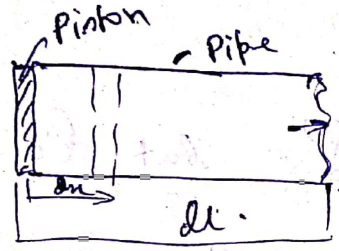
Velocity of pressure or sound wave in a fluid.

The term sonic or acoustic velocity refers to the speed at which small pressure disturbances are propagated in the form of wave through a fluid.



Duct Area A - Piston slides without friction. At rest. Pres P density ρ. After disturbance pres $(P+dP)$ density $(\rho+d\rho)$.

During time dt, piston moves dx & pressure wave moves dl.

$$dl = a\,dt$$
$$dx = dv \times dt.$$

dv = velocity of gas particle.

$$(p+dp)A - pA = \text{mass} \times \text{acc}^n = \frac{\rho . A\, dl}{dt} \times dv$$

$$dP = \rho . dl . \frac{dv}{dt} = \rho\, a\, dv$$

$$dP = \rho\, a\, dv. \implies dv = \frac{dP}{a\rho}$$

Mass of gas before disturbance = After disturbance

$$\rho . A . dl = (\rho + d\rho) A\, (dl - dx)$$

$$\therefore \rho A\, a\,dt = (\rho+d\rho) A\, (a\,dt - dv . dt)$$
$$= (\rho+d\rho) A\,dt\, (a - dv)$$

dividing both side by $A\,dt$

$$\therefore \rho a = \rho a - \rho\, dv + a\,d\rho - \underbrace{d\rho\, dv}_{\text{neglected as very small.}}$$

$$\therefore a\, d\rho = \rho\, dv \implies dv = a . \frac{d\rho}{\rho}$$

or $$\frac{dP}{a\rho} = a . \frac{d\rho}{\rho}$$

or $$a^2 = \frac{dP}{d\rho} \implies a = \sqrt{\frac{dP}{d\rho}}$$

speed of wave propagation.

Sonic vel. for a adiabatic process.

The propagation of minor disturbance through air can be thought to be a reversible adiabatic process. [No heat transfer as at control surface as no temperature gradient]. The departure from thermodynamic equilibrium is neglected

$\frac{p}{\rho^{\gamma}} = c. \rightarrow \ln p = \ln c + \gamma \ln \rho$

on integratic

$$\frac{dp}{p} = \frac{dc}{c} + \gamma . \frac{d\rho}{\rho} \quad \text{but } dc = 0$$

$$\therefore \frac{dp}{p} = \gamma . \frac{d\rho}{\rho} \qquad \therefore \frac{dp}{d\rho} = \gamma . \frac{p}{\rho}$$

$\therefore$ sonic velocity $a = \sqrt{\frac{dp}{d\rho}} = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\gamma R T}$

Bulk modulus of elasticity $K = - \frac{dp}{dV/V} = \frac{dp}{d\rho/\rho}$.

$$\therefore a = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\gamma R T}.$$

Conclusion

(1) → sonic vel depends on change in density with press.

for gases $\frac{d\rho}{dp}$ is large hence 'a' is small

for fluids $\frac{d\rho}{dp}$ is small & 'a' is large.

a at $15^{\circ}C$ air - 340 m/sec water - 1440 m/s.

(ii) higher K → higher sonic vel.

higher β_c (co-eff of compressibility) → lower 'a'!

(iii) Sonic vel increases with growth in temp.

(iv) As molecular wt ~~decrease~~ increases, sonic vel. decreases ∴ 1400 m/s in H_2 and drops to 100 m/s for some refrigerant.

Compressible factors.

The important parameter that determine the compressibility effect of flow is Mach no.

Mach no. = Flow velocity / sound velocity.

$$\frac{\text{Inertia force}}{\text{Elastic force}} = \frac{\rho A V^2}{K \cdot A} = \frac{V^2}{K/\rho} = \frac{V^2}{a^2} = M^2$$

$$\therefore M = \sqrt{\frac{\text{Inertia force}}{\text{Elastic force}}} = \frac{V}{a} = \text{Mach no.}$$

It is the important criterion for assessing the compressibility.

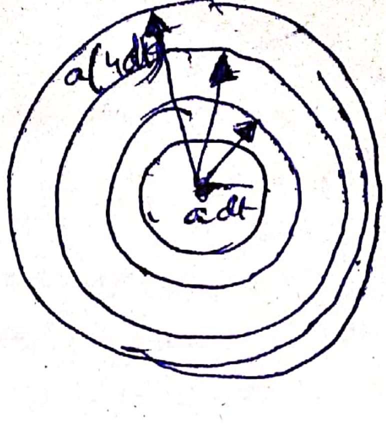
		Hypersonic – Rockets
3.0	5.0	
		Supersonic — CONCORDO, GNAT
1.2	1.1	transonic (Compressor blades)
.8	.9	
		Subsonic → Aircraft, Turbomachine
.3	.3	
		Incompressible — Fan, Blower, Hyd.

Som & Biswas

Mach Cone, Mach angle & Mach line.

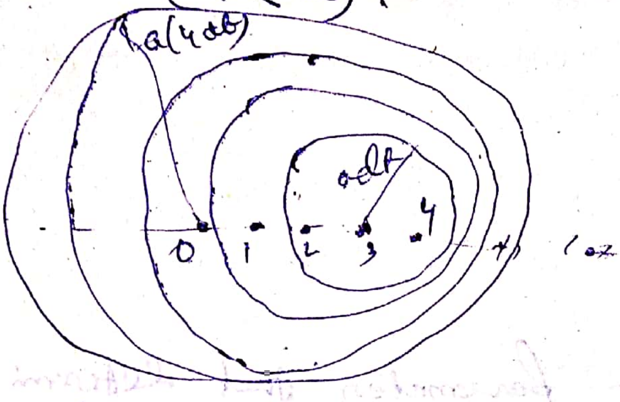
Consider a small projectile, say an ~~aerof~~ air-foil that can creates pressure disturbance.

If projectile at rest the disturbance ~~with~~ (pressure wave) with travel with sonic vel. a to all direction & ~~with~~ give that can be represented by concentric circles.

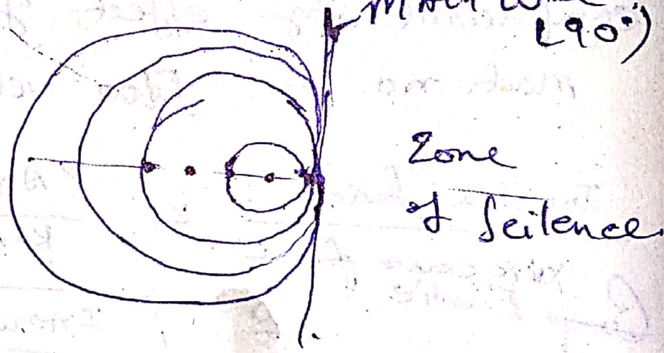


Subsonic ~~vel.~~ Flow

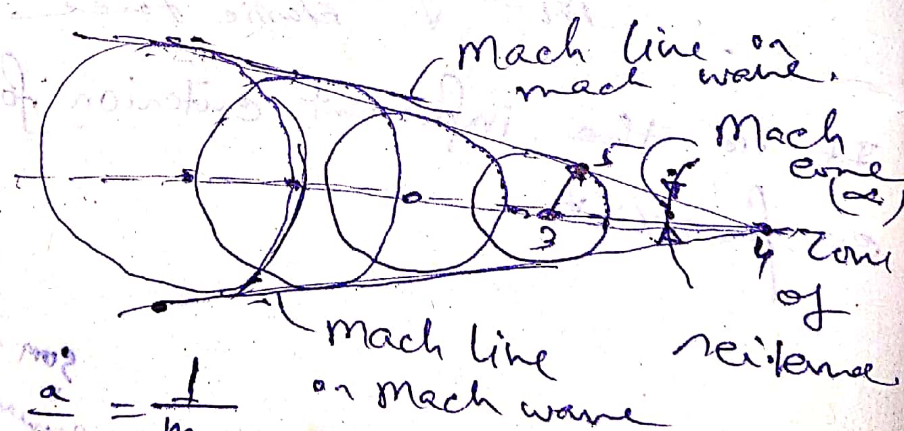
If the projectile moves with vel V ($V<a$).



Sonic ~~vel.~~ Flow



Supersonic Flow



Mach angle α

$$\sin\alpha = \frac{\text{Distance } 3\text{-}5}{\text{Distance } 3\text{-}4} = \frac{a\,dt}{V\,dt} = \frac{a}{V} = \frac{1}{M}$$

Problems:

Basic equations for 1-D Compressible ~~flo~~ flow.

① Continuity eqⁿ :-

$$\rho A V = c.$$

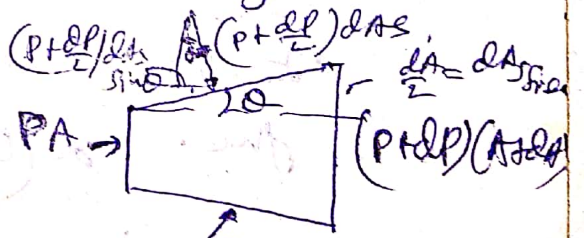
or $\ln\rho + \ln A + \ln V = \ln c.$

upon differenciation. $\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = \frac{dc}{c} = 0$

or $\therefore \boxed{\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0}$

This is differential form of continuity eqⁿ of 1-D steady compressible flow.

② Momentum eqⁿ. → Consider steady fluid ~~1D~~ flow through pipe line whose c/s area is gradually varying.



Press at inlet $= pA$

press at outlet $= (p+dp)(A+dA)$

Avg force normal to pipe wall $(p+\frac{dp}{2})dA_s$

Component along x-direction $(p+\frac{dp}{2})dA_s \sin\theta$

$$= \left(p+\frac{dp}{2}\right) dA.$$

For ideal flow there is no frictional force or shear stress.

Force along x - Direction.

$$= pA - (p+dp)(A+dA) + \left(p+\frac{dp}{2}\right)dA = -A\,dp$$

$$-A\,dp = \text{mass flow rate} \times \text{change in vel.}$$
$$= (\rho A V)\,dV$$

or $$\boxed{\frac{dp}{\rho} + V.dV = 0}$$ → This is called momentum or dynamic eqn or Euler eqn.

Integration of this gives Bernoulli's eqn. For compressible flow it is possible if press has a particular relation with density. This occurs only for homogeneous fluid.

For isentropic flow $\frac{P}{\rho^{\gamma}} = C$ ⇒ $P = \rho^{\gamma} C$

$dP = C\gamma\rho^{\gamma-1}\,d\rho$

or $$\frac{C.\gamma.\rho^{\gamma-1}\,d\rho}{\rho} + V\,dV = 0$$

upon integration

$$C.\frac{\gamma}{\gamma-1}\rho^{\gamma-1} + \frac{V^2}{2} = C.$$

or $$\frac{\gamma}{\gamma-1}.\frac{P}{\rho} + \frac{V^2}{2} = C.$$ ⇒ Bernoulli's eqn (Nm/kg).

Energy eqn:

$$E + \frac{P}{\rho} + \frac{V^2}{2} + q = (E+dE) + \left[\frac{P}{\rho} + d\left(\frac{P}{\rho}\right)\right] + \frac{(V+dV)^2}{2} + W_s.$$

if $q = 0$, $W_s = 0$

then: $$d\left(E + \frac{P}{\rho}\right) + V.dV = 0$$

$$E + \frac{P}{\rho} = h$$

$$\therefore\ dh + V\,dV = 0$$

integration $$h + \frac{V^2}{2} = C$$

$h = C_v T + RT$

$= C_p T = \frac{\gamma}{\gamma-1} RT$

$$\therefore\ \frac{\gamma}{\gamma-1}RT + \frac{V^2}{2} = C.$$

$$\left[\frac{\gamma}{\gamma-1}.\frac{P}{\rho} + \frac{V^2}{2} = C\right]$$

This eqⁿ is Bernoulli's eqⁿ for gases.

Problems

~~ISEN~~

ISENTROPIC FLOW RELATIONS

• Consider steady ~~flow of a~~ frictionless flow of a compressible fluid over a body. (No change in elevation)

$$V\,dV + dh = 0$$

or $$h + \frac{V^2}{2} = C = h_0$$ → Stagnation enthalpy

or $$C_p T + \frac{V^2}{2} = C_p \cdot T_0$$ → Stagnation Temperature

h_0 = stagnation enthalpy (or total head). The enthalpy when flow is brought to zero. (No heat transfer or work done). T_0 is the corresponding temp. This is called total temp.

$$T_0 = T + \frac{V^2}{2C_p}.$$

Static temp T is taken by a ~~to~~ thermometer which ~~flows~~ moved with the fluid at same vel. of the fluid.

T_0 can be measured with a thermometer kept at rest. The flowing fluid strikes it and comes to rest. The term $\frac{V^2}{2C_p}$ is called the velocity or dynamic temperature.

$\frac{V^2}{2C_p}$ ~~T_0~~ or $(T_0 - T) = \frac{V^2}{2C_p}$.

Fluid at rest $T_0 = T$ → Static temp.

$\therefore \frac{T_0}{T} = 1 + \frac{V^2}{2C_pT} = 1 + \frac{V^2}{2 \cdot \frac{\gamma}{\gamma-1} RT}$

$= 1 + \frac{\gamma-1}{2} \cdot \frac{V^2}{\gamma RT} = 1 + \frac{\gamma-1}{2} \frac{V^2}{a^2}$

$= 1 + \frac{\gamma-1}{2} \cdot M^2$

For a reversible adiabatic process

$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}}$ $\quad \frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{1}{\gamma-1}}$

$\therefore \frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2$ $\quad \frac{P_0}{P} = \left[1 + \frac{\gamma-1}{2} M^2\right]^{\frac{\gamma}{\gamma-1}}$

$\frac{\rho_0}{\rho} = \left[1 + \frac{\gamma-1}{2} \cdot M^2\right]^{\frac{1}{\gamma-1}}$

P_0 is called the stagnation pressure and ρ_0 is known as stagnation ~~of~~ density.

For a given stagnation condition, Pressure, temp & density decreases as Mach number increases.

For isentropic behavior of of a fluid, the stagnation fluid properties remain constant.

$\frac{T_2}{T_1} = \frac{1 + \frac{\gamma-1}{2} \cdot M_1^2}{1 + \frac{\gamma-1}{2} M_2^2}$ $\quad$ as $T_1\left[1 + \frac{\gamma-1}{2} M_1^2\right] = T_2\left[1 + \frac{\gamma-1}{2} M_2^2\right]$

$= T_0$

$=$ Stagnation temp.

Like wise $\frac{P_2}{P_1} = \left[\frac{1 + \frac{\gamma-1}{2} \cdot M_1^2}{1 + \frac{\gamma-1}{2} M_2^2}\right]^{\frac{\gamma}{\gamma-1}}$ $\quad \frac{\rho_2}{\rho_1} = \left[\frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2}\right]^{\frac{1}{\gamma-1}}$

Problem

Compressibility Correction Factor

For reversible (frictionless) adiabatic process, the relation between stagnation pressure p_0 and static pressure p is given by

$$\frac{p_0}{p} = \left[1 + \frac{\gamma - 1}{2} M\right]^{\frac{\gamma}{\gamma - 1}}.$$

Expanding according to binomial theorem becomes,

$$\frac{p_0}{p} = 1 + \frac{\gamma}{2}M^2 + \frac{\gamma}{8}M^4 + \frac{\gamma(2-\gamma)}{48}M^6 + .$$

$$p_0 - p = p\frac{\gamma}{2}M^2\left[1 + \frac{M^2}{4} + \frac{2-\gamma}{24}M^4 + \cdots\right]$$

$$p\frac{\gamma}{2}M^2 = p \cdot \frac{\gamma}{2} \times \frac{V^2}{a^2} = \frac{p}{2} \times \gamma \cdot \frac{V^2}{\gamma \cdot p/\rho} = \frac{1}{2}\rho V^2$$

$$\therefore \quad \frac{p_0 - p}{\frac{1}{2}\rho V^2} = 1 + \frac{M^2}{4} + \frac{2-\gamma}{24}M^4 + \cdots \quad \cdots (1)$$

for incompressible flow $\quad \frac{p}{\rho} + \frac{V^2}{2} = \frac{p_0}{\rho_0} + \frac{V_0^2}{2}$

but $V_0 = 0$

$$\therefore \quad \frac{p_0 - p}{\frac{1}{2}\rho V^2} = 1 \quad \cdots (2)$$

Comparing this two equations we get.

$1 + \frac{M^2}{4} + \frac{2-\gamma}{24}M^4 + \cdots$ is compressibility factor.

M	.1	.2	.3	.4	.5	.6
CF	1.003	1.010	1.023	1.041	1.064	1.393

for $M = .2$ compressibility affects stagnation pressure by less than 1%.

for $M = .3$ it is about 2.3%.

$.5$ 6.4%.

So if $M < 0.3$ fluid is treated as incompressible.