Analysis & design of turbomachines is essentially based on knowledge of forces exerted on or by moving fluid.

Force is caused by change in momentum of fluid jet. (Pump, Turbine.)
Jet means fluid stream issuing from a nozzle with high vel i.e high K.E. Vane – Flat or curved plate fixed to the rim of a wheel.

$$\longrightarrow F = \frac{d}{dt}(mv) = m\cdot\frac{dv}{dt} + v\cdot\frac{dm}{dt}. \quad dm = 0$$

$$\therefore F\cdot dt = m\,dv. \quad \text{or} \quad F\int_0^t dt = m\int_{v_1}^{v_2} dv.$$

$$\text{or } F(t-0) = m(v_2 - v_1)$$

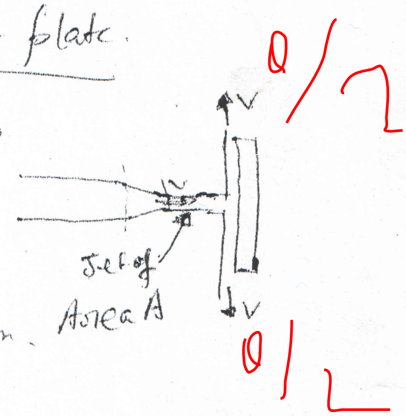
$$\therefore F\cdot t = m(v_2 - v_1).$$

$$\text{or } F = \dot{m}(v_2 - v_1) = \rho Q\cdot(v_2 - v_1) = \rho\,\cancel{AV_1(V_2-V_1)}$$

The above eq^n is the force exerted by body on the fluid.

∴ Force given by fluid [Newton third law] is $F = \dot{m}(v_1 - v_2) = \rho Q(v_1 - v_2)$.

Jet impingement on a stationary flat plate.

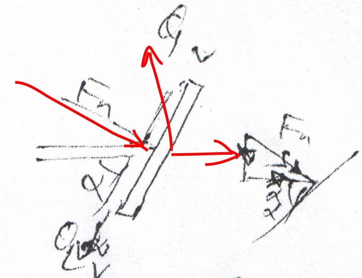


Assumption → i) No friction between plane and jet,
ii) No energy loss due to impact.
iii) Jet moves on and off the plate with same velocity, and vel. distribution is uniform.
iv) No variation in elevation.
v) Const. press (Atms).

$$F_n = \rho AV(V_1 - V_2) \qquad V_1 = V \qquad V_2 = 0$$

$$\therefore F_n = \rho AV^2.$$

For inclined plate.



Initial Vel $V_1 = V\sin\alpha$ Final $V_2 = 0$

∴ Normal force on plate : $\rho AV(V\sin\alpha - 0) = \rho AV^2\sin\alpha.$

$$F_x = F_n\sin\alpha = \rho AV^2\sin^2\alpha.$$

$$F_y = F_n\cos\alpha = \rho AV^2\sin\alpha\cos\alpha.$$

Resultant Force in the direction of parallel to the plate is zero, [Since no Frictional Force]

∴ Final momentum rate − initial momentum rate = Force

$(\rho Q_1 V - \rho Q_2 V) - \rho Q V \cos\alpha = 0$

∴ $Q_1 - Q_2 = Q\cos\alpha$ but $Q_1 + Q_2 = Q$

∴ $Q_1 = \frac{1}{2} Q(1+\cos\alpha)$ $Q_2 = \frac{Q}{2}(1-\cos\alpha)$

∴ ratio of discharge $= \frac{Q_1}{Q_2} = \frac{1+\cos\alpha}{1-\cos\alpha}$.

when $\cos\alpha = 0$ ~~or~~ $\alpha = 90^\circ$ $Q_1 = Q_2$

__Again__

$F_x = \rho Q V - (\rho Q_1 V\cos\alpha - \rho Q_2 V\cos\alpha) = \rho V\{Q - (Q_1 - Q_2)\cos\alpha\}$.

$F_y = \rho Q_1 V\sin\alpha - \rho Q_2 V\sin\alpha = \rho V(Q_1 - Q_2)\sin\alpha$.

$F = \sqrt{F_x^2 + F_y^2} = \rho V\left[(Q_1-Q_2)^2 + Q^2 - 2Q(Q_1-Q_2)\cos\alpha\right]^{1/2}$

Substituting $Q_1 = \frac{Q}{2}(1+\cos\alpha)$ & $Q_2 = \frac{Q}{2}(1-\cos\alpha)$

gives. $F = \rho Q V\sin\alpha. = \rho A V^2 \sin\alpha$.

Resultant Force direction:

$\tan\theta = \frac{F_y}{F_x} = \frac{\rho V(Q_1-Q_2)\sin\alpha}{\rho V\{Q - (Q_1-Q_2)\cos\alpha\}}$

substituting Q_1 & Q_2 we get $\tan\theta = \cot\alpha = \tan(90-\alpha)$

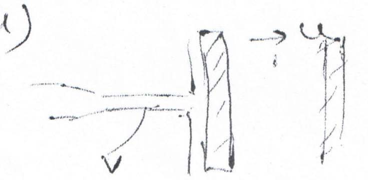
∴ resultant force is normal to the plate.

& depends on Q_1 & Q_2.

__Moving Flat Plate.__

Jet vel $= V$ Plate vel $= u$ in direction of jet.

∴ Vel of jet relative to plate $= (V-u)$



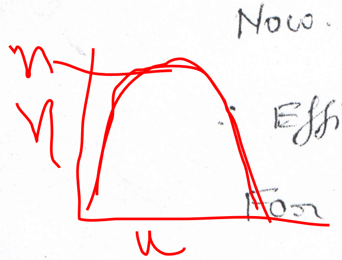
∴ $F = \rho Q\{(V-u) - 0\}$.

$Q = A\cdot(V-u)$

∴ $F = \rho A(V-u)^2$

Work done by jet on plate $= \rho A(V-u)^2\cdot u$.

Now. K.E. of jet $= \frac{1}{2} m V^2 = \frac{1}{2}\cdot\rho AV\cdot V^2 = \frac{1}{2}\rho AV^3$



∴ Efficiency of the system $\eta = \boxed{\frac{\rho A(V-u)^2 u}{\frac{1}{2}\rho AV^3} = \frac{2}{V^3}\left[V^2u - 2u^2V + u^3\right]}$

For max^m. $\eta \rightarrow \frac{d\eta}{du} = 0$ gives $(V-u)(V-3u) = 0$.

For $V-u=0$ $V=u$ and for this work done by jet is zero.

$\therefore$ Max^m. work done $= \rho A (V - V/3)^2 \cdot \frac{V}{3} = \frac{4}{27} \rho A V^3$

Max^m. $\eta = \frac{4/27 \, \rho A V^3}{\frac{1}{2} \rho A V^3} = \frac{8}{27}$

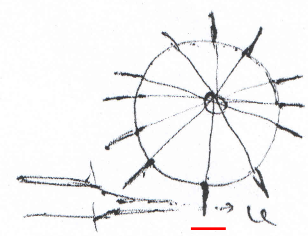
For inclined plate. $F_n = \rho A (V-u) [(V-u) \sin\alpha - 0]$

$= \rho A (V-u)^2 \sin\alpha$

F_x Work done $= F_x . u = \rho A (V-u)^2 . \sin^2\alpha . u$

Series of plates on a wheel.

$F_n = \rho A V [(V-u) - 0] = \rho A V (V-u)$



$\therefore$ Work done $= \rho A V (V-u) . u$

K.E of issuing jet $= \frac{1}{2} m V^2$

$= \frac{1}{2} (\rho A V) V^2 = \frac{1}{2} \rho . A V^3.$

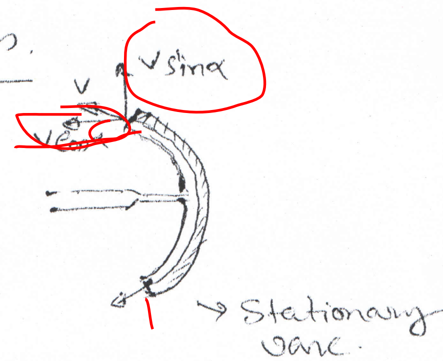
Efficiency of the system $(\eta) = \frac{\text{Work done on wheel}}{\text{K.E of Jet}}$

$= \frac{\rho A V (V-u) . u}{\frac{1}{2} \rho A V^3} = \frac{2u(V-u)}{V^2}.$

now For max^m η. $\frac{d\eta}{du} = \frac{2}{V^2}(V - 2u) = 0$ $\therefore V = 2u$ or $u = \frac{V}{2}$.

$\therefore \eta_{max} = \frac{2 \cdot \frac{V}{2}\left(V - \frac{V}{2}\right)}{V^2} = \frac{1}{2} = 50\%$

Impact of Jet on curved vanes.



$F = \dot{m}(V_1 - V_2)$

$= \rho A V[V - (-V\cos\alpha)] = \rho A V^2(1+\cos\alpha)$

If vel at outlet is kV [because of loss due to friction]

$\therefore \quad F = \rho A V^2(1+k\cos\alpha)$

if $\alpha = 90^\circ$ & $k=1$ $\quad F = \rho A V^2$.

if $\alpha = 0^\circ$ & $k = 1$ $\quad F = \rho A V^2(1+\cos 0^\circ) = 2\rho A V^2$.

which is twice that exerted by a flat plate.

Jet striking at the centre of a moving vane.

Single vane moving with velocity u.

$F_n = \rho A(V-u)^2(1+\cos\alpha)$

Work done $= F_n \cdot u = \rho A(V-u)^2 \cdot \cancel{\cos\alpha}(1+\cos\alpha) \cdot u$.

K.E. of jet $= \frac{1}{2}(\rho A V)V^2 = \frac{1}{2}\rho A V^3$

$$\eta = \frac{2(V-u)^2(1+\cos\alpha)u}{V^3} = \frac{2(1+\cos\alpha)}{V^3}(V-u)^2 u.$$

For max^m η

$\frac{d\eta}{du} = 0$ $\quad$ or $\quad \frac{d}{du}(V^2u - 2Vu^2 + u^3) = 0$

or $\quad V^2 - 4Vu + 3u^2 = 0$

$\therefore (V-u)(V-3u) = 0$

$\therefore$ For $\underline{V=u}$ work $= 0$

$\therefore$ $V = 3u$. or $u = V/3$.

$\eta_{max} = \frac{8}{27}(1+\cos\alpha)$

For semicircular vane $\alpha = 0^\circ$ $\cos\alpha = 1$ $\eta_{max} = \frac{16}{27} = .592$

~~Ser~~

Series of vanes → ie vane on wheel.

$F_n = \rho A V[(V-u) - (-(V-u)\cos\alpha)] = \rho A V(V-u)(1+\cos\alpha)$.

$$\therefore \eta = \frac{\rho A V.(V-u)(1+\cos\alpha).u}{\frac{1}{2}\rho A V^3} = \frac{2}{V^2}(1+\cos\alpha)(V-u)u.$$

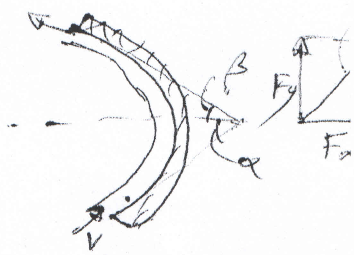
for η_{max} $\frac{d\eta}{du} = 0$ $\quad \therefore V - 2u = 0$ $\quad V = 2u$. $\quad u = V/2$

$\therefore \eta_{max} = \frac{1+\cos\alpha}{\ldots}$ when $\alpha = 0^\circ$ $\eta = 100\%$

$\alpha = 0°$ means semicircle. Use - Pelton wheel.
α - 10° to 20° to avoid jet impingement on next wheel

$\therefore F = \rho A (V-u)^2 (1 + k \cos\alpha)$ - single vane (moving)
$= \rho A V (V-u)(1 + k \cos\alpha)$ - series of vanes (moving).

Jet striking tangentially at one tip



① Stationary vane

$F_x = \rho A V [V\cos\alpha - (-V\cos\beta)]$
$= \rho A V^2 (\cos\alpha + \cos\beta)$

$F_y = \rho A V (V\sin\alpha - V\sin\beta)$
$= \rho A V^2 (\sin\alpha - \sin\beta)$

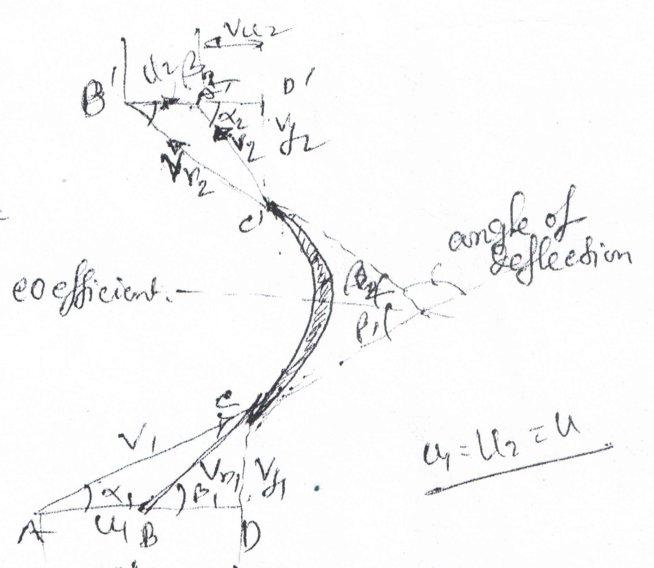
Resultant force F on the vane $= \sqrt{F_x^2 + F_y^2}$

$\phi = \tan^{-1}(F_y/F_x)$

For symmetric vane $\alpha = \beta$ $F_x = 2\rho A V^2 \cos\alpha$
$F_y = 0$

for semicircular vane $\alpha = \beta = 0$ $F_x = 2\rho A V^2$ $F_y = 0$.

② moving vane



For smooth shockless flow
$V_{r_1} = V_{r_2}$. But in practice
$K = V_{r_2}/V_{r_1}$ = Blade velocity coefficient.

ABC → inlet vel. triangle
A'B'C' → Outlet " "

i) V_1 & V_2 are absolute vel.

ii) Component of absolute vel. in the direction of motion.
- whirl or tangential component (V_u)

iii) Component of vel. in perpendicular direction of motion - flow component (V_f).

The change in whirl component produce the force to rotate the vane.

$F = \rho A V_{r_1} [V_{u_1} - (-V_{u_2})] = \rho A V_{r_1} [V_{u_1} + V_{u_2}]$

∴ General form of expression $F = m(V_{u1} \pm V_{u2})$

where m is the mass of liquid impinging upon the vane.

Work done $= m(V_{u1} \pm V_{u2}) \times u$

Vane efficiency $\eta = \frac{m(V_{u1} \pm V_{u2}) \times u}{\frac{1}{2} m . V_1^2} = \frac{2(V_{u1} \pm V_{u2}) \times u}{V_1^2}$

Since vane is frictionless $\frac{1}{2} m (V_1^2 - V_2^2) = m (V_{u1} \pm V_{u2}).u$

∴ $(V_1^2 - V_2^2) = 2(V_{u1} \pm V_{u2}).u$.

∴ Vane efficiency $\eta = \frac{V_1^2 - V_2^2}{V_1^2} = 1 - \left(\frac{V_2}{V_1}\right)^2$

For a series of vanes mounted on a wheel.

$u = \omega \times r = \frac{2\pi N}{60}.r.$

But ~~inlet~~ radius of inlet and outlet is different so

$u_1 = \omega . r_1$ and $u_2 = \omega r_2$

This diff. in velocity gives rise to axial force which should be taken to a thrust bearing.

Angular momentum at inlet $= m V_1 \cos\alpha_1 r_1 = m V_{u1} . r_1$

Angular momentum at " $= m . V_{u2} . r_2$

∴ change in angular momentum $= m(V_{u1} r_1 - V_{u2} r_2)$

∴ rate of change in angular mom $= \dot{m}(- - - - -)$

$= \rho Q (V_{u1} r_1 - V_{u2} r_2)$

Work done = Torque × Angular vel ~~momentum~~ $= \rho Q (V_{u1} r_1 - V_{u2} r_2) . \omega$

$= \rho Q (V_{u1} . u_1 - V_{u2} . u_2)$

Euler's momentum equation.