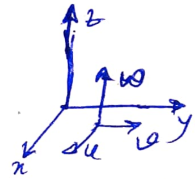
Fluid Kinematics.

It describes geometry of the fluid motion.
Parameters :- Displacement, velocity and acceleration or ~~energies~~ other quantities ~~[illegible]~~ derived from displacement and time.
In the analysis, the force or energies responsible for accelerating or decelerating the flow are not considered.
From velocity distribution, pressure distribution can be computed and force acting on it can be computed.



Description of Fluid Flow

The motion of a fluid particle is expressed quantitavely in terms of velocity.

$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k} \qquad |\vec{V}| = [u^2+v^2+w^2]^{1/2}$$

Velocity components are functions of both position and time. $\vec{V} = f(\vec{r}, t)$.

where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

The velocity components are then defined as

$$u = \frac{dx}{dt} = u(x, y, z, t) \qquad v = \frac{dy}{dt} = v(x, y, z, t)$$

$$w = \frac{dz}{dt} = w(x, y, z, t).$$

The relative position of fluid particles is not fixed but varies with time. At a given instant each individual particles has its own velocity and accn, and these variables change ~~with both~~ both with respect to position and time. Hence for complete description of fluid flow, one has to observe the motion of fluid particles at various positions at successive instants of time.

Ex: $u = 6xy^3+t \qquad v = 3yz+t^2+5 \qquad w = 2+3ty$ m/sec

The Expression for velocity vector at point P(4, 1, 2) m at t = 3 sec. :

$u = 6\times4\times1^2+3 = 27$ m/s. $\quad v = 3\times1\times2+3^2+5 = 20$ m/s.

$w = 2+3\times3\times1 = 11$ m/s.

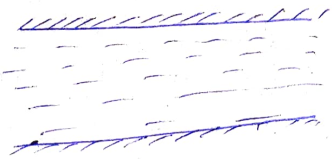
$\vec{V} = 27i + 20j + 11k$

Magnitude of velocity ~~at~~ at pt. P

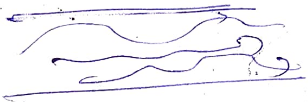
$$V = \sqrt{u^2+v^2+w^2} = \sqrt{27^2+20^2+11^2} = 35.35 \text{ m/s.}$$

Classification of fluid flows.

A ① Laminar – A laminar flow is characterized by smooth flow of one lamina of fluid over other. Fluid elements move in well defined path, and they retain the same relative position a successive cross section of the flow passage. Laminar flow is also called stream line or viscous flow. This type of flow generally occurs when flow vel. is low of the fluid is highly viscous.



② Turbulent → Fluid element move in erratic or ~~up~~ unpredictable path. Individual fluid particles are subjected to fluctuating transverse velocity so the motion is not rectilinear. Eddy may form. The random eddying ~~motto~~ motion is called turbulence. Turbulent flow generally prevails ~~over~~ in rivers, canals and in the atmosphere.



② Steady flow → Fluid parameter at any pt. on the c/s do not vary with time. However the parameters ~~from~~ may vary at different c/s. Thus vel, press, temp & density etc. are function of only location, not time.

$\vec{V} = V(\vec{s})$, $\vec{a} = a(\vec{s})$ Fun… steady

~~Example~~ $P = P(x, y, z)$

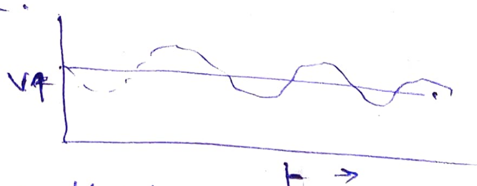
$$\frac{\partial P}{\partial t} = 0$$

Ex:- Liquid from vessel with constant heads

So properties can be varies along space.

Examples:- Liquid efflux from a vessel in which constant level is maintained. Flow of water in a pipe line due to a centrifugal pump being run at uniform rotational speed.

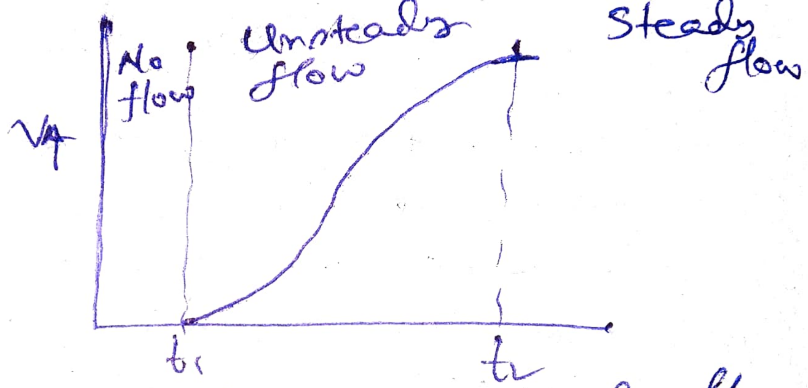
In turbulent flow ~~one~~ there are always small instantaneous fluctuations of velocity due to erratic motion of fluid particle. The flow still can be stated as steady if fluctuations ~~are~~ average equally on both side and mean temporal velocity $V = \frac{1}{t}\int_0^t u\,dt$ does not change with time.



Flow is unsteady when $\frac{\partial P}{\partial t} \neq 0$

Ex:- Liquid falling under gravity out of an opening at the bottom of a vessel.

* Flow of fluid in reciprocating pump.



Development of steady flow in a pipe.

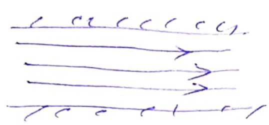
Proper choosing of frame of reference make a unsteady flow steady

Uniform flow

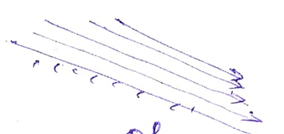
If, at any given time the flow parameters (~~press~~ p, V, ρ, ~~μ~~ and T) remains constant throughout the flow field then the flow is uniform. $\frac{\partial \rho}{\partial s} = 0$ ad $\frac{\partial \rho}{\partial t} \neq 0$ [~~s~~ denotes the distance measured from some fixed point on the path of flow.

A uniform flow is prescribed only in term of velocity rather than in terms of all dependent variables. The velocity vector must be same ~~for~~ at all sections, ~~it~~ for uniformity.

$\left.\begin{array}{l} \vec{V} = V(t) \\ \vec{a} = a(t) \end{array}\right\}$ Eulerian



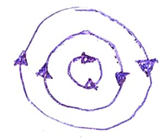
Flow between parallel plates.



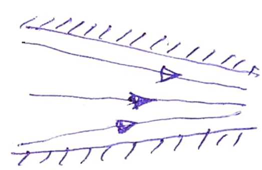
Open channel flow.

Flow is non-uniform if there is a change in flow parameters from one section to another. Space rate of change of flow parameters then does not vanish; $\frac{\partial \rho}{\partial s} \neq 0$.

Ex:-



Vortex Flow



Converging Flow.

So

Steadiness refers to no change with time and uniformity refers to no change with space. Steadyness and uniformity of flow can co-exists independently or in combination:

1. Steady uniform flow. e.g., the flow at constant rate in a pipe line of constant c/s area. $\vec{V} = \text{Constant}$
2. non-steady uniform flow: The flow at an increasing or decreasing rate through a pipe line of constant c/s area. $\vec{V} = V(t)$
3. Steady non-uniform flow:- Flow at steady or constant rate through a converging and diverging pipe. $\vec{V} = V(\vec{s})$
4. non-steady non uniform flow - Flow at increasing or decreasing rate through a converging or $\vec{V} = V(\vec{s}, t)$

diverging pipe.

Compressible and Incompressible Flow.

Incompressible → If the ~~density~~ change ~~in~~ in density of the fluid due to variation of pressure and temp. ~~are~~ is insignificant in the flow field then the flow is incompressible.

Practically any liquid can be regarded as incompressible, because change in P and temp ~~as~~ has little effect on volume.

Exception :- Water is subjected to severe acc^n such as water hammers that cause ~~compression.~~ compression.

But when change is appreciable it is call compressible. The gases are compressible fluid. The density variation is much when subjected to high press & temp. When density variation is small (e.g. flow of air in a ventilating system, flow of gas in pipe or channel), it can be treated as incompressible.

$$\text{Mach number } (M) = \frac{\text{Local velocity of fluid}}{\text{Sonic velocity in fluid}}$$

a measure of relative importance of compressibility.

If $M < 0.3$ → compressibility effect of ignored.
* $M < 1$ → Subsonic flow. * Sonic flow: $M=1$
$M > 1$ → Supersonic flow * Hyper sonic flow: $M > 5$

Isothermal; Adiabatic and Isentropic Flow.

Isothermal flow → no change in temp, while flowing from one section to another.

Adiabatic → No heat transfer.

Isentropic → Reversible adiabatic process is called isentropic.

Pressure Flow: In pressure flow the fluid motion is bounded by solid walls on all the sides and free surface of liquid does not exists, i.e, the liquid surface is not exposed to atmospheric or any other constant pressure. A pipe flow is pressure flow as pressure gradient exists along the pipe length.

In Pressure less flow also known as gravity flow, the fluid is bounded by three sides.

Ideal and Real flow.

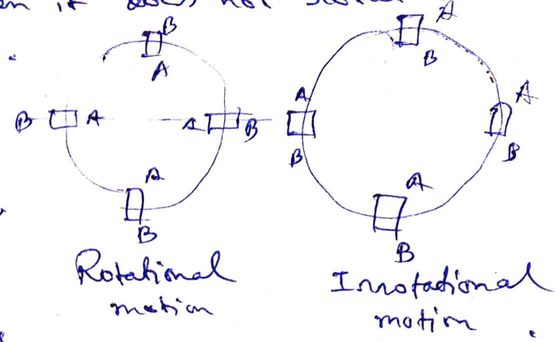
Ideal or frictionless flow where no shear stress is presumed to ~~exit~~ exist between two adjascent fluid layer and the boundary. Hence the fluid should be non-viscous, or the vel. grad normal to the direction of flow is zero. Only normal stresses can exist in ideal flows.

For real flows shear stress comes into account when particles in motion. These stresses oppose sliding of one layer over another. Real flow situations are characterized by the frictional resistance to fluid motion.

Rotational and Irrotational Flow.

A Rotational flow exists when the fluid particles rotate about their own mass centre while moving along a stream line.

Flow is irrotational when it does not rotate about their mass centre.



Rotational motion

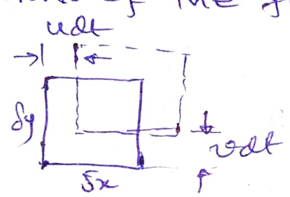
Irrotational motion.

Ex:- Motion of a liquid in a rotating tank is an example of rotational motion, where velocity varies with distance from the centre.

A vortex or whirlpool which develops above a drain in the bottom of a stationary tank represents an irrotational motion.

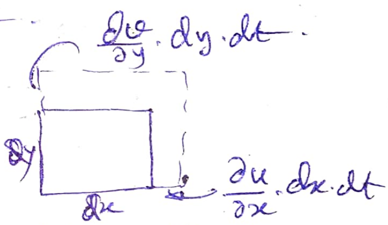
Fluid can have translatory or rotary motion.

(i) During linear or pure translation, the dimensions of the fluid ~~the~~ element do not change.



Linear ~~tran~~ slation.

The velocity in x and y direction are udt and v dt.

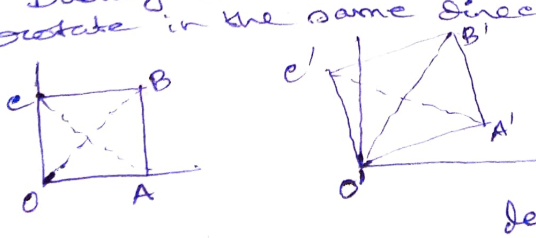


Linear Deformation

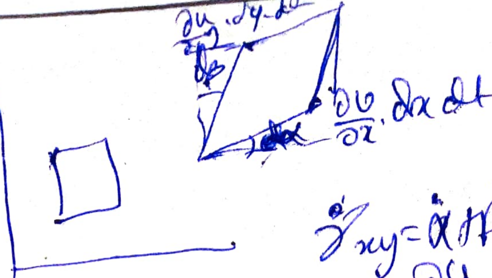
(ii) During linear deformation the fluid undergoes ~~in~~ a change in its shape, however no change occurs in the direction of principal ~~axis~~. of fluid elements.

The deformation in x-direction is $\frac{\partial u}{\partial x}.dx\,dt$ and in y-direction is $\frac{\partial v}{\partial y}.dy.dt$. In both the above cases the displacement is parallel to the original position.

(iii) During rotation, both axis of the fluid element rotate in the same direction.



detail later. under potential flow



$\frac{d\alpha}{dt} =$

$d\alpha \cdot dx = \frac{\partial v}{\partial x} \cdot dx\, dt$

$\frac{d\alpha}{dt} = \frac{\partial v}{\partial x}$.

$d\beta \cdot dy = \frac{\partial u}{\partial y} \cdot dy \cdot dt$

$\frac{d\beta}{dt} = \frac{\partial u}{\partial y}$

$\dot{\gamma}_{xy} = \dot{\alpha} + \dot{\beta}$

$= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y}$

$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$

$= \frac{1}{2}(\dot{\alpha} - \dot{\beta}) \rightarrow$ Anticlockwise (+)ve

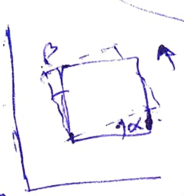
$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$

$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$

① If $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

Then $\omega_z = \frac{\partial v}{\partial x}$

$\dot{\gamma}_{xy} = 0$



$\dot{\alpha} = \dot{\beta}$

$\vec{\omega} = \frac{1}{2}(\nabla \times \vec{V})$ curl

pure rotation

(Solid body rotation.

② $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$ $\quad \dot{\gamma}_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = 2\frac{\partial u}{\partial y} = 2\frac{\partial v}{\partial x}$

but $\omega_z = 0$ [only deformation no rotation]

Irrotational flow or motion.

For 3D case. $\dot{\epsilon}_x = \frac{\partial u}{\partial x}$, $\dot{\epsilon}_y = \frac{\partial v}{\partial y}$, $\dot{\epsilon}_z = \frac{\partial w}{\partial y}$ $\quad u = u(x,y,z)$, $v = v(x,y,z)$, $w = w(x,y,z)$

Vorticity $\vec{\Omega} = 2\vec{\omega} = \nabla \times \vec{V}$

Imaginary line on which the vorticity vector $\vec{\Omega}$ at that point tangent to each pt is called vortex line.

$\vec{\Omega} \times d\vec{s} = 0$ $\quad \frac{dx}{\Omega_x} = \frac{dy}{\Omega_y} = \frac{dz}{\Omega_z}$.

$\Omega_x = 2\omega_x$ $\quad \Omega_y = 2\omega_y$ $\quad \Omega_z = 2\omega_z$.

Polar co-ordinate $\Omega_z = 2\omega_z = \frac{\partial V_\theta}{\partial r} - \frac{1}{r}\frac{\partial V_r}{\partial \theta} + \frac{V_\theta}{r}$.

$\Omega_r = 2\omega_r = \frac{1}{r}\frac{\partial V_z}{\partial \theta} - \frac{\partial V_\theta}{\partial z}$

$\Omega_\theta = 2\omega_\theta = \frac{\partial V_r}{\partial z} - \frac{\partial V_z}{\partial r}$.

(iv). During angular or shear deformation. OABC may take the position OA'B'C'.

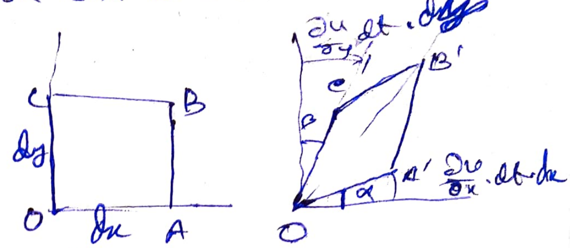
Along OA.

Angular change $\frac{\partial v}{\partial x} \cdot dx \cdot dt / dx$

$= \frac{\partial v}{\partial x} \cdot dt$

Along OC.

$\frac{\partial u}{\partial y} \cdot dy \cdot dt / dy = \frac{\partial u}{\partial y} \cdot dt$



Angular or shearing deformation of a fluid element.

Treating clockwise rotation as negative, the average angular displacement or mean rotation of of fluid element is equals $\frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dt$.

$\therefore$ the rate of rotation $\omega = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$.

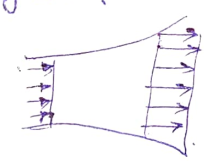
If $\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$ then there will be no rotation of the diagonal though the sides may have ~~rotation~~ angular deformation. Then the fluid is irrotational.

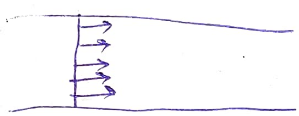
A steady irrotational flow is known as potential flow.

<u>One - D flow</u>

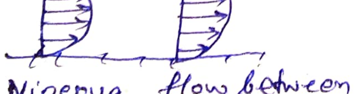
Variation of parameters occurs in one direction only. No variation of properties at any cross section normal to the flow.

The streamlines are straight & pararallel.

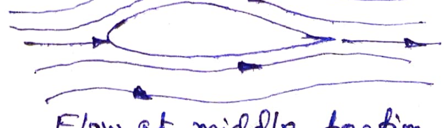




In 2-D the flow parameters vary along two direction. This occurs in actual fluid flow condition.



Viscous flow between parallel plates & ducts.



Flow at middle portion of an aeroplane wing.

For 3-D flow, properties of fluid varies in all three directions. The streamlines are spaced curves. Example :- Flow in a river, flow within fluid machine. Flow at inlet of a nozzle. etc.

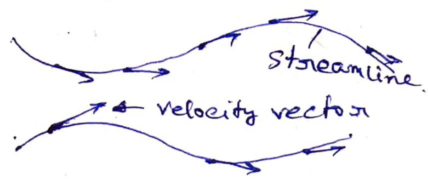
A fluid flow with symmetrical velocity profiles about the axis is called an axisymmetric flow. Such a flow is 2D because in cylindrical co-ordinate the vel. gradients exist only in the axial and radial direction. Variables do not change in circumferential direction.

Steady $V = f(x, y, z)$ Unsteady $V = f(x, y, z, t)$

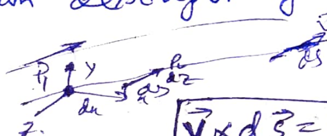
Streamlines :- [Eulerian concept of fluid flow] A streamline is an imaginary line drawn in the flow field in such a manner such that the vel. vector of the fluid at each and every pt. on the streamline is tangent to the streamline at that instant.

$$\begin{vmatrix} i & j & k \\ u & v & w \\ \delta x & \delta y & \delta z \end{vmatrix} = 0$$

Consequently a tangent to the curve at any point gives the direction of velocity vector at the point at that instant. Streamlines are ~~the~~ thus equivalent to an instantaneous snapshots indicating direction of ~~an~~ number of fluid particles at an ~~instan~~ instant of time. It is Eulerian description of ~~motion~~ motion of fluid.



~~dt~~ at an interval dt, a fluid travels ds. ~~$ds = dx\,i + dy\,j + dz\,k$~~



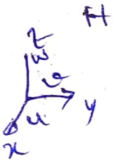
$$\boxed{\vec{V} \times d\vec{s} = \vec{0}}$$

$$\therefore \quad u = \frac{dx}{dt} \quad v = \frac{dy}{dt} \quad w = \frac{dz}{dt}$$

$\vec{V} = u\,i + v\,j + w\,k$

$\vec{s} = dx\,i + dy\,j + dz\,k$

Hence eqn of general streamline will be



$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}.$$

For 2D. $\dfrac{dx}{u} = \dfrac{dy}{v}$ or $\dfrac{dy}{dx} = \dfrac{u}{v}$

$u\,dy - v\,dx = 0$
$u\,dz - w\,dx = 0$
$v\,dz - w\,dy = 0$

Thus slope of a plane streamline equals the ratio of velocity components.

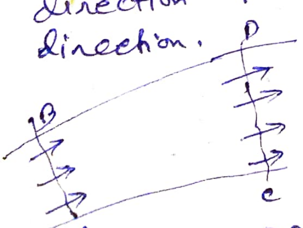
For cylindrical co-ordinate $dr : r\,d\theta = V_r : V_\theta$

The ~~important~~ important characteristics of streamlines are

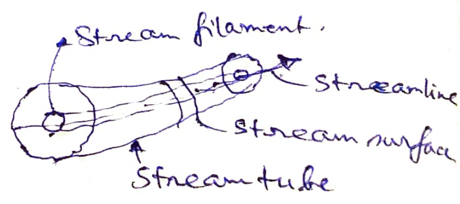
(i) Streamlines do not cross each other. Otherwise a fluid particle will have two velocities at pt. of intersection ~~within~~ i which is physically impossible.

ii) There can not be any movement of fluid particles across the streamline. The flow is only along streamline and donot cross it. Hence streamlines can be considered as rigid ~~streamlines~~ boundaries.

iii) Streamline spacing varies inversely as the velocity, converging streamlines at any particular direction indicates accelerated velocities in that direction.



AC & BD are streamlines.



For c/s AB & CD, Mass is same. flo
So flow = AV = constant. (As area increases velocity decreases.

A grouping of neighbouring streamlines forming a cylindrical passage with elementary c/s area is known as stream filament. A no. of stream filament makes stream tubes. Surface of stream tube is stream surface. There is no flow across the stream surface.

The shape of stream tube changes from one instant to another due to change in position of the streamlines. Example - pipes, nozzles.

Pathline is Lagrangian method in describing fluid flow. It show path of different fluid particle as a function of time.

A pathline represents the trace on trajectory of a fluid particle over a period of time. Ex: The path traced by single smoke particle. This is lagrangian description of motion. Pathline involves a variation of time while streamline is for a particular instant of time.

(pathline)

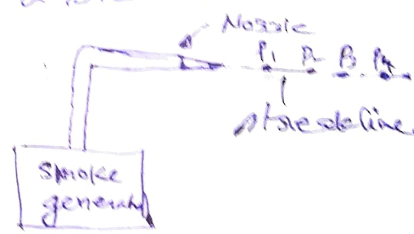
$\vec{V} = V(\vec{S}, t)$

$\int \frac{d\vec{S}}{dt} = \int \vec{V}(\vec{S}, t)$

$\vec{S} = f(\vec{S}_0, t)$

Pathlines & streamlines are identical in a steady flow.

A streak line is instanteneous picture of the positions of all the fluid particles have passed through a fixed point in the flow field.



For a steady flow, streamline streakline & pathline are same if they originate from same point.

$S_0 = F(\vec{S}_1, t)$

$S = f(S_0, t)$

$\vec{S} = f[F(\vec{S}_1, t), t]$

Stagnation pt. → Fluid remain static i.e $u = 0$, $v = 0$.

A timeline is a set of fluid particles that form a line at a given instant.

Problem :- Determine the eqⁿ of a streamline for 2D flow field for which ~~vel.~~ velocity components are given by

i) $u = a$ $v = a$ $[a \neq 0]$ The streamline passes through (1, 3).

Soln: $\frac{dx}{u} = \frac{dy}{v}$ or $dy = dx \Rightarrow y = x + c$ (after integration)

$3 = 1 + c \Rightarrow c = 2$

$\therefore \boxed{y = x + 2}$ - reqd eqⁿ.

St line with slope $\theta = 45°$ an intercept of 2.

ii) $u = -\frac{y}{b^2}$, $v = \frac{x}{a^2}$ and passing through $(a, 0)$.

$\frac{dx}{u} = \frac{dy}{v}$ $\therefore$ $\frac{dy}{dx} = \frac{v}{u} = \frac{x}{a^2} \times -\frac{b^2}{y}$.

$\therefore \frac{x\,dx}{a^2} + \frac{y\,dy}{b^2} = 0$.

after integration $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2c$.

$\frac{a^2}{a^2} + \frac{0}{b^2} = 2c$. $c = \frac{1}{2}$

$\therefore \boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$ → elliptic stream line.

iii) $V_r = \frac{\cos\theta}{r^2}$, $V_\theta = \frac{\sin\theta}{r^2}$ passing through $r = 2$ and $\theta = \pi/2$

$\frac{dr}{r\,d\theta} = \frac{V_r}{V_\theta} = \frac{\cos\theta}{\sin\theta} \Rightarrow \frac{dr}{r} = \cot\theta \cdot d\theta$.

after integration $\ln r = \ln \sin\theta + \ln c$.

$r = c\sin\theta$ at $r = 2$ $\theta = \frac{\pi}{2}$

$\therefore 2 = c \cdot \sin\frac{\pi}{2} \Rightarrow c = 2$.

$\therefore \boxed{r = 2\sin\theta}$ required eqⁿ.

Ex. The vel. distribution of a 3D flow in $\vec{V} = ax\hat{i} + ay\hat{j} - 2az\hat{k}$

Find The eqⁿ of streamline passes through position vector $\bar{r} = 2i + 2j + 4k$

Soln: $u = ax$, $v = ay$, $w = -2az$ and we have to find the eqⁿ of streamline passing through 2, 2, 4,

$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \Rightarrow \underset{(1)}{\frac{dx}{ax}} = \underset{(2)}{\frac{dy}{ay}} = \underset{(3)}{\frac{dz}{-2az}}$.

considering (1) & (2)

$\frac{dx}{x} = \frac{dy}{y}$ or $\ln x = \ln y + \ln c'$

$x = yc$. $x = 2, y = 2$ $c = 1$

$\therefore \boxed{x = y}$

Considering ① & ③ $-2\int \frac{dx}{x} = \frac{1}{-2}\int \frac{dz}{z}$

$\therefore$ $-2\ln x = -\frac{1}{2}\ln z + \ln c \Rightarrow x = \frac{c}{\sqrt{z}}$.

$\therefore$ ~~$x^{-2} = zc$~~.

$x=2$ $z=4$ gives $c=4$

$\therefore$ $\boxed{x = \frac{4}{\sqrt{z}}}$ --- (B)

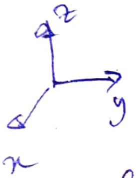
combining (A) & (B). $\boxed{x = y = \frac{4}{\sqrt{z}}}$ $\Rightarrow$ eqn of streamline.

Acceleration of Fluid Particles.

At a given instant each individual particle has its own velocity and acceleration, and these variables change both with respect to time and position. Hence for complete description of fluid flow, one has to observe the motion of fluid particles at various positions and at successive instant of time.

~~1~~ Lagrangian method.

→ Description of individual fluid particles during their course of motion. Observer travel with particle. $\varsigma = (\vec{S_0}, t)$



Let original position ~~at time t_0~~ is x_0, y_0, z_0 ~~a, b, c~~, and after time interval t it becomes x, y & z.

The kinematic flow pattern is then fully described if $x = x(x_0, y_0, z_0, t)$; $y = y(x_0, y_0, z_0, t)$ and $z = z(x_0, y_0, z_0, t)$ are known. at $t = t_0$

Initial space co-ordinates x_0, y_0, z_0 & t are called lagrangian variable. $\vec{V} = \left(\frac{d\vec{S}}{dt}\right)_{S_0}$

These velocities $u = \left(\frac{dx}{dt}\right)_{x_0 y_0 z_0, t}$ $v = \left(\frac{dy}{dt}\right)_{x_0 y_0 z_0, t}$ $w = \left(\frac{dz}{dt}\right)_{x_0 y_0 z_0}$

$\vec{\varsigma} = \varsigma(\vec{S_0}, t)$

$S = ix + jy + kz$

$a_x = \frac{du}{dt} = \frac{d^2x}{dt^2}$ $a_y = \frac{dv}{dt} = \frac{d^2y}{dt^2}$, $a_z = \frac{dw}{dt} = \frac{d^2z}{dt^2}$

all the kinematic parameter is a function of $(x_0\, y_0\, z_0, t)$ i.e. identity & time;

Eulerian Method

In this method attention is focussed on the motion of properties of different fluid particles as they pass fixed points in space. The components of velocity vector are function of space co-ordinates and time.

$u = u(x, y, z, t)$

$v = v(x, y, z, t)$

$w = w(x, y, z, t)$.

$\vec{V} = V(\vec{S}, t)$

$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$

$\vec{S} = \hat{i}x + \hat{j}y + \hat{k}z$

$V = V(\vec{S}, t)$

$\vec{a} = a(\vec{S}, t)$

Thus $V(x,y,z,t) = \vec{i}\cdot u(x,y,z,t) + \hat{j}\,v(x,y,z,t) + \hat{k}\,w(x,y,z,t)$.

Several other quantities known as *kinematic properties*, by mathematically manipulating the velocity field.

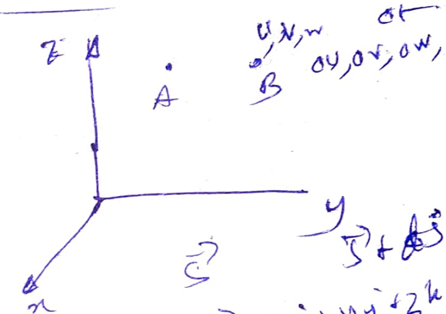
a) Displacement Vector $r = \int v\,dt$.

b) Acceleration: $a = \frac{dV}{dt}$

c) Volume flow rate $Q = \int (V\cdot n)\,dA$

d) Volume expansion rate $\frac{1}{V}\cdot\frac{dV}{dt} = \nabla\cdot V$

e) Local angular velocity $\omega = \frac{1}{2}(\nabla\times V)$



$\vec{s} = xi + yj + zk$

$d\vec{s} = \delta x\,i + \delta y\,j + \delta z\,k$

$\frac{d\vec{s}}{dt} = V(\vec{s},t)$ | $\frac{dx}{dt} = u(x,y,z,t)$ |

$u = u(x,y,z,t)$

$u + \delta u = u(x + \delta x, y + \delta y, z + \delta z, t + \delta t)$

Expanding the R.H.S in the form of Taylor series.

we get

$$u + \delta u = u(x,y,z,t) + \frac{\partial u}{\partial x}\cdot\delta x + \frac{\partial u}{\partial y}\cdot\delta y + \frac{\partial u}{\partial z}\cdot\delta z + \frac{\partial u}{\partial t}\cdot\delta t + O(H)$$

$\delta x = u\cdot\delta t$, $\delta y = v\cdot\delta t$, $\delta z = w\cdot\delta t$.

$$\therefore \frac{u + \delta u - u}{\delta t} = \frac{\partial u}{\partial x}\cdot\frac{\delta x}{\delta t} + \frac{\partial u}{\partial y}\cdot\frac{\delta y}{\delta t} + \frac{\partial u}{\partial z}\cdot\frac{\delta z}{\delta t} + \frac{\partial u}{\partial t} + O(H)$$

$$\therefore \frac{\delta u}{\delta t} = u\cdot\frac{\partial u}{\partial x} + v\cdot\frac{\partial u}{\partial y} + w\cdot\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} + O(H)$$

if $\delta t \to 0$ $\quad \left.\frac{\delta u}{\delta t}\right|_{\delta t\to 0} = \frac{Du}{Dt} \quad \left.O(H)\right|_{\delta t\to 0} = 0$

The vel. component of flow field can be represented by three component scalar equation.

so. $u = \frac{dx}{dt}$, $v = \frac{dy}{\partial t}$, $w = \frac{dz}{\partial t}$.

Since vel. of a fluid element is or particle is a function of both position and time, the accn is given by.

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial u}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial u}{\partial z}\cdot\frac{dz}{dt}$$

$$= \frac{\partial u}{\partial t} + u\cdot\frac{\partial u}{\partial x} + v\cdot\frac{\partial u}{\partial y} + w\cdot\frac{\partial u}{\partial z} = \frac{Du}{Dt}$$

$$= \frac{\partial u}{\partial t} + (\vec{V}\cdot\nabla)u$$

$\frac{D}{Dt}$ → $\boxed{\text{substantial derivative}}$ or material "

similarly. ∇ = divergency = $\left(\frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k\right)$

and $\frac{D}{Dt} = \frac{\partial}{\partial t} + u\cdot\frac{\partial}{\partial x} + v\cdot\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$.

similarly $a_y = \frac{Dv}{Dt} = \frac{\partial v}{\partial t} + u\cdot\frac{\partial v}{\partial x} + v\cdot\frac{\partial v}{\partial y} + w\cdot\frac{\partial v}{\partial z} = \frac{\partial v}{\partial t} + (\vec{V}\cdot\nabla)v$

$a_z = \frac{Dw}{Dt} = \frac{\partial w}{\partial t} + u\cdot\frac{\partial w}{\partial x} + v\cdot\frac{\partial w}{\partial y} + w\cdot\frac{\partial w}{\partial z} = \frac{\partial w}{\partial t} + \vec{V}\cdot\nabla$

thus $a = \frac{D\vec{V}}{Dt} = \frac{\partial\vec{V}}{\partial t} + \vec{V}\cdot\nabla\cdot\vec{V}$

the terms. $\frac{\partial u}{\partial t}$, $\frac{\partial v}{\partial t}$, $\frac{\partial w}{\partial t}$ represents local accn [change of flow with time at each pt. in the flow field]

Next terms $\cancel{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial t}}$ $(\vec{V}\cdot\nabla)\vec{V}$ is known as convective derivative, which is time rate of change due to change in position in the field.