Flow through open Channels.

Artificial or natural flow passages ~~that have~~ through which liquid flows with its free surface exposed to atmospheric pressure or a constant pressure, is called open channels. Partly-filled culverts and pipelines are also ~~called~~ treated as open channel.

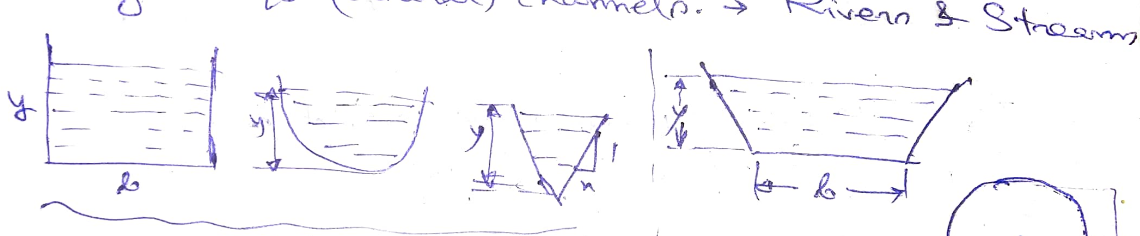
Group (i) open & covered channels,
ii) regular & irregular c/s channels.

Open channels :- Irrigation canals, water falls, rivers, streams and flumes.

Covered channels :- Public water supply, sewage lines.

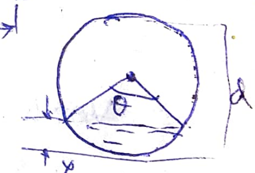
Regular (c/s) artificial channels → Irrigation canals, sewerage pipes,

Irregular c/s (natural) channels. → Rivers & Streams



Exponential

$A = K y^m$.

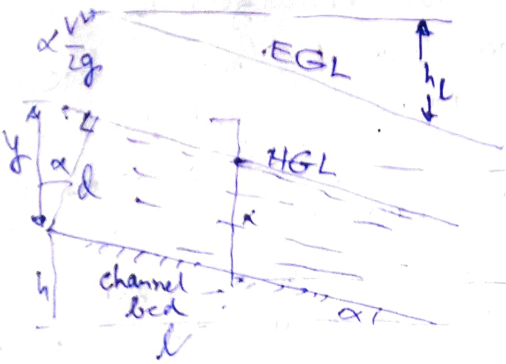


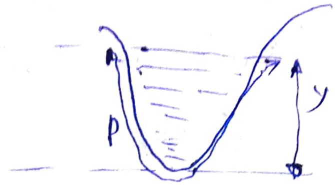
Non-exponential

$A \neq K y^m$.

The relations of channel flow is derived from experiments as many variables affect the channel flow.

Terms related to channel flow





Depth of flow (y) – vertical distance from free liquid surface to channel bed.

Depth of flow section (d) – Depth of liquid measured normal to the direction of flow.

$d = y\cos\alpha$. for very small slope $\cos\alpha \simeq 1 \Rightarrow d \simeq y$.

or $d \simeq y$

Top width (T) → width of the channel ~~surface~~ section at free liquid surface

~~Wet~~ Wetted Area (A). → c/s area of channel normal to the direction of flow.

Channel slope (α) – Angle of channel bed and FLS with horizontal.

$\sin\alpha = \frac{h}{l}$ h = vertical fall in length l of channel.

α is small $\therefore$ $\tan\alpha = \sin\alpha = h/l$.

α = 1 in 5000 to 10000 in open earthen channels.

1 in 100000 in large rivers.

Wetted perimeter and Hydraulic mean depth :-

channel lining (sides and base) that comes in direct contact with the liquid stream, is called the wetted perimeter (P).

hydraulic mean depth (y_m) = $\frac{\text{Wetted area }(A)}{\text{Wetted perimeter }(P)}$.

(i) Rectangular. $y_m = \frac{A}{P} = \frac{by}{b+2y}$

(ii) Pipe running ~~full~~ full $= \frac{(\pi/4)d^2}{\pi d} = \frac{d}{4}$

(iii) Pipe not running full $y_m = \frac{A}{P}$

$$= \frac{(r^2/2)(2\theta - \sin 2\theta)}{2r\theta}$$

Hydraulic Depth = D = $\frac{\text{Wetted area}}{\text{Top width}} = \frac{A}{T}$.

Apparently it represents ~~the~~ a rectangular area (T×D) equivalent to cross sectional area of the flow.

Hydraulic gradient line (HGL) & Energy gradient line (EGL).

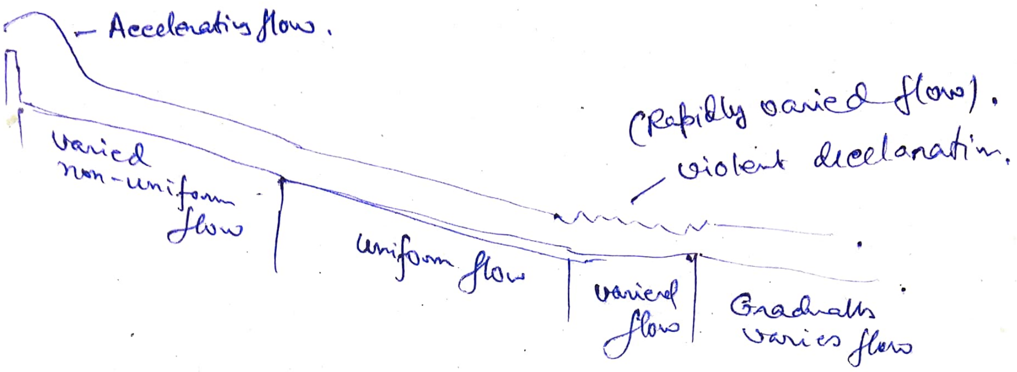
HGL represents pressure at various c/s of the channel. For open channels it ~~is equal~~ coincides. to the FLS.

EGL indicates total energy of the liquid. It lies above HGL and the distance is $\alpha \cdot \frac{v^2}{2g}$.

α = K.E. correction factor = 1.1 to 1.2

The slope of EGL (hydraulic slope). $= \frac{h_f}{l} = \frac{\text{head lost due to friction}}{\text{length of channel in which loss occurs}}$.

Type of flow



Laminar and turbulent flow. → Depends on Reynolds number,

$$Re = \frac{\rho \cdot V \cdot y_m}{\mu}.$$

$Re < 500$ laminar

$500 < Re < 2000$ transitional

$Re > 2000$ ~~tur~~ turbulent.

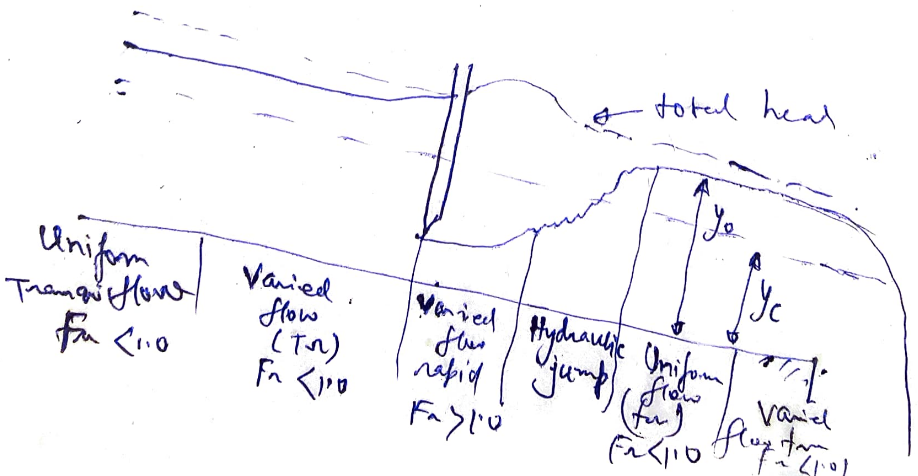
Tranquil & rapid flow. → Flow is due to gravity.

∴ Froude number $F_r = \frac{V}{\sqrt{g y_m}}$ · for analysis of open channel flow.

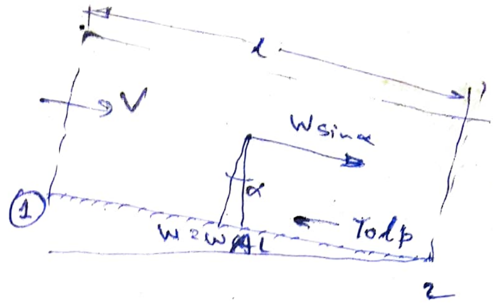
i) $F_r < 1.0$ – tranquil or stream flow. Velocity low. Disturbance can propagate to upstream. The nature of flow is governed by downstream conditions.

ii) $F_r = 1.0$. → Critical flow. Flow vel. is equal to the vel of an elementary wave.

iii) $F_r > 1.0$ Rapid or shooting flow where flow vel is very high. Disturbance in downstream can't travel upstream.



CHEZY EQUATION



Pressure P_1 & P_2 acting at two ends.
depth of liquid is constant.
Pressure distribution is hydraustatic.

Force producing motion is the component of wt.
$= \gamma A l \sin\alpha$. γ = sp. gravity.

Frictional resistance is $\tau_0 l P$ — τ_0 = boundary shear stress per unit area

For steady flow the friction force
= wt. of liquid mass acting along the fluid moth.

$\tau_0 lP = \gamma A l \sin\alpha$.

$\tau_0 \propto V^2 \Rightarrow \tau_0 = f V^2$.

$\therefore f V^2 l P = \gamma A l \sin\alpha$.

$V = \sqrt{\frac{\gamma}{f}} \times \sqrt{\frac{A}{P}\sin\alpha}$

$\frac{A}{P} = y_m$ = hydraulic mean depth.
$\sin\alpha = s$ = slope

$= \sqrt{\frac{\gamma}{f}}\sqrt{y_m s}$
$= C\sqrt{y_m s}$

$C = \sqrt{\frac{\gamma}{f}}$ is a variable depending on roughness of channel surface and ~~flow~~ the flow Reynolds number.

So. $V = C\sqrt{y_m s}$ is known as Chezy equation developed by Antoine de Chezy (French Engg 1775)

Discharge. $Q = A \cdot C \cdot \sqrt{y_m s} = K\sqrt{s}$. where $K = AC\sqrt{y_m}$.

The faction K is the conveyance of the channel section. It is a measure of carrying capacity of the channel. For a channel of constant slope, the conveynance is directly proportional to discharge.

Emperical relations for Chezy Constant c.

c is a variable. Its value depends on Re, and surface roughness. c has a dimension of $L^{1/2}/T$ so its numerical value depends on units employed.

Some emperical relations of Chezy coefficient are as follows. (These are based on experimental evidenses).

* The Manning formula $c = y_m^{1/6}/N$.

  y_m = hydraulic mean depth N = Manning Const (depends on roughness)

  Application is wide, best for turbulent fully developed flow. (It is the case of most channel flow).

* The Bazin formula $c = \frac{87}{1+(K/\sqrt{y_m})}$.

  K = Bazin constant. (depends on roughness)

  Not accurate for large channels.

* The Kutter formula $c = \frac{23+(0.00155/s)+(1/N)}{1+[23+0.00155/s](N/\sqrt{y_m})}$

  N = Kutter constant (depends on type of the channel).

  Give satisfactory result for variety of flow situation. (Tables & charts are available).

Problem:- Calculate flow rate and conveyance for a rectangular channel of 5 m wide for uniform flow at a depth of 1.5 m. The bed slope is 1 vertical to 1000 horizontal. Coment on state of flow. $C = 50\ m^{1/2}/s$.

Soln:- Flow area $A = by = 5 \times 1.5 = 7.5\ m^2$.

$P = b + 2y = 5 + 2 \times 1.5 = 8\ m.$

$y_m = \frac{A}{P} = 0.9735\ m.$

$V = C\sqrt{y_m s} = 50\sqrt{0.9375 \times (1/1000)} = 1.53\ m/s.$

Discharge $Q = AV = 7.5 \times 1.53 = 11.48\ m^3/s.$

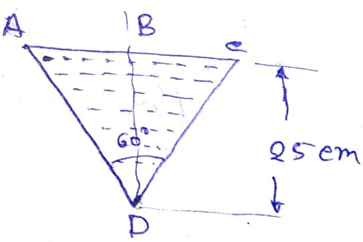
Conveyance $K = CA\sqrt{y_m} = 50 \times 7.5 \times \sqrt{.9375} = 363.09$ (m^3/s) ??

Froude number $F_r = \frac{V}{\sqrt{gy}} = \frac{1.53}{\sqrt{9.81 \times 1.5}} = 0.399.$

$F_r < 1.0$ = Flow is tranquill.

2. A triangular gutter, whose sides ~~are~~ include an angle of 60°, conveys water at a uniform depth of 25 cm. If the discharge is 0.04 m^3/s, workout the bed gradient of the trough. $C = 52\ m^{1/2}/sec.$

Soln. $AD = CD = \frac{BD}{\cos 30^\circ} = \frac{.25}{.866} = .288\ m$



$AC = 2\,BD \tan 30^\circ = 2 \times 0.025 \times 0.5774 = 0.288\ m.$

Area of flow $A = \frac{1}{2} AC \times BD = \frac{1}{2} \times 0.288 \times 0.25 = 0.036\ m^2.$

$P = AD + CD = 0.576\ m$

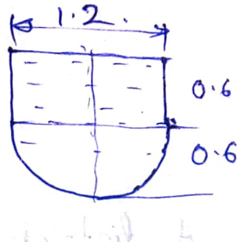
$y_m = \frac{A}{P} = \frac{0.036}{.576} = 0.0626\ m.$

$Q = AV = AC\sqrt{y_m s} \Rightarrow s = \frac{Q^2}{A^2 C^2 y_m} = \cancel{Q}$

$= \frac{0.04}{0.036^2 \times 52^2 \times 0.0626} = \frac{1}{137}.$ Ans.

Q.3 A channel having ~~semicircular~~ bottom of 1.2 m diameter and two sides as vertical when depth of flow is 1.2 m. $C = 68\ m^{1/2}/s$. $s = 1$ in 950. Calculate discharge.

Soln:-



$$A = 1.2 \times 0.6 + \frac{1}{2} \times \pi \times (0.6)^2 = 1.285\ m^2$$

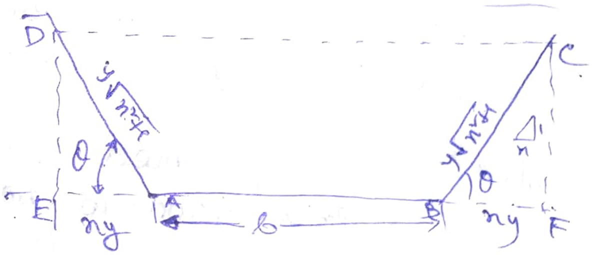
$$P = 0.6 \times 2 + \pi \times 0.6 = 3.085\ m$$

$$y_m = \frac{A}{P} = 0.4165\ m.$$

$$Q = AV = AC\sqrt{y_m s} = 1.83\ m^3/s.$$

Q.4 An irrigation canal of trapezoidal channel has ~~de~~ bed ~~depth~~ width of 3.5 m and bed slope of 1 in 1600, depth of flow = 1.5 m and the side ~~#~~ slope of the channel is $\frac{1}{2}$ (ie 1 vertical to 2 horizontal). Determine the average flow velocity and the discharge carried by the channel. [Use Bazin formula with $K = 1.54$. Also compute the average shear stress at the channel boundary.

Soln:-



$AB = b$, $DE = y$

Horizontal distance $EA = BF = ny$

n = slope 1 : n

Top width $CD = AB + 2BF$
$= b + 2ny$

Slant height $AD = BC = \sqrt{y^2 + n^2y^2} = y\sqrt{(n^2+1)}$

$$P = DA + AB + BC = b + 2y\sqrt{n^2+1}$$

$$= 3.5 + 2 \times 1.5\sqrt{(2^2+1)} = 10.21\ m.$$

$$A = \frac{\text{top width} + \text{bottom width}}{2} \times \text{height}.$$

$$= \frac{(b+2ny)+b}{2} \times y = y(b+ny) = 1.5(3.5 + 2 \times 1.5)$$

$$= 9.75\ m^2$$

$$y_m = \frac{A}{P} = 0.955\ m.$$

Bazin formula for Chezy coff. $C = \dfrac{87}{1 + K/\sqrt{y_m}}$

$$= \frac{87}{1 + (1.54/\sqrt{0.955})} = 33.77.$$

Flow velocity $V = C\sqrt{y_m s} = 33.77 \times \sqrt{.955 \times (1/1600)}$

$$= 0.825\ m/sec.$$

Discharge $Q = AV = 9.75 \times 0.825 = 8.04\ m^3/sec.$

ii) Under equilibrium. $\tau_0 l P = \gamma A l \sin\alpha.$

$\tau_0 = \gamma \cdot \frac{A}{P} \sin\alpha = \gamma \cdot y_m \cdot s.$

$= 9810 \times 0.955 \times \frac{1}{1600}$

$= 0.596\ N/m^2$

Prob: A district has a driamage area of 10 sq. km with a population of 4000 per sq. km. The daily water supply is 175 l per head. During dry weather. it is found that 8 % of total water supply passes along the sewer between the hours 12 noon & 1 pm.

Assuming max^m rainfall of 2 cm in 24 hours over the whole area, determine the dia of the circullar pipe which will carry max^m dry weather flow and the rainfall without the sewer becoming more than half full. slope 1:2500, $C = 60\ m^{1/2}/s.$

Soln:- Drainage area $= 10$ sq. km $= 10 \times 10^6\ m^2$

population $= 4000 \times 10 = 40000$

Daily water supply $= 175$ l/head $= 0.175 \times 10^{-3}\ m^3$ per head.

Daily " " to district $= 175 \times 10^{-3} \times 40000$

$= 7000\ m^3$

water consumption between 12 noon to 1 pm

$= 0.08 \times 7000 = 560\ m^3/hr = 0.155\ m^3/s.$

$\therefore$ Rate of dry ~~flo~~ weather flow $= 0.155\ m^3/s.$

Rainfall $=$ 2 cm in 24 hr. $= \frac{0.02 \times 10 \times 10^6}{24 \times 3600}$

$= 2.315\ m^3/s.$

Total discharge which sewer to convey

$= 0.155 + 2.315\ m^3/sec.$

Flow Area $= \frac{1}{2} \times \frac{\pi}{4} d^2 = \frac{\pi}{8} d^2.$

$P = \frac{1}{2} \times \pi d = \frac{\pi d}{2}$

$y_m = \frac{A}{P} = \frac{\pi/8\, d^2}{\pi d/2} = \frac{d}{4}$

$Q = A\, C \cdot \sqrt{y_m\, s}$

$\therefore\ 2.47 = \frac{\pi}{8} d^2 \times 60 \times \sqrt{\frac{d}{4} \times \frac{1}{2500}} = 0.2335\, d^{5/2}$

$\therefore\ d = \underline{2.56\ m}$. (Reqd dia of sewer.)

Economic section for max^m discharge

$Q = AC\sqrt{\left(\frac{A}{P}\right)s}$. $\left[Q = K\sqrt{\frac{1}{P}}\right.$ when $K = AC\sqrt{As}$ → Q is max^m for P is min^m $\left.\right]$

For a given slope (s), roughness & c/s area the flow will be max^m when y_m is max^m i.e when P is min^m.

The most economic shape must simulate;

- max^m discharge for a given c/s area.
- min^m ~~extra~~ excavation & lining for designed amount of discharge.
- Least P for min^m resistance to flow, and optimum discharge.

The channel c/s corresponding to minimum perimeter for a given flow area is called most economical.

Rectangular

$A = by \quad P = b + 2y$.

$P = \frac{A}{y} + 2y$

$\frac{dP}{dy} = -\frac{A}{y^2} + 2 = 0$; or $A = 2y^2$ or $by = 2y^2$ or $\underline{b = 2y}$.

$\therefore y_m = \frac{A}{b+2y} = \frac{by}{b+2y} = \frac{2y \cdot y}{4y} = \frac{y}{2}$

i.e inscribed circle is tangent to the bed and the sides.

~~TRAP~~

Prob^n: A rectangular channel of 8 m wide and 1.5 m deep has a slope of 0.001 in 1 and lined with smooth concrete plaster. It is desired to enhance discharge by keeping the amount of lining same. Find the new dimensions & % increase in discharge. Take Manning constant $N = 0.015$.

Original channel. $A = by = 12\ m^2$ $P = b + 2y = 11\ m$.

$y_m = \frac{A}{P} = 1.09\ m$. $C = \frac{(y_m)^{1/6}}{N} = \frac{(1.09)^{1/6}}{0.015} = 67.64$

$Q = AV = 12 \times 67.64\sqrt{1.09 \times 0.001} = 26.79\ m^3/s$.

P.T.O

For new shape $\therefore P = b' + 2y' = 4y' = 11\ m$ $\quad y' = 2.75\ m$.

$b' = 2 \times y' = 5.5\ m$.

$A' = ~~b~~\ b'y' = 15.125\ m^2$

$y_m' = \frac{A'}{P} = \frac{y'}{2} = \frac{2.75}{2} = 1.375\ m$.

Chezy Constant $C' = \frac{(y_m')^{1/6}}{N} = 70.28$. $\quad Q' = A'v' = 15.125 \times 70.28 \times \sqrt{1.375 \times 0.001}$

$= 39.42\ m^3/s$.

% increase $= \frac{39.42 - 26.79}{26.79} = \underline{47.1\ \%}$.

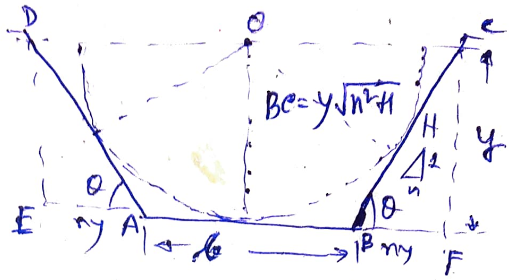
Most economical Trapezoidal channel.

$A = y(b+ny)$

$P = b + 2y\sqrt{(n^2+1)}$

Eliminating b we get

$P = \frac{A}{y} - ny + 2y\sqrt{n^2+1}$



Optimum c/s of a trapezoidal channel.

For given A, ~~and~~ maximization of flow requires minimization of P.
minimization of P implies that $\frac{\partial P}{\partial y} = 0$; $\frac{\partial P}{\partial n} = 0$

$\frac{\partial P}{\partial y} = 0 \Rightarrow -\frac{A}{y^2} - n + 2\sqrt{n^2+1} = 0$

$y\sqrt{(n^2+1)} = \frac{b+2ny}{2}$ or $2y\sqrt{n^2+1} = b+2ny$

i.e. The sloping sides equals half the top width. If this condition is applied then side slope is fixed.

$y_m = \frac{A}{P} = \frac{(b+ny)y}{b+2y\sqrt{n^2+1}} = \frac{(b+ny)y}{b+b+2ny} = \frac{(b+ny)y}{2(b+ny)} = \frac{y}{2}$

$\therefore$ For max^m Discharge $y_m = y/2$

$\therefore$ Hydraulic mean depth is equal to half of the flow depth.

θ = angle of slope O = centre of the top width.

OH = a perpendicular to the sloping side BC.

$\triangle$ OCH is then right angled triangle with $\angle OCH = \theta$

$\sin\theta = \frac{OH}{OC}$ $OH = OC\sin\theta$

$\triangle BFC$ $\sin\theta = \frac{y}{y\sqrt{n^2+1}} = \frac{1}{\sqrt{n^2+1}}$

$OH = OC\frac{1}{\sqrt{n^2+1}}$

For trapezoidal channel of most economic c/s half the top width ~~ofc~~ OC equals length of the sloping side. i.e. $OC = \frac{b+2ny}{2} = y\sqrt{n^2+1}$.

$\therefore OH = \frac{y\sqrt{n^2+1}}{\sqrt{n^2+1}} = y$ = depth of flow.

So with centre at O and radius = y we can draw a circle. ~~with~~ The three sides ~~with~~ will be tanjent of the circle. So most efficient trapezoidal channel is a half hexagon.

Best side slopes for the most economic trapezoidal section

* The condition is $\frac{dp}{dn}=0$

$\therefore \quad \frac{d}{dn}\left[\frac{A}{y} - ny + 2y\sqrt{n^2+1}\right] = 0$

$-y + 2y + \frac{1}{2}(n^2+1)^{-1/2} \times 2n = 0$

or $\quad -y + 2ny \times \frac{1}{\sqrt{n^2+1}} = 0$

or $\sqrt{n^2+1} = 2n \quad$ or $n = \frac{1}{\sqrt{3}}$.

$\therefore \tan\theta = \frac{1}{n} = \sqrt{3} = \tan 60^\circ \quad \therefore \theta = 60^\circ$.

Again $\quad y\sqrt{n^2+1} = \frac{b+2ny}{2}$

for $n = \frac{1}{\sqrt{3}}$ we get $b = \frac{2y}{\sqrt{3}}$

$\therefore$ Perimeter $P = 3b$

$A = y(b+ny) = y\left(\frac{2y}{\sqrt{3}} + \frac{1}{\sqrt{3}}y\right) = \frac{3y \cdot y}{\sqrt{3}}$
$= \sqrt{3}y^2$

$P = \sqrt{3}y$

A trapezoidal channel having the side slope equal to 60° with horizontal and laid on a slope of 1 in 750, carries a discharge of $10\ m^3/s$. Find the width at base and depth of flow for most economic c/s. $C = 66\ m^{1/2}/s$.

Soln: for trapezoidal section of optimum c/s the relation $\frac{b+2ny}{2} = y\sqrt{n^2+1}$; $y_m = \frac{y}{2}$

$\tan\theta = \tan 60^\circ = \sqrt{3} = \frac{1}{n}$.

$$\therefore \frac{b + \frac{2}{\sqrt{3}}y}{2} = y\frac{2}{\sqrt{3}} \Rightarrow \boxed{b = \frac{2y}{\sqrt{3}}}$$

$$A = (b+ny).y = \sqrt{3}\,y^2$$

$$V = c\sqrt{y_m s} \quad \therefore \quad Q = AV\,\cancel{\sqrt{y_m s}} = A\,c\sqrt{y_m s}$$

$$10 = \sqrt{3}y^2 \times 66\sqrt{\frac{y}{2} \times \frac{1}{750}}$$

$$= 2.95\,y^{5/2}$$

$$\therefore y = (10/2.95)^{2/5} = 1.63\,m$$

$$b = \frac{2y}{\sqrt{3}} = \frac{2 \times 1.63}{\sqrt{3}} = 1.88\,m$$