Fluid Statics.

Fluid statics is the study of fluid at rest. The concept includes ~~ati~~ situations where fluids are either actually at rest or undergo uniform acceleration in a container or rotates as a solid mass. No shear force is then present as the fluid particles do not move with respect to one another.

Pressure → A fluid element or mass is essentially acted upon by two categories of forces. (a) Body force → includes gravitational force, electric or magnetic. Magnitude of these forces proportional to the mass of the fluid.
(b) Surface forces → Action of surrounding fluid on element under consideration through direct contact. These are press. (Normal force) and shear (tangential force). In fluid at rest there is no relative motion between layers of fluid. The vel. gradient is zero and hence there is no shear in fluid. So there is no tangential force. Hence, only normal force is present. This normal surface force is called the pressure force.

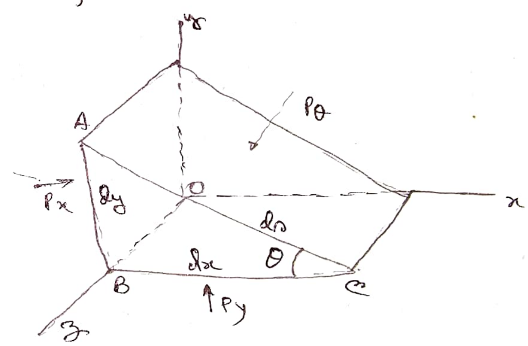
$p = \frac{dF}{dA}$ or $p = \frac{F}{A}$ F = total force & A = total area.

Dimension = FL^{-2} Unit N/m^2 (Pascal) $1\,atm = 101.325\ kPa$.

Pascals Law of hydrostatics.

"Intensity of pressure at a pt. in a fluid at rest is same in all direction".

Force on face AB $= p_x \times$ Area
$= p_x \times (dy \times 1)$
$= p_x\, dy$
[∵ the fluid element has unit depth]



Force on face BC $= P_y \cdot dx$
" " AC $= P_\theta \cdot ds$.

Wt. of element $= \frac{1}{2} \times dx \cdot dy \cdot 1 \times w$
and this acts through C.G. of element. w = sp. wt. of fluid.

Resolving forces in x direction. $P_x \cdot dy = P_\theta \cdot ds \sin\theta = P_\theta \cdot dy$ [∵ $dy = ds \sin\theta$]
$\Rightarrow P_x = P_\theta$

Resolving forces in y direction - $P_y dx = \frac{1}{2} \cdot w \cdot dx \cdot dy + P_\theta \cdot ds \cos\theta$.

If the element is very small then dx & dy is very small. So the product of $dx \cdot dy$ can be be neglected & $dx = ds \cdot \cos\theta$.

∴ $P_y = P_\theta$ ∴ $P_x = P_y = P_\theta$. [Proved]

The result is independent of θ and it follows the pressure acts actually in all directions in a stationary fluid.

$P_\theta = P_x = P_y$ ⟹ these are basically ~~ste~~ stresses normal stresses

and position $\sigma_\theta = \sigma_x = \sigma_y = -p$.

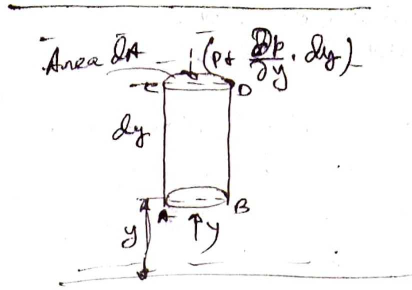
Pressure - Density - Height Relationship :

Pressure force on bottom face $AB = p.dA$

" " " top " $= \left(p+\frac{\partial p}{\partial y}dy\right).dA$

Weight of fluid element $= w.\, dA\, dy$

$w =$ sp wt.



For equilibrium.

$$p\,dA - \left(p+\frac{\partial p}{\partial y}.dy\right)dA - w\,dA\,dy = 0$$

or $\frac{\partial p}{\partial y} = -w$ [Since

$\therefore$ or $\frac{dp}{dy} = -w$ [$\because$ we are considering variation in pressure in y direction]

$\therefore$ $dp = -w.dy$ if $dy = 0$ then $dp = 0$.

$\because$ Pressure intensity is not changed if no change in elevation.

For incompressible fluid, if the fluid is homogeneous of constant specific wt

$\therefore$ $p_2 - p_1 = -w(y_2 - y_1)$ or $\frac{p_1}{w} + y_1 = \frac{p_2}{w} + y_2$

Here y = elevation head

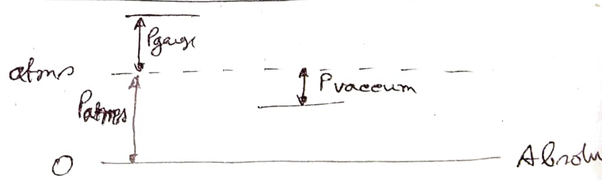
and $\frac{p}{w}$ = pressure head. $\left(\frac{p}{w}+y\right)$ = piezometric head.

Evidently at every point in a homogeneous fluid at rest, the piezometric head is constant.

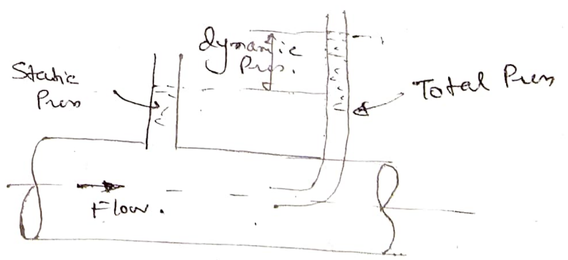
Atmospheric Pressure, Absolute pres, Gauge Pres & Vaccum.

$$P_{abs} = P_{atm} + P_{gauge}.$$

Or $\cancel{P_{atm}} = P_{abs} = P_{atm} - P_{vaccum}.$



Static Pressure & Total Pressure



Static pres is defined as the force per unit area acting on the wall by a fluid at rest ~~rest~~ or flowing parallel to the wall in a pipe line.

Total or stagnation pressure is defined as the pressure that would be obtained if the fluid stream were brought to rest isentropically.

The difference between ~~static and~~ total and the static pressure gi the pres. due to vel. of fluid, known as dynamic press.

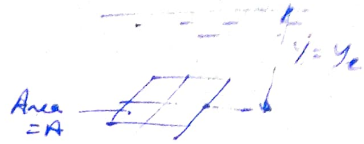
Dynamic pres = Total Pres − Static pres.

$$\frac{V^2}{2g} = P_{tot} - P_{stat}.$$ [when P_{tot} & P_{stat} expressed in of head]

expressed in Head.

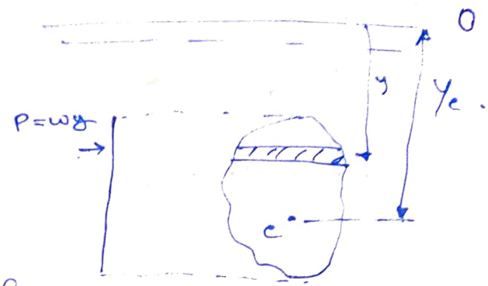
Force on a horizontal submerged plane surface.

$F = p \cdot A = w \cdot y \cdot A = w \cdot y_e \cdot A$
$= wA \cdot y_e$.



y_e = depth of the C.G. of the submerged surface ~~on the~~ below the free surface of the liquid.

Force on vertical plane Submerged Surface.



A plane surface of arbitrary shape immersed vertically in a static mass of fluid. Depth of liquid varies from pt. to pt. & hence pressure intensity is not constant over the entire surface.

Consider an elementary strip of area dA at a depth of y, from the free surface of liquid and parallel to it. Pressure $p = wy$.

∴ Force on strip $dF = p \cdot dA = w \cdot y \cdot dA$.

∴ Total Pressure $F = \int dF = \int w\, y\, dA = w \int y\, dA = w \cdot A \cdot y_e$.

[$\int y\, dA$ = moment of surface about the free liq. surface. $= A \cdot y_e$]

Depth of centre of pressure → Centre of press defines the pt. of application of total ~~pressure~~ force on the surface. Its location can be calculated by principle of moment. [Sum of the moment of resultant force about an axis is equal to the sum of components about the same axis]

Total force on strip $dF = w \cdot y\, dA$.

moment of force about free liq. surface $= w \cdot y^2\, dA$ [$dF \times y$]

Sum of all such moments $= \int w \cdot y^2\, dA = w \cdot I_0$

[$I_0 = \int y^2\, dA$ = moment of Inertia of surface about free liq. surface.]

Moment of total force $= wAy_e \times y_p$ (y_p = centre of press)

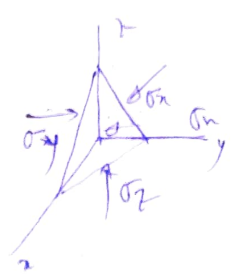
∴ $wAy_e \cdot y_p = wI_0$

∴ $y_p = \frac{I_0}{Ay_e} = \frac{I_e + A \cdot y_e^2}{Ay_e} = y_e + \frac{I_e}{Ay_e}$

[I_e = MOI about centroid of surface.]

∴ $y_p - y_e = I_e / Ay_e$ which is always positive

∴ y_p is below the Y_e.



$F_y = \sigma_y \cdot \frac{dx \cdot dz}{2} - \sigma_n \cdot dA \cos\beta = 0$
$(dx \cdot dz/2)$

$F_x = \sigma_x \cdot \frac{dy\, dz}{2} - \sigma_n \cdot dA \cos\alpha = 0$
$(dy\, dz/2)$

$F_z = \sigma_z \cdot \frac{dx\, dy}{2} - \sigma_n \cdot dA \cos\gamma - \frac{1}{6} \rho g\, dx\, dy\, dz \ldots$
$(dx\, dy/2)$ neglected.

∴ $\sigma_x = \sigma_y = \sigma_z = \sigma_n$.

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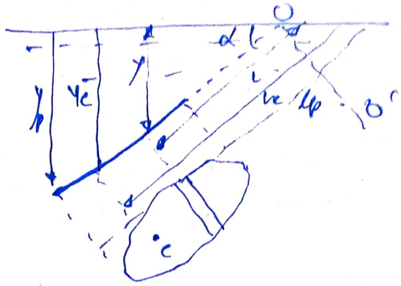
Force on Inclined submerged plane surface.



$$dF = p.dA = w.y.dA$$
$$= w.\, l \sin\alpha .\, dA$$
$$F = \int dF = \int w.\, l \sin\alpha \, dA$$
$$= w \sin\alpha \int l\, dA.$$
$$= w \sin\alpha .\, A.\, l_c.$$
$$= w.A.(l_c \sin\alpha)$$
$$= w A y_c.$$

Moment of pressure force dF about OO' $= dF \times l = w l \sin\alpha \, dA.$

$\therefore$ Sum of all moments $= \int w \sin\alpha .\, l^2 dA = w \sin\alpha \int l^2 dA$

$= w \sin\alpha .\, I_o$

Moment of total force on O-O' is $w A y_c \times l_p.$

$$\therefore \quad l_p = \frac{w \sin\alpha .\, I_o}{w A y_c} = \frac{I_o \sin\alpha}{A y_c}.$$

$I_o = I_c + A.\, l_c^2$ $\quad \therefore$ and $l_p = y_p / \sin\alpha.$

$$\therefore \quad y_p = \frac{(I_c + A.\, l_c^2)\sin^2\alpha}{A y_c} \quad \text{but } l_c = \frac{y_c}{\sin\alpha}.$$

$$\therefore \quad y_p = \frac{I_c \sin^2\alpha}{A y_c} + \frac{A.\, y_c^2}{A y_c} = y_c + \frac{I_c \sin^2\alpha}{A y_c}$$

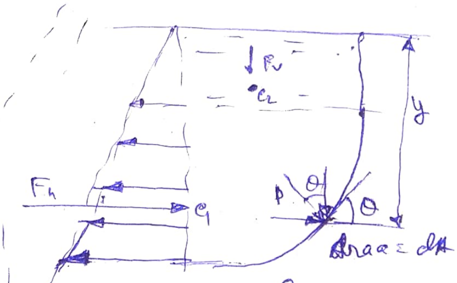
$$\therefore \quad \boxed{y_p - y_c = \frac{I_c . \sin^2\alpha}{A y_c}}$$

For vertical surface $\alpha = 90° \Rightarrow \sin\alpha = 1.$

For horizontal surface $\alpha = 0° \Rightarrow \sin\alpha = 0.$

$\Rightarrow y_p = y_c.$

Force on curved Submerged Surfaces



Elementary area dA lying at vertical depth y below free surface of liquid. If p = normal pressure intensity at that area then. $dF = p.dA = w.y\, dA$ and Total force on curved surface is $F = \int w y\, dA$. Since for a curved surface direction of pressure force varies from point to pt. then straight forward integration procedure can no longer be applied.

C_1 = Centre of pressure on projected area

C_2 = Centre of volume ABCDEF

Computation of total pressure on curved surface is then made possible by assessing the pressure forces acting on projected horizontal & vertical planes.

$dF_h = dF \sin\theta = p\, dA \sin\theta$ $\quad \therefore F_h = \int dF_h = w \int y\, dA \sin\theta \rightarrow$ This is total pres. force on projected area of vertical plane.

$dF_v = dF \cos\theta = p.\, dA.\cos\theta$ $\quad \therefore F_v = \int dF_v = w \int y.\, dA \cos\theta \rightarrow$ Total pres. Force on projected area of curved surface on horizontal plane.

$F = \sqrt{F_h^2 + F_v^2}$ and acting at an angle $\theta = \tan^{-1}\left(\frac{F_v}{F_h}\right)$