What is a Fluid?

A fluid may be defined as substance which deforms continuously under the action of shear force (or stress), regardless of its magnitude. A small stress which appears as negligible in magnitude can cause the deformation of the fluid.

Ideal Fluid - A fluid is said to be ideal when it is incompressible and non-viscous (inviscid). Further an ideal fluid has no surface tension. $\mu = 0$ $\rho = \text{Constant}$ $K = -\left(\frac{dp}{dv/v}\right)_T = \infty$ $\sigma = 0$

Ideal fluids are imaginary and do not exists in nature. K = bulk modulus. $K = \rho\left(\frac{\partial P}{\partial \rho}\right)_T$.

Properties of Fluid

Fluid Continuum → Although fluid consist of discrete molecules, analysis of fluid ~~mechanics~~ flow problem is made by concept that treat the fluid as continuous media. The liquid molecules are so closely spaced that strong intermolecular cohesive force compel the fluid to behave as a continuous mass.

Density → Defined by mass/unit volume.

At any pt. $\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V}$ when Δm = mass contained in a small volume ΔV

Specific Gravity → = ~~Ratio of two densities~~ (numerically)

It is the numerical ratio of two densities, and water ~~is~~ (at 4°C) is taken as reference. It is dimensionless and have no unit.

So. sp. gravity = $\frac{\text{Density of any substance}}{\text{Density of water at } 4^\circ C}$

Specific Weight → (w) It is the wt. of a substance/unit volume.

It is the force exerted by ~~the~~ gravity on the unit volume of fluid. Unit is Force/unit vol. i.e. N/m^3.

$\gamma = \rho \cdot g$ $\left[\frac{W}{V} = g \cdot \frac{m}{V} = \rho \cdot g\right]$

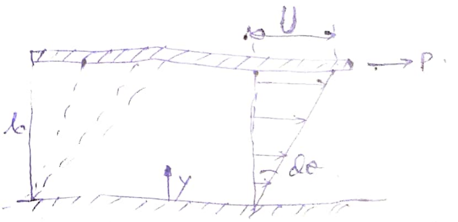
Viscosity → It is the property of fluid by virtue of which a fluid offers resistance to deformation under influence of shear force.

So long as the shear force exists the deformation will continue, and rate of deformation depends on magnitude of shear force. The molecular friction and shear resistance within the fluid opposes such continuous deformation.

$U = b \cdot \frac{d\theta}{dt}$ $\left[\frac{U}{b} = \text{rate of angular deformation} = \frac{d\theta}{dt}\right]$

or. $u = y \cdot \frac{d\theta}{dt}$

or $\frac{d\theta}{dt} = \frac{u}{y} = \frac{U}{b}$

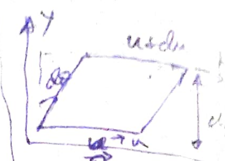


where u is the vel. at a distance of y from the stationary plate.

If A = area of the moving plate and F is the force reqd to give cont. vel. of U.

then $\frac{F}{A} \propto \frac{d\theta}{\partial t}$

or $\tau = \mu \cdot \frac{d\theta}{\partial t} = \mu \cdot \frac{u}{y} = \mu \cdot \frac{U}{b}$.



$dy \times d\theta = du\,dt$

or $\frac{du}{dy} = \frac{d\theta}{\partial t}$

in differential form $\tau = \mu \cdot \frac{du}{dy}$

So. shear stress is dependent on fluid deformation or vel. gradient at right angle to the direction of velocity. This is known as Newton's law of viscosity (or fluid friction).

For a given shear force acting on a fluid element is, the rate at which the fluid deforms is inversely proportional to the viscosity.

Unit of viscosity → $\mu = \frac{\tau}{\partial U/dy} = \frac{F}{L^2} * [L/T \times \frac{1}{L}] = FT/L^2 \quad [\frac{N\text{-}s}{m^2}]$

$F = \frac{mL}{T^2} \quad \therefore \mu = \frac{M}{LT}$

∴ Unit S.I. $\mu = \frac{N\text{-}s}{m^2} = Pa\text{-}s$

μ = Co-efficient of dynamic viscosity = Pa-s

1 Poise = $\frac{1\ gm}{cm\text{-}sec} = \frac{1\ dyne\text{-}s}{cm^2} = 0.1$ Pa-s.

cP = centipoise = $\frac{1}{100}$ poise.

μ_{water} = 1.0 cP = 10^{-3} N-s/m² at 20°c. $\mu_{air} = 0.0181$ cP at 20°c

Specific Viscosity → It is the ratio of viscosity of fluid to the viscosity of water at 20°c.

Kinematic Viscosity (ν) = $\frac{\text{Dynamic Viscosity}}{\text{Density of Fluid}} = \frac{\mu}{\rho}$ is comparison of inertia & viscosity

Unit = $\frac{mass/m^2}{Time} = [\frac{m^2}{T}]$ S.I. → m²/sec. cgs - cm²/sec

ν_{water} = 1.0 cSt (Centistoke) = 1×10^{-6} m²/s. [Stoke = cm²/s.]

= 1.0 mm²/sec.

ν_{air} at 20°c. = 15 cSt = 15 mm²/sec.

Kinematic viscosity is more for air.

Note :→ Viscosity of any fluid is temperature dependent.

Viscosity of liquid decreases with increase in temperature while viscosity of gas increases with the increase in temperature.

For liquid $\mu_t = \frac{\mu_0}{1 + At + Bt^2}$ [μ_t = viscosity at t°c, μ_0 = " at 0°c]

Water { μ_0 = 0.0179 poise, A = 0.03368, B = 0.000221

For gas $\mu_t = \mu_0 + \alpha t - \beta t^2$

Air $\mu_0 = 1.7 \times 10^{-5}$ N·s/m², $\alpha = 0.56 \times 10^{-7}$, $\beta = 0.1189 \times 10^{-9}$

with increase in press viscosity increases

$\mu_p = \mu_0 \exp[K(p - p_0)]$

Compressibility & Bulk Modulus.

Coefficient of Compressibility $(\beta_c) = -\frac{1}{V}\left(\frac{dV}{dP}\right)$ Compressibility is the ability to change its volume under pressure.

$-ve$ sign indicates is change in Pressure is $(+ve)$ the change in volume is $(-ve)$ or vice versa.

Bulk modulus of elasticity $(K) = \frac{1}{\beta_c} = -\frac{dp}{(dV/V)} = \frac{\text{Compressive stress}}{\text{Unit volumetric strain.}}$

again- $m = \rho V$ or $dm = \rho \cdot dV + V \cdot d\rho$

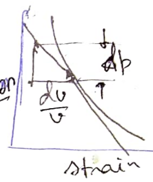
Since mass is constant $dm = 0$ $\therefore -\frac{dV}{V} = \frac{d\rho}{\rho}$

$\therefore K = \frac{dp}{(d\rho/\rho)}$ $\frac{d\rho}{\rho}$ = relative change in density of fluid.

For isothermal process $pV = \text{const}$. $\therefore p\,dV + V\,dp = 0$. $\therefore -\frac{dV}{V} = \frac{dp}{p}$

$\therefore \underline{K = p}$.

For adiabatic process. $pV^{\gamma} = \text{const}$. gives. $\underline{K = \gamma p}$.



$K_{water} = 20 \times 10^8\ N/m^2$ and $K_{air} = 1.05 \times 10^5\ N/m^2$

Water $-dV = \frac{V\,dp}{K} = \frac{1 \times 10 \times 10^5}{20 \times 10^8} = \frac{1}{2000}\ m^3$ [air is 20000 times more compressible than water]

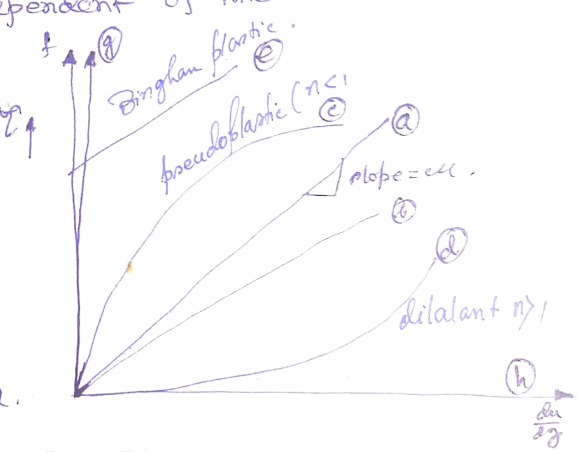
Thermal Expansion:

Co-efficient of thermal expansion $(\beta_{ex}) = \frac{1}{V}\left(\frac{dV}{dT}\right)_p$ [Relative increase in vol. per degree rise in temperature during an isobaric process]

$\beta_{ex} = 14 \times 10^{-6}$ at $0°C$ and 1 bar
$= 700 \times 10^{-6}$ at $100°C$ and 100 bar.

Newtonian & Non-Newtonian Fluid

A fluid which obeys Newton's law of viscosity [i.e. $\tau = \mu \cdot \frac{du}{dy}$] is known as Newtonian fluid. Newtonian fluid have certain constant viscosity i.e. viscosity is independent of shear stress or independent of velocity gradient.



a, b → Newtonian fluid.
Fluid a is more viscous than fluid b.

c → Pseudo plastic
d → Dilatant
e → Ideal Plastic
f → ideal solid
g → real solid, h → Ideal fluid.

Newtonian fluid → Example - water air kerosene. thin lubricating oil etc

General Form $\tau = K\left(\frac{du}{dy}\right)^n + B$ if $K = \mu$ & $n = 1$ → Newtonian $B = 0$

Otherwise → Non-newtonian. [eg. Blood, Thick lubricating oil, slurry & contain suspensions]

if $n < 1$ → Pseudo-plastic. — The coefficient is smaller at higher velocity gradient. and curve becomes flatter at higher velocity gradient - Ex:- milk, blood, clay liquid cement etc
Thixotropic $B = 0$

if $n > 1$ → Dilatant. → Shear rate increases with higher vel. grad.
Ex:- Concentrated sugar & Starch soln. (~~Thixotropic fluid~~)
Rheopectic $B = 0$

Other Properties → Surface tension and capilarity

Cohesion & Adhesion → Represents the adhering ~~of~~ or clinging of the fluid molecules to the solid surface with which they come in contact

Force with which the neighbouring or adjacent fluid molecules are attracted towards each other.

$$\text{Relative Density} = \frac{\text{Weight of body}}{\text{Wt. of equal volume of water}}$$

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* for ideal fluid $k=0$ & $B=0$ so $\tau=0$

* for Bingham fluid or ideal plastic $n=1$ $B=\tau_0$.

$$\tau = \tau_0 + \mu \frac{du}{dy}.$$

Ex: water ~~air~~ suspension of clay and flyash, sewage, sludge etc.

Power Law model or Oswald - de Waele model → $\tau = m\left(\frac{du}{dy}\right)^n = \underbrace{m\left(\frac{du}{dy}\right)^{n-1}}_{\mu_{app}} \cdot \frac{du}{dy}$

m = ~~Flow~~ fluid consistency index.

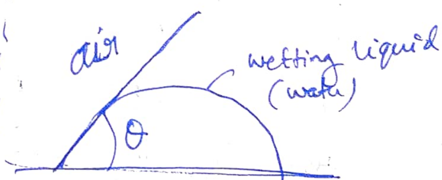
n = Flow behaviour index.

SURFACE TENSION & CAPILARITY

Cohesion & adhesion.

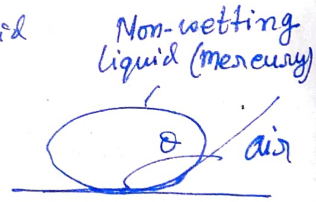
Cohesion → Forces between like molecules

Adhesion → Forces between unlike molecules



$\theta < \frac{\pi}{2}$

Adhesive force is more than cohesive force.



$\theta > \frac{\pi}{2}$

cohesive molecular force greater than adhesive force.

~~Contact~~ Liquid-gas interface with solid surface.

$\theta < \frac{\pi}{2}$ for wetting surfaces and wetting increases as $\theta \to 0$.

For not wetting liquid $\theta > \frac{\pi}{2}$. The contact angle depends on nature and type of liquid, solid surface and its cleanliness.

For pure water $\theta = 0$

For contaminated " $\theta = 25^\circ$ or more

For mercury $\theta = 130^\circ$ to 150°.

Cohesive force is equal in all direction. The molecules lying at the surface have a net attraction to pull them into the interior of the liquid mass. A quantum of of energy (work) is thus expensed to bring the molecule to the free liquid surface which then acts like an elastic or stretched membrane. Energy expended per unit area of surface is called surface tension.

Unit = $N \cdot m/m^2 = N/m$ = (Force / unit length) $\boxed{\frac{M}{T^2} \text{ or } \frac{F}{L}}$

symbol σ → (Surface tension occurs at the interface of a liquid and gas or at interface of ~~is called surface tension; designated~~ two liquids.

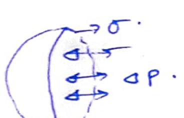
Value of surface tension depends on
(i) nature of the liquid
(ii) nature of surrounding matter which may be a solid liquid or gas.
(iii) K.E. and hence temp. of liquid molecules.
(surface tension reduces with increase in temp as cohesive forces reduces with temp.)

(iv)
Increase in temperature reduces the inter molecular cohesive force and thus surface tension reduces.

At critical pt. where liquid and vapour phase coincides, value of σ becomes zero.

σ_{air}	$= 0.073$ N/m
σ_{water}	$= 0.480$ N/m

Pressure inside a water droplet and soap bubble.



A water drop of dia d is cut into two halves.
On one half. pressure force $(p_i - p_o)\frac{\pi}{4}d^2$
Tensile force due to surface tension.
$= \sigma \times \pi d$

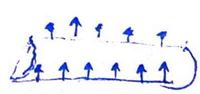
$$\therefore (p_i - p_o)\frac{\pi}{4}d^2 = \sigma \times \pi d.$$

$$\Rightarrow (p_i - p_o) = 4\sigma/d. = 2\sigma/r.$$

For soap bubble surface tension force will act on both the surfaces and accordingly. $(p_i - p_o)\cdot\frac{\pi}{4}d^2 = 2\cdot\sigma\cdot\pi d$,

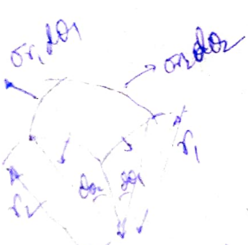
$$\therefore (p_i - p_o) = 8\sigma/d = \frac{4\sigma}{r}.$$

Liquid jet of dia d and length l.



pressure force $(p_i - p_o) \times l d = \sigma \times 2l$

$$\therefore (p_i - p_o) = \frac{2\sigma}{d}.$$



$$2\sigma r_2 d\theta_2 \sin\frac{d\theta_1}{2} + 2\sigma r_1 d\theta_1 \sin\frac{d\theta_2}{2} = (p_i - p_o) r_1 r_2 d\theta_1 d\theta_2$$

$$\sin\frac{d\theta_1}{2} \approx \frac{d\theta_1}{2}$$

$$\Delta P = \frac{\sigma}{r_1} + \frac{\sigma}{r_2}$$

on sphere $\Delta P = \frac{2\sigma}{r}$
Soap bubble : $\Delta P = \frac{4\sigma}{r}$.

(6) Problems: Viscosity

(2.11) ② Two horizontal flat plates are placed 0.15 mm apart and the space between them is filled with an oil of viscosity 1 poise. The upper plate of area 1.5 m² is required to move with a speed of 0.5 m/s relative to the lower plate. Determine the necessary force & power need to maintain this speed.

Viscous shear stress $\tau = \mu \frac{du}{dy}$ $\mu = 1$ poise $= \cdot 1$ N s/m² $du = 0.5$ m/s, $dy = 0.15$ mm $= \cdot 15 \times 10^{-3}$ m

$\therefore \tau = \frac{\cdot 1 \times 0.5}{\cdot 15 \times 10^{-3}} = 333.3$ N/m²

$F = \tau \times A = 333.3 \times 1.5 = 500$ N,   Power $= F \times V = 500 \times 0.5 = 250$ W $= 0.25$ KW

(2.16) ③ A cylinder of dia 15 cm and wt. 90 N slides a distance of 12.5 cm in a lubricated pipe. The clearance between the cylinder and pipe is 2.5×10^{-5} m. The cylinder is noted to decelerate at a rate of 0.6 m/s², when speed is 6 m/s. Calculate the viscosity of oil used.

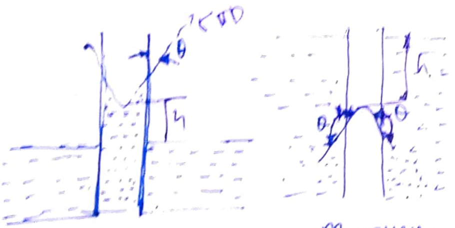
Soln: $\tau = \mu \cdot \frac{du}{dy} = \mu \cdot \frac{u}{t}$.   Force = stress × Area.

$F = \mu \cdot \frac{u}{t} \times \pi d l = \frac{\mu \times 6}{2.5 \times 10^{-5}} \times \pi \times 0.15 \times 0.125 = 14130\,\mu$ N

Σ Force = Mass × Acc'n

$\therefore 90 - 14130\mu = \frac{90}{9.81} \times (-0.6) \Rightarrow \mu = 6.76 \times 10^{-3}$ N s/m²

Capillarity



Water   Mercury

Adhesion between glass and water is more than cohesion.

Cohesion between mercury and glass is more than adhesion. Convex me

Wt of liquid rise or lowered
= ~~Weight of liquid raised or lowered~~

$= \left(\frac{\pi D^2}{4} \times h\right) \times \rho g$ ($\frac{\pi D^2}{4}$ → area; h → rise or fall; ρg → Sp. wt.)

Vertical component of surface tension force $= \sigma \cos\theta \times$ circumference.
$= \sigma \cos\theta \times \pi d$

$\therefore \frac{\pi}{4} d^2 h \rho g = \sigma \cos\theta\, \pi d$.

$h = \frac{4\sigma \cos\theta}{w d}$

Gas Laws (Thermodynamic Relations).

$PV = mRT$ or $Pv = RT$

R = Characteristic gas Constant $= \frac{\bar{R}}{M.W}$ $\bar{R} = 8314$ N-m/kg mol-K $= 8.314$ KJ/kg mol-K

① Isobaric Process (P = Const.)
$\frac{v}{T} = \frac{1}{\rho T}$ = Const. or $\frac{V}{T}$ = Const. (Charles law)

② Isothermal. PV = Const. or $Pv = \frac{P}{\rho}$ = Const. (Boyle's Law)

③ Reversible adiabatic PV^{γ} = Const or Pv^{γ} = Const.

Problem (2.13) → The clearance space between a shaft and a concentric sleeve has been filled with a Newtonian fluid. The sleeve attains a speed of ~~60 cm/s~~ 60 cm/s when a force of 500 N is applied to it parallel to the shaft. What force is needed if it is desired to move the sleeve with a speed of 300 cm/s. ?

Ans:-

$\tau = \mu \cdot \frac{du}{dy}$. film thickness t is small.

$\therefore \frac{F}{A} = \tau = \mu \cdot \frac{u}{t}$. $F = \mu A \frac{u}{t}$

or $F \propto u \Rightarrow \frac{F_1}{u_1} = \frac{F_2}{u_2}$ $F_2 = \frac{F_1 \cdot u_2}{u_1}$.

$= \frac{500 \times 300}{60}$

$= 2500$ N.

(2.15) Prob: A dash pot of 10 cm diameter and 12.5 cm long slides vertically down in a 10.5 cm diameter cylinder. The oil filling the annular space has a viscosity of 0.80 poise. Find the speed with which the piston slides down if load on the piston is 10 N.

Shear Force = shear stress × Area

$= \mu \cdot \frac{u}{t} \times (\pi d l) = 0.08\ (N\text{-}s/m^2)\left(\frac{u}{0.00025} \times \pi \times .1 \times .125\right)$

$\therefore 10\ N = (0.08 \times u \times \pi \times 0.1 \times 0.125) / 0.00025$

$u = 0.796$ m/s.

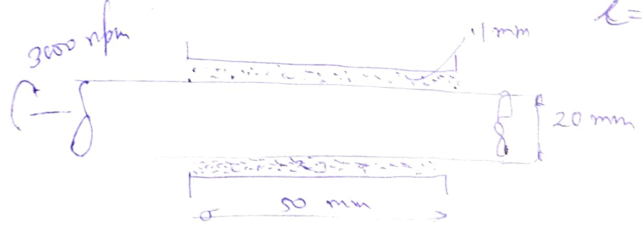
(2.24) Problem

In a 50 mm long journal bearing arrangement, the clearance between the two at concentric condition is 0.1 mm. The shaft is 20 mm in dia and rotate at 3000 rpm. The dynamic viscosity of the lubricant used is 0.01 Pa-s and the velo variation of the lubricant is linear. Considering the lubricant to be Newtonian, calculate the frictional torque the journal has to overcome, and corresponding power loss.

$\tau = \mu \cdot \frac{du}{dy} = \mu \cdot \frac{u}{t}$ $\mu = 0.01$ Pa.s = .01 N s/m²

$d = .02$ m
$l = .05$ m

$F = \mu \cdot \frac{u}{t} \times \pi d l$.



$u = \frac{\pi d N}{60} = \frac{\pi \times .02 \times 3000}{60}$

$= 3.14$ m/s

$t = .01$ m = .0001 m

$\therefore$ ~~Frictional Torque~~ $F = \left(.01 \times \frac{3.14}{.0001} \times 0.02 \times \pi \times .05\right)$

$\therefore$ Torque $= F \times r = \left(.1 \times \frac{3.14}{0.0001} \times 0.02 \times \pi \times 0.05\right) \times 0.01$

$= 9.85 \times 10^{-3}$ Nm.

$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 3000}{60} = 314$ rad/s.

$\therefore P = T \times \omega = (9.85 \times 10^{-3}) \times 314 = 3.09$ Nm/sec. = 3.09 Watt