INTERFERENCE OF LIGHT   Problem set - 1

① Two coherent sources of same frequencies have intensities I_0 and $2I_0$.

Let the maximum and minimum intensity I_1 and I_2 respectively.

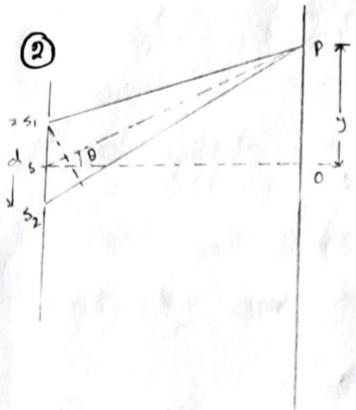
$$\therefore \frac{I_1}{I_2} = \frac{(\sqrt{I_0}+\sqrt{2I_0})^2}{(\sqrt{2I_0}-\sqrt{I_0})^2} = \frac{I_0(\sqrt{2}+1)^2}{I_0(\sqrt{2}-1)^2}$$

$$\text{or } \frac{I_1}{I_2} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}} = \frac{(3+2\sqrt{2})^2}{3^2-(2\sqrt{2})^2} = \frac{17+12\sqrt{2}}{9-8}$$

$$\text{or } \frac{I_1}{I_2} = (17+12\sqrt{2})$$

$\therefore$ The ratio of maximum to minimum intensity in their interference pattern is $(17+12\sqrt{2})$

②



We know $d = 0.1\,mm$

$D = 1.2\,m$

$\lambda = 600\,nm$

(a) Here $\theta = \angle PSO = 0.8°$

from ΔPOS, $\tan\theta = \frac{PO}{SO} = \frac{y}{D}$

or $y = D\tan\theta$

$= 1.675\,cm$

Now path difference $= S_2P - S_1P = \delta$

we know, $S_2P^2 = D^2 + (\frac{d}{2}+y)^2$

$S_1P^2 = D^2 + (y-\frac{d}{2})^2$

$\therefore 2D\delta = 2yd$ or $\delta = \frac{yd}{D} = \frac{16.75 \times 0.1}{1200}\,mm = 1.395\times10^{-3}$

$\therefore$ phase difference $= \frac{2\pi}{\lambda}\cdot\delta \times$ path diff

$= \frac{2\pi}{600\times10^{-6}} \times 1.395\times10^{-3} = 4.65\pi$

(b) Here, $y = 4.00$ mm

or $\delta = \frac{yd}{D} = \frac{4 \times 0.1}{1200}$ mm $= \frac{1}{3000}$ mm

$\therefore$ Phase difference $= \frac{2\pi}{600 \times 10^{-6}} \times \frac{1}{3000} = \frac{10\pi}{9}$

(c) Phase difference $(\phi) = \frac{\pi}{3}$ rad

$\therefore$ path difference $d = \frac{\lambda}{2\pi} \cdot \frac{\pi}{3} = \frac{\lambda}{6}$

$\therefore \frac{yd}{D} = \frac{\lambda}{6}$

or $d \tan\theta = \frac{\lambda}{6}$ or $\tan\theta = \frac{1}{6} \times \frac{600 \times 10^{-6}}{0.1}$

or $\theta = 0.05729°$

$\therefore$ Angle of diffraction $= 0.05729°$

(d) if $d \tan\theta = \frac{\lambda}{4}$

or $\tan\theta = \frac{600 \times 10^{-6}}{4 \times 0.1}$

or $\theta = \tan^{-1}(0.0015) = 0.08594°$

(e) Spacing between two adjacent bright fringes

$\beta = \frac{D\lambda}{d} = \frac{1200 \times 600 \times 10^{-6}}{0.1}$

$= 7.2$ mm

(f) Distance between 4th bright fringe and the centre $= 4\beta = 4 \times 7.2 = 28.8$ mm

③ Let θ_m = Maximum angle

$\theta_m = 30^\circ$; $d = 0.32$ mm and $\lambda = 500$nm

Now $\frac{y}{D} = \tan\theta_m = \tan 30^\circ$

or $y = D/\sqrt{3}$

$\beta = \frac{D\lambda}{d}$

$\therefore \frac{y}{\beta} = \frac{D}{\sqrt{3}} \cdot \frac{d}{D\lambda} = \frac{d}{\sqrt{3}\lambda}$

$= \frac{0.32}{\sqrt{3} \times 500 \times 10^{-6}} = 369.5$

$\therefore$ Bright fringes on ~~ors~~ one side $= (369.5 - 1.5)/2$

$= 184$

$\therefore$ Total bright fringes $= 368 + 1 = 369$

④ ~~XA~~ We have $\lambda = 500$ Å $= 500 \times 10^{-7}$ mm

$\beta = 5$mm and $D = 1$m $= 1000$mm

We know, $\beta = \frac{D\lambda}{d}$

or $5 = \frac{1000 \times 500 \times 10^{-7}}{d}$

or $d = 0.01$

$\therefore$ Separation between two slits $= 0.01$mm

⑤ Refractive index of the meterial $(\mu) = 1.5$

and wavelength of light $\lambda = 5500$ Å

Let the separation between two slits be d and the distance between slits and screen be D.

$\therefore$ Fringe width $(\beta) = \frac{D\lambda}{d}$

$\therefore$ Displacement $(x) = 5\beta = \frac{5D\lambda}{d}$

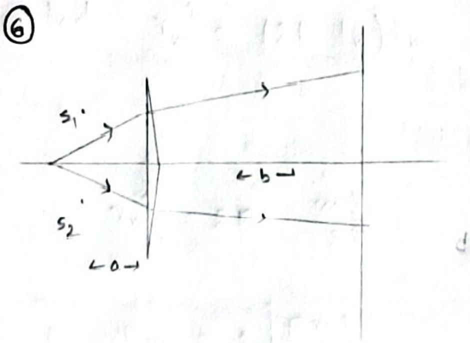
$\therefore \frac{D}{d}(n-1)t = \frac{5D\lambda}{d}$

or $5\lambda = (n-1)t$

or $t = \frac{5\lambda}{n-1} = \frac{5 \times 5500}{1.5-1} = 55 \times 10^{-5}$ cm

$\therefore$ change in optical path $= 0.5 \times 55 \times 10^{-5} = 27.5 \times 10^{-5}$ cm

(6)



Refractive index $(n) = 1.732$

Apex angle $(\alpha) = 0.85°$

$= 0.0148$ rad

wavelength $(\lambda) = 6563$ Å

$a = 25.0$ cm

$b = 75$ cm

We know $d = 2a(n-1)\alpha$

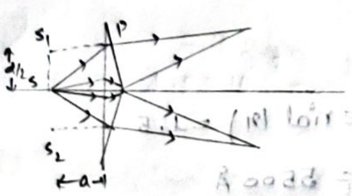
$= 2 \times 25 \times (1.732 - 1) \times 0.014835$ cm

$= 0.5429$ cm

$\therefore$ Fringe separation $(\beta) = \frac{(a+b)}{d}\lambda$

$= \frac{100 \times 6563 \times 10^{-8}}{0.5429} = 0.012$ cm

(7) (a)



Now, the apex angle α is very small.

$\therefore D = (n-1)\alpha$

From ΔSS_1P ; $\angle S_1PS = D$

$\tan D = \frac{d/2}{a}$

As $D \to 0$; $\tan D \simeq D$

$\therefore D = \frac{d/2}{a}$

or $\frac{d}{2a} = (n-1)\alpha$ ($n \to$ refractive index)

$\therefore d = 2a(n-1)\alpha$

(b) Here $\frac{b}{a} = 20$, $\lambda = 5893\,Å$; $\beta = 0.1$ cm ; $n = 1.5$

We know that $d = 2a(n-1)\alpha$

and $\beta = \frac{(b+a)\lambda}{d}$

$\therefore \frac{(b+a)\lambda}{\beta} = 2a(n-1)\alpha$

or $\frac{5893\times10^{-8}}{0.1}(20+1) = \alpha$

or $\alpha = 0.012375° = 0.709$ radian

(8) Separation between slits and focal plane $= (a+b) = 1$ m

We know $d_1 = 4.05$ mm ; $d_2 = 2.9$ mm and $\lambda = 5893\,Å$

So, $d = \sqrt{4.05\times2.9}$ mm $= 0.342$ cm

$\therefore$ Distance between two consecutive bright bands

$\beta = \frac{D\lambda}{d} = \frac{(a+b)\lambda}{d}$

$= \frac{100\times5893\times10^{-8}}{0.342}$ cm

$= 0.017$ mm

(9) We have refractive index $\eta = \frac{4}{3}$

Let us consider the case for m^{th} dark fringe

It will be produced when,

$2\eta t_m = m\lambda$

$\therefore t_m = \frac{m\lambda}{2\eta}$

[Let thickness at mth dark band → t_m

R → radius of curvature of lense]

And $r_m^2 = (2R - t_m)t_m \simeq t_m \times 2R$

$\therefore r_m^2 = 2R \cdot \frac{m\lambda}{2\eta}$

$D_m = 2\sqrt{\frac{Rm\lambda}{\eta}}$

In case of air $D_a = 2\sqrt{Rm\lambda}$

" " " liquid $D_l = 2\sqrt{\frac{Rm\lambda}{4/3}} = \frac{\sqrt{3}}{2} D_a$

the diameter will change is a proportion $\sqrt{3}/2$

(10) We know $r_{15} = 0.8\,cm$; $r_{10} = 0.2\,cm$; $R = 100\,cm$

$\therefore$ wavelength $\lambda = \frac{r_{15}^2 - r_{10}^2}{500}$ cm

$= \frac{(0.8)^2 - (0.2)^2}{500} = \frac{0.6}{500} = \frac{6}{5000}$

$= 1.2 \times 10^{-3}$ cm $= 0.0012$ cm

(11) We have the wavelengths $\lambda_1 = 4000\,\mathring{A}$

and $\lambda_2 = 4002\,\mathring{A}$

$\therefore$ When, maxima due to λ_1 is overlapped with the minima due to λ_2, the rings will disappear.

In this situation,

$$\left(m+\frac{1}{2}\right)\lambda_1 = m\lambda_2$$

or $4000m + 2000 = 4002m$

or $m = 1000$

$\therefore$ thickness of the air film where 1000th dark ring for λ_2 is formed $(t) = \frac{m\lambda_2}{2\eta}$

$$= \frac{1000 \times 4002}{2\eta} = 0.02 \text{ cm}$$

$\therefore$ $r = \sqrt{2Rt}$

$$= \sqrt{2 \times 400 \times 0.02} = 4 \text{ cm}$$

(12) Number of fringes collapsed $(N_2) = 100$

Displacement $(d_0) = 0.0489$ mm.

$\therefore$ wavelength $(\lambda) = \frac{2d_0}{N_0}$

$$= \frac{2 \times 0.0489}{100} = 9780 \text{ Å}$$

(13) Two given wavelengths $\lambda_1 = 4882$ Å ; $\lambda_2 = 4886$ Å

Let, the mirror moves d distance between two positions of disappearance of the fringes,

$\therefore$ The condition $\rightarrow$ $\frac{2d}{\lambda_1} - \frac{2d}{\lambda_2} = n/2$

($n \rightarrow$ odd integer)

For the distance between disappearance

$$\frac{2d}{\lambda_1} - \frac{2d}{\lambda_2} = 1 \quad \text{or} \quad 2d \cdot \frac{4}{4882 \times 4886} = 1$$

or $d = 0.0298$ cm