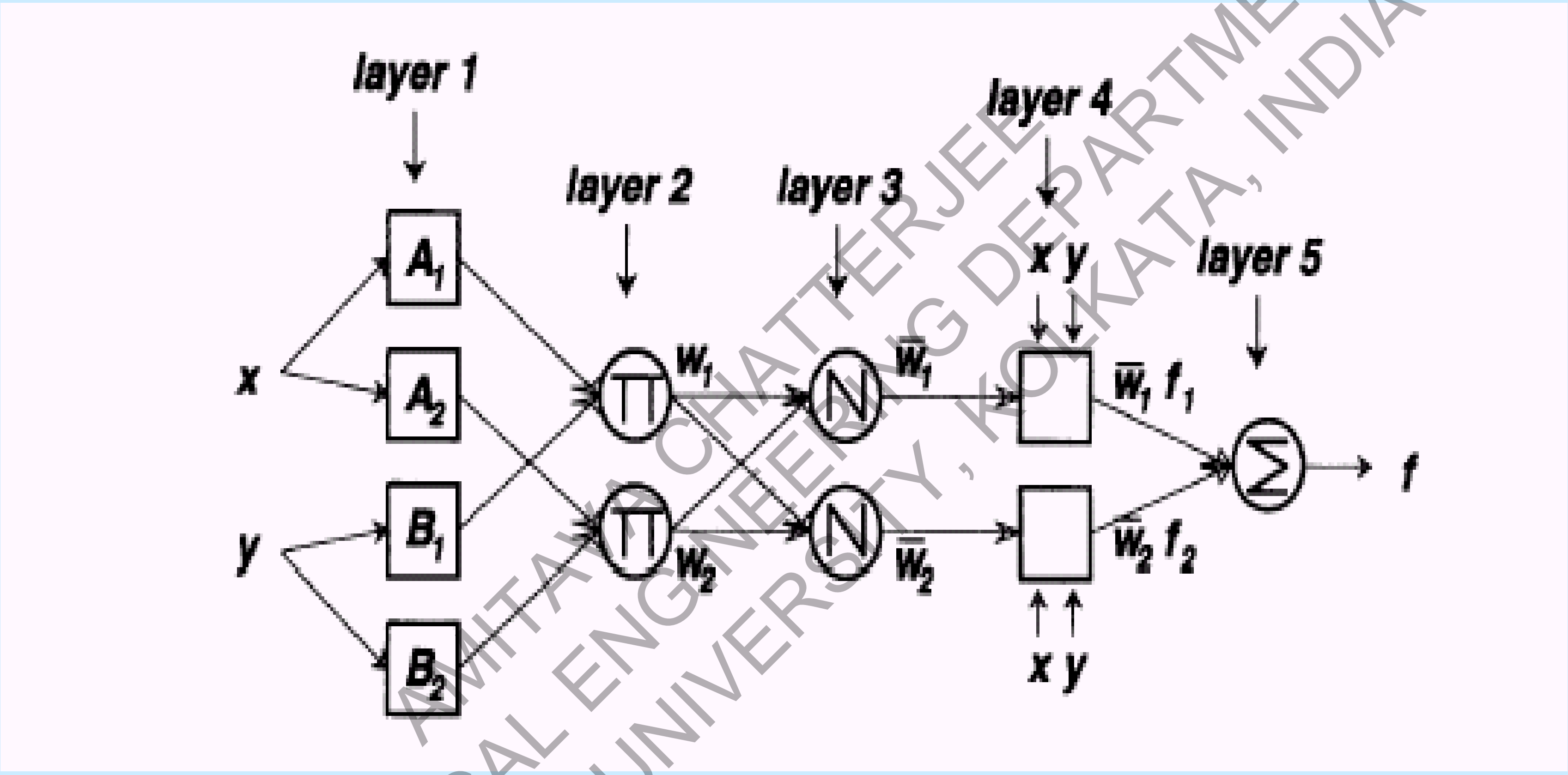


A two-input one-output first-order Sugeno fuzzy model with two rules

ADAPTIVE FUZZY SYSTEMS

- **Structure/architecture adaptation**
- **Parameter adaptation**
- **Simultaneous structure and parameter adaptation**
- **Online and offline adaptation**

NEURO-FUZZY SYSTEMS



Equivalent ANFIS architecture for the Sugeno-system

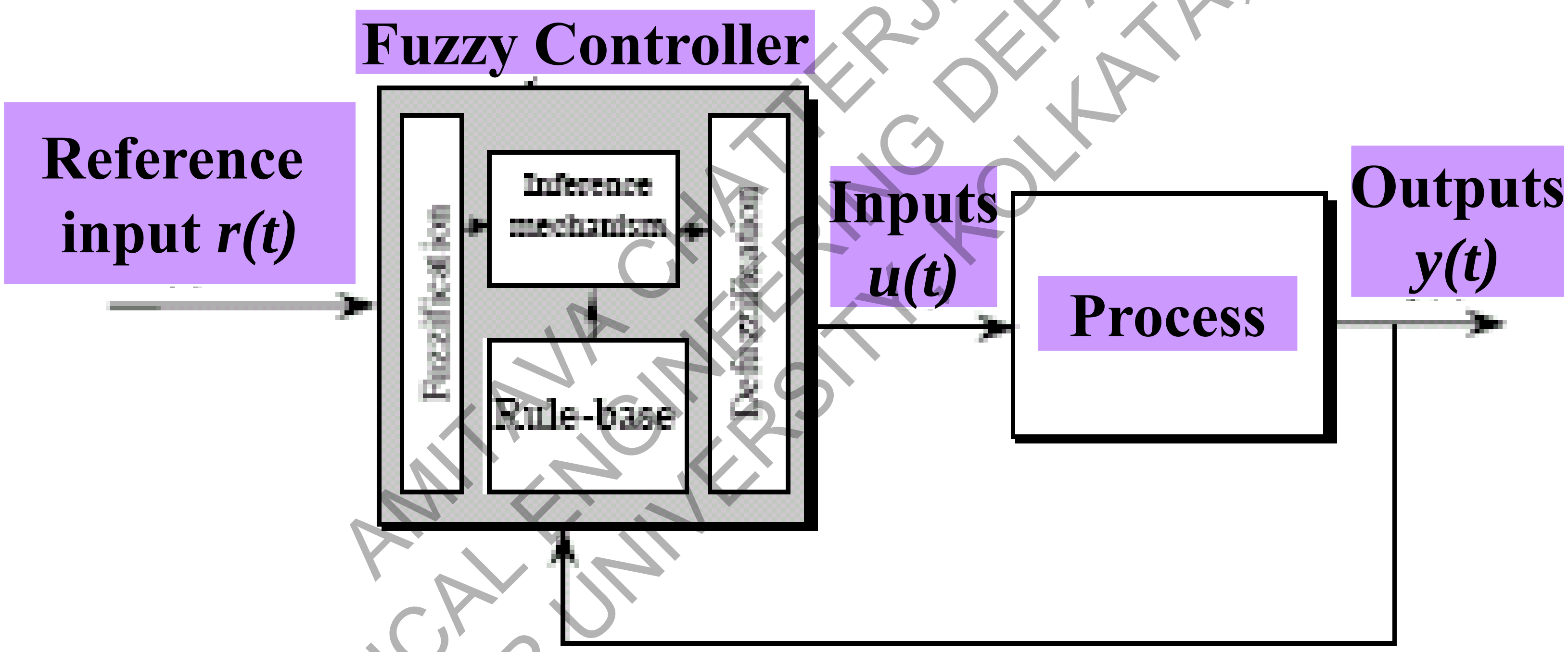
ANFIS: Adaptive Neuro-Fuzzy Inference System

FUZZY CONTROL

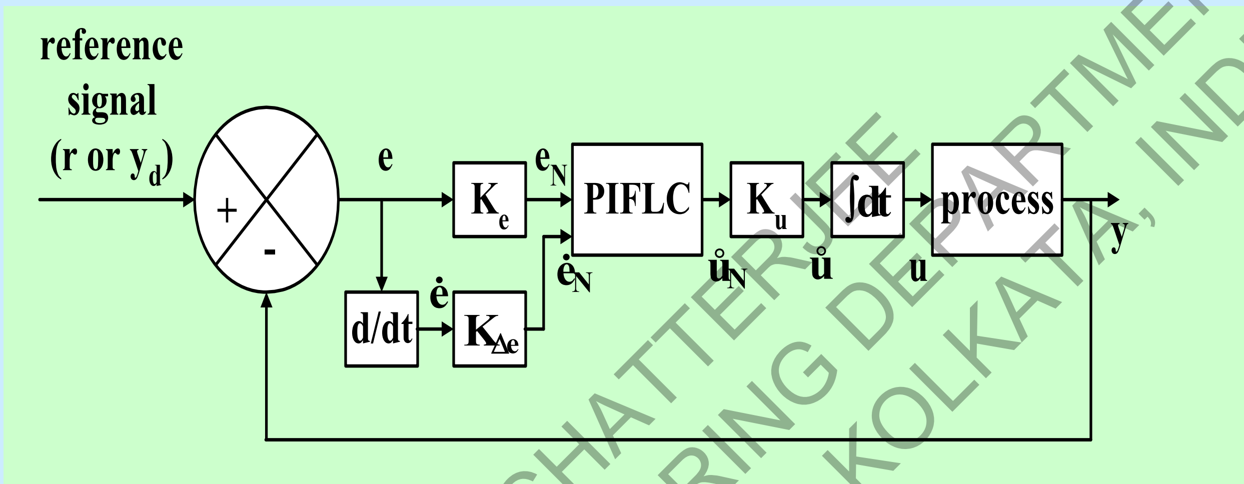
Why?

- A practical alternative for a variety of challenging control applications
- A convenient method for constructing nonlinear control methodologies by using heuristic control knowledge
- Reduction of development and maintenance time
- simultaneous achievement of system identification and control
- Better performance in controlling dynamic and/or ill-defined processes
- Marketing and patents

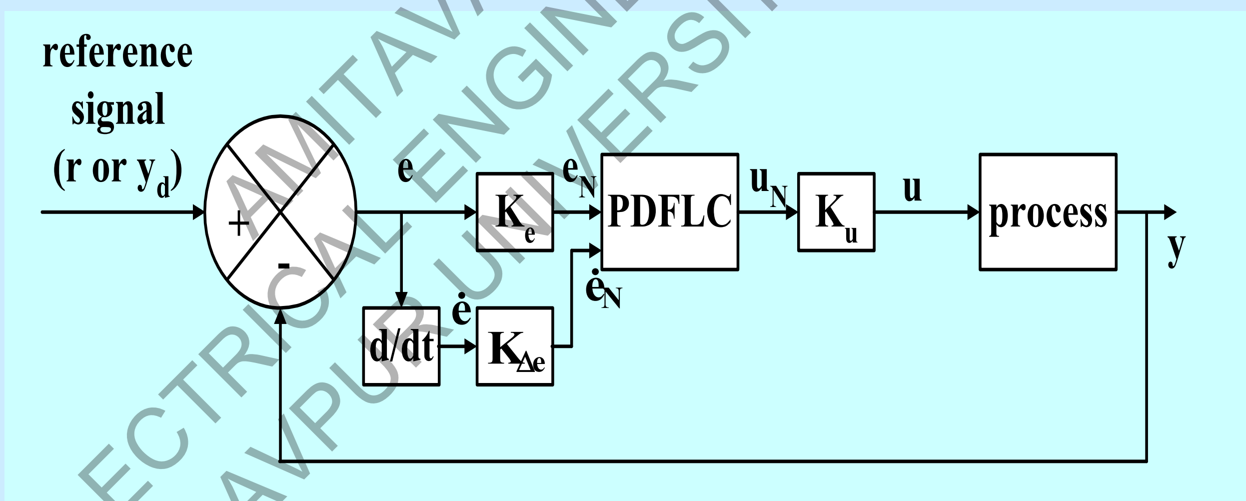
SCHEMATIC REPRESENTATION OF A FUZZY CONTROLLER



FUZZY CONTROLLERS IN CONTINUOUS-TIME SYSTEMS

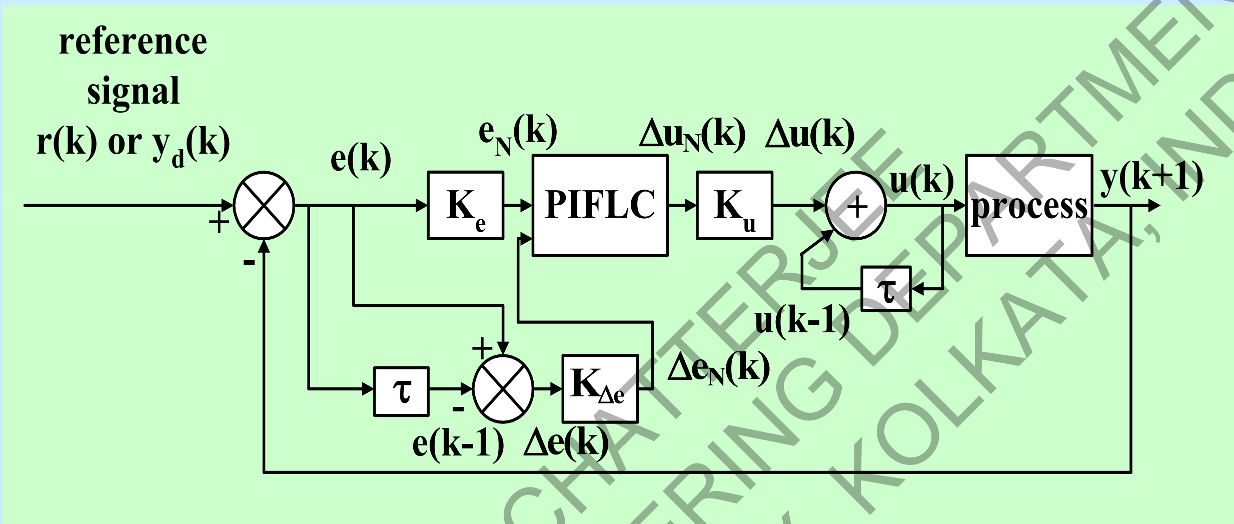


PI - Configuration

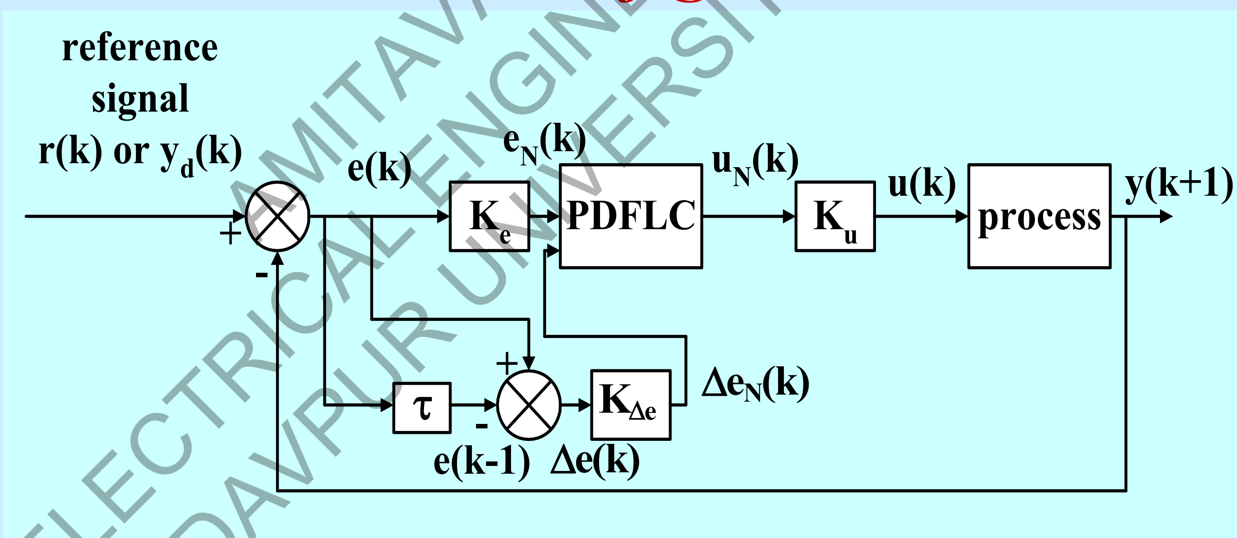


PD - Configuration

FUZZY CONTROLLERS IN DISCRETE-TIME SYSTEMS

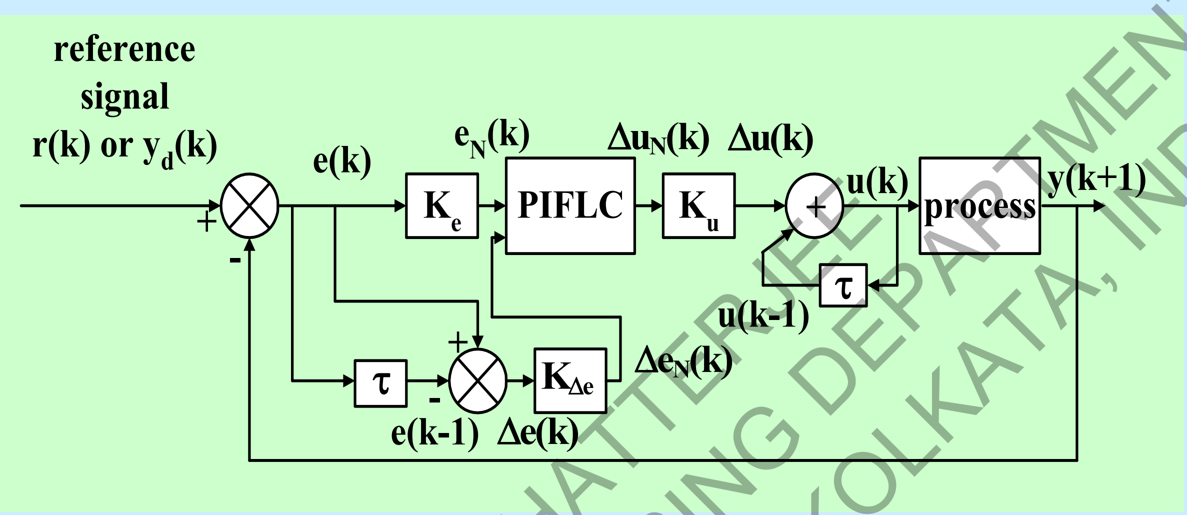


PI - Configuration

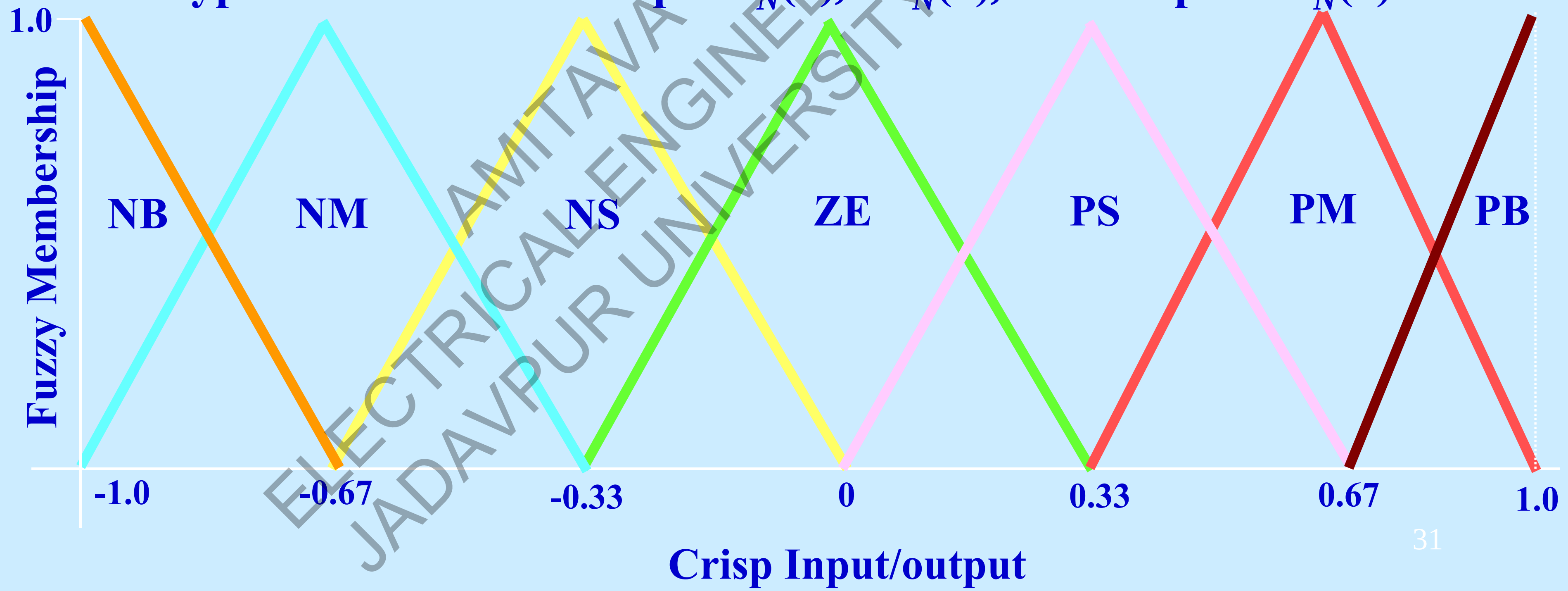


PD - Configuration

DESIGN OF DISCRETE-TIME FUZZY PI-CONTROLLERS

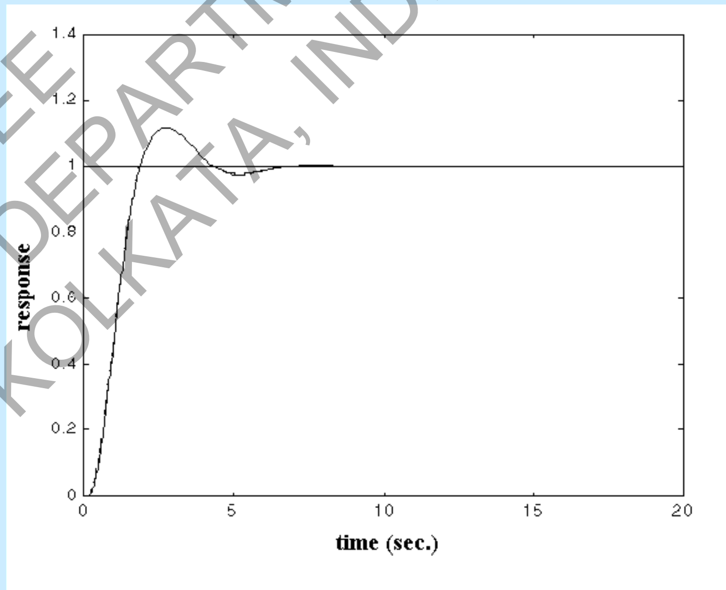


Typical MFs for each input $e_N(k)$, $\Delta e_N(k)$, and output $\Delta u_N(k)$



A REPRESENTATIVE RULE-BASE FOR FUZZY PI-CONTROLLER

$\Delta e_N \rightarrow$ $e_N \downarrow$	NB	NM	NS	ZE	PS	PM	PB
NB	NB	NB	NB	NB	NM	NS	ZE
NM	NB	NB	NB	NM	NS	ZE	PS
NS	NB	NB	NM	NS	ZE	PS	PM
ZE	NB	NM	NS	ZE	PS	PM	PB
PS	NM	NS	ZE	PS	PM	PB	PB
PM	NS	ZE	PS	PM	PB	PB	PB
PB	ZE	PS	PM	PB	PB	PB	PB



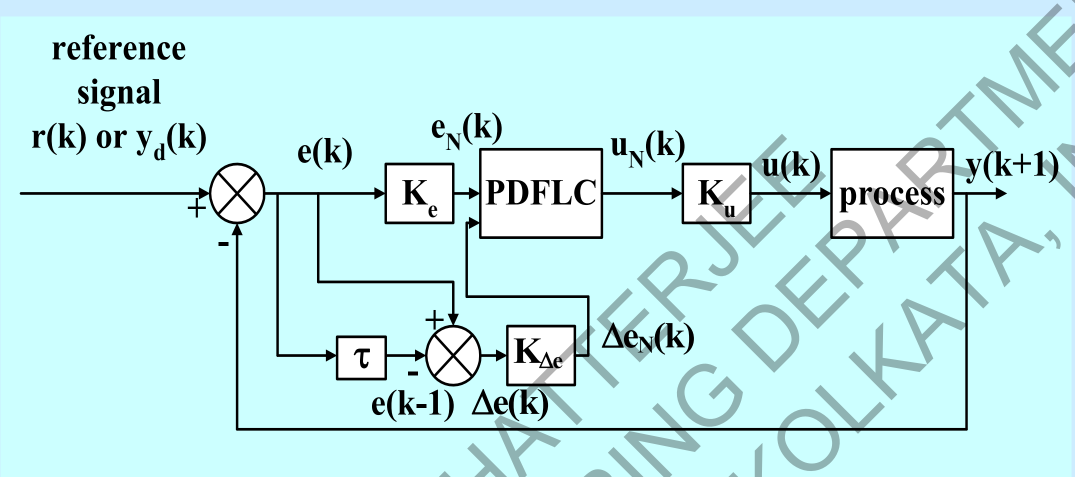
A typical output response curve

Input **1 :** $e_N(k) = K_e(y_d(k) - y(k))$**, fuzzified using 7 triangular MFs, in the range [-1, 1]**

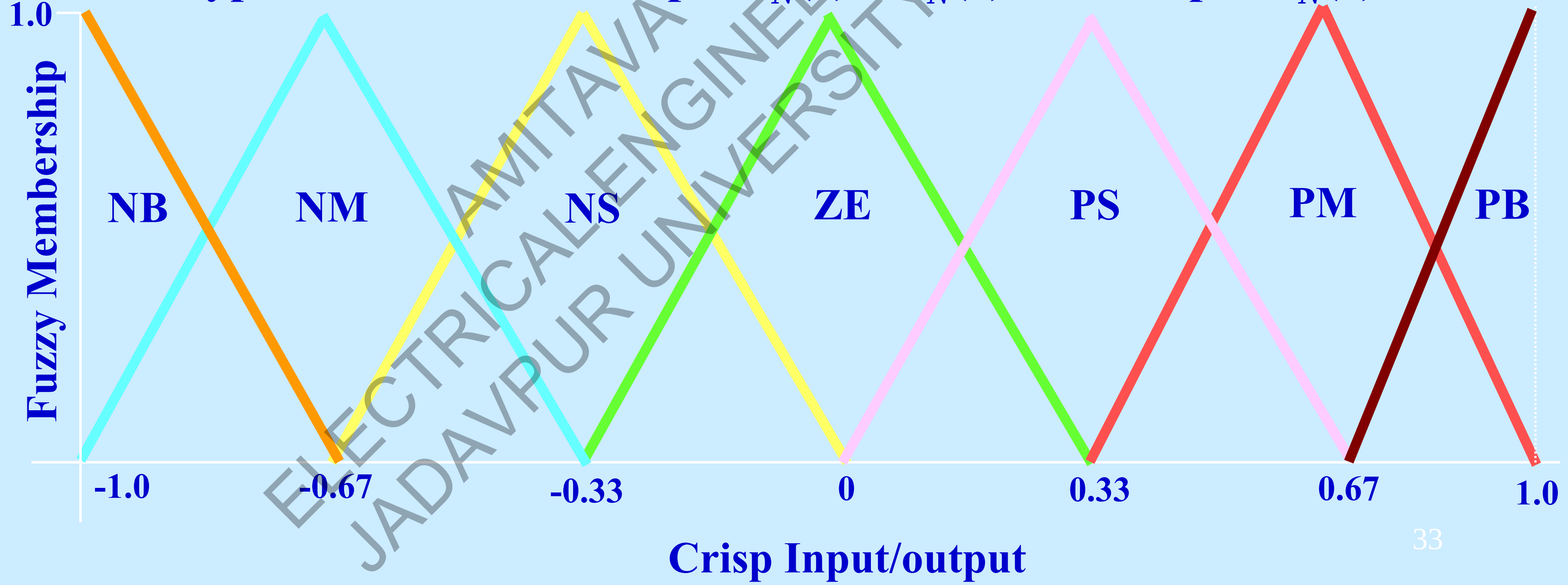
Input **2 :** $\Delta e_N(k) = K_{\Delta e}(e(k) - e(k\text{-}1))$**, fuzzified using 7 triangular MFs, in the range [-1, 1]**

Output **:** $\Delta u_N(k)$**, fuzzified using 7 triangular MFs, in the range [-1, 1]**

DESIGN OF DISCRETE-TIME FUZZY PD-CONTROLLERS

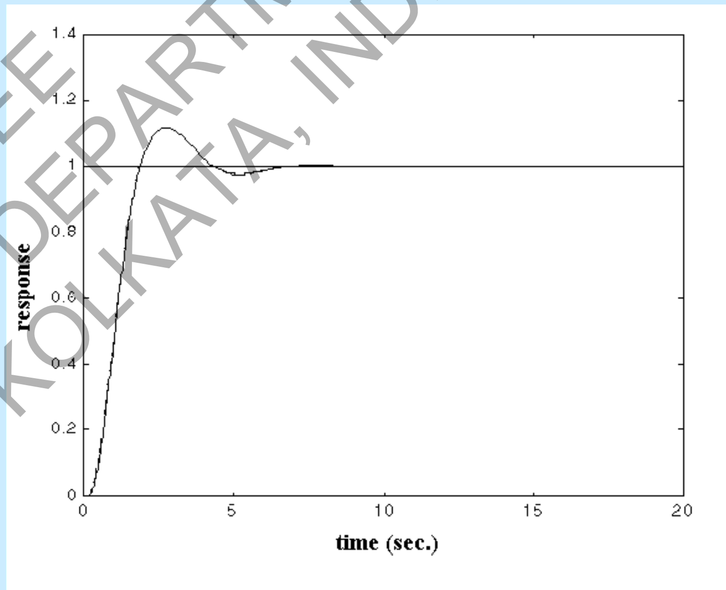


Typical MFs for each input $e_N(k)$, $\Delta e_N(k)$, and output $u_N(k)$



A REPRESENTATIVE RULE-BASE FOR FUZZY PD-CONTROLLER

$\Delta e_N \rightarrow$ $e_N \downarrow$	NB	NM	NS	ZE	PS	PM	PB
NB	NB	NS	PS	PB	PB	PB	PB
NM	NB	NM	ZE	PM	PM	PB	PB
NS	NB	NM	NS	PS	PM	PB	PB
ZE	NB	NM	NS	ZE	PS	PM	PB
PS	NB	NB	NM	NS	PS	PM	PB
PM	NB	NB	NM	NM	ZE	PM	PB
PB	NB	NB	NB	NB	NS	PS	PB



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Input **2 :** $\Delta e_N(k) = K_{\Delta e}(e(k) - e(k-1))$**, fuzzified using 7 triangular MFs, in the range [-1, 1]**

Output **:** $u_N(k)$**, fuzzified using 7 triangular MFs, in the range [-1, 1]**

TWO IMPORTANT DESIGN CRITERIA FOR AN FLC

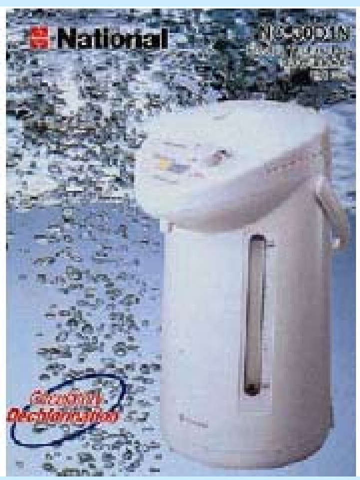
- **Choice of Membership Functions (MFs)**

- **Choice of Scaling Factors (SFs)**

TWO EXAMPLES OF POPULAR HOME APPLIANCES

• Panasonic ®/ National ® Fuzzy Logic Rice Cooker

Fuzzy logic controls the cooking process, self-adjusting for rice and water conditions



•National ® Deluxe Electric Fuzzy Logic Thermo Pot

Fuzzy logic controls the production of clean boiled water on demand for making tea

FUZZY SYSTEMS AND FUZZY CONTROL

References

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Thank You…