* shaft encoders

Controller output

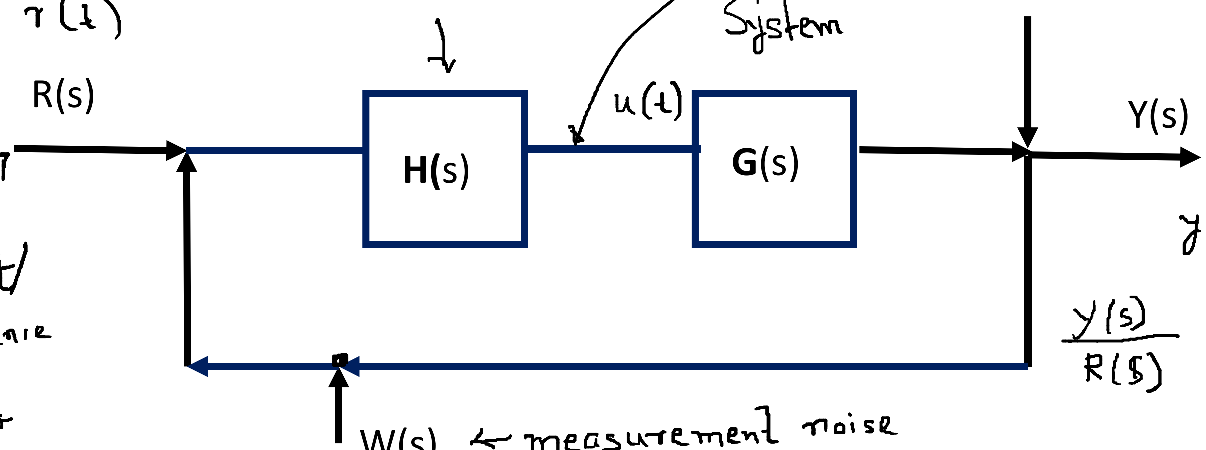
Disturbance

controller

System

$r(t)$

$u(t)$



set point / reference

Controlled variable / output variable

$y(t)$

← measurement noise

$$\frac{Y(s)}{R(s)} = \frac{G(s)H(s)}{1+G(s)H(s)}$$

$$\frac{Y(s)}{D(s)} = \frac{1}{1+G(s)H(s)}$$

$$\left|\frac{Y(s)}{W(s)}\right| = \frac{G(s)H(s)}{1+G(s)H(s)}.$$

~~Characteristics of a co~~

What should a controller achieve?

a. Good Tracking. ⟶ $y(t)$ should track $r(t)$

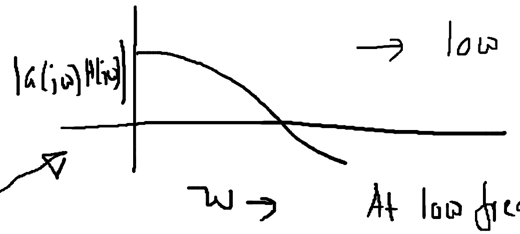
b. Good Disturbance rejection ⟶ $y(t)$ should not be affected by $D(t)$.

c. Good Noise rejection ⟶ $y(t)$ should not be affected by $W(t)$.

d. Robustness (parametric robustness and stability robustness)

$$\frac{Y(s)}{R(s)} = \frac{G(s)H(s)}{1+G(s)H(s)}.$$

$|G(j\omega)H(j\omega)| \rightarrow$



$\rightarrow$ low pass filter.

As $\omega \uparrow\uparrow$ $|G(j\omega)H(j\omega)| \downarrow\downarrow$

At low frequency.

$$\left|\frac{Y(j\omega)}{R(j\omega)}\right| \approx \frac{|G(j\omega)H(j\omega)|}{|G(j\omega)H(j\omega)|} \approx 1$$

$$\frac{Y(j\omega)}{R(j\omega)} = \frac{G(j\omega)H(j\omega)}{1+G(j\omega)H(j\omega)} \rightarrow \underline{\text{input sensitivity}}$$

s or complementary sensitivity

At high frequency $\left|\frac{Y(j\omega)}{R(j\omega)}\right| \approx \frac{0}{1+0} \approx 0$

$$\frac{Y(j\omega)}{D(j\omega)} = \frac{1}{1+G(j\omega)H(j\omega)} \rightarrow \text{Output sensitivity or}$$

$\rightarrow$ Now ~~complementary~~ sensitivity

$\Rightarrow$ At high frequency $\frac{1}{1+G(j\omega)H(j\omega)} \rightarrow \frac{1}{1+0} = 1$

But at low frequency $\frac{1}{1+G(j\omega)H(j\omega)} \rightarrow \frac{1}{G(j\omega)H(j\omega)} \rightarrow 0$

$\downarrow$ this is large

<u>Moral of the story.</u>

Design $G(j\omega)H(j\omega)$ as low pass filter.

Noise rejection $\frac{Y(s)}{W(s)} \rightarrow$ similar to $\frac{Y(s)}{R(s)}$. $\left|\frac{Y(j\omega)}{W(j\omega)}\right|$ is <u>low</u> as $W(j\omega)$ ~~has a~~ is associated with higher frequencies.

Robustness

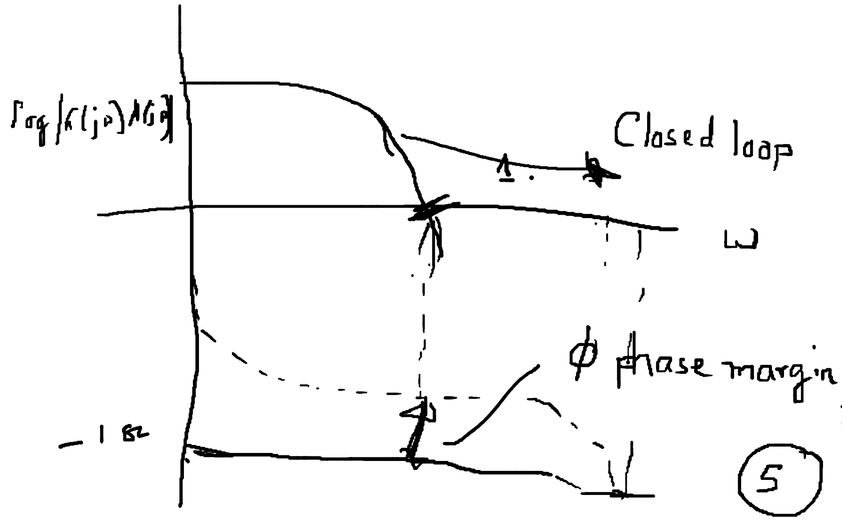
We have designed $G(j\omega)H(j\omega)$ to be a low pass filter.

Thus, as $\omega\uparrow$ $|G(j\omega)H(j\omega)|\downarrow$. What happens to the phase? i.e. $\angle G(j\omega)H(j\omega)$?

$$\frac{K}{1+j\omega T}$$

$$1+G(j\omega)H(j\omega)$$

$$1 \quad -1+j0$$



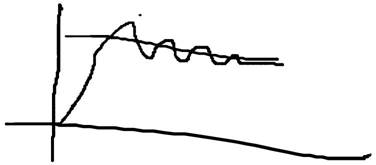
K $\quad 20\log_{10}(K)$ $\quad 1\angle 0$

System should have good GM and PM

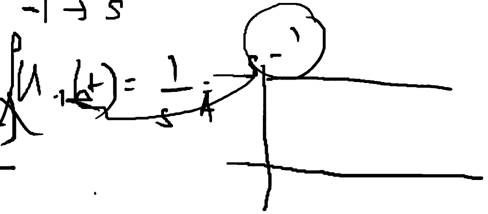
The controller characteristics should remain ~~constant over~~ same over an interval of parameter variations.

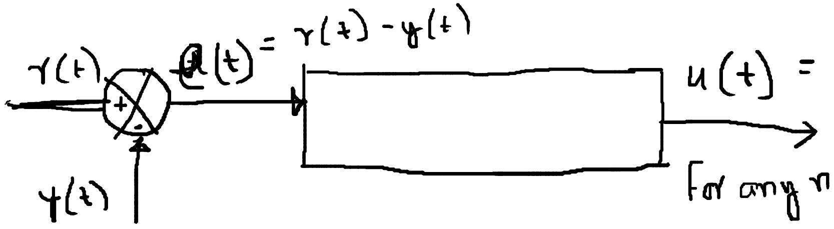
Resilience

5 Controller



A <u>PID controller</u> : ~~We~~ $H(s) = k_p + \frac{k_i}{s} + k_D s$.



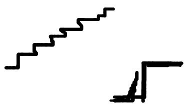


$$u(t) = k_p e(t) + k_i \int_0^t e(t)\,dt \;\; k_D \frac{de(t)}{dt}$$

For any natural system, lets say $\underline{r(t) = A u_{-1}(t)}$. $A u_{-1}(t) = A$ for $t \geq 0$

At $t=0$, what is $u(t)$? System has a delay.
So in the worst case $e(t) = r(t)$ and $\frac{de(t)}{dt}$ is very ~~the~~ large at $t=0$. $k_D \frac{de(t)}{dt}$ is called the derivative kick.

$k_i \int \underline{e(t)}\,dt$

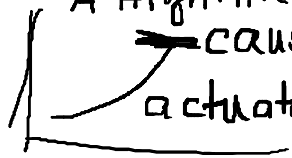


$T_i = \frac{1}{k_i}$ $\quad k_i = \frac{1}{T_i} \times \frac{1}{k_p f_i}$

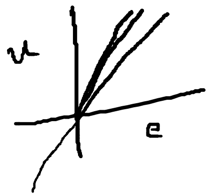
$T_D = k_D$

$e(t)$

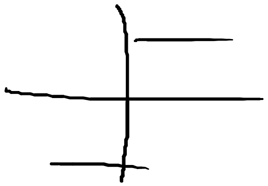
→ Integral windup.
A High integral gain ~~=~~ causing actuator saturation



proportional band.
$\Delta P = \frac{100}{K}$ %.

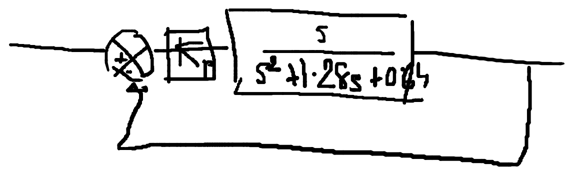


⇒



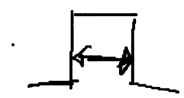
Bang Bang Control
Z-N

$$G_p(s) = \frac{5}{s^2 + 1.28s + 0.64}$$



As k increases, the closed system becomes more and more underdamped.

$$G_c(s) = k_p + \frac{k_i}{s} + k_D s \Rightarrow G_c(s) = \frac{k_p s + k_i + k_D s^2}{s}$$ → High pass filter



$$G_c(s)\,G_p(s) = \frac{(k_p s + k_i + k_D s^2)}{s} \cdot \frac{5}{s^2 + 1.28s + 0.64}$$