Fluid Dynamics.

It is the study of fluid motion that involves forces of action and reaction. [i.e forces causing accⁿ or decⁿ]

Dynamics of fluid motion is governed by the Euler equation (momentum ~~eqⁿ~~ principal) and ~~Bernoulli's~~ Bernoulli's eqⁿ (energy principal). The's momentum and energy principals are derived from Newton's 2nd law of motion. [F=ma].

Concepts

* System - fixed mass, system boundary flexible.
* Control volume - Fixed volume in space bounded by the control surface. Some part of ~~c/s~~ c/s is ~~ptg~~ physical boundary and others are called cross section (c/s).

Energy and its forms.

Energy represents the capacity to produce a change in existing system i.e. capacity to do work.

Energy of a system :- ① Stored energy like K.E, PE and internal energy and ② Energy in transit :- Heat & work.

* P.E. or datum energy → $= mgZ$.
* KE. → $dF = m.\frac{dv}{dt}$

$$\delta W = dF \times ds = m.\frac{dv}{dt}.ds = m.v.dv.$$

$$W = m\int_1^2 v\,dv = \frac{1}{2}mv^2.$$

Internal energy :- Energy stored within ~~mass~~ the system due to spacing in fluid molecules. It comprises of K.E. due to molecular agitation and P.E. due to attractive and repulsive forces between the molecules ~~of~~ or atoms constituting the fluid mass.

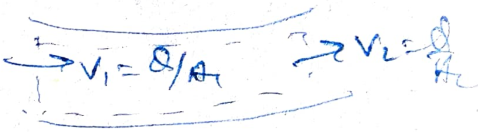
Heat & Work.

Flow work - is a measure of the work required to push a fluid mass across the control surface at the entrance and exit cross-section.

Flow work = force × displacement

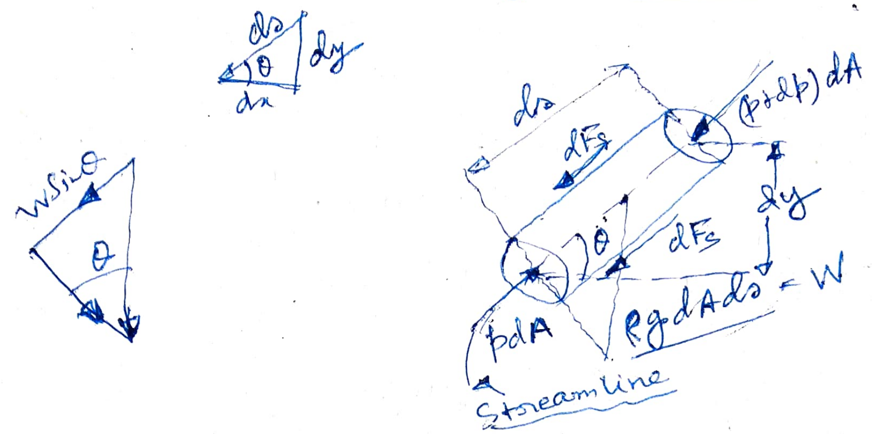
$= p_1 A_1 V_1 . dt$

$= p_1 \forall_1 dt$ $\quad \forall_1 = \text{Vol.}$



$\therefore \frac{\text{Flow work}}{\text{unit time} \times \text{unit mass}} = p_1 \vartheta_1 = \frac{p_1}{\rho_1}$

Euler Equation along a streamline



Euler's equation applies Newton's 2nd law of motion. Element of fluid moving within a stream tube. Elemental area dA.

Normal force:

p is at upstream & $p+dp$ in the downstream.
Net pressure force acting on the element.

$$pdA - (p+dp)\,dA = -dp \cdot dA.$$

Tangential force → due to viscous shear.

$dF_s = \tau \cdot dP \cdot ds$; $\quad dP$ = perimeter.

τ = shear stress. $\quad dP \cdot ds$ = total area of the streamtube.

The sum of all shear force is a measure of energy loss due to friction.

Body force: $= \rho g \cdot dA \cdot ds \cdot \sin\theta$.

$= \rho g\, dA \cdot dy$.

The resulting force is = mass × accn.

$$-dp\,dA - \rho g \cdot dA\, dy - \tau \cdot dP\, ds = \rho \cdot dA\, ds\, a_s.$$

Now $a_s = \dfrac{\partial u}{\partial t} + u \cdot \dfrac{\partial u}{\partial s}$ [Since one dimensional]

For steady flow $\dfrac{\partial u}{\partial t} = 0$. $\therefore a_s = u \cdot \dfrac{du}{ds}$ [Since 1-D]

$$\therefore -dp\,dA - \rho g\, dA\, dy - \tau\, dP\, ds = \rho\, dA \cdot u\, du$$

Deviding throughout by the fluid mass $\rho\, dA\, ds$ give

$$u \cdot \frac{du}{ds} + \frac{1}{\rho} \cdot \frac{dp}{ds} + g \cdot \frac{dy}{ds} = -\frac{\tau}{\rho}\frac{dP}{dA}$$

[This is Euler equation of motion]

(i) $u\frac{du}{ds}$ is a measure of convective accⁿ as it move from one region to another region of different velocity. It is a measure of K.E.

ii) $\frac{1}{\rho}\cdot\frac{dp}{ds}$ - force/unit mass caused by pressure distribution.

iii) $g\cdot\frac{dy}{ds}$ gravitational force per unit ~~mass~~ mass.

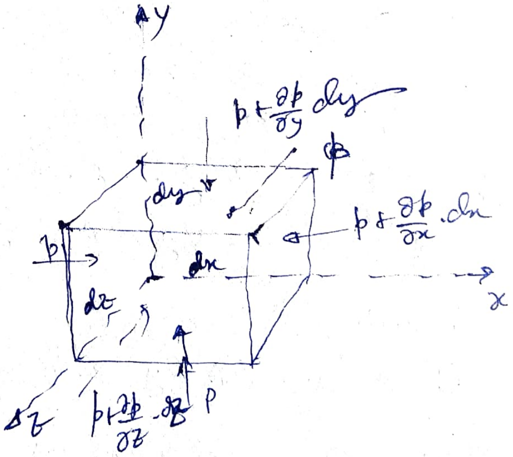
iv) $-\frac{\tau}{\rho}\cdot\frac{dP}{dA}$ → force/unit mass caused by friction.

for ideal fluid $\tau = 0$

∴ The eqⁿ reduces to $u\cdot du + \frac{dp}{\rho} + g\,dy = 0$

The above equations have been set up considering flow within a stream tube.

Euler's Equation in Cartesian Co-ordinates.



① Net pressure force in x-direction

$= p\cdot dy\cdot dz - \left(p+\frac{\partial p}{\partial x}\cdot dx\right)dy\,dz$

$= -\frac{\partial p}{\partial x}\cdot dx\cdot dy\cdot dz.$

② Body force - Let R be the body force per unit mass of fluid having component X, Y, Z in x, y, and z direction respectively.

Body force along X-direction $X\rho\,dx\,dy\,dz$

* Inertia force

Along x- co-ordinate the inertia force is

mass × acceleration $= \rho\,dx\,dy\,dz\,\frac{du}{dt}\left(\frac{DU}{Dt}\right.$

∴ $X\rho\,dx\,dy\,dz - \frac{\partial p}{\partial x}\cdot dx\cdot dy\,dz = \rho\cdot dx\cdot dy\cdot dz\cdot\frac{du}{dt}\left(\frac{DU}{Dt}\right)$

$\therefore X - \frac{1}{\rho}\cdot\frac{\partial p}{\partial x} = \frac{du}{dt}\left(\frac{DU}{Dt}\right)$ [each term is Force per unit mass i.e accn.]

$\therefore X - \frac{1}{\rho}\cdot\frac{\partial p}{\partial x} = \frac{\partial u}{\partial t} + u\cdot\frac{\partial u}{\partial x} + v\cdot\frac{\partial u}{\partial y} + w\cdot\frac{\partial u}{\partial z}$.

Similarly

$Y - \frac{1}{\rho}\frac{\partial p}{\partial y} = \frac{\partial v}{\partial t} + u\cdot\frac{\partial v}{\partial x} + v\cdot\frac{\partial v}{\partial y} + w\cdot\frac{\partial v}{\partial z}$

$Z - \frac{1}{\rho}\cdot\frac{\partial p}{\partial z} = \frac{\partial w}{\partial t} + u\cdot\frac{\partial w}{\partial x} + v\cdot\frac{\partial w}{\partial y} + w\cdot\frac{\partial w}{\partial z}$

$$\boxed{\frac{\partial \vec{V}}{\partial t} + \vec{V}(\nabla\cdot\vec{V}) = -\frac{1}{\rho}\nabla\cdot p + (X+Y+Z)\vec{X}}$$

→ Euler equn.

Each term is accn i.e force/unit mass

Considering the flow steady and multiplying with dx, dy & dz respectively we get

$X\,dx - \frac{1}{\rho}\frac{\partial P}{\partial x}\cdot dx = u\cdot\frac{\partial u}{\partial x}\cdot dx + v\cdot\frac{\partial u}{\partial x}\cdot dx + w\cdot\frac{\partial u}{\partial z}\cdot dx$.

$Y\,dy - \frac{1}{\rho}\frac{\partial P}{\partial y}\cdot dy = u\cdot\frac{\partial v}{\partial x}\cdot dy + v\cdot\frac{\partial v}{\partial y}\cdot dy + w\cdot\frac{\partial v}{\partial z}\cdot dy$

$Z\cdot dz - \frac{1}{\rho}\frac{\partial P}{\partial z}\cdot dz = u\cdot\frac{\partial w}{\partial x}\cdot dz + v\cdot\frac{\partial w}{\partial y}\cdot dz + w\cdot\frac{\partial w}{\partial z}\cdot dz$.

Energy per unit mass

For a streamline $\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$.

$\therefore u\,dy = v\,dx$; $v\,dz = w\,dy$; $u\,dz = w\,dx$

$X\,dx - \frac{1}{\rho}\frac{\partial P}{\partial x}\cdot dx = u\cdot\frac{\partial u}{\partial x}\cdot dx + u\cdot\frac{\partial u}{\partial y}\cdot dy + u\cdot\frac{\partial u}{\partial z}\cdot dz$

$u\cdot\frac{\partial u}{\partial x} = \frac{1}{2}\frac{\partial u^2}{\partial x}$

$\therefore X\,dx - \frac{1}{\rho}\cdot\frac{\partial P}{\partial x}\cdot dx = \frac{1}{2}\left[\frac{\partial}{\partial x}(u^2)dx + \frac{\partial}{\partial y}(u^2)dy + \frac{\partial}{\partial z}(u^2)dz\right] = \frac{1}{2}d(u^2)$

$\therefore Y\cdot dy - \frac{1}{\rho}\frac{\partial P}{\partial y}\cdot dy = \frac{1}{2}d(v^2)$

$Z\,dz - \frac{1}{\rho}\frac{\partial P}{\partial z}dz = \frac{1}{2}d(w^2)$.

Adding $X\,dx + Y\,dy + Z\,dz - \frac{1}{\rho}\left(\frac{\partial P}{\partial x}\cdot dx + \frac{\partial P}{\partial y}dy + \frac{\partial P}{\partial z}\cdot dz\right)$

$= \frac{1}{2}d(u^2) + \frac{1}{2}d(v^2) + \frac{1}{2}d(w^2)$

$X\,dx + Y\,dy + Z\,dz - \frac{1}{\rho}\cdot dp = \frac{1}{2}d(V^2)$

* if $X = 0 = Z$, $Y = -g$ for static fluid $V = 0$

$-g\,dy - \frac{1}{\rho}dp = 0 \Rightarrow dp = -\rho g\,dy = -\gamma\,dy$

$\int^2 dp = -\gamma\int^2 dy \quad p_2 - p_1 = -\gamma(y_2 - y_1) = \gamma(y_1 - y_2)$

Bernoullies Theorem: Integration of Euler's equation for 1 D flow.

It ~~state~~ relates velocity, pressure and elevation changes of a fluid in motion.

$$\frac{1}{2} dv^2 + \frac{dp}{\rho} + g\,dy = 0$$

$$\int \frac{1}{2} dv^2 + \int \frac{dp}{\rho} + g \int dy = \text{const}$$

$$\therefore \frac{v^2}{2} + \frac{p}{\rho} + gy = \text{Constant} \Rightarrow \text{Bernoulli's eq}^n.$$

[Swiss Mathematician Daniel Bernoulli (1700-1782)]

The constant of integration varies from streamline to streamline but ~~var~~ remain constant along a streamline for a steady incompressible and frictionless flow.

~~desig~~ The unit of above eq^n is $\frac{N \cdot m}{kg}$ i.e. energy per unit mass.

~~a~~ deviding by ~~ρ~~ g we get

$$\frac{p}{\gamma} + \frac{v^2}{2g} + y = H \Rightarrow \text{energy per unit weith} \left(\frac{m \cdot N}{N}\right).$$

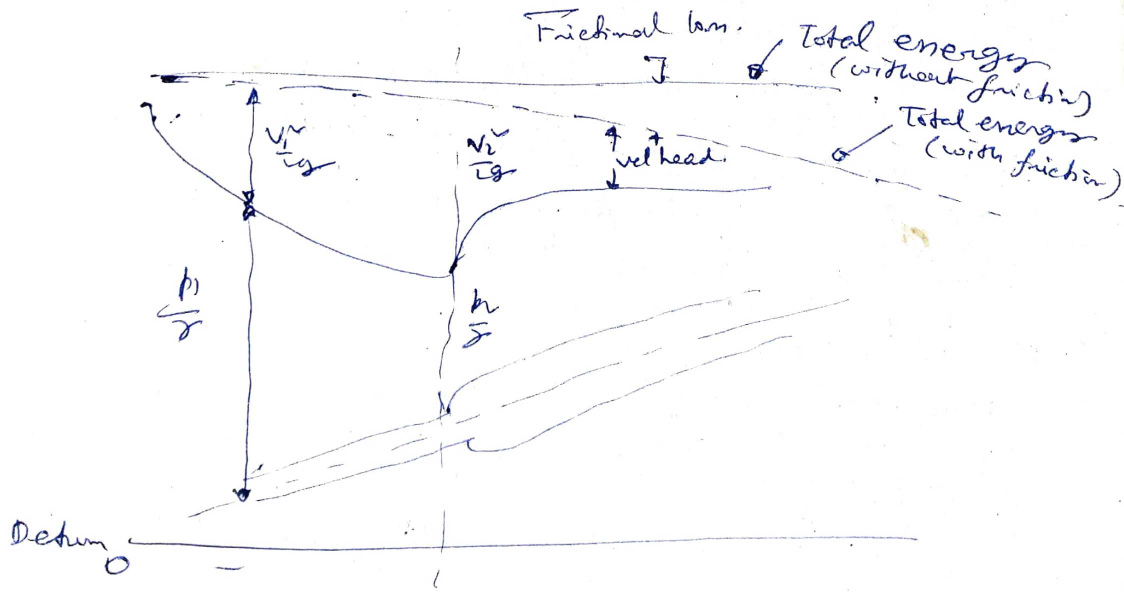
and has physical dimension of length. (m)

$\frac{v^2}{2g}$ = velocity head, $\frac{p}{\gamma}$ = pressure head or static head.

y = elevated head, position head, potential head,

The sum H ⇒ total or hydrodynamic head.

[It is sum of ~~vel~~, ~~ki~~ potential, kinaetic and pressure energy].



Bernoulli's eqⁿ can also be written as

$$\rho \cdot \frac{V^2}{2} + p + \gamma y = \text{Constant.}$$ each for $\frac{N \cdot m}{m^3}$

i.e energy / unit vol.

[This form is applied to gas flow]

Assumptions of Bernoulli's eqⁿ.

1) Flow is steady, incompressible, and 1-D.

② Flow is continuous ~~and velocity is~~.

③ Fluid is ideal.

④ Only gravity and pressure forces are present. [i.e no energy from Heat or work

[The eqⁿ holds for frictionless (inviscid) and constant density fluid. The flow must be steady and the relation holds for single streamline]

For gases density is very small. Hence variation of gz is much less compared to $\frac{p}{\rho}$ and hence the eqⁿ becomes

$$\frac{p}{\rho} + \frac{u^2}{2} = \text{Constant}$$

$$p + \frac{\rho u^2}{2} = \text{Constant.}$$

The term $\frac{u^2}{2}$ represents kinetic energy of a fluid per unit mass. gz represents the static energy due to change in elevation.

$\frac{p}{\rho}$ → corresponds to the work that would be done by the pressure force if fluid is moved from a pt. where the ~~pres~~ pressure is p to one where the pressure is zero.

KE Correction factor

The discharge through a small element of δA c/s is $u\delta A$ and mass flow rate $\rho u \delta A$. The KE passes through the element is $\frac{1}{2}(\rho u \delta A) u^2$ & for the ~~who~~ whole c/s is $\int \frac{1}{2} \rho u^3 dA$ & total mass flow rate $\int \rho u dA$. ~~If ρ is constant.~~

Now. K.E per unit mass = $\dfrac{\int \frac{1}{2} \rho u^3 dA}{\int \rho u dA}$

if ρ is constant then if $\frac{\int \rho U^3 dA}{2/\rho\, dA}$ Unless U is const. over entire c/s then this exp does not corresponds to $U^2/2$

$$\therefore \alpha = \frac{1}{A}\int\left(\frac{U}{\bar{U}}\right)^3 dA \;*$$

SSSF

$$m_1\left(p_1v_1 + u_1 + \frac{V_1^2}{2} + gy_1 + q\right)$$
$$= m_2\left(p_2v_2 + u_2 + \frac{V_2^2}{2} + gy_2 + w\right)$$

$m_1 = m_2$ $\quad q = 0$ $\quad \rho_1 = \rho_2$ (incompressible)

$u_1 = u_2 \rightarrow$ no frictional loss $\quad w = 0$

$$\frac{p_1}{\rho_1} + \frac{V_1^2}{2} + gy_1 = \frac{p_2}{\rho_2} + \frac{V_2^2}{2} + gy_2 \Rightarrow \left(\frac{p}{\rho} + \frac{v^2}{2} + gy = c\right)$$

$$\therefore \left(\frac{p}{\gamma} + \frac{v^2}{2g} + y\right) = \text{Constant}.$$

Problem: A pipe of 12.5 cm in diameter is used to transport oil of relative density 0.75 under a pressure of 1 bar. If the total energy relative to a datum plane 2.5 m below the centre of the pipe is 20 Nm/N, work out the flow rate of oil.

Ans: $20 = \frac{v^2}{2g} + \frac{p}{w} + y = \frac{v^2}{2\times 9.81} + \frac{1\times 10^5}{0.75\times 9810} + 2.5$

$V = \sqrt{2\times 9.81\times 3.91} = 8.76.$

discharge $Q = AV = \frac{\pi}{4}\times(0.125)^2 \times 8.76 = 0.1074\ m^3/s.$

$*\ \alpha$ = KE correction factor $= \frac{1}{A}\int_A \frac{U^3}{\bar{U}}\, dA.$

Actual KE $= \alpha \frac{v^2}{2}$ and $\alpha = \frac{1}{A}\int_A U$

α can never be less than Unity.

A pump delivers water through a ~~150 mm~~ 150 mm dia pipe. Inlet A, 225 m dia ~~m~~ $\bar{U} = 1.35$ m/s. press = 150 Hg vacuum. The pump outlet B is 600 mm above A and 150 mm dia. At C, 5 m above B the gauge press = 35 kPa. if friction in pipe BC is 2.5 kW (dissipation) and power reqd to drive the pump is 12.7 kW, calculate overall η of the pump.

at A $U_A = 1.35$ m/s. $U_B = V_C = U_A \times \frac{\text{Area } A}{\text{Area } C}$

$$= 1.35 \times \left(\frac{225}{150}\right)^2 = 3.038 \text{ m/sec.}$$

$$\therefore \frac{P_A}{\rho} + \frac{1}{2}U_A^2 + gz_A + \frac{\text{Energy added to pump/time}}{\text{Unit mass/time}}$$

$$- \frac{\text{Energy lost/time}}{\text{Unit mass/time}} = \frac{P_C}{\rho} + \frac{1}{2}V_C^2 + gz_C.$$

67.27.

Momentum of fluid in motion (Impulse-momentum)

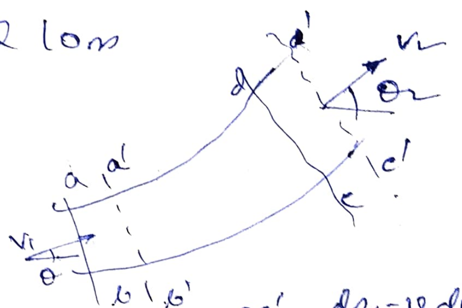
$$F = \frac{d}{dt}(mV) = m\frac{dv}{dt} + v\underbrace{\frac{dm}{dt}}_{=0} = m\frac{dv}{dt}.$$

"The time rate of change in momentum is proportional to the impressed force and takes place in the direction in which the force acts."

$F\,dt$ represents impulse of applied for.

Application:

1) Force on pipe bends and transitions.
2) Force exerted by a fluid jet striking against fixed or moving blades.
3) Force on propeller blades.
4) Jet Propulsion
5) To calculate head loss



mass within $aa'bb'$ = $cdc'd'$
$= \rho_1 A_1 ds_1 = \rho_2 A_2 ds_2$

$\therefore$ Change in momentum

$aa' = ds_1 = V_1 dt$
$dd' = ds_2 = V_2 dt$.

$= (\rho_2 A_2 V_2 dt)V_2 - (\rho_1 A_1 V_1 dt)V_1$

$A_1 V_1 = A_2 V_2 = Q$
$\rho_1 = \rho_2 = \rho$

$= \rho Q (V_2 - V_1) dt$

$F\,dt = \rho Q (V_2 - V_1) dt$

$F = \frac{\gamma Q}{g}(V_2 - V_1)$. $\frac{\gamma Q}{g} = \rho Q$ = mass flux.

$\therefore F_x = \frac{\gamma Q}{g}(V_2 \cos\theta_2 - V_1 \cos\theta_1)$
$F_y = \frac{\gamma Q}{g}(V_2 \sin\theta_2 - V_1 \sin\theta_1)$

Force exerted by the pipe bends on the fluid.

Force exerted by the fluid on

$F_x = \frac{\rho Q}{g}(V_1 \cos\theta_1 - V_2 \cos\theta_2$

$F_y = \frac{\rho Q}{g}(V_1 \sin\theta_1 - V_2 \sin\theta_2)$.

Total $F_x = \frac{\gamma Q}{w}(V_1 \cos\theta_1 - V_2 \cos\theta_2) + p_1 A_1 \cos\theta_1 - p_2 A_2 \cos\theta_2$

Bernoulli's eqⁿ with Head loss

Head loss occur mainly due to friction while fluid flows through a pipe or duct. This can be represented in modified Bernoulli's eqⁿ as.

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_f.$$

where h_f represents the frictional work done per unit weight of a fluid ~~e~~ element while flowing from pt 1 to pt 2. It is basically the loss of total ~~e~~ mechanical energy.

For inviscid fluid h_f or $h_L = 0$

Total head loss in a pipe or duct system

$$h_L = \Sigma h_{major\ losses} + \Sigma h_{minor\ loss}$$

$$= f \cdot \frac{L}{D_h} \cdot \frac{v^2}{2g} + \Sigma k \cdot \frac{v^2}{2g}$$

L = length of duct (m)
f = friction coefficient
v = flow velocity m/s
g = accⁿ due to gravity m/s²

D_h = hydraulic diameter

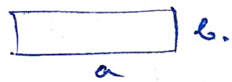
$$= \frac{4A}{P}$$

A = area (m²) of duct c/s
P = wetted perimeter (m)

For circular pipe $D_h = 4 \cdot \frac{\pi D^2}{\pi D 4} = D$

For annulus. $D_h = \frac{4(\pi D_o^2 - \pi D_i^2)}{4 \cdot \pi D_o + \pi D_i} = (D_o - D_i).$

For rectangular ~~tube~~ Duct



$$D_h = \frac{4ab}{2(a+b)} = \frac{2ab}{(a+b)}$$

The hydraulic diameter is not same as equivalent diameter. It is the diameter of a pipe that give same pressure loss as an rectangular duct.

Major loss occur due to friction. This depend on flow velocity, pipe or duct length, pipe dia and a friction factor based on roughness of the pipe and type of flow (flow is laminar or turbulent

$h_f = f \cdot \frac{L}{D_h} \frac{V^2}{2g}$ → known as Darcy-Weisbach eqn. and valid for steady fully developed incompressible flow.

$$\frac{p_1}{\gamma} + \alpha\frac{V_1^2}{2g} + y_1 = p_2/\gamma + \frac{V_2^2}{2g} + y_2 + h_{Loss}.$$

if $V_1 = V_2$ & $y_1 = y_2$

then $h_{loss} = \frac{p_1}{\gamma} - \frac{p_2}{\gamma} = \frac{1}{\gamma}(p_1 - p_2) = h_{static\,pt\,1} - h_{static\,pt\,2}$

friction coefficient $f = 64/Re$ for laminar flow.

∴ $Re < 2300$.

for transition flow. $2300 < Re < 4000$ the flow varies between laminar & turbulent flow and friction factor cannot be determined;

For turbulent flow: f depends on Re and roughness of duct or pipe wall:

$$f = \phi\left(Re, \frac{\epsilon}{D_h}\right)$$

ϵ = roughness and D_h = hydraulic dia.

$\frac{\epsilon}{D_h}$ = relative roughness.

Relative Roughness is determined by the experiment and expressed in m.

The friction factor can be determined by Colebrook equation

$$\frac{1}{f^{1/2}} = -2\log_{10}\left[(2.51/(Re\, f^{1/2})) + \frac{\epsilon}{D_h}/3.72\right]$$

A graphical representation of the above eqn is known as Moody diagram.