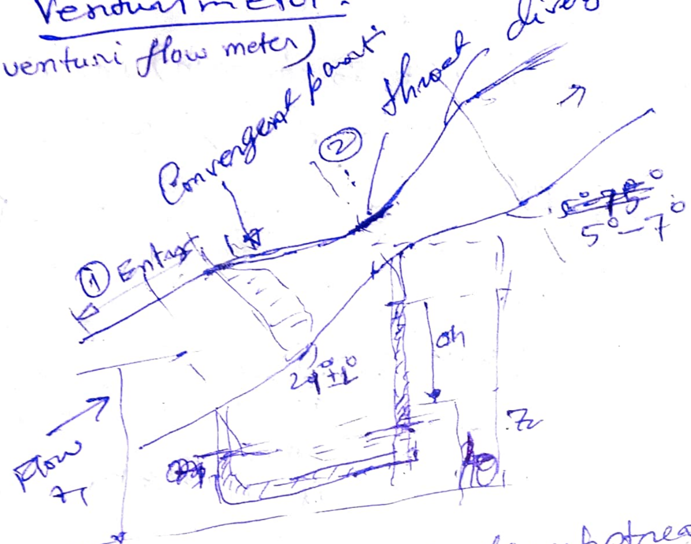
Flow Measurement.

Almost all the flow measuring devices in closed conduits are based on idea that when reduction in cross-sectional area is introduced in a pipe, the flow accelerates with corresponding drop in pressure.

The devices those are mostly used are venturimeter, Orificemeter and flow nozzles.

Venturimeter.
(or venturi flow meter)

Invented by Clements Herschel (1887) named after Italian engr. Venturi



** Such installed that upstream part of flow takes place through short conical portion, Angle of converging part is higher and length is short. The diverging portion is longer in length and angle is small (5°–7°) to avoid any loss of energy due to formation of eddy or any flow separation.

Velocity change takes place. Vel is max^{m} at throat and press is min at throat.

Steady, ideal, 1D flow of fluid,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\therefore \frac{V_2^2 - V_1^2}{2g} = \frac{p_1 - p_2}{\rho g} + z_1 - z_2$$

$$Q = V_1 A_1 = V_2 A_2$$

$$\therefore V_1 = \frac{V_2 A_2}{A_1}$$

$$\therefore \frac{V_2^2}{2g}\left(1 - \frac{A_2^2}{A_1^2}\right) = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right)$$

$$V_2 = \frac{1}{\sqrt{1 - \frac{A_2^2}{A_1^2}}} \sqrt{2g\,(h_1^* - h_2^*)}$$

h_1^* & h_2^* = piezometric pressure head

$$Q = A_2 V_2 = \frac{A_2}{\sqrt{1 - \frac{A_2^2}{A_1^2}}} \cdot \sqrt{2g\,(h_1^* - h_2^*)} = K\sqrt{h_1^* - h_2^*}$$

The press difference between ① & ② is measured by manometer.

From figure we can see that.

$$p_1 + \rho g (z_1 - h_0) = p_2 + \rho g (z_2 - h_0 - \delta h) + \delta h \cdot \rho_m \cdot g$$

$$\therefore (p_1 + \rho g z_1) - (p_2 + \rho g z_2) = (\rho_m - \rho)\, g\, \delta h.$$

$$\therefore \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = \left(\frac{\rho_m}{\rho} - 1\right)\delta h = P_h.$$

$= h_1^* - h_2^*$ = Difference in piezometric head.

[ρ_m = density of fluid in manometer; ρ = density of fluid in of flow]

$$\therefore Q = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g\left(\frac{\rho_m}{\rho} - 1\right)\delta h} = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g P_h}$$

δh measured will be always higher ~~than the~~ for real fluid the ideal fluid. / because of frictional losses in addition to change in momentum.

So above eqn always overestimate the flow rate.

$$\therefore Q_{actual} = C_d \times Q$$

C_d = co-efficient of discharge

$$C_d = \frac{\text{Actual rate of discharge}}{\text{Theoretical rate of discharge}}$$

C_d = 0.95 to .98 for a venturimeter.

Constructional features of ventury.
Straight pipe preceeding venturimeter should be 5 to 10 dia without any fittings, misalignment or other source of large scale turbulence. Straight vanes at upstream can be placed to reduce turbulence

Converging Conical section has an convergence angle of $21 \pm 2°$. Velocity increases and pressure decreases.

Throat :- minimum area of cylindrical section. Vel. maximum, pressure minimum. The throat dia between $\frac{1}{2}$ to $\frac{1}{4}$ of inlet dia. Length of throat equals its Diameter.

Divergent section: Recovery of K.E by conversion to pressure takes place. Angle 5° to 7° to accomplish maxm recovery of K.E. At large angle flow separates from boundary.

Pressure taps are single hole or piezometric ring. [Small sized are made of brass or bronze] less than 5 cm.]

Smooth fi

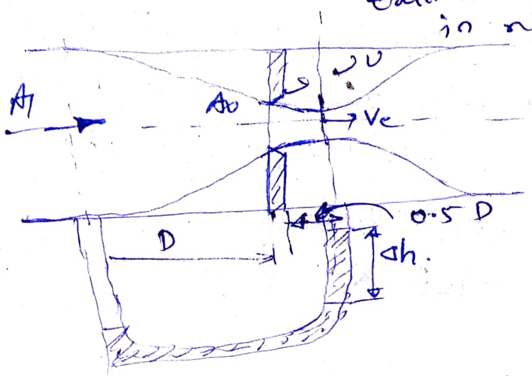
~~Adv~~

Adv:- ① High press recovery
② smooth surface so no wear & tear.
③ Suited for large flow

Disadv → Cost high, space requirement high. generally not useful below 7.5 cm dia pipe.

Orificemeter →

A thin circular metalic plate with a hole in it. It is held by two flanges. call, Minimum pressure exists at vena-contracta which is not within orifice.



Location of vena contracta depends on Re, $\frac{d}{D}$ Area between orifice & pipe, roughness & compressibility of the fluid.

upstream pipe diameter = 10 D. (at least)

C_d varies with type of orifice, pipe size, orifice size, Re, & location of the pressure connection.

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_e}{\rho g} + \frac{V_e^2}{2g}$$

$V_1 A_1 = V_e A_e$.

$$V_e = \sqrt{\frac{2(p_1 - p_e)}{\rho(1 - A_e^2/A_1^2)}}$$

Actual $V_e = C_v \cdot \sqrt{\frac{2g(\frac{\rho_m}{\rho} - 1)\delta h}{(1 - A_e^2/A_1^2)}}$

$Q = A_e V_e = A_1 V_1 = A_2 V_2$

$Q = A_e \cdot V_e$

Coefficient of Contraction $C_c = \frac{A_e}{A_0}$

$$C_d = C_v C_c \cdot \frac{\sqrt{1 - (\frac{A_0}{A_1})^2}}{\sqrt{1 - (\frac{A_0}{A_1})^2 C_c^2}}.$$

$$Q = C_c \cdot A_0 C_v \sqrt{\frac{2g(\frac{\rho_m}{\rho} - 1)\delta h}{1 - C_c^2 \frac{A_0^2}{A_1^2}}}$$

$$Q = \frac{C_d A_0}{\sqrt{1 - (\frac{A_0}{A_1})^2}} \cdot \sqrt{2gh}$$

$$= C_v \cdot C_c \cdot A_0 \cdot \sqrt{\frac{2g}{(1 - C_c^2 \frac{A_0^2}{A_1^2})}} \sqrt{(\rho_m/\rho - 1)\delta h}$$

$$h = \left(\frac{\rho_m}{\rho} - 1\right)\delta h.$$

$$= C \sqrt{\left(\frac{\rho_m}{\rho} - 1\right)\delta h}$$

$$C = C_d \cdot A_0 \sqrt{\frac{2g}{(1 - C_c^2 \frac{A_0^2}{A_1^2})}}$$ where $C_d = C_v \cdot C_c$.

Value of C depend upon the orifice duct area and Re.

$C_d = 0.60$ to 0.65

Flow nozzle

$C_d = .70$ to $.80$

Prob: Find the discharge of water flowing through a pipe of ϕ 30 cm placed in an inclined position where a venturimeter is inserted, having a throat dia of 15 cm. (30 × 15 cm. venturimeter)
(3) The difference of press between the main and throat is measured by a liquid of sp gr 0.6 in an inverted U-tube which gives a reading of 30 cm. The loss of head is 0.2 times the kinetic head of pipe.

Problem: A venturimeter with 200 mm at inlet and 100 mm throat is laid with axis horizontal and is used for measuring the flow of oil of
(2) sp. gravity 0.8. The difference of levels in the U-tube differential manometer reads 180 mm of mercury whilst 11.52×10^3 kg of oil is collected in 4 minutes. Take sp. gravity of mercury as 13.6.

Soln: $A_1 = \frac{\pi}{4}(.2)^2 = 0.0314\ m^2$; $A_2 = \frac{\pi}{4}(0.1)^2 = 0.00785\ m^2$

$\therefore \frac{A_1}{A_2} = \frac{0.0314}{0.00785} = 4$.

Piezometric head $P_h = h\,(S_m - 1) = 0.18\left[\frac{13.6}{0.8} - 1\right]$
$= 2.88$ m of oil.

$\therefore$ Mass of oil $= 11.52 \times 10^3$ kg in 4 minute.

Discharge $Q = \frac{11.52 \times 10^3}{4 \times 60} \times \left(\frac{1}{800}\right) = 0.06\ m^3/sec$

$Q = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g P_h}$

$= C_d \cdot \frac{A_1}{\sqrt{(A_1/A_2)^2 - 1}} \sqrt{2g P_h}$

or $0.06 = C_d \cdot \frac{0.0314}{\sqrt{4^2 - 1}} \sqrt{2 \times 9.81 \times 2.88} = 0.0609\ C_d$.

$\therefore C_d = \frac{0.06}{0.0609} = 0.985$

Prob: (3) A horizontal venturimeter with 30 cm inlet and 10 cm throat is used for measuring the flow of water through a pipeline. If press in pipe is 1.5 kPa [150] and the vacuum press at the throat is 40 cm of mercury, calculate the rate of flow. It may be presumed that 5% of differential head is lost between pipe main and the throat section. Also make calculation for discharge coefficient. Take sp wt. of water = 10 KN/m³

Soln:- $A_1 = \frac{\pi}{4}(0.3)^2 = 0.07065\ m^2$ $A_2 = \frac{\pi}{4}(0.1)^2 = 0.00785\ m^2$

$\frac{A_1}{A_2} = 9$; Pressure $\frac{p_1}{\gamma} = \frac{150}{10} = 15$ m of water.

$\frac{p_2}{\gamma} = 40$ cm of mercury vacuum. $= 40 \times 13.6$
$= 5.44$ m of H_2O

Head loss between inlet and throat

$h_f = 5\%$ of differential head.

$= \frac{5}{100}[15 - (-5.44)] = 1.022$ m of H_2O.

Applying Bernoulli's eqn between inlet and throat section -

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_f.$$

$$\therefore \frac{(V_2^2 - V_1^2)}{2g} = \frac{p_1}{\gamma} - \frac{p_2}{\gamma} + (z_1 - z_2) - h_f.$$

$$= 15 - (-5.44) + 0 - 1.022 = 19.418 \text{ m of } H_2O$$

From continuity eqn $V_1 A_1 = V_2 A_2$

$$\therefore V_1 = V_2 \cdot \frac{A_2}{A_1} = \frac{V_2}{9}.$$

$$\therefore \frac{V_2^2 - (V_2/9)^2}{2g} = 19.418 \Rightarrow V_2 = 19.64 \text{ m/s}$$

Discharge Q = Area × Vel = 0.00785 × 19.64 = .154 m³/s.

This is the actual discharge since head loss between head and throat has been considered.

$$\therefore Q = C_d \cdot \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \cdot \sqrt{2gh} = \frac{C_d \cdot A_1}{\sqrt{\left(\frac{A_1}{A_2}\right)^2 - 1}} \sqrt{2g(p_1 - p_2)/\gamma}$$

$$0.154 = C_d \cdot \frac{0.07065}{\sqrt{9^2 - 1}} \cdot \sqrt{2 \times 9.81 \times 20.44} = 0.158\ C_d.$$

$$\therefore C_d = \frac{0.154}{0.158} = 0.975.$$

Problem → Find the discharge of water flowing through a pipe of 30 cm dia placed in an inclined position where a venturimeter is inserted, having throat diameter of 15 cm. The diff ~~between~~ of pressure between the main & throat is measured by a

(4) liquid of sp. gr. 0.6 in an inverted U tube, which gives reading of 30 cm. The loss of head ~~is~~ between the main and the throat is 0.2 times the kinetic head of pipe.

Soln:- $A_1 = \frac{\pi}{4}(0.3)^2 = 0.07065\ m^2$ $A_2 = \frac{\pi}{4}(0.15)^2 = 0.01766\ m^2$

Applying Bernoulli's eq^n between inlet & throat.

we get. $\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_f$.

$$\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = \frac{p_1 - p_2}{\gamma} + z_1 - z_2 - h_f$$

But $\frac{p_1 - p_2}{\gamma} + z_1 - z_2 = (h_1^* - h_2^*) = h(1 - S_m)$

$= 0.3(1 - 0.6) = .12$ m of H_2O.

$$h_f = 0.2 \times \frac{V_1^2}{2g}$$

$$\therefore \frac{V_2^2}{2g} - \frac{V_1^2}{2g} = 0.12 - 0.2 \times \frac{V_1^2}{2g}$$

$$\therefore \frac{V_2^2}{2g} - 0.8 \cdot \frac{V_1^2}{2g} = 0.12$$

$$V_1 = \frac{A_2}{A_1} \cdot V_2 = \frac{0.01766}{0.07065} V_2 = \frac{V_2}{4}$$

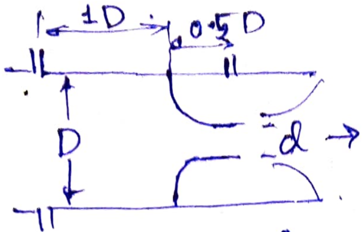
$$\text{or } \frac{V_2^2}{2g} - \frac{0.8}{2g} \times \left(\frac{V_2}{4}\right)^2 = 0.12 \Rightarrow \frac{0.95\ V_2^2}{2g} = .12$$

or $V_2 = 1.574$ m/sec.

$\therefore$ Discharge $Q = V_2 \times A_2 = 0.01766 \times 1.574\ m^3/sec$

$= 0.0278\ m^3/sec.$

Flow Nozzle.



A flow nozzle comprises a smooth, gradual contraction to throat, followed by a free, uncontrolled expansion back to the original pipe flow area.

There is no provision for orderly transformation of vel. into static pressure. the nozzle has a pressure loss of 80 to 90% of the differential pressure obtained.

The discharge co-efficient of a flow nozzle depends on smoothness of approach to tangency, length of the cylindrical portion of the nozzle and location of the pressure taps.

Usually made of gun metal, stainless steel or monel metal.

Adv. ① cheaper than a venturimeter. Can be installed anywhere without difficulty.
② increased Cd than orifice.
③ Length is less than venturimeter.
④ widely accepted for high pres/high temp flow nozzle. Good for slurry.

Disadv. ① Pres recovery is poor. Can't be used at where available pressure head is low or where pressure recovery is must.
② Compared to orifice meter it is difficult to install & costly.
③ Not available over 120 cm pipe.