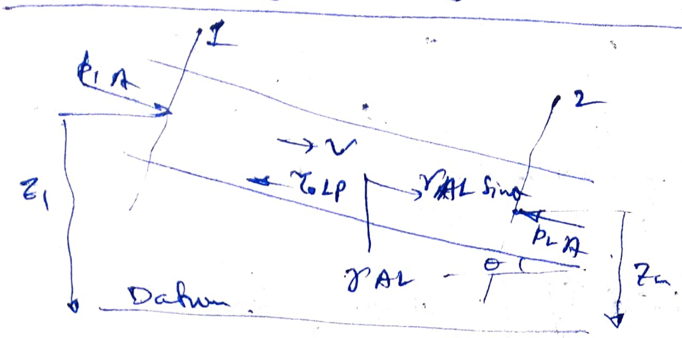
Derivation of Darcy-Weisbach eqn.



Wall shear stress $\tau_0 = \lambda \cdot \frac{\rho}{2} v^2$

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_{Loss} \quad \cdots (1)$$

$$\therefore h_L = \frac{p_1 - p_2}{\gamma} + z_1 - z_2.$$

Linear momentum eqn. $\Sigma F = 0$ P = perimeter

$$(p_1 - p_2)A + \gamma AL \sin\theta - \tau_0 LP = 0$$

Since $L \sin\theta = z_1 - z_2$.

we get $\frac{p_1 - p_2}{\gamma} + z_1 - z_2 = \tau_0 \frac{LP}{\gamma A} \quad \cdots (2)$

Comparing (1) & (2) we get $\tau_0 \frac{LP}{\gamma A} = h_{Loss}$.

$$\therefore H_L = \frac{\tau_0 LP}{\gamma A} = \lambda \cdot \frac{\rho}{2} v^2 \cdot \frac{LP}{\gamma A} \Rightarrow$$

$$= \lambda \cdot \frac{1}{2} \frac{v^2 \cdot L \times 4}{g \cdot D_h}$$

$D_h = \frac{4A}{P}$

$$\boxed{\frac{P}{A} = \frac{4}{D_h}}$$

$$\boxed{\frac{P}{A} = \frac{\pi D}{\frac{\pi}{4} D^2} = \frac{4}{D}}$$

$$\boxed{\frac{P}{A} = \frac{4}{D_h}} \quad = 4\lambda \cdot \frac{L}{D_h} \cdot \frac{v^2}{2g}.$$

For pipe. $\lambda = \frac{f}{4}$ or $f = 4\lambda$.

$$\boxed{\therefore H_L = f \cdot \frac{L}{D_h} \cdot \frac{v^2}{2g}}$$

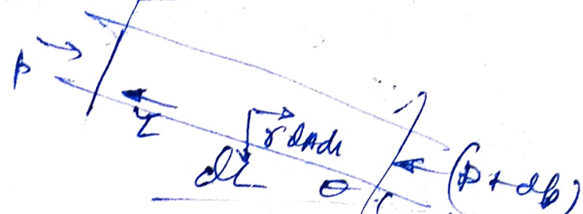
$\boxed{\lambda \text{ is found by experiments.}}$ for different kind of flow.

Define Reynold's no. $= \frac{\text{Inertia force}}{\text{Viscous force}}$.

For Laminar viscous flow $h_f = \frac{32 \mu v l}{\rho g d^2} = f \frac{l}{d} \frac{v^2}{2g}$

gives $\begin{cases} f = 64 \frac{\mu}{\rho d v} \\ \quad = 64/Re \end{cases}$

Velocity distribution for laminar flow of viscous fluid through a pipe



$$p\,dA - (p+dp)\,dA + \gamma\,dA\,dl\sin\theta - \tau\,2\pi r\,dl = 0$$

Thin concentric cylinders slides over one another.

Laminar steady flow.

Consider a circular element of r and c/s of dA.

$$\tau = -\frac{\pi r^2}{2\pi r}\left[\frac{d}{dl}(p+\gamma z)\right] \quad \left[\because dA = \pi r^2 \ \& \ dl\sin\theta = dz\right]$$

$p + \gamma z = p^*$ = piezometric head.

$$\tau = -\frac{r}{2}\frac{dp^*}{dl} = -\mu\cdot\frac{dv}{dr} \quad \text{[as per Newtons Law]}$$

-ve as v decreases as r increases from centre

$$\therefore \frac{dv}{dr} = \frac{r}{2\mu}\frac{dp^*}{dl}$$

$$dv = \frac{r}{2\mu}\frac{dp^*}{dl}\cdot dr$$

$$\therefore v = \frac{1}{2\mu}\cdot\frac{dp^*}{dl}\cdot\frac{r^2}{2} + c$$

at $r = R$ $v = 0$

$$\therefore c = -\frac{1}{2\mu}\frac{dp^*}{dl}\cdot\frac{R^2}{2}$$

$$\therefore v = -\frac{1}{2\mu}\left(\frac{dp^*}{dl}\right)\left(\frac{R^2-r^2}{2}\right)$$

This is velocity distribution of laminar flow in a circular pipe and it is parabolic in nature (flow is fully developed)

for max^m vel. $\frac{dv}{dr} = 0$ $\therefore -\frac{1}{4\mu}\frac{dp^*}{dl}(-2r) = 0$

$\Rightarrow r = 0$ at centre.

$$v_{max} = -\frac{1}{4\mu}\left(\frac{dp^*}{dl}\right)\cdot R^2 = \frac{\Delta p\,R^2}{4\mu l}$$

$$-\frac{dp^*}{dl} = \frac{\left(\frac{p_2}{\gamma}+z_2\right)-\left(\frac{p_1}{\gamma}+z_1\right)}{l_2-l_1} = \frac{\Delta p}{l}$$

$$\boxed{v_{max} = \frac{\Delta p\,R^2}{4\mu l}}$$

To find out Q, δh & av. velocity $\bar{v}$

Consider elementary ring of radius 'r' and thickness dr.

$$dQ = v\,(2\pi r\,dr) \qquad Q = AV$$

$$Q = \int_0^R v\,2\pi r\,dr = \int_0^R \frac{1}{4\mu}\left(-\frac{dp^*}{dl}\right)(R^2-r^2)\cdot 2\pi r\,dr$$

$$= \frac{1}{8\mu}\left(-\frac{dp^*}{dl}\right)\pi R^4$$

$$\boxed{Q = \frac{1}{8\mu}\cdot\frac{\delta p}{l}\cdot\pi R^4} \qquad \boxed{\text{as } -\frac{dp^*}{dl} = \frac{\delta p}{l}}$$

$R = \frac{d}{2}$

$$Q = \frac{\pi d^4\,\delta p}{128\,\mu L} = \frac{\pi d^4}{128\,\mu l}\cdot\gamma\cdot\left(\frac{\delta p}{\gamma}\right)$$

$\frac{\delta p}{\gamma} = \delta h$ = drop in piezometric head $= h_f$.

$$\boxed{\delta p = \frac{32\,\mu\,V_{av}\cdot l}{d^2}}$$

$$= \frac{\pi d^4 \gamma}{128\,\mu l}\cdot\delta h$$

$$\boxed{\delta h = \frac{128\,\mu\,l\cdot Q}{\pi\,\gamma\,d^4}}$$

This is known as Hagen - Poiseuilli's eqn (Expt 1839, 1840 (indep)). Analytical proof by G.H. Wiedemann in 1856.

Av. velocity $\bar{v} = \frac{Q}{\pi R^2} = \frac{\frac{1}{8\mu}\cdot\frac{\delta p}{l}\,\pi R^4}{\pi R^2} = \frac{1}{8\mu}\frac{\delta p}{l}\cdot R^2$

$$\frac{\bar{v}}{v_{max}} = \frac{\frac{1}{8\mu}\frac{\delta p}{l}\cdot R^2}{\frac{\delta p\,R^2}{4\mu l}} = \frac{1}{2} \qquad \boxed{v_{max} = 2\bar{v}}$$

For laminar flow $\quad h_f = f\cdot\frac{l\,v^2}{2gd} = \frac{128\,\mu\,l\,Q}{\gamma\,\pi\,d^4}$

$\cancel{h_f = f\cdot\frac{l v^2}{2gd}}$

But $v = \frac{Q}{A} = \frac{4Q}{\pi d^2}$ and $\gamma = \rho g$

$$f\cdot l\cdot\left(\frac{4Q}{\pi d^2}\right)^2\frac{1}{2gd} = \frac{128\,\mu\cdot l\,Q}{\rho g\cdot\pi\,d^4}$$

$$f = \frac{64\,\mu}{\rho\,d\,v} = \frac{64}{\frac{\rho v d}{\mu}} = \frac{64}{Re}.$$

$$\boxed{f = \frac{64}{Re}}$$

Equating Darcy-Weisbach eqn & Hagen Poiseulle's eqn

For turbulent flow

1. Blasius eqn $\quad f = \frac{0.3164}{(Re)^{.25}} \qquad 20\times10^3 < Re < 8$

2. Mikuradse eqn $\quad f = 0.0032 + \frac{.221}{Re^{.237}} \qquad 2\times10^4 < Re < 2$

3. Schiller eqn $\quad f = .005 + \frac{.396}{Re^{0.3}} \qquad 2\times10^4 < Re < 2\times10^6$

Minor losses in pipes

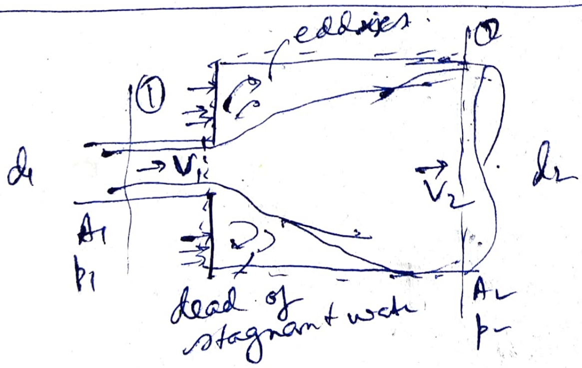
This type of loss occur due to change of c/s, bends, valves and fittings of all kinds. The losses are ~~similar in~~ smaller in comparison to major loss (i.e head loss due to friction). For long ~~ff~~ pipe this loss is neglected. For short pipe this ~~can~~ should be taken care of. The minor loss are due to sudden change in vel., either in magnitude or in direction. This change induce large scale of turbulence which ~~into~~ generate heat.

Experimentally it is found that this minor losses are proportional to mean velocity head.

~~Loss~~ i.e $h_L = K \cdot \frac{v^2}{2g}$. K = minor loss coefficient. At high values of Re K becomes constant.

~~K becomes constant~~.

Loss due to sudden enlargement



$V_1 > V_2$ [applying continuity eq^n]

p = mean press of the eddying fluid over the the annular face GP.

Neglect shear force.

Consider cv - BCDEFG.

External forces acting on the control volume in the direction of flow. [but $p' = p_1$] - experimental evidence

$$F = p_1 A_1 - p_2 A_2 + p'(A_2 - A_1) = (p_1 - p_2) A_2$$

p' = mean pressure of eddying fluid over annular face

Applying momentum eq^n.

$$\rho Q (V_2 - V_1) = (p_1 - p_2) A_2$$

or $$p_1 - p_2 = \frac{\rho Q}{A_2}(V_2 - V_1) = \rho V_2 (V_2 - V_1) \quad \text{--- (1)}$$

Applying Bernoulli's eq^n

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_L$$

or $h_L = \frac{p_1 - p_2}{\gamma} + \frac{1}{2g}(V_1^2 - V_2^2)$ as $[z_1 = z_2]$ ... (2)

$= \frac{\rho V_2(V_2 - V_1)}{\gamma} + \frac{1}{2g}(V_1^2 - V_2^2)$

$= \frac{(V_1 - V_2)^2}{2g}$

$$\boxed{h_L = \frac{V_1^2}{2g}\left(1 - \frac{A_1}{A_2}\right)^2 = \frac{V_2^2}{2g}\left(\frac{A_2}{A_1} - 1\right)^2}$$

$V_1 A_1 = V_2 A_2$, $V_2 = \frac{A_1}{A_2} V_1$

First obtained by:- Borda (1733-39) & LMN Carnot (1753-1823)

This is known as Borda-Carnot head loss.

For a particular type of pipe given $L = K \cdot \frac{V^2}{2g}$.

Exit loss.

For large reservoir $A_2 \to \infty$.

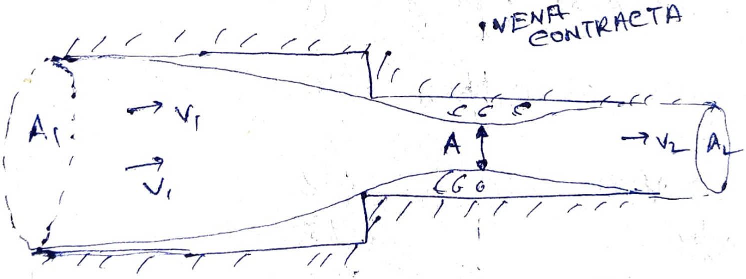
∴ $h_L = \frac{V_1^2}{2g}$ ⇒ known as exit loss of pipe.

i.e. $K_{exit} = 1.0$

Other terms synonymous with the loss of head due to abrupt enlargement are,

- Eddy loss because of expansion loss, is expended exclusively on eddy formation.
- Shock loss.

Sudden Contraction.



i) There is little loss of head between entrance and vena contracta, as chance of boundary layer separation is minimum.

ii) divergent flow downstream causes loss of energy due to formation of eddys.

$h_L = \frac{(V_c - V_2)^2}{2g}$ V_c = vel at vena contracta.

$\frac{A_c}{A_2} = C_c$ $A_c = C_c A_2$ $A_2 V_2 = A_c V_c = C_c A_2 V_c$

or $V_c = \frac{V_2}{C_c}$

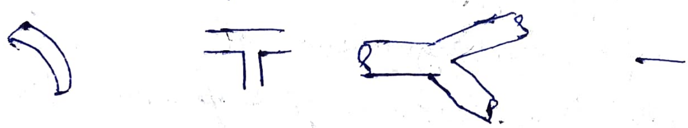
$$h_L = \left[\left(\frac{v_2}{C_c}\right) - v_2\right]^2 / 2g = \left(\frac{1}{C_c} - 1\right)^2 \frac{v^2}{2g} = K_{con}\frac{v^2}{2g}$$

K_{con} depends on the rate of contraction i.e. on area ratio A_2/A_1 and to some extent ~~of~~ on the flow Reynolds number.

If A_1 is very large then $K_{con} \approx 0.5$. Then it is called the pipe entrance loss.

Sharp edge mouthpiece $K_{ent} = 0.5$
Re-entrant or Borda mouthpiece $K_{ent} = 1.0$
Rounded or Bell-mouthed entrance $K_{ent} = 0.05$

Bend – Total angle of bend is a function of total angle ~~ondoy~~ bend and ratio r/d
For 90° smooth bend minimum loss is experienced with ~~2.5-5~~ $r/d = 2.5 - 5$.



Prob:- ~~Water is flowing through a 20 cm Dia pipe with friction factor f=0.04 The area~~

A_2/A_1	0	0.04	0.16	0.36	0.64	1
K_{ent}	0.5	0.45	0.38	.28	.14	0

Approximate values of k for commercial pipe fitting.

Globe valve wide open	10
Gate valve	
wide open	0.2
$\frac{3}{4}$ open	1.15
$\frac{1}{2}$ "	5.6
$\frac{1}{4}$ "	24
Pump foot valve	1.5
90° elbow (threaded)	0.9
45° " "	0.4
Side out of T junction	1.8

Prob

Determine the head (energy) loss for a flow of 140 L/s of oil, $\nu = 0.00001\ m^2/s$. through 400 m of 200 mm CI pipe. $\epsilon = 0.25$.

$$R = \frac{4Q}{\pi D \nu} = \frac{4 \times .14}{\pi \times .2 \times .00001} = 89,127.$$

$$\frac{\epsilon}{D} = .25/200 = .00125$$

From Moody diagram $f = 0.023$

$$f = \frac{1.325}{\left[\ln \frac{\epsilon}{3.7D} + 5.74/R^{0.9}\right]^2}$$

$10^{-6} \le \epsilon \le 10^{-2}$ & $5000 \le Re \le 10^8$

by formula $f = 0.0234$

$$\therefore h_f = f \cdot \frac{L}{D} \frac{v^2}{2g} = .023 \times \frac{400}{.2} \times \left[\frac{.14}{\frac{\pi}{4}(0.2)^2}\right]^2 \times \frac{1}{2 \times 9.806} =$$

$= 46.58$ N.m/N (m).

Prob

A pipe of 25 cm dia & 60 m long conveys water at a velocity of 3.0 m/s. Find the head lost due to friction. ~~by using~~

$\nu = 0.01$ stoke $= 0.01 \times 10^{-4}\ m^2/s$.

$$Re = \frac{vd}{\nu} = \frac{3 \times 0.25}{.01 \times 10^{-4}} = 7.5 \times 10^5 \Rightarrow \text{turbulent}.$$

$$f = \frac{.3164}{Re^{.25}} = .01075 \text{ (Blasius eqn)}$$

$$\therefore h_f = f \cdot \frac{l}{d} \cdot \frac{v^2}{2g} = 1.183\ m.$$

Oil of $\nu = 10^{-5}\ m^2/s$ flows at steady rate through a CI pipe of 100 mm dia and $\epsilon = .25$ mm. Length of pipe $= 120$ m & head loss $= 5$ m of oil. Find flow rate.

Soln: $\frac{\epsilon}{D} = .0025$ gen $f = 0.026$

$$5 = .026 \times \frac{120}{.10} \times \frac{v^2}{2 \times 9.81}$$ or $V = 1.773$ m/s.

$$Re = \frac{VD}{\nu} = \frac{1.773 \times .10}{10^{-5}} = 1.773 \times 10^4.$$

at this Re & $\frac{\epsilon}{D} = .0025$ data $f = .0316$.

$$5 = .0316 \times \frac{120}{.1} \times \frac{v^2}{2 \times 9.81}$$ $V = 1.608$ m/s.

afo $Re = 1.608 \times 10^4$ at this Re & $\frac{\epsilon}{D}$ $- f = .0318$ (% error is less)

$V = 1.608$ $Q = VA = .013\ m^3/s.$

Prob: Determine the size of a G.I. pipe needed to transmit water at distance 180 m with a head loss = 9 m ($\nu = 1.14\times10^{-6}$ m²/s. $\epsilon = .15$ mm)

Soln: $9 = f\cdot\frac{180}{D}\cdot\left(\frac{.085}{\pi D^2/4}\right)^2\cdot\frac{1}{2\times 9.81}$ .. or $D^5 = .012 f$

$Re = \frac{.085 D}{\frac{\pi D^2}{4}\cdot 1.14\times10^{-6}} = 9.49\times10^4\cdot\frac{1}{D}$.

Guess $f = .024 \Rightarrow D = .196$ m & $Re = 4.84\times10^5$

$\therefore \frac{\epsilon}{D} = \frac{.15}{.196}\times10^{-3} = .00076$.

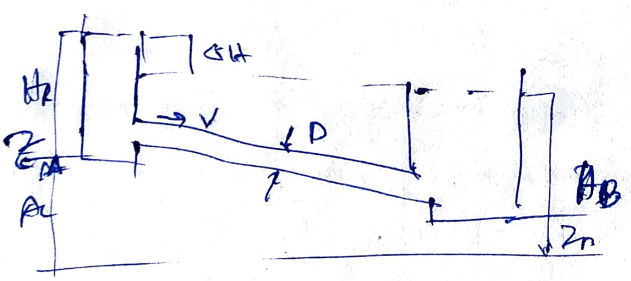
at this Re & $\frac{\epsilon}{D}$ $f = .019$

Recalculate D & Re $D = .187$ m $Re = 5.07\times10^5$.

at this $Re = 5.07\times10^5$ & $\frac{\epsilon}{D}$ $\left(\frac{.15}{187} = 0.0008\right)$

$f = .0192$.

(Error in negligible) $\therefore D = 187$ mm $= .187$ m



$h_f = \frac{.5V^2}{2g} + f\cdot\frac{L}{D}\cdot\frac{V^2}{2g} + \frac{V^2}{2g} = \left(1.5 + f\frac{L}{D}\right)\frac{V^2}{2g}$

entry loss   exit loss

$h_f = \left[8\cdot\left(1.5 + f\cdot\frac{L}{D}\right)\cdot\frac{1}{\pi^2 D^4 g}\right]Q^2 = RQ^2$

R = flow resistance.

$\Delta H = RQ^2$

$Q^2 = i$ $v = R\cdot i$

$\Delta H = V$