Linear Quadratic Regulator Design by Particle Swarm Optimization for a Flexible Link Robot Manipulator

Thesis Submitted in the partial fulfillment of the requirements for the degree of

MASTER OF ELECTRICALENGINEERING

Submitted By

NAMRAJIT DEY

Examination Roll number: M4ELE1611
Registration Number: 128901 of 14-15

Under the Guidance of
DR. SMITA SADHU

Electrical Engineering Department
Faculty Council of Engineering and Technology
JADAVPUR UNIVERSITY
KOLKATA- 700032

May, 2016

Faculty Council of Engineering and Technology
JADAVPUR UNIVERSITY
KOLKATA- 700032

Certificate of Recommendation

This is to certify that **Mr. Namrajit Dey (M4ELE1611)** has completed his dissertation entitled, **"*Linear Quadratic Regulator Design by Particle Swarm Optimization for a Flexible Link Robot Manipulator*"**, under the direct supervision and guidance of **Dr. Smita Sadhu,** Electrical Engineering Department, Jadavpur University. I am satisfied with his work, which is being presented for the partial fulfillment of the degree of **Master of Electrical Engineering** of Jadavpur University, Kolkata-700032.

Dr. Smita Sadhu
Professor, Electrical Engineering Department
Jadavpur University, Kolkata – 700 032

Dr. Swapan Kumar Goswami
Head, Electrical Engineering Department
Jadavpur University, Kolkata – 700 032

Dr. Sivaji Bandyopadhyay
Dean, Faculty of Engineering & Technology
Jadavpur University, Kolkata – 700 032

Faculty Council of Engineering and Technology
JADAVPUR UNIVERSITY
KOLKATA- 700032

Certificate of Approval *

The foregoing thesis is hereby approved as a creditable study of Master of Electrical Engineering and presented in a manner satisfactory to warrant its acceptance as a pre-requisite to the degree for which it has been submitted. It is understood that by this approval the undersigned do not necessarily endorse or approve any statement made, opinion expressed or conclusion therein but approve this thesis only for the purpose for which it is submitted.

Final Examination for
Evaluation of the Thesis

____

____

____

Signature of Examiners

** Only in case the thesis is approved*

Declaration of Originality and Compliance of Academic Ethics

I hereby declare that this thesis contains literature survey and original research work by the undersigned candidate, as part of his **Master of Electrical Engineering** studies.

All information in this document has been obtained and presented in accordance with academic rules and ethical conduct.

I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name (Block Letters) : **NAMRAJIT DEY**

Exam Roll Number : M4ELE1611

Thesis Title : **Linear Quadratic Regulator Design by Particle Swarm Optimization for a flexible link Robot Manipulator**

Signature with Date:

Dedicated to
my "Beloved parents"

ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to everyone who has helped me throughout the completion of my thesis.

First and foremost I express my thanks and gratitude to my guide, Prof. Smita Sadhu, Department of Electrical Engineering, Jadavpur University, Kolkata, for her invaluable guidance, suggestions and encouragement throughout my project, which helped me in successfully completing it. You gave me freedom to choose a project of my own choice and a chance to get knowledge from a prestigious institution like Jadavpur University, Kolkata. I shall consider this as an inspiration to extend my work in this field.

I specially thank Emeritus Prof. T. K. Ghosal, Jadavpur University Kolkata for his valuable inputs which helped me in understanding concepts better. I am also thankful to Prof. Samar Bhattacharya, Prof. Gourhari Das, Dr. Madhubanti Maitra and Dr. Rajit Kumar Barai, Jadavpur University Kolkata for giving me a good theoretical and practical knowledge of various control subjects that helped me throughout the thesis work.

Now, I would like to thank pursuing PhD scholar Mr. Nilanjan Patra and my Jadavpur University batchmates Mr. Rudrashis Majumder, Ms. Aiman Javed who helped directly and indirectly and motivated me throughout my thesis completion in Jadavpur University.

Finally, I would like to thank my parents and friends for their unconditional support and love.

Date:
Namrajit Dey
Jadavpur University, Kolkata

ABSTRACT

In this thesis, Pole placement with a degree of stability and damping is first studied to design state feedback controller. Continuous time Algebraic Riccati Equation (CARE) as a generalized eigenvalue- eigenvector problem is analyzed and solved in aforesaid pole placement. Afterwards, an intelligent and very well-known swarm intelligence method Particle Swarm Optimization (PSO) has been studied for calculation of state feedback gains which satisfied pole placement with similar objective.

However, some authors have shown that pole-placement may suffer from poor robustness when dynamic perturbations in its state-space formulation are introduced or in-case zeros located far from imaginary axis. So, application of Linear Quadratic Regulator (LQR) to obtain state feedback gains is considered. Accordingly, LQR cost matrices Q and R are optimized using PSO technique which eliminated trial-error approach of weight selection in LQR design. A simple flexible link two degree robot manipulator is considered to verify all above studies. Overall, better tracking performance was achieved in PSO based designed LQR.

KEYWORDS: Flexible link Robot-Manipulator, Pole placement, Linear Quadratic Regulator, Particle Swarm Optimization.

Contents

Index **Page No.**

5. Chapter 5: Pole Placement within specified Disk

6. Chapter 6 : Particle Swarm Optimization

7. Chapter 7: Application in a Robot Manipulator

8. Chapter 8: Discussions and Conclusions

Chapter 1
Introduction

1.1 Background

Mechatronics is the backbone of many real world examples of control system technology. Starting from robotics to automated machineries in widely varying industries like food processing, packaging, construction equipment manufacturing and so on. Even in IC packaging and large scale electronic manufacturing has been time and cost saving with the help of automation industry. Mechatronics and its related fields have growing importance day by day in modern world. Mainly motivated by South East Asian countries like Japan, North Korea, South Korea India is now producing state-of the art technologies for a rapid growth in manufacturing industry. While selecting a mechatronic system in our main area of research, importance must be given on trajectory tracking as well as reduction of noise and harmonics to maximize the system performance. Also the response of the system must successfully track the reference input in order to get desired closed loop response. In the context of the control problem, the basic idea is not only to stabilize a plant but also to involve the achievement of some desired performance specifications,such as bandwidth, disturbance rejection, noise reduction, reference tracking and so on. In our case a double link flexible robot manipulator which replicates many of the real world examples was considered as the plant for investigation. In order to attain closed loop stability with desired performance attainment, PID controller is not of correct approach even if it works perfectly in various cases. So, in order to obtain optimal control Linear Quadratic Control is considered as a tool to obtain reference tracking as well as performance optimization. But choosing of cost matrices Q, R needs to be done in trial and error approach in conventional LQR method. Therefore, an intelligent search technique, Particle Swarm Optimization is considered to get optimize Q, R values. The results of search values and how it works in a closed loop system environment is the need and motivation of this dissertation.

1.2 Motivation of the thesis

In the context of the control problem, the basic idea is not only to stabilize a plant but also to involve the achievement of some desired performance specifications,such as bandwidth, disturbance rejection, noise reduction, reference tracking andso on. For those purposes, modern control design methods have been extensivelyused to acquire a great deal of fundamental and also empirically based knowledge ofthe systems. Linear optimal control is one approach which often gives the designersatisfactory results with respect to the stability and the performance of the controlledsystems. One advantage of it is that the mathematical optimization methods are adopted so that a control law for a linear system can be readily derived based on a prescribed objective function. The resulting computational procedures may then often be applied to nonlinear systems. Moreover, if the plant states are allavailable, good robustness properties of the optimal regulators used can be clearly revealed in terms of the stability margins.

A famous example is full state feedback design with linear quadratic regulator (LQR) approach. As the state space representation is a natural way for system description by LQR the system performance can be managed and the plant inputs and the control input can be synthesized using an optimal control law by solving the Riccati equation. Meanwhile, the guaranteed stability robustness is automatically provided by LQR, unlike pole placement techniques, for instance. Moreover, another interesting analysis is the asymptotic behavior of the LQR as the weights Q, R approach the extreme cases. This further indicates that weight selection techniques can be incorporated to improve the performance in the cheap control regulators on various systems.

Now, in order to get optimal performance criterion which ensures desired performance various performances indexes like Integral Absolute Error (IAE), Integral of Squared Error (ISE) etc. were considered. Moreover, designing cost matrices Q, R matrices involving those criteria requires intelligent search techniques like min-max algorithm, GA algorithm etc. The perfect regulation problemwas also investigated in the context of the cheap regulator problem and the cheap servomechanism problem even for systems with non-minimum behaviors. As is known, the right half plane (RHP) zeros of the open loop system always exert some limitations on the overall performance by the analysis of the sensitivity function and the complementary sensitivity function. However, the limitations on the cheap regulators can be directly characterized by the complex plane plots of the RHP zeros. Considering open loop RHP zeros, design of LQR suffered with some limitations in stated feedback gain values. So closed loop pole locations such as dominant poles and non-dominant poles are important to compensate behavior of RHP zeros.

All these notes were to be taken care of by intelligent selection of Q, R cost matrices in LQR so that system can achieve tracking as well some desired system specifications regarding disturbance rejection, noise rejection can be achieved by the designed LQR.

So, the primary motivation behind the dissertation is as follows-

- Existing pole-placement techniques and application in non-minimum phase system to investigate performance of a closed loop system with a state feedback controller including integral control for better tracking and disturbance rejection.

- A generalized eigen value-eigen vector problem form of Continuous Algebraic Riccati Equation (CARE) which gives an easy approach of P matrix evaluation.

- Analytical LQR design and its performance subject to plant dynamics uncertainties and input/output disturbance investigation.

1.3 Problem Statement

Based on the above motivation, the present work builds upon Particle Swarm optimization based LQR design and its application in a two-link flexible robot manipulator. The thesis also involves a comparative study between direct state feedback gains calculation by intelligent pole placement and another way of state feedback gains calculation by optimization of weighting matrices Q, R in the same PSO route. Then, it investigates robustness of designed in presence of plant uncertainties and input/output disturbances.

1.4 Contributions of thesis

This dissertation has some following salient contributions as follows-

- LQR design for a flexible robot manipulator using an intelligent and evolutionary search technique known as Particle Swarm Optimization. While application of PSO, some PSO parameters like weight selection, velocity range selection and correctness of out of bounds values of particle positions were considered which directed better optimization.

- The robot manipulator performance between analytical design of LQR and PSO based design of LQR was compared. This successfully shows PSO gives a better result in better selection of Q, R matrices which further help the control system designer to achieve desired performance criteria as well as attain perfect tracking in presence of plant parameter perturbation and input disturbances.

1.5 Organization of thesis

Chapter-1: This chapter includes background of work, motivation and research objectives along with the flow of work.

Chapter-2: This chapter includes literature review, how the research work has been forwarded in this field.

Chapter-3: This chapter involves analytical LQR design method. A simple 2^{nd} order system and its traditional LQR design using odesolver in MATLAB have been represented.

Chapter-4: This chapter deals with Integral action in state feedback control action, how it is important to make closed loop system enable in tracking in presence of plant parameter perturbations and input/output disturbances.

Chapter-5: This chapter states about pole placement, a well-known method in feedback gains calculation. Also how a generalized eigen value problem resulted into pole placement within a disk specified by stability margins.

Chapter-6: This chapter represents Particle Swarm Optimization and its application in State feedback controller gains calculation.

Chapter-7: This chapter finally deals with application of PSO approach in both state feedback method and LQR method in a Robot Manipulator system. There comparisons have been represented.

Chapter-8: This chapter deals with overall conclusions and discussions about future scope of work.

Chapter 2
Literature Review

2.1 Introduction

The foregoing thesis has taken progress as first detailed study on existing pole placement techniques and stability considerations associated with it. Then particle swarm optimization used as an evolutionary search mechanism has been studied. Thereafter optimal control design for linearized systems with help of linear quadratic regulator has been studied. Secondly, all above mentioned control design procedures have been applied to a two-degree flexible robot manipulator and results has been studied and compared. Various existing books, journals, conference papers have been studied regarding this.

2.2 Solution Domain Literature Review

Modern control design methods have been extensively used to acquire a great deal of fundamental and also empirically based knowledge of the systems. Linear optimal control is one approach which often gives the designer satisfactory results with respect to the stability and the performance of the controlled systems. One advantage of it is that the mathematical optimization methods are adopted so that a control law for a linear system can be readily derived based on a prescribed objective function. The resulting computational procedures may then often be applied to nonlinear systems. Moreover, if the plant states are all available, good robustness properties of the optimal regulators used can be clearly revealed in terms of the stability margins. A famous example is full state feedback design with linear quadratic regulator (LQR) design. As the state space representation is a natural way for system description by LQR the system performance can be managed and the plant inputs and the control input can be synthesized using an optimal control law by solving the Riccati equation. Meanwhile, the guaranteed stability robustness is automatically provided by LQR, unlike pole placement techniques, for instance.

2.2.1 *Pole Placement Technique*

The idea of pole placement technique in closed loop design in illustrated in [Fallside 1977]. It ensures only modification of characteristics equation in closed loop and thereby solving it to get desired closed loop pole locations. [Bogachev 1979] also refers an interesting analytic design of control system using the closed loop poles in a specified region. State feedback problem has been presented in [Kuo 1980]. [Furuta 1987] has shown a generalized eigen value approach of pole placement within a specified disk. It also specifies the condition of D-stability i.e limitations in pole placement within disk to maintain root locus in the left side of imaginary axis in s-plane. [Misra 1996] has shown an interesting way to attach desired stability parameters like damping and absolute stability to pole placement and accordingly place poles inside a specified disk. Integral action in closed loop attains better tracking performance in presence of parametric uncertainties. [Ramli 2007] represents comparison in state feedback control action without and with integral action.

2.2.2 *Numerical Solution of Algebraic Riccati Equation*

[Anderson 1979] derived a set of assumptions and conditions upon the co-efficient of algebraic Riccatieqation. [Mori 1980] proposed to use the discrete Riccati equation to determine the control law of the continuous system. [Pappas 1980] has first shown numerical solution method of algebraic Riccati equation. [Vaughn 1970] has shown a direct non-recursive method to approach the numerical solution of discrete algebraic Riccati equation. But it involved direct inverse calculation of state transition matrix; if state matrix is ill-conditioned numerical difficulties arise. Also transition delays associated with state matrix arises numerical difficulties as shown in [Gould 1969]. On the other hand, [Bialkowski 1978] has shown how singular state transition matrices may appear, in which case it is impossible to find inverse of state transition matrix.

Meanwhile [Moler 1973] proposes an algorithm for numerical computation of matrix eigen value problems. Another method known as Scur method was proposed by [Laub 1979] to numerically solve algebraic riccati equation. [Dooren 1981] has also shown a numerical method for solving algebraic RiccatiEquatins. However our present discussion is based on generalized eigen value-eigen vector approach of numerical solution as proposed by [Pappas 1980]

2.2.3 *Particle Swarm Optimization*

[Kennedy 1995] proposes Particle Swarm Optimization technique inspired by fish schooling and bird flocking. It shares similarities with other evolutionary techniques such as GA and DE is represented in [Kennedy 1997]. Further developments regarding PSO and its objective function to be optimized has been discussed in [Shi 1998] . Guaranteed convergence and weight selection in PSO was discussed in [Peer 2003]. [Gao 2004] referred to application of PSO in machine-learning. In our dissertation [Gaing 2004], application of PSO in AVR controller has been used to develop algorithms of PSO design in state-feedback control system.

2.2.4 *Linear Quadratic Regulator*

Mathematics of LQR has been studied from [Naidu 2002]. The perfect regulation problem was investigated in the context of theregulator problem and the servomechanism problem even for systems with non-minimum behavior. As is known, the right half plane (RHP) zeros of the open loop systemalways exert some limitations on the overall performance by the analysis of thesensitivity function and the complementary sensitivity function [Freudenberg 1985]. However, the limitations on regulators can be directly characterizedby the complex plane plots of the RHP zeros. Anderson andMoore [Anderson 1979] have shown that LQR can have attractive stability margins, i.e. infinitegain margin, phase margins of 60 degree and gain margin of 0.5 for single input single output (SISO) plants, based on the return difference equality [Kalman 1964].All the works above have shown that LQR possesses excellent stability robustness and ideal asymptotic responses, especially in non-minimum phase systems. However, [Soroka, 1984] argued that the optimal LQR may suffer from poor stability robustness when, even, small changes occur particularly in the input matrix. Their examples show that the expectation of stability properties of LQR may be destroyed.

Thus, it leads us to think about whether perturbations considered in designing traditional LQR or not? Hence, we further need to investigate when perturbations happen in the state space form of the plant, how would the perturbations affect the closed-loop transfer function? Also PSO results better optimization of Q, R weight matrices. So whether PSO based LQR can perform well in terms of plant perturbations or not.

2.3 Problem Domain Literature Review

2.3.1 *Flexible Link Robot Manipulator*

Based on the above research motivations a single link two degree robot manipulator system model was considered to check above study of PSO based Linear Quadratic Control design. Following research articles have helped in better understanding of the plant model and its working.

Flexible manipulators have drawn attention to control system research because of their flexibility compared to the rigid ones. [Subudhi 2002] has represented dynamic modeling and control of manipulator with mathematics of joints and links. Further [Guirrez 1998] has implanted PID based control design for single link flexible manipulator system. An approach of intelligent PSO based direct state feedback design is implemented in [Solihin 2009]. Thereby our objective is to perform the same idea regarding LQR design since LQR also attains desired performance indices. In this thesis, the plant model is taken from [Quanser Manuals] for rotary flexible link.

Chapter 3

Traditional LQR Design in MATLAB

3.1 Introduction

In this chapter, we present the closed-loop optimal control of linearplants or linearized systems with quadraticperformance index or measure. Thisleads to the Linear Quadratic Regulator (LQR) system dealing with stateregulation, output regulation, and tracking. Overall, we areinterested in the design of optimal linear systems with quadratic performanceindices. This chapter is inspired by [Naidu 2002]

3.2 Problem Formulation

We discuss the plant and the quadratic performance index with particular reference to physical significance. This helps us to obtain some elegant mathematical conditions on the choice of various matrices in the quadratic cost functional. Consider a Linear Time Invariant System given by-

$\dot{x}(t) = Ax(t) + Bu(t)$ ……………………………………………………………..3.2
$Y(t) = Cx(t)$…………………………………………………………………………….....3.3

and the cost functional (CF) or Performance Index (PI) is given by –

$J(x,u) = J(x(t_0), u(t), t_0$

$$= \left(\frac{1}{2}\right)[z(t_f) - y(t_f)]^T F(t_f)[z(t_f) - y(t_f)]$$

$+(1/2)\ \int_{t_0}^{t_f}[[z(t) - y(t)]^T Q[z(t) - y(t)] + u(t)^T Ru(t)]dt$ ………………………………3.4

where$A \in R^{n\times n}, B \in R^{n\times m}, C \in R^{p\times n}, x(t) \in R^n, y(t) \in R^p\ , z(t) \in R^p and\ u(t) \in R^m$
We assume that control is unconstrained and $0 < m \leq p \leq n$. All these states/outputs are completely observable. Here $z(t)$ represents reference variable and $F(\,n \times n)$ denotes steady state cost matrix.

1. **State Regulator-** When we try to keep system state $x(t) = 0$ and output $y(t) = x(t)$.

2. **Output Regulator-** When we try to keep output $y(t) = 0$.

3. **State Tracking System-** The desired reference state is non-zero and the error $z(t) - y(t)$ is to be made zero.

For a infinite LQR problem $t_f$tends to infinity. This means steady state cost matrix is irrelevant to this case i.e$F(t_f) = 0$. So J integral refers to only part one of equation 3.4

3.3 LQR Cost Matrices

3.3.1 State/Error Weight Matrix ($\boldsymbol{Q}$)

In order to keep error small and error squared non-negative, the integral of expression $1/2e(t)^T Qe(t)$ should be non-negative and small. Thus Q must be positive semi-definite.

3.3.2 Input Weight Matrix ($\boldsymbol{R}$)

Higher cost required for larger control effort. Control cost has to be a positive quantity. R should be positive definite.

3.3.3 Terminal Cost Matrix ($\boldsymbol{F}$)

In order to achieve perfect tracking $e(t)$ steady state or terminating time t_f is required to be as small as possible. Fshould be semi-definite.

Also, $Q = Q^T \geq 0$and $R = R^T \geq 0$, $F = F^T \geq 0$ i.e all are symmetric as well.

3.4 A case study

The detail mathematics of LQR design in MATLAB has been explained in [Naidu 2002] . Here we present a sample rotating inverse pendulum system [Solihin 2009] considered and its response in LQR design-

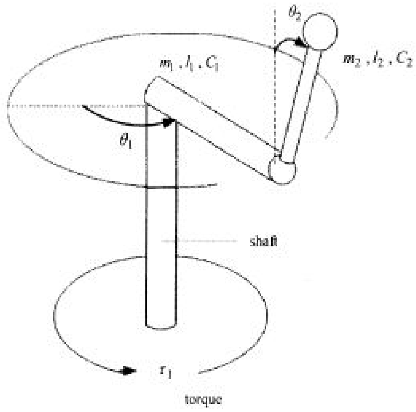

Fig: 3.1 Rotating Inverse Pendulum Model (Source:[Solihin 2009])

Rotating inverse pendulum is a very common plant of dynamic control. The figure 3.1 represents two sub-systems, one is a DC motor shaft rotating the base another is the inverse pendulum. The LTI model is given by-

$$\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & -7.1247 & -0.0963 & 0\\ 0 & 59.3267 & 0.0741 & 0\end{bmatrix}\begin{bmatrix}\theta_1\\ \theta_2\\ \dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 7.0002\\ -5.3847\end{bmatrix}u \qquad \text{.......................................}3.5$$

Where θ_1 represents angle of rotary base and θ_2 represents angle of the inverse pendulum.u represents the input voltage applied to DC motor. LQR is designed for the system described by 3.5 and results are as follows-

The state feedback controller obtained by traditional LQR is given by

K = [-10.0000 -116.4506 -6.3457 -15.7502]

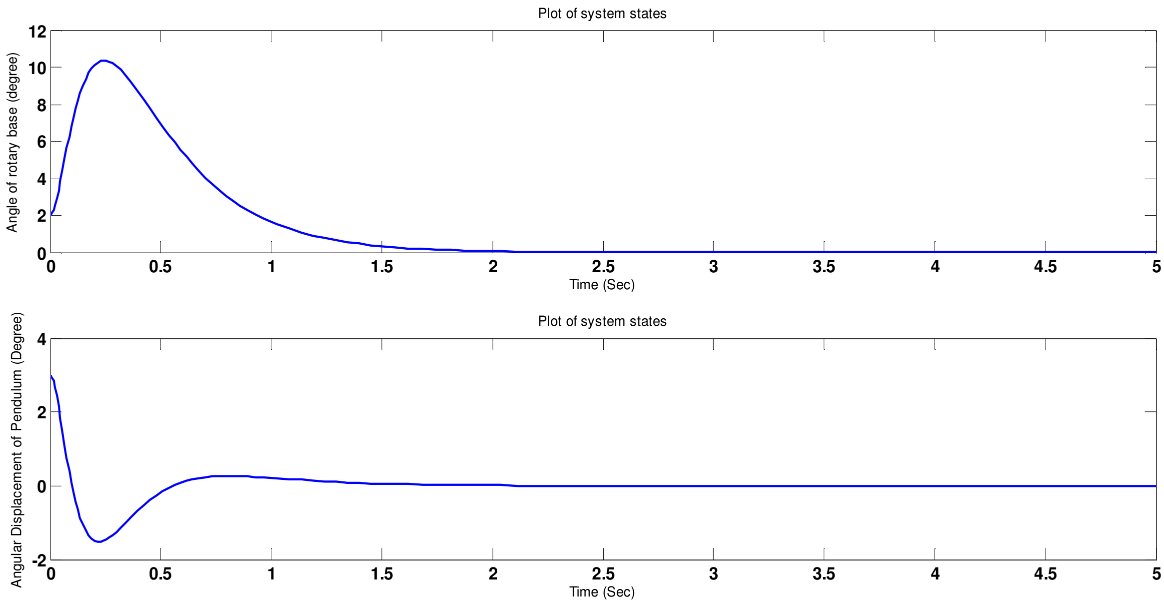

Fig 3.2: Simulation trajectories in MATLAB LQR

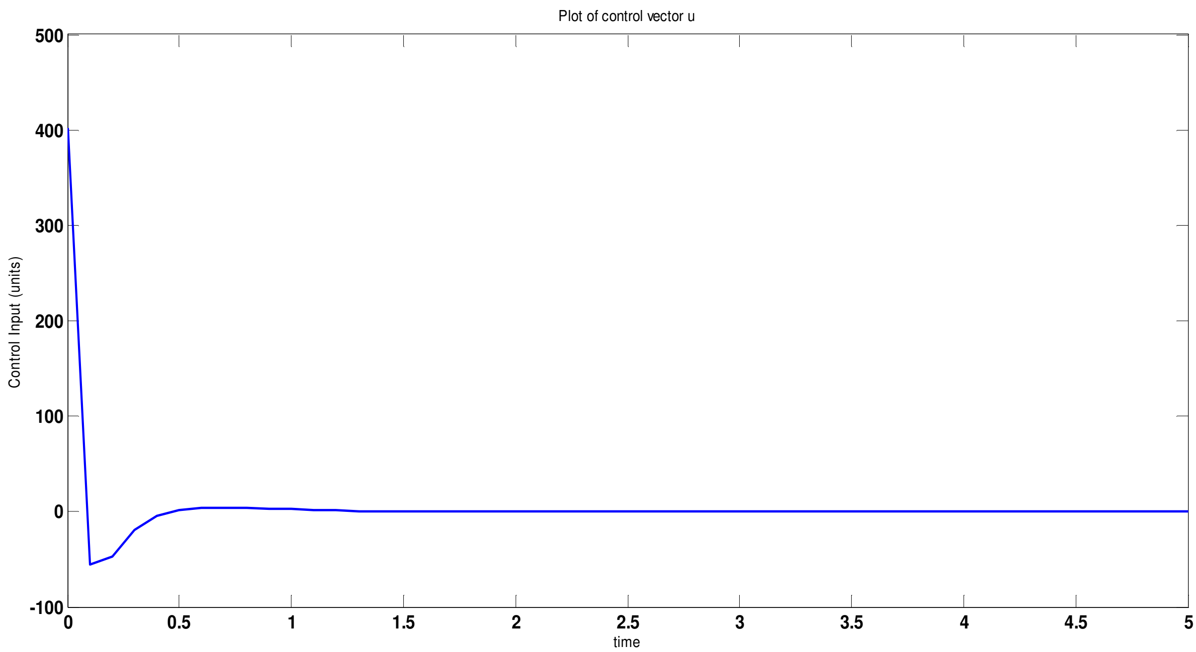

Fig 3.3: Control input in MATLAB LQR

Chapter 4

Integral Action in State Feedback

4.1 Introduction

In the last chapter regarding plant model and the problem statement, state feedback was considered to design the overall closed loop system. In this chapter we present a brief review of state feedback and why integral action has been introduced intentionally in the system. As a result the plant order got increased by one, it will be discussed elaborately throughout the chapter.

4.2 Regulation and Tracking using state feedback

Considering the open loop state equation as follows-

$\dot{x}(t) = Ax(t) + Bu(t)$……………………………………………………………4.1

$y(t) = Cx(t)$……………………………………………………………………....4.2

We apply the control law $u(t) = Nr(t) - Kx(t)$ and obtain the closed loop state equation as-

$\dot{x}(t) = (A - BK)x(t) + BNr(t)$……………………………………………… …..4.3

If only the system is controllable then only we are able to place closed loop poles i.e. eigen values of $A - BK$ in our desired location. Here, $\bar{N}$is the feed forward compensation gain required to track the input effectively as shown through an example in later section of this chapter.

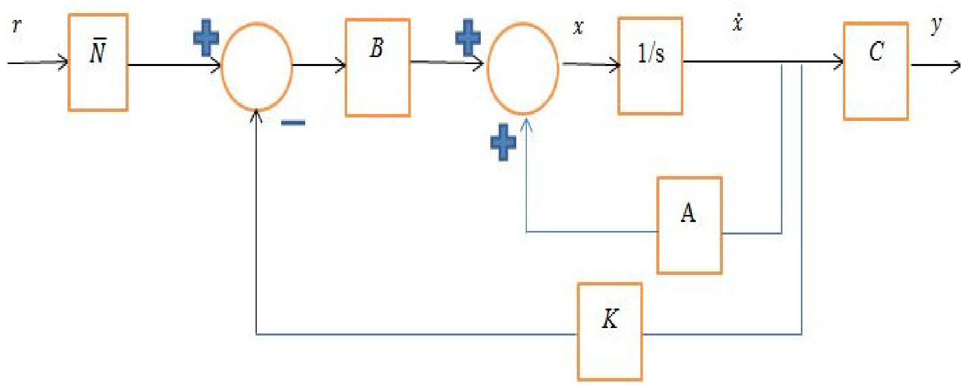

Fig 4.1: Feedforward gain$\bar{N}$ to attain reference tracking

Two typical control problems of interest exist.

1. The regulator problem, in which $r = 0$ and we try to attain $\lim_{t\to\infty} y(t) = 0$. This is purely a stabilization problem.

2. The tracking problem, in which$y(t)$ is specified to track $r(t) \neq 0$. When $(t) = R$, it is similar to a regulator problem and otherwise it is more of a challenging problem to track varying $r(t)$ also known as servomechanism type problem.

4.3 Integral Action in State Feedback

A robust approach to achieve a perfect tracking in closed loop system has been described in this section [Ramli 2007]. The output $y(t)$ is taken and fed back to the reference $r(t)$ to get the error signal denoted by $e(t) = r(t) - y(t)$. Now the error signal $e(t)$ is passed through an integrator along with an integral gain ki such that $\lim_{t\to\infty} e(t) = 0$ and $y(t)$ successfully tracks $r(t)$.

This can be achieved by all the following conditions-

1. Under parameter uncertainties in the actual plant.
2. Under input and/or output disturbances.

So, the introduction of integral action not only ensures perfect tracking of the reference input but also ensures the state feedback controllers' robust performance in presence of parameter uncertainties and input-output disturbances.

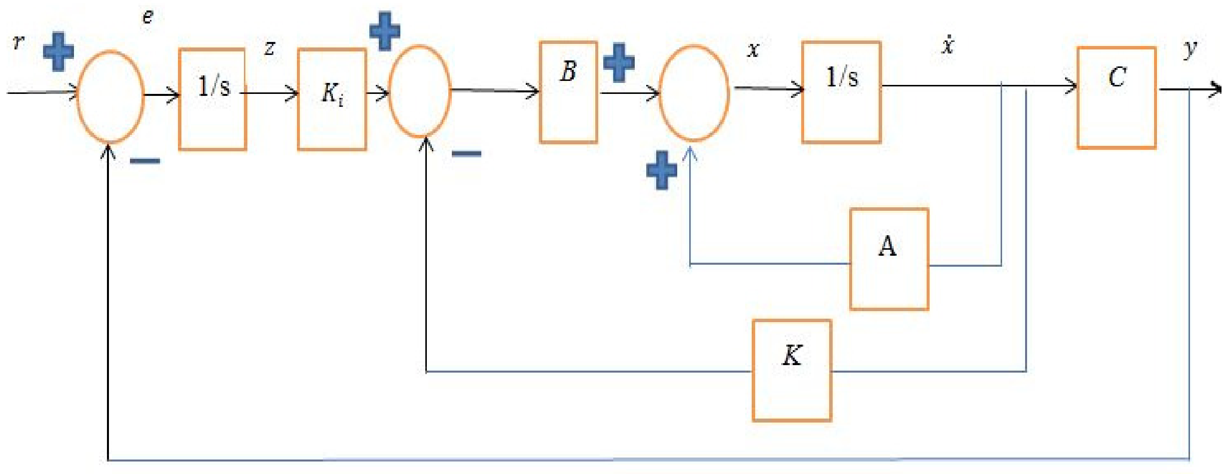

Fig: 4.2 Closed Loop Integral Control

The main idea in the addition of integral action is to augment the plant with an extra state: the integral of the tracking error, considered as $z(t)$ such that $e(t) = \dot{z}(t)$.

$e(t) = \dot{z}(t) = r(t) - y(t) = r(t) - Cx(t)$...4.4

And the control law $u(t)$ for the augmented plant is given by –

$u(t) = -[(k \quad ki] \begin{bmatrix} x(t) \\ z(t) \end{bmatrix}$...4.5

The resulting augmented system is given by-

$\begin{bmatrix} \dot{x}(t) \\ \dot{z}(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} - \begin{bmatrix} B \\ 0 \end{bmatrix} [k \quad ki] \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$ 4.6

Comparing the state feedback equation 4.6 for closed loop system with 4.3 we have augmented matrix co-efficients as-

$Aa = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}, Ba = \begin{bmatrix} B \\ 0 \end{bmatrix}, Ka = [k \quad ki]$

$\begin{bmatrix} \dot{x}(t) \\ \dot{z}(t) \end{bmatrix} = (Aa - Ba * Ka) \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$..4.6

And the output equation is given by-

$y(t) = [C \quad 0] \begin{bmatrix} x(t) \\ z(t) \end{bmatrix}$...4.7

As a result of above operation system characteristics equation got multiplied by s ,i.e plant order got increased by 1 without any change in the co-efficient values of characteristics equation.

4.4 A particular case study

We reviewed the state feedback design procedure with the following 2nd order system.

$A = \begin{bmatrix} -10 & 1 \\ -0.02 & -2 \end{bmatrix} B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ and $C = [1 \quad 0]$

1. First we checked open loop response with respect to a unit step input.
2. Obtained eigen values of A i.e open loop poles.

Open loop poles were located at $s = -9.99, -2.00$.

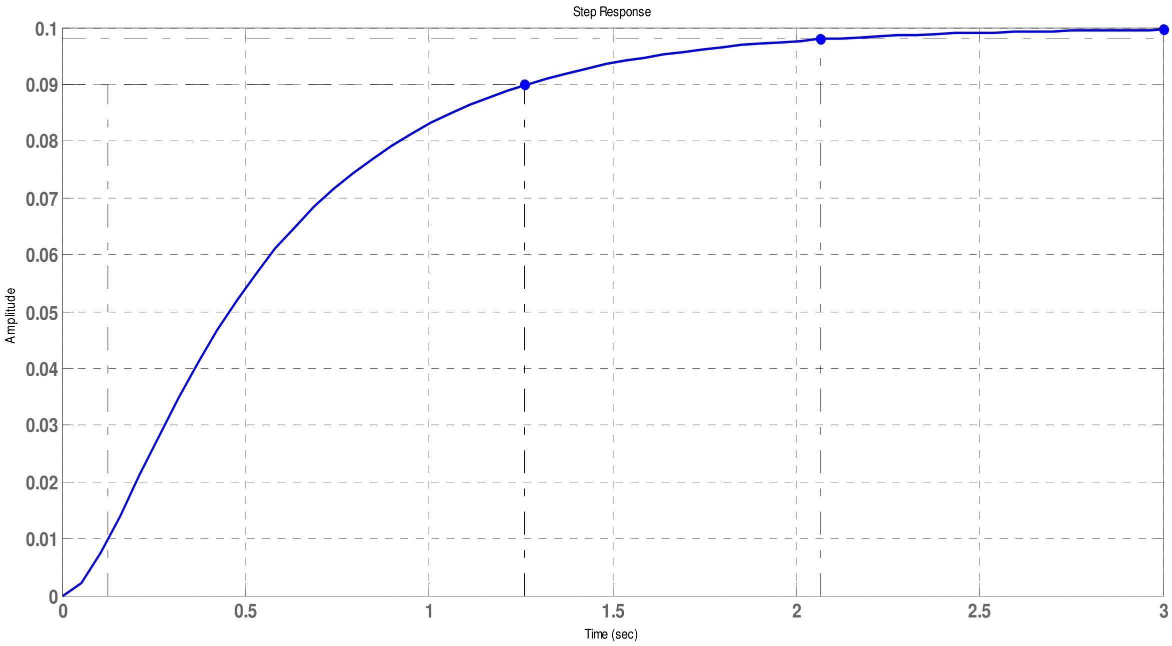

Fig:4.3- Open loop response of system

From the open loop response it was found that steady state value reaches only upto 0.1 and it took settling time more than 2 Sec and rise time more than 1 sec.

3. Next we checked the controllability of the system and it was found to be controllable.

   $Q = [B \quad AB] = \begin{bmatrix} 0 & 2 \\ 2 & -4 \end{bmatrix}$has a full rank equal to 2.

4. Now in order to place closed loop poles at our desired pole locations, corresponding gain matix$K$ was calculated. Closed loop poles were placed at $s = -5 \pm j$

$K$was obtained as $[12.99 \quad -1]$ followed by[Ramli 2007] .

5. Then closed loop response against same unit step input was observed.

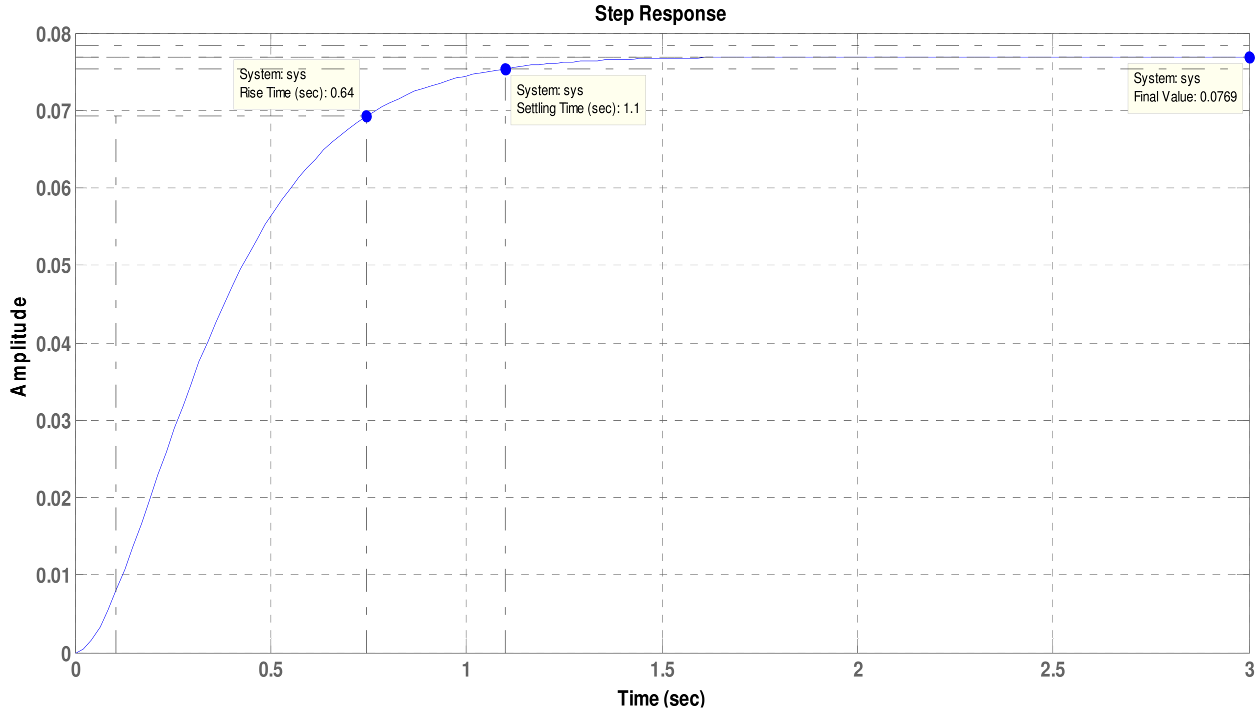

Fig:4.4- Closed loop response of system

Steady state value reaches only up to near 0.08, so feed forward gain $\bar{N}$ is necessary to achieve zero steady state error. Though rise time & settling time with state feedback reduced.

6. In order to track the reference input i.e unit step input, feed forward gain $\bar{N}$ was calculated as [Ramli 2007]. Refer to Figure 4.1

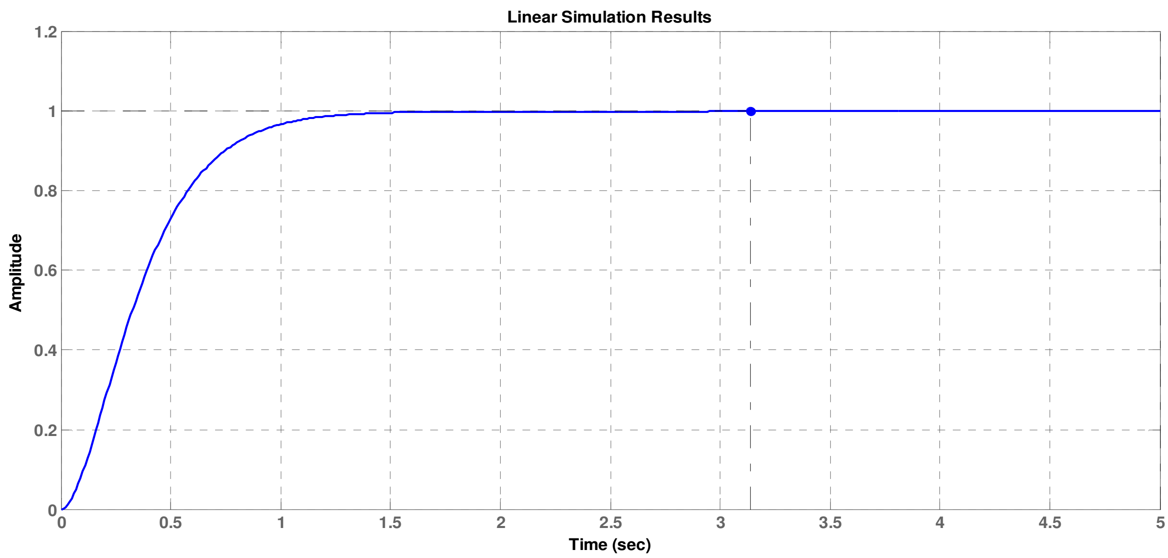

Fig. 4.5Closed Loop response with gain $\bar{N}$ (Without any integral action)

Steady state value reaches to 1, thereby achieving zero steady state error.

7. The robustness of above designed state feedback controller was checked by introducing some parameter uncertainty within the plant as follows-

$$A = \begin{bmatrix} -10 & 1 \\ -0.02 & -2 \end{bmatrix} + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

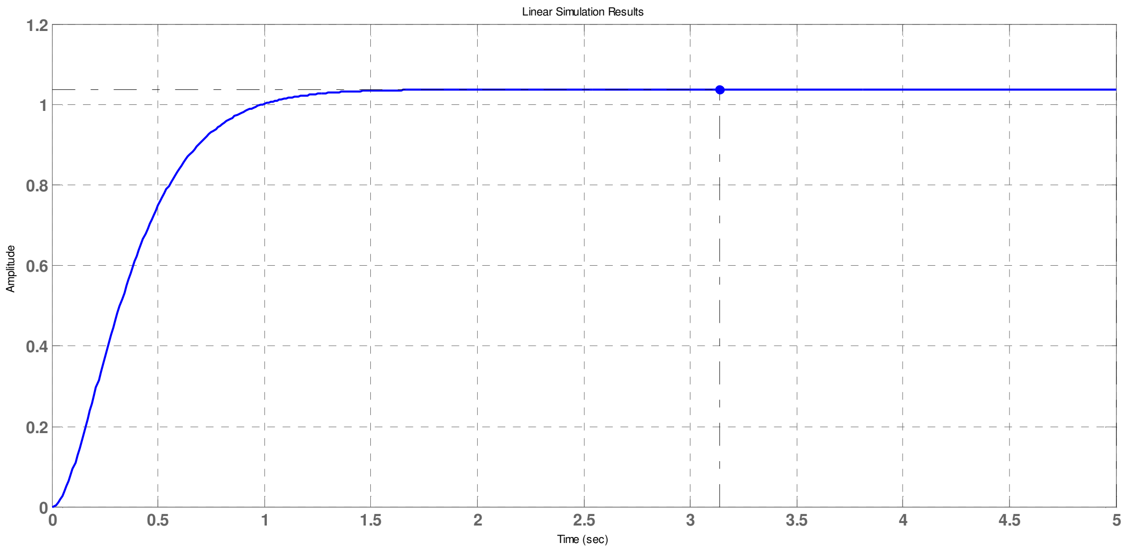

Fig. 4.6 Closed Loop response with presence of plant disturbances (Without any integral action)

Steady state value reaches to 1.04, failing to achieve zero steady state error. So, integral action is necessary to reduce steady state error in presence of step input.

8. To apply integral action on state feedback augmented system was considered as explained earlier. Augmented matrices Aa, Ba were calculated as per equation 4.6 and correspondingly input matrix was considered as $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ instead of B. Also, the pair (Aa, Ba) was found controllable. The augmented gain Ka was found as [22 2 -78]

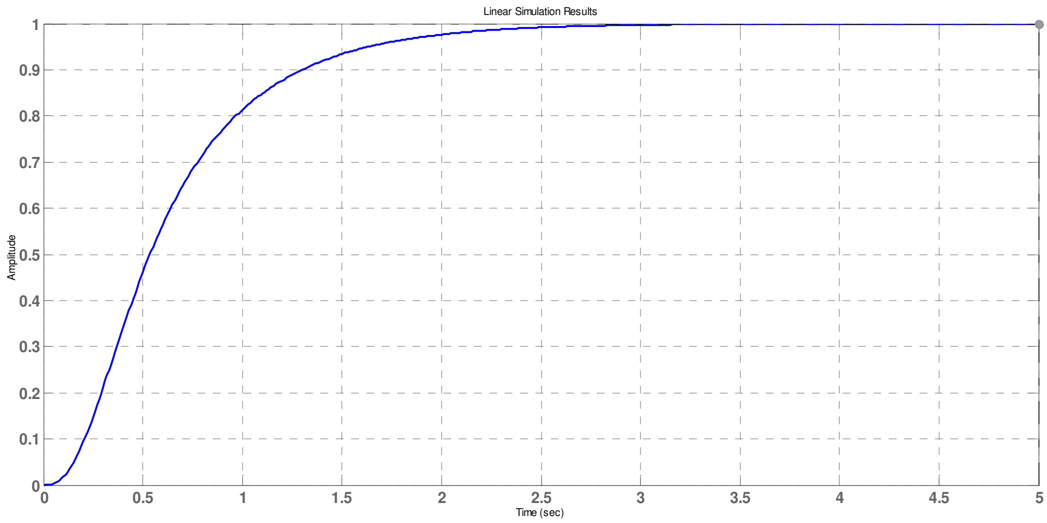

Fig. 4.7 Closed Loop response with presence of plant disturbances (With integral action)

Steady state value reaches perfectly to 1. So, integral action has successfully reduced steady state error even in presence of parameter disturbance in plant.

9. Lastly, in order to verify the effectiveness of the integral action, it was applied an impulse disturbance d=1.2 at time t=3 sec. and corresponding response was observed.

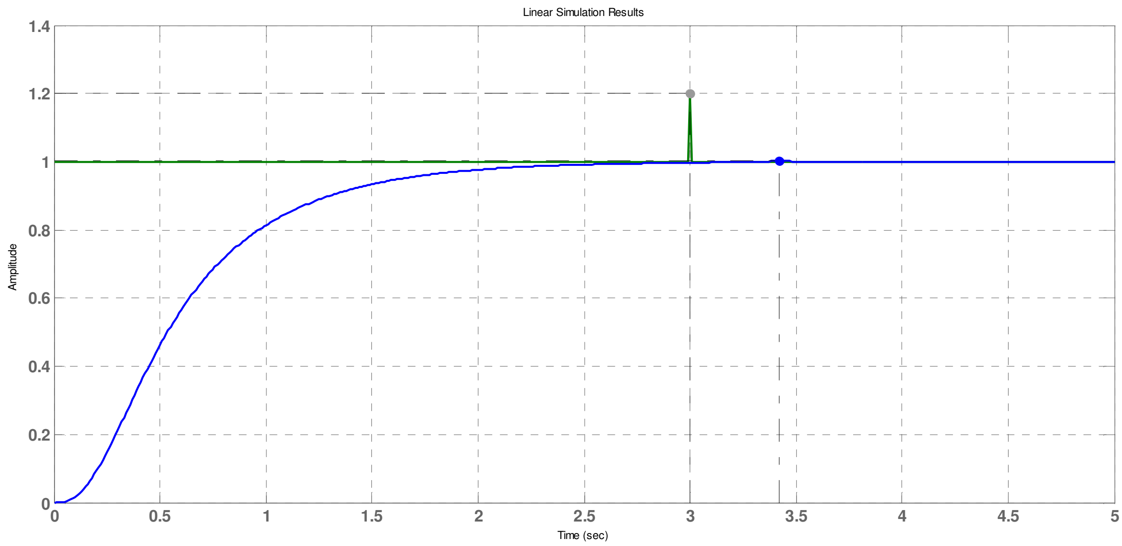

Fig. 4.8 Closed Loop response with presence of input disturbances (With integral action)

Steady state value reaches perfectly to 1. The input disturbance has negligible effect on the plant output steady state tracking.

Chapter 5

Pole placement within specified disk

5.1 Introduction

In order to design LQR for manipulator system with intentional integral action to state feedback as mentioned in chapter 4, a predefined relative stability and damping was considered. This type of problem is known as pole placement within a specified disk. [Furuta 1987] Pole placement is a well-known approach to design control systems. This pole placement approach of LQR design builds on [Furuta 1987] & [Misra 1996]. As followed from [Furuta 1987], a generalised eigen value problem was required to solve in order to place closed loop poles in desired locations and the relative stability was observed. The detailed solution method is described in later section of this chapter followed by [Pappas 1980].After solution it was ensured that in relation to optimal control and robustness all poles lied within the specified disk even after introduction of parameter perturbation in the plant.

5.2 Problem Statement and analysis

A continuous system defined by-

$\dot{x}(t) = Ax(t) + Bu(t)$ ……………………………………………………………….5.1
$Y(t) = Cx(t)$…………………………………………………………………………….5.2

was considered where u is an m-dimensional input vector, x is an m dimensional state vector, and y is p dimensional output vector, and A, B, C are constant matrices of appropriate dimensions. It was also assumed that pair (A, B) is controllable.

The problem is to determine state feedback $u = Kx$ such that all poles of the closed loop system are assigned in the disk D with centre at $-\alpha + j0$ and radius r.

Now we present conditions which must be satisfied in order to place closed loop poles within the desired disk. Proofs of these conditions have been given in [furuta].

<u>Theorem 1:</u>Consider the following equation :

$-\alpha(A + BK)^{*}P - \alpha P(A + BK) + (A + BK)^{*}P(A + BK) + (\alpha^{2} - r^{2})P = -Q$……………..5.3

Where Q is positive definite. Then the eigen values of $(A + BK)$ are within a specified disk D if and only if there exists a positive definite solution of P satisfying the equation 5.3.

In order to choose a method that satisfies the problem, following another theorem based on discrete Riccati equation was considered.

Theorem 2:The state feedback law given by $u = Kx = -(r^2R + B^TPB)^{-1}B^TP(A - \alpha I)x$ assigns all the closed loop poles of a continuous system described by 6.1 within the disk D where P is a positive definite symmetric solution of the discrete Riccati Equation

$$P = \left(\frac{A-\alpha I}{r}\right)^T P\left(\frac{A-\alpha I}{r}\right) + Q - \left(\frac{A-\alpha I}{r}\right)^T PB(r^2R + B^TPB)^{-1}B^TP\frac{(A-\alpha I)}{r} \text{......................}5.4$$

** **Note** that both theorems refer to the same problem of closed loop pole assignment since a predefined disk D in-case of continuous system implies to a unit disk D for discrete type system to attain the same criteria of stability. Also in discrete case, for a causal system, poles should lie within unit disk in order to satisfy the condition of stability i.e ROC must include unit circle.

Let $\lambda_i (i = 1,2,3,4, \dots n)$ be an eigen value of modified system $(A + BK)$ and corresponding eigen vector be v_i then following can be written-

$$\left(\lambda_i I - \left(\frac{A-\alpha I}{r}\right) + B(r^2R + B^TPB)^{-1}B^TP\frac{(A-\alpha I)}{r}\right)v_i = 0 \text{...}5.5$$

$$Pv_i = \left(\frac{A-\alpha I}{r}\right)^T P\lambda_i v_i + Qv_i \text{ ....}5.6$$

Defining u_i as $u_i = Pv_i$ we can arrange equations 6.5 and 6.6 as a generalized eigen value eigen vector problem such that-

$$\begin{bmatrix} \left(\frac{A-\alpha I}{r}\right) & 0 \\ -Q & I \end{bmatrix} \begin{bmatrix} v_i \\ u_i \end{bmatrix} = \lambda_i \begin{bmatrix} I & Br^{-2}R^{-1}B^T \\ 0 & \left(\frac{A-\alpha I}{r}\right) \end{bmatrix} \begin{bmatrix} v_i \\ u_i \end{bmatrix} \text{..}5.7$$

Equation 5.7 represents a well-known generalised eigen vector -eigen value problem of the form as shown below

$Mz = \lambda Lz$, where λ is an eigen value and z is corresponding eigen vector. Solving the above problem for eigen values to be less than 1, P was found as-

$P = [u_1, u_2, u_3, \dots, u_n][v_1, v_2, \dots, v_n]^{-1}$……………………………………………5.8

5.3 Solution of Riccati Equation by Generalized Eigenvalue Approach

5.3.1 Brief Introduction

As described in earlier section the problem of closed loop pole placement within a disk D for a continuous system is similar to the same in a discrete type system for pole placement within a unit disk considering stability and convergence criteria. In [Pappas 1980], two methods have been used to solve a discrete- time algebraic riccati equation, generalized eigen value-eigen vector approach and schur vector approach. In our case generalized eigen value eigen vector approach was considered to solve the equation mentioned in 5.4 and thereby pole placement was attained as desired by calculating control law u from the solution of P.

Before entering the solution approach, here we define the standard discrete type algebraic riccati equation which is to be solved by generalised eigen value approach as described by [Pappas 1980].

$X = A^T XA + Q - A^T XB(R + B^T XB)^{-1} B^T XA$…………………………………………5.9

Now this equation will be referred in next discussion many times as our ultimately objective to be solved by generalized eigen value eigen vector approach.

5.3.2 Solution of GEP

Consider the generalized eigen value problem (GEP), $Mz = \lambda Lz$…………………………5.10

The generalized eigen values of the equation 5.10 is comparable to our problem of equation 5.7.The generalized eigen values are the roots of generalized characteristic equation $\det(M - \lambda L) = 0$. For each eigen value λ , a non-zero vector satisfying 5.10 will be called a generalized eigen vector corresponding to λ. If a solution λ has a multiplicity of $r > 1$, then the set of vectors $\{z_1, z_2, z_3 \dots\dots z_l\}$ satisfying

$(M - \lambda L)z_k = Lz_{k-1}$, $k = 2,3, \dots l; l \leq r$ will be called a chain of generalised eigen vectors.

Now a few theorems have been mentioned here to represent the nature of eigen values of te above problem which will give an idea about the solution domain of λ. Proofs have been mentioned in reference [Pappas 1980].

Theorem 3: Let $A, B, U, V \in C^{n\times n}$ with U and V non-singluar. Then the eigen values of the problems $Az = \lambda Bz$ and $UAVz = \lambda UBVz$ are the same.

Theorem 4:None of the eigen values of the problem $Mz = \lambda Lz$ lies on the unit circle.

Theorem 5: Consider the generalised eigen value problem $Mz = \lambda Lz$. If $\lambda \neq 0$ is an eigen value, then $1/\lambda$ is also an eigen value with the same multiplicity.

Theorem 6: Consider the generalized eigen value problem $= \lambda Lz$. If $l = 0$ is an eigen value with multiplicity$r$, there is only $2n - r$ finite eigen values for this problem. We may say that the r missing eigen values are 'infinite' eigen values i.e reciprocals of '0'.

To summarize the concepts originated from theorems 4,5,6 we can arrange the eigen values of our problem in following way-

$$0,0,\ldots\ldots 0, \lambda_{r+1}, \ldots\ldots.., \lambda_n, \frac{1}{\lambda_n}, \frac{1}{\lambda_{n-1}}, \ldots.,.\frac{1}{\lambda_{r+1}}, \infty, \ldots..\infty, \infty$$

$$r \qquad\qquad n-r \qquad\qquad n-r \qquad\qquad r$$

With $0 < |\lambda_i| < 1, where\ i = r + 1, \ldots..n.$

Theorem 7: For the problem $Mz = \lambda Lz$, let U be the $2n \times n$ matrix of the generalized eigen vectors and generalized principal vectors corresponding to the n stable eigen values. Then

$MU = LUS$, where S is the $n \times n$ Jordan canonical form, corresponding to all $\lambda_i, 0 < |\lambda_i| < 1$.

Consider that, $0 < |\lambda_i| < 1$ only refers to our stabilizing solution of riccati equation described in 6.9. Correspondingly the matrix U, a basis for stable eigen space can be partitioned into two $n \times n$ submatrices

$$U = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

The above observation for any set of n generalized eigen values and generalized principal eigen vectors and a corresponding matrix S in Jordan canonical form is used next proof of lemmas that follows to the solution of riccati equation 5.4

Lemma 1: All solutions of the riccati equation described by 5.9 are in the form $X = PQ^{-1}$, where $\begin{bmatrix} P \\ Q \end{bmatrix}$ is a set of generalised eigen vectors and generalised principal vectors of problem $Mz = \lambda Lz$.

Lemma 2:Consider any set of n generalized eigen vectors and generalised principal eigenvectors $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$, and a corresponding matrix S in Jordan Canonical form. Assume V_1 is invertible. Then-

(a) $X = V_2 V_1^{-1}$solves 5.9 assuming $(R + B^T XB)$ is invertible.

(b) $X = X^T \geq 0$if and only if S is stable i.e (A, C) is completely reconstructible where $Q = C^T C$.

Further the condition of positive definite solution of X, i.e $X = X^T \geq 0$, that V_1 must be invertible and (A, C) should be completely constructible for a stable S has been proved in [Anderson 1979].

5.4 D-Stability Margin Analysis

The D-pole assignment problem mentioned in this chapter in previous section has some limitations in presence of plant parameter perturbations. But uncertainty of the plant model can be well treated in terms of its limitations known from this section. if a control law $u = \overline{K}Kx$ is supplied to the plant where $\overline{K}$ is modelled form of plant parameter perturbation then the limitation of $\overline{K}$, both in terms of gain margin and phase margin is defined by following.

If riccati equation 5.9 has a positive definite solution with quadratic cost-efficient matrix $R = diag(r_1, r_2, \dots, r_m)$ then as shown in [], $\overline{K} = \{\overline{k_1}, \overline{k_2}, \overline{k_3}, \dots \overline{k_m}\}$ has limitations given by

$\frac{1}{a_i+1} \leq \overline{k_i} \leq \frac{1}{1-a_i}$,and $|\emptyset_i| \leq 2sin^{-1}(a_i/2)$, where $i = 1,2, \dots m$

And $a_i = (\frac{r^2 r_i}{r^2 r_i + \lambda_m})$ where λ_m is the maximum eigen value of $B^T PB$.

***Note-** As radius of desired disk $r \to 0, a_i \to 0$, the gain and phase margin disappear. The closed loop poles try to locate as nearer to each other as they can. This affects in the robustness of the system. So, it is necessary to have r as large as possible.

5.5 Some Case Studies

Case 1: To check the application of GEP in D-pole assignment, we took an example from [] of the following system-

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \boldsymbol{u}$$

We have to apply state feedback to place the closed loop poles within a disk of radius r =6 and Centre at $c = -\alpha + j0$ such that $\alpha = 2$.

Solution: Consider R =0.1 and $Q = I_3$

Obtained $K = \begin{bmatrix} 0.7124 & -7.3180 & 0.4568 \\ -29.3239 & 29.3850 & -10.8350 \end{bmatrix}$

And the resulted closed loop poles were $\{-6, -5.22 \pm 0.37j\} \in D$

Case 2: Considering to achieve desired stability margin and degree of damping as per specified we took this system and designed state feedback to place closed loop poles within disk D

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \mathbf{0} & \mathbf{2} & \mathbf{1} \\ \mathbf{2} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \boldsymbol{u}$$

We have to design LQR for the above system where closed loop system have an absolute stability of $s = -2$ and damping ratio will be $\varepsilon = 0.707$.

Solution: Consider R=diag [10, 10] and $Q = I_3$

From the calculation of disk centre and disk radius mentioned in 6.4 we obtained disk centre at $c = -6.4824 + j0$ and radius of 4.8284.

We obtained $K = \begin{bmatrix} -2.6933 & -6.0559 & 1.6811 \\ 2.8002 & -1.5169 & -8.5272 \end{bmatrix}$

And resulted closed loop poles were placed at $\{-6.8284, -3.3239 \pm 0.5243j\} \in D$

In the system defined in case 2 we have D-stability margin in terms of gain margin and phase margin as $0.5137 \leq k_i \leq 18.69$ and $|\emptyset_i| \leq 56.49^{\circ}$

So, if the plant parameter perturbation is beyond the gain margin as specified above, closed loop poles may not be assigned within the disk any more. This is the limitation of D-pole assignment.

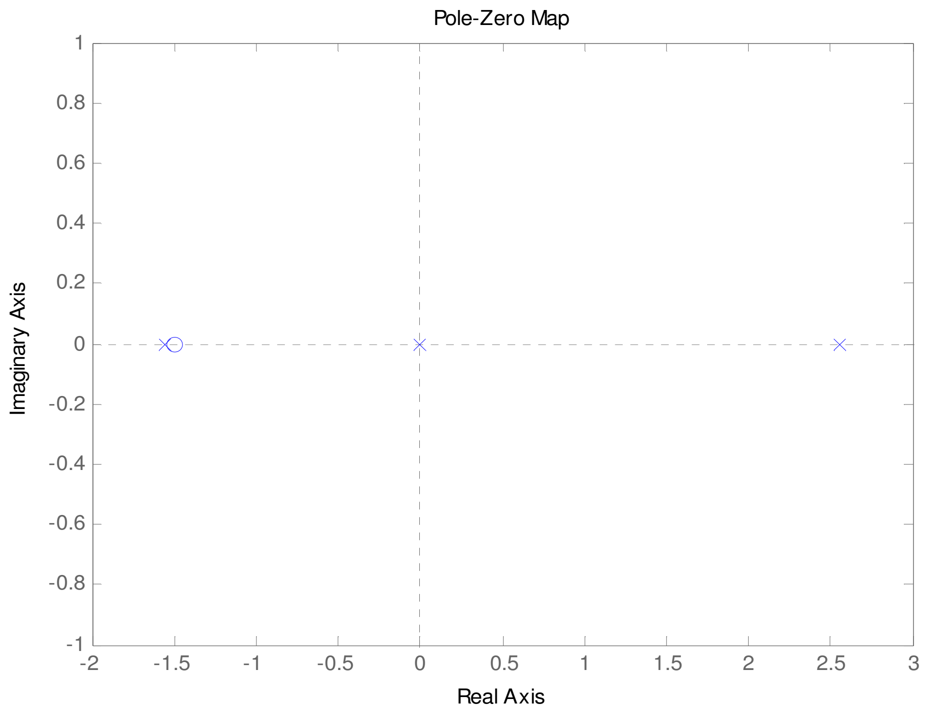

Fig5.1: Open Loop Poles in case study 2

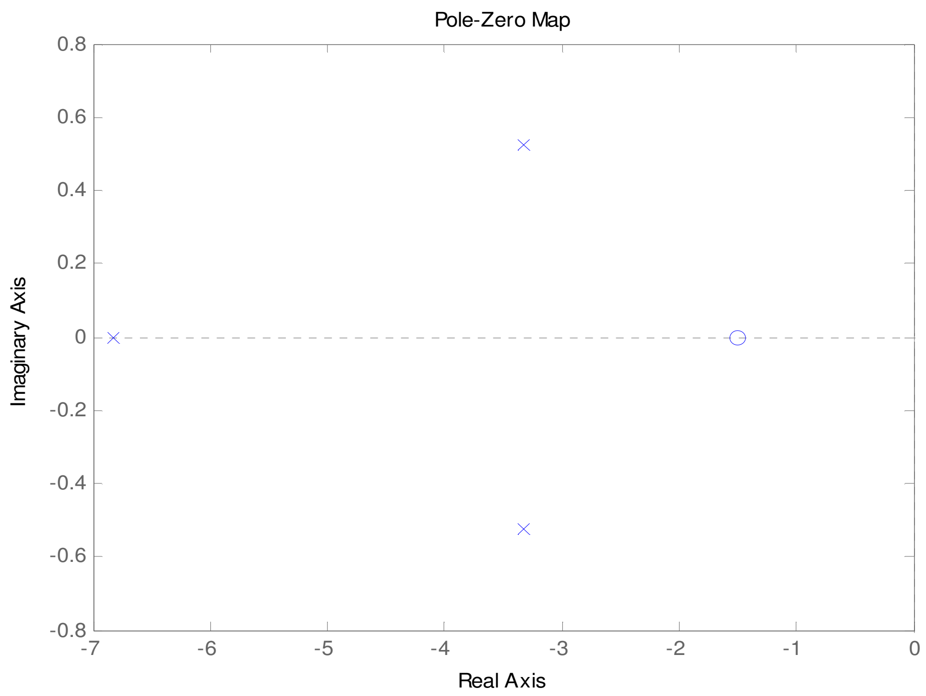

Fig5.2: Closed Loop Poles in case study 2

5.6 Conclusions and discussions

- In case study 1, we obtained pole placement for any specified disk given by its radius and its centre.

- In case study 2, some interesting system parameters are linked to disk specification. The required degree of damping and stability has been considered to specify the dimension of disk. In this case, desired stability parameters are directly introduced in Riccati Equation.

- While solving Riccati Equation, a generalized eigenvalue-eigenvector problem is considered. Since stable eigen values are less than 1, sorting them and thereby designing closed loop poles helps us to attain desired degree of stability.

Chapter 6
Particle Swarm Optimization

6.1 Introduction

In order to design state feedback controller gains for linear systems subject to a quadratic performance index, the controller gains were generated from judicious selection of a pair of symmetric matrices Q and R as mentioned previously. Conventionally, controller gains were obtained by pole placement approach or LQR design via Riccati equation solving. Particularly choosing Q and R before solving Riccatieqation was done by trial and error approach. So, it is proposed to proceed LQR design with some intelligent parameter selection method such as PSO. In this chapter the step by step approach of LQR design has been elaborately discussed [Gaing 2004]

6.2 Particle Swarm Optimization Overview

- Evolutionary computational technique based on the movement and intelligence of swarms looking for the most fertile feeding location.
- It was developed in 1995 by James Kennedy and Russell Eberhart, motivated by social behaviour of bird flocking and fish schooling[Kennedy 1995]
- Simple algorithm, easy to implement and few parameters to adjust mainly the velocity.
- A "swarm" is an apparently disorganized collection (population) of moving individuals that tend to cluster together while each individual seems to be moving in a random direction.
- It uses a number of agents (particles) that constitute a swarm moving around in the search space looking for the best solution.
- Each particle is treated as a point in a D-dimensional space which adjusts its "flying" according to its own flying experience as well as the flying experience of other particles.
- Each particle keeps track of its coordinates in the problem space which are associated with the best solution (fitness) that has achieved so far. This value is called *pbest.*[Gaing 2004]
- Another best value that is tracked by the PSO is the best value obtained so far by any particle in the neighbors of the particle. This value is called *gbest*. [Gaing 2004]
- The PSO concept consists of changing the velocity (or accelerating) of each particle toward its *pbest* and the *gbest* position at each time step.

6.3 Particle Swarm Optimization Algorithm

For each particle
Initialize particle with feasible random number
End
Do

For each particle
Calculate the fitness value
If the fitness value is better than the best fitness value (*pbest*) in history
Set current value as the new *pbest*
End

Choose the particle with the best fitness value of all the particles as the *gbest*

For each particle
Calculate particle velocity according to velocity update equation

Update particle position according to position update equation
End

While maximum iterations or minimum error criteria (if mentioned) is not attained.

6.4 Solution steps in PSO

- **6.4.1**

Problem Perspective-

Here we have to stabilize a double arm flexible manipulator system which is inherently stable by placing closed loop poles within a region specified by two disks, refer chapter 5. Therefore, we have to optimize the closed pole loop distances from the desired region of stability margin. Those disks were chosen by specified damping and stability margin as specified by designer. Since the system has one zero located at far to the right of jω axis, we have to compensate the effect of that open loop zero by properly placing closed loop dominant and non-dominant poles at strategic locations in s-plane. Also while running iterations in PSO, the distance from the pole to the desired disk region is minimized such that closed loop poles are located within the disk region as specified in chapter 5.

- **6.4.2 PSO approach to find optimal gain values**

The searching process of PSO-PID parameter is considered as follows-

Step 1)

Specify the lower and upper bounds of the 5 controller gain parameters $[k1, k2, k3, k4\ and\ ki\ ]$ and initialize randomly the individuals of the population including searching points, velocities, *pbest*s, and *gbest*s.

Step 2)

For each initial individual of the population, find the closed loop system and thereby obtain the closed loop pole locations.

Step 3)

Calculate the evaluation value of each individual closed loop pole location for every population. Among them select the particle with smallest distance criteria as the positional best for that particle.The intelligent particle selection logic is based as follows [Solihin 2009]-

If a dominant λcl does not lie within disk d1

Performance criterion J is big (ignored);

Else if a non-dominant λcl does not lie within region disk d2-d1

Performance criterion J is big (ignored);

Else

Performance criterion for that λcl is evaluated as- $J = ((x - c1)^2 + y^2) + ((x - c2)^2 + y^2)$;

Where c1 and c2 respectively denotes centre of disk 1 and disk 2 and (x, y)denotes that closed loop pole location in s-plane.

Step 4)

Compare each individual's evaluation value$J$ with its $pbest$. The best evaluation value among the $pbests$is denoted as$gbest$.

Step 5)

Modify the member velocity of each individual K according to the following equation-

$$v_{j,g}^{t+1} = w \times v_j^t + c1 \times rand() \times (pbest_{j,g} - k_{j,g}^t) + c2 \times rand() \times (gbest_g - k_{j,g}^t)$$

j= 1, 2, 3…. to N(Population size)

g=1, 2, 3…….to D (No. of parameters in each generation, here D=5 since we are evaluating K_1, K_2, K_3,K_4,and K_i these 5 parameters)

t= Iteration number

rand () refers to a function which returns a random number between 0 and 1.

w=inertia weight and $k_{j,g}^{t}$ represents value of $k_g$parameter of j^{th} generation of t^{th} iteration.

The constants c1 and c2 represent the weighting of the stochastic acceleration terms that pull each particle toward and positions.Lowvaluesallowparticlestoroamfarfrom the target regions before being tugged back. On the other hand, high values result in abrupt movement toward, or past, target regions. Hence, the acceleration constants c1 and c2 were often set to be 1.05 according to past experiences.

Step 6)

Here we specify bounds on each particle velocity

If $v_g^{t+1} > v_g^{max}$, then $v_g^{t+1} = v_g^{max}$else if $v_g^{t+1} < v_g^{max}$, then .$v_g^{t+1} = v_g^{min}$.

Step 7)

Modify member position of each individual from step 5

$k_{j,g}^{t+1} = k_{j,g}^{t} + v_g^{t+1}$ such that $k_{j,g}^{min} < k_{j,g}^{t+1}$<$v_{j,g}^{max}$, bounds on each particle position.

Step 8)

If the number of iterations reaches the maximum, then go to Step 9. Otherwise, go to Step 2.

Step 9)

The individual that generates the latest*gbest* is an optimal controller parameter. The corresponding evaluation value of distance is the optimal distance from the closed loop pole location at region (d2-d1) and the disk centres.

- **6.4.3 Selection of PSO parameters**

To start up with PSO, certain parameters need to be defined. Selection of these parameters decides to a great extent the ability of global minimization. The maximum velocity affects the ability of escaping from local optimization and refining global optimization. The size of swarm balances the requirement of global optimization and computational cost. Initializing the values of the parameters is as per table.6.1.

Table 6.1: Selection of PSO parameters

Population Size (N)	10
No. of Iteration (t)	50
Velocity constant c1	1.05
Velocity constant c2	1.05
Initial Velocity	25% of parameter size

The algorithm described above is described in a Robotic Manipulator and results of direct feedback gain design by pole placement and that by PSO based LQR are discussed and analysed in next chapter.

Chapter 7
Application in Robot Manipulator

7.1 Introduction

In previous chapter, we discussed the algorithm of Particle Swarm of optimization. Its application to control system design is shown here. A simple two degree robot manipulator has been considered as model plant [Solihin 2009] Then both Pole-placement based feedback controller design and LQR based feedback controller design has been performed. Thereby, a comparative study has been shown in this chapter.

7.2 Rotary Flexible Link Manipulator

The setup of a rotary flexible link manipulator can be described as below. It consists of a strain gauge which is mounted at the clamped end of a thin stainless steel flexible link. The output is an analog signal proportional to deflection of the link [Solihin 2009] A DC motor helps the flexible link to rotate from one end in a horizontal plane. The motor end is instrumented with a strain gauge to detect the deflection of the tip of the flexible link. The rotary flexible link is an ideal experiment intended to represent a similar system used in an aircraft, or a robot. Figure 7.1 shows a sample model which can be used in experiment.

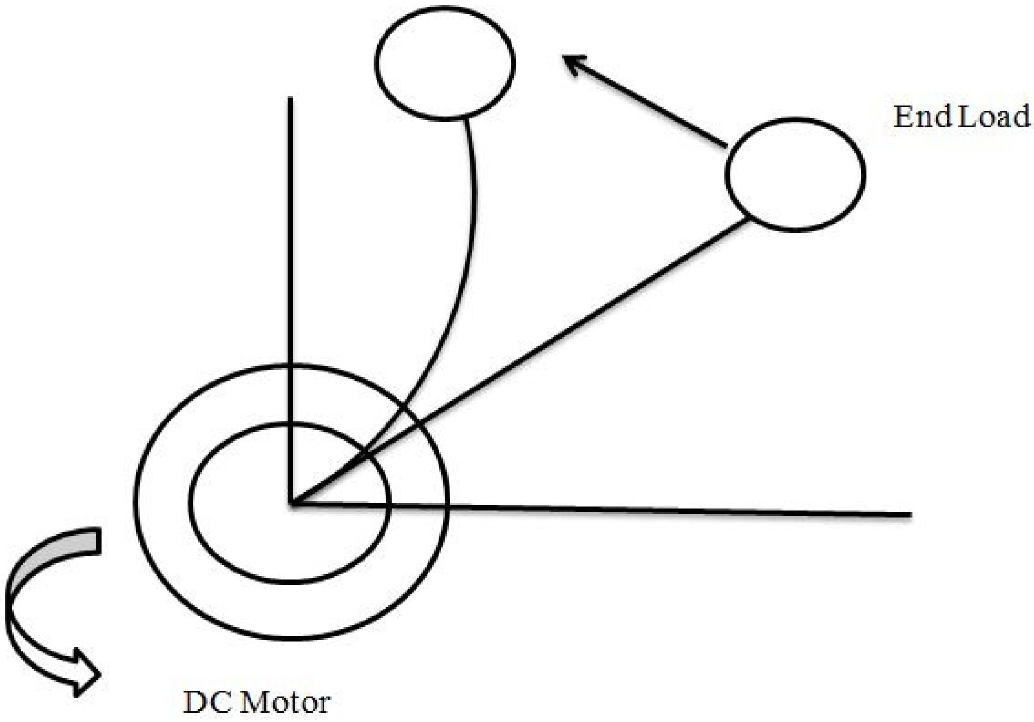

Figure 7.1 : Flexible link manipulator

7.3 Linearized Model of Plant

A linearized dynamic model of the system is developed using Euler-Lagrange formulation. Detail derivation of model is not discussed here since our main objective is controller design for the plant. The model is taken from Quanser Laboratory Manual of rotary flexible link experiment [Quanser Manuals].

The schematic diagram is depicted blow in 7.2. The input of the system is voltage input V to the end-effector (DC Motor). The outputs are motor angular position (θ) and arm deflection angle(α).

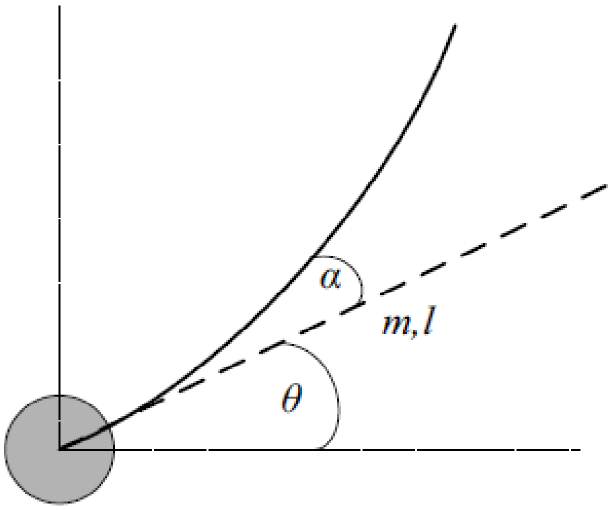

Figure 7.2 Schematic diagram of flexible link (Source: [Solihin 2009])

The system parameters taken into consideration are taken from TABLE 1 as given in [Solihin 2009] Following the parametric calculations the state space space representation of the system including the actuator dynamics is given by-

$$\begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \\ \ddot{\theta} \\ \ddot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 520.07 & 1.49 & 0 \\ 0 & -875.91 & -1.4961 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1.97 \\ -1.97 \end{bmatrix} V \quad \ldots\ldots 7.1$$

7.4 Results of PSO Design in direct state -feedback

It was seen that the error value tends to decrease for a larger number of iterations. As such the algorithm was restricted to 100 iterations for beyond which there was only a negligible improvement. Based on PSO for the application of the LQR, following values were obtained as follows from the table.

Table 7.1 Resulting state feedback gains using PSO

	K1	***K2***	***K3***	***K4***	*Ki*
PSO State-feedback controller	-17.6600	-462.8100	26.2000	-6.9900	-116.8000
LQR based state- feedback controller	18.3133	-104.3913	20.2485	8.2473	-100.0000

7.4.1 Time Response-comparative analysis

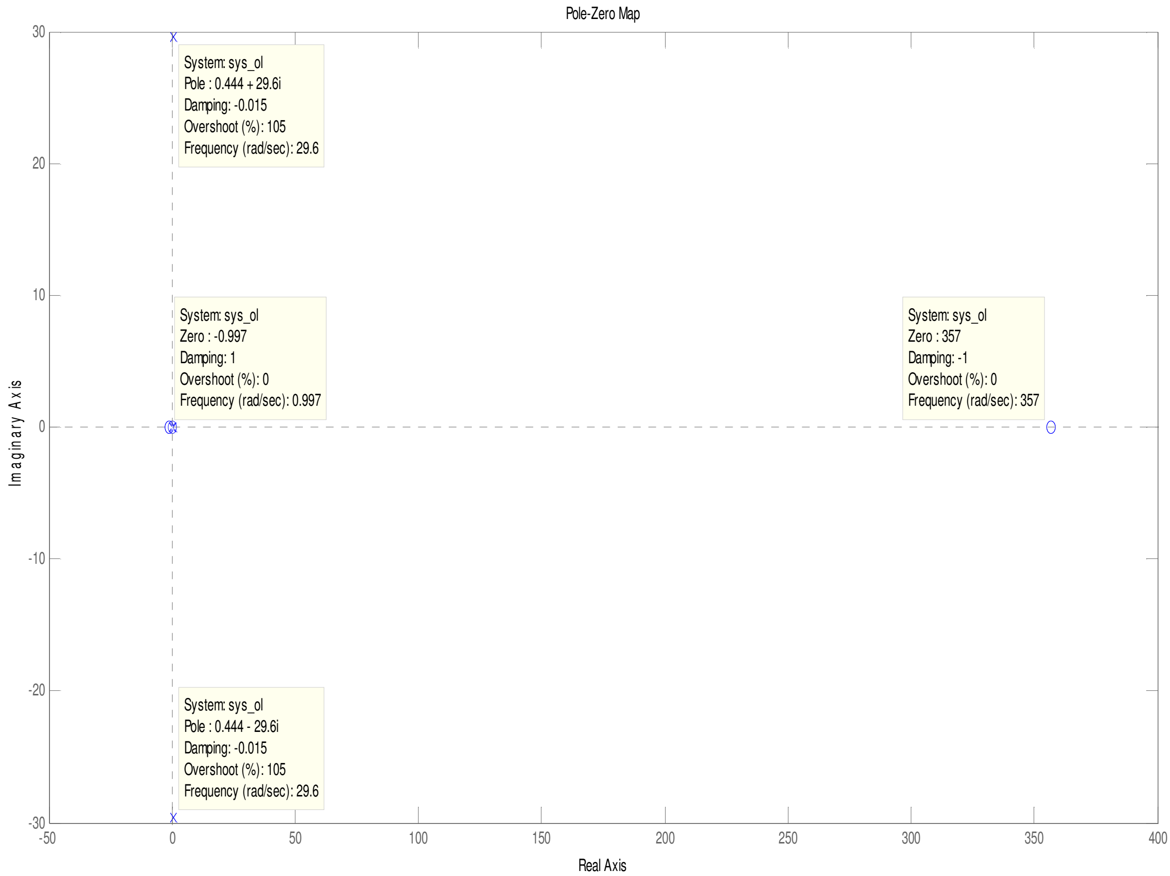

Figure 7.3 Open Loop Pole-Zero Plot

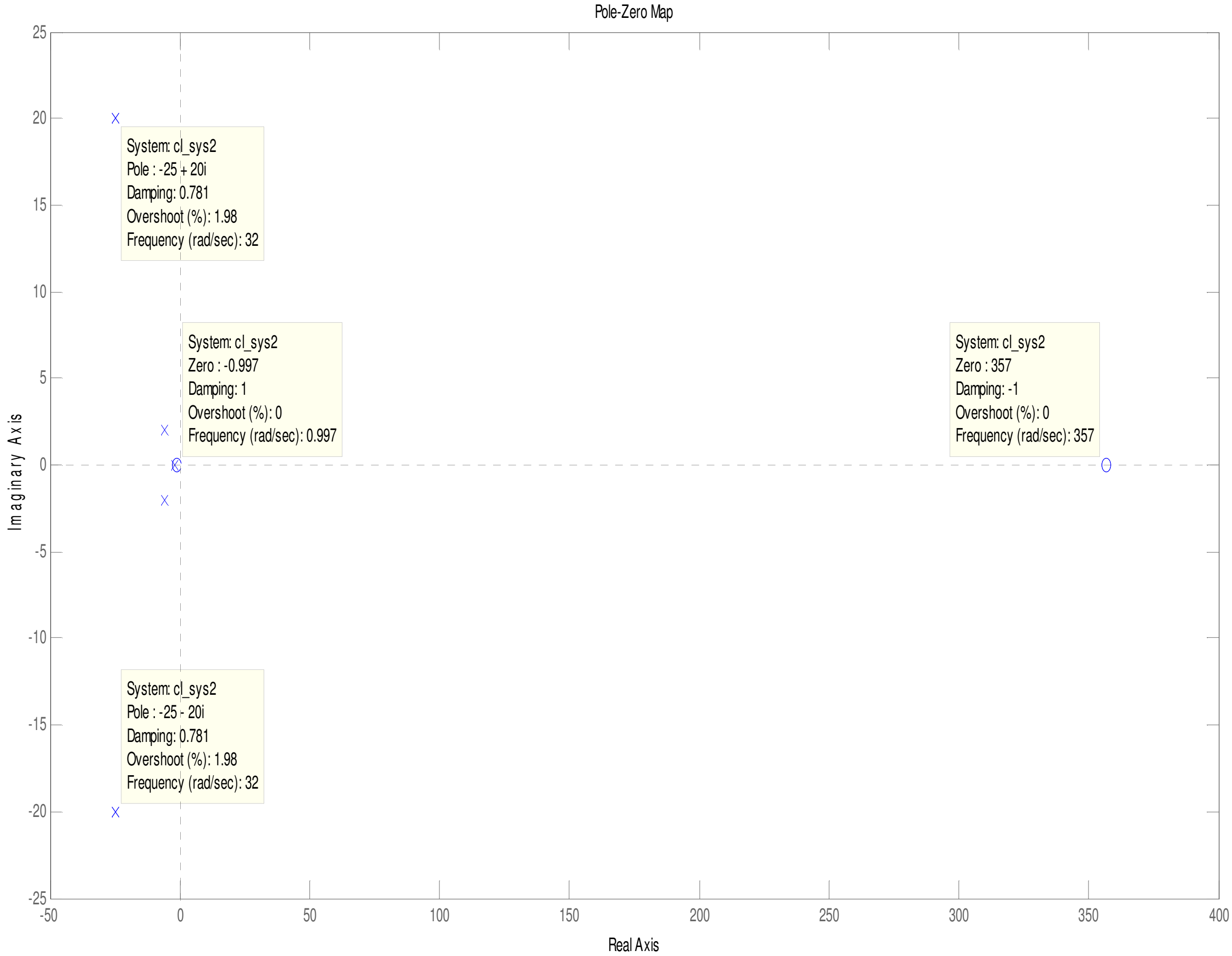

Figure 7.4 Closed Loop Pole-Zero Plot

Referring to chapter 5, to design closed loop system subject to given stability margins, closed loop were to be placed inside a disk region as specified in [Solihin 2009] The figure 7.4 shows how closed loop poles are placed satisfying disk conditions. In later discussions we will find that in LQR design also, we will find closed loop poles placed inside the disk region by PSO algorithm instead of direct pole-placement.

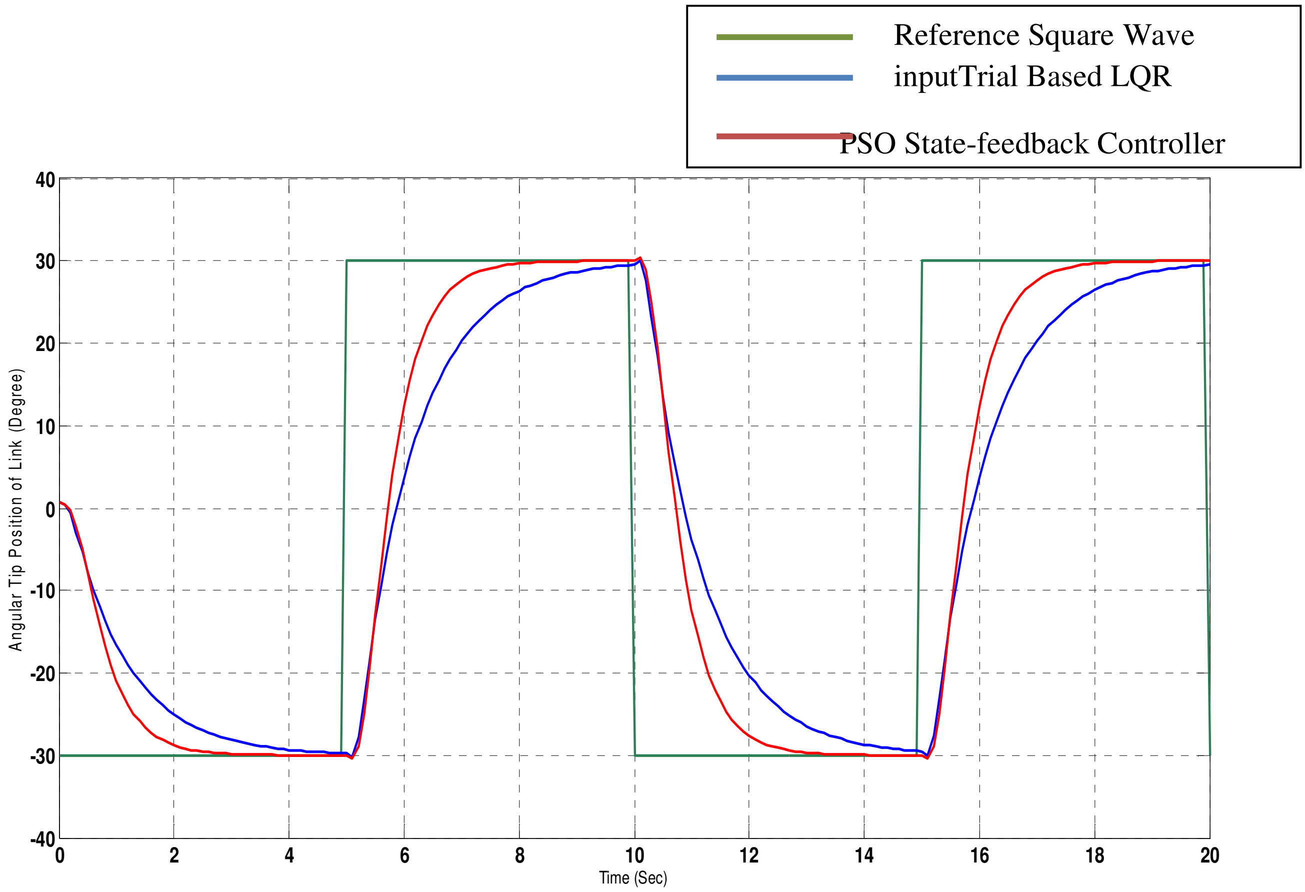

Figure 7.3 Angular Tip Position of Manipulator

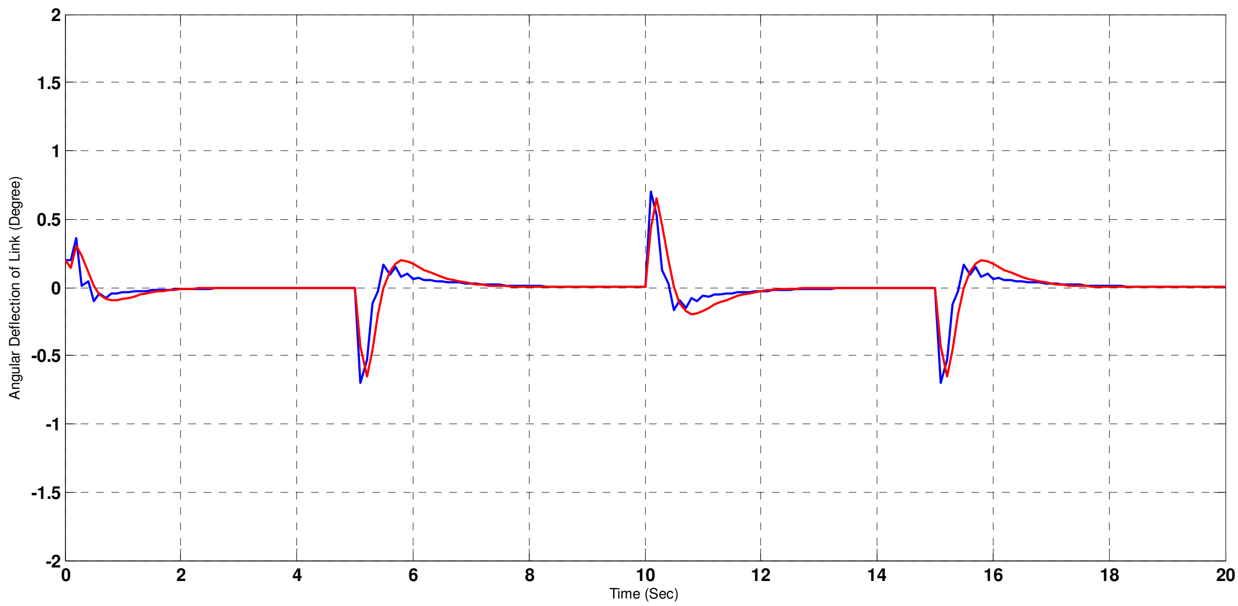

Figure 7.4 Deflection angle of Robot Manipulator

7.5 Results of PSO design in LQR

Q=[	144	0	0	0;
	0	144	0	0;
	0	0	34	0;
	0	0	0	10377]

R= 1.2

Closed loop poles are located inside disk region as following figure-

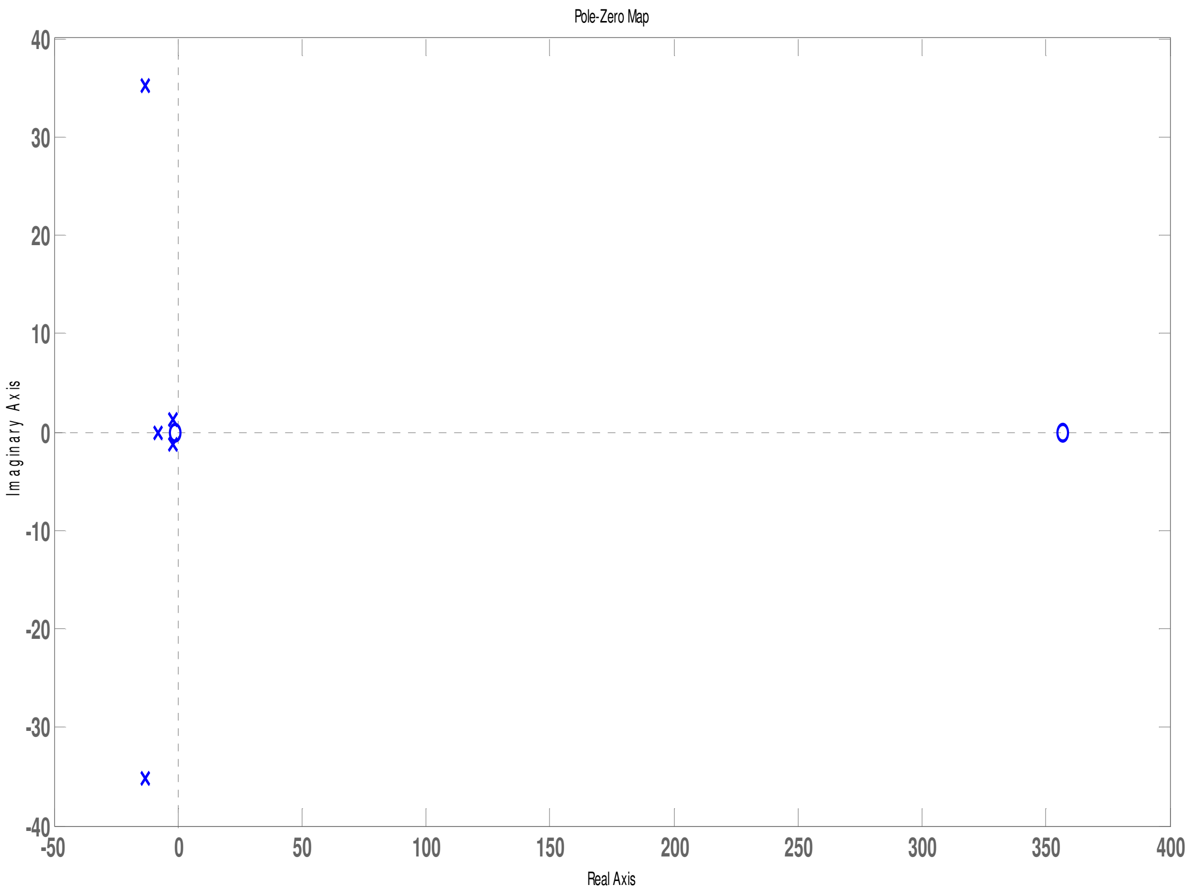

Fig 7.5: Closed loop poles and zeros in PSO based LQR

7.5.1 Time Response simulation in LQR

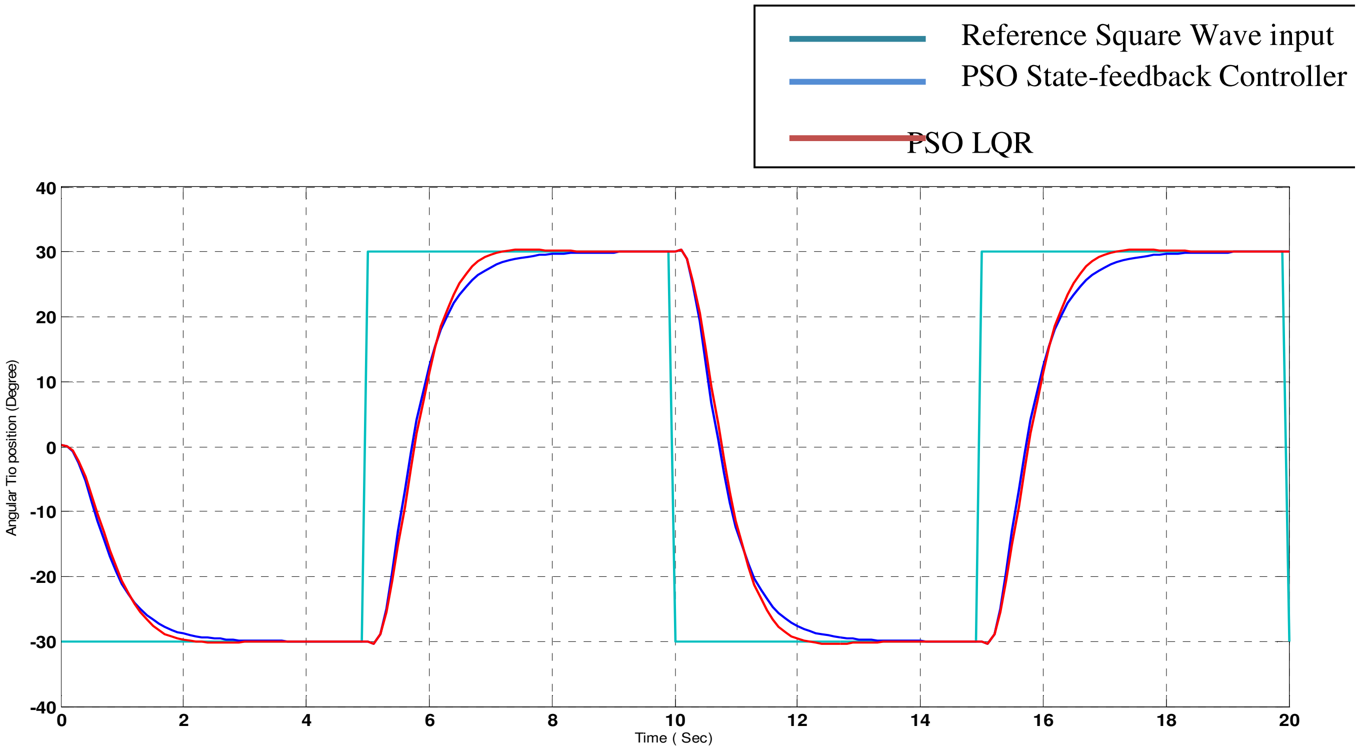

Figure 7.6 Angular Tip Position of Manipulator

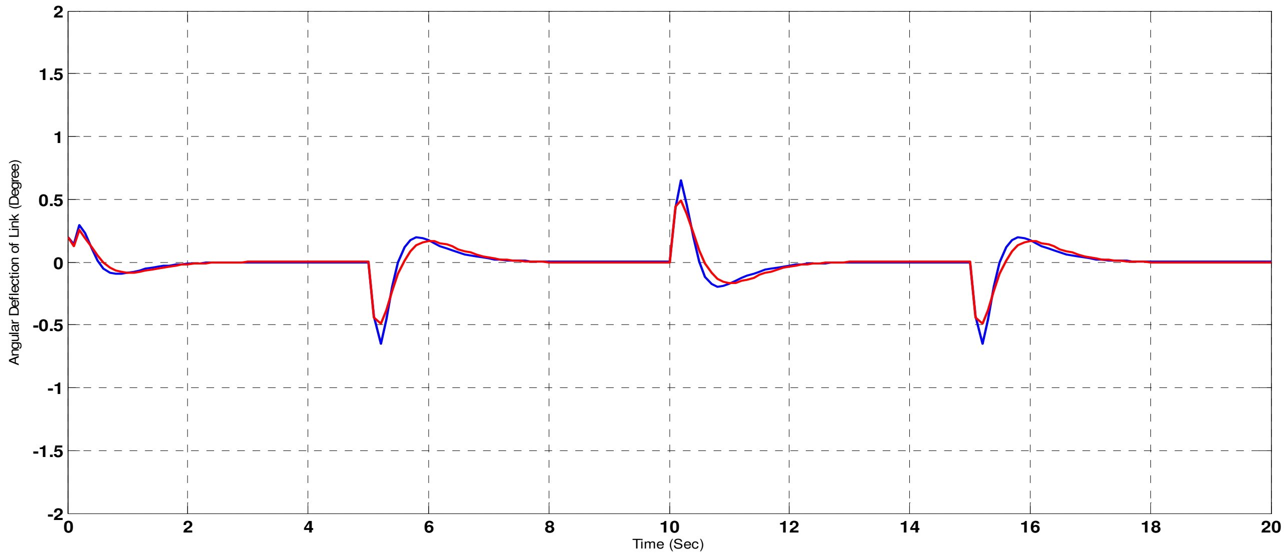

Figure 7.7 Deflection angle of Robot Manipulator

7.6 Conclusions and discussions

1. State feedback results obtained by PSO shows better performance in reference tracking as it can be seen from figure 7.3

2. The deflection angle of robot manipulator should be ideally zero even in presence of variation in inputs. So, a square wave response has been shown in figure 7.4

3. While trial based LQR shows harmonics and transients in deflection angle response, PSO attained smoother response and shows less deflection from attaining steady state zero.

4. Finally Figure 7.6 and 7.7 shows comparison between PSO based LQR and PSO based state feedback controller responses. PSO based LQR is superior in terms of minimizing noise and attaining better tracking of square wave reference.

.

Chapter 8
Discussions and Conclusion

8.1 Discussions

The objective of this thesis is to study different closed loop control system design techniques in view of tracking performance. Linear optimal control is one approach which often gives the designer satisfactory results with respect to the stability and the performance of the controlled systems. One advantage of it is that the mathematical optimization methods are adopted so that a control law for a linear system can be readily derived basedon a prescribed objective function. The resulting computational procedures may then often be applied to nonlinear systems. Moreover, if the plant states are all available, good robustness properties of the optimal regulators used can be clearly revealed in terms of the stability margins. Another famousexample is full state feedback controller design in pole-placement. As the state space representation is a natural way for system description by LQR the system performance can be managed and the plant inputs and the control input can be synthesized usingan optimal control law by solving the Riccati equation. Meanwhile, the guaranteed stability robustness is automatically provided by LQR, unlike pole placement techniques.

LQR design involves calculation of an optimal performance index referred to physical properties of the system. As for instance, we shall want to stabilize a closed loop plant using minimum control input as well as minimum state transition cost. In order to attain steady state, both control input cost and state transition cost should be minimized to get optimal state feedback controller. This is taken care by LQR. But LQR method suffers from time delay in response, poor robustness properties in presence of parametric uncertainties in plant. Also Q, R matrices are required to specify in LQR. Therefore the designer has no direct control upon state feedback gains and thereby closed loop eigenvalues.

Considering above details, an intelligent selection of Q, R matrices is necessary in order to achieve desired tracking performance. In this thesis, Particle Swarm Optimization algorithm has been used to solve algebraic Riccati equation with a view to optimize a performance index based upon closed loop pole locations.

8.2 Conclusions

Following the various results we can conclude that-

- PSO based state feedback controller can attain tracking performance better compared to trial based LQR. In terms of regulating deflection arm angle of a robot manipulator, LQR designed by conventional method shows transients in its response. This problem is eliminated by smoother response in PSO based state feedback controller.

- Further results of PSO design in MATLAB have shown that PSO based LQR shows better tracking performance compared to PSO state feedback controller. It refers to the fact that LQR method of closed loop control design involves minimizing a performance criteria defined by physical requirements of plant like reference tracking, disturbance rejection etc.

- Results in chapter 4 show that, while considering state feedback design, integral action in error signal gives better tracking performance in presence of plant parameter perturbations and input-output disturbances.

8.3 Future Scope of work

- Robustness analysis of LQR.

- Optimality check of LQR. Whether PSO based LQR is optimal or not?

- Investigation of noise reduction, disturbance rejection and parametric uncertainties in plant performance with designed LQR.

Bibliography

[Fallside 1977] F. Fallside, Ed., *Control System Design by Pole-Zero Assignment*. New York: Academic, 1977.

[Bogachev 1979] A. V. Bogachev, V. V. Grigorev, V. N. Drozdov, and A. N Korovyakov, "Analytical design of controllers from root indicators,"*Automate., Remote Contr.,* vol. 40 no. 8, pp 21-28, 1979.

[Kuo 1980] B.C Kuo, *Digital Control Systems*. New York: Holt, Rinehart, and Winston, 1980.

[Furuta 1987] K.Furuta and S. B. Kim, "Pole assignment in a specified disk," *Automatic Control, IEEE Transactions on* 32, no. 5,423-427,1987.

[Misra 1996] Misra,P. (1996). LQR design with prescribed damping and degree of stability. Proc. of IEEE International Symposium on Computer-Aided Control System Design, pp. 68-70.

[Ramli 2007] M. S.Ramli, M. F. Rahmat and M. S. Najib. "Design and Modeling of Integral Control State-feedback Controller for Implementation on Servomotor Control," In *6th WSEAS International Conference on Circuits, Systems, Electronics, Control & Signal Processing, Cairo, Egypt*. 2007.

[Anderson 1979] B. D. O. Anderson and J. B. Moore, *Optimal filtering*. Englewood Cliffs, NJ: Patience-Hall, 1979.

[Mori 1980] Y. Mori and Y. Shimemura, "On a design method for feedback control law to locate the eigenvalues in a specified region," SICE (in Japanese), vol. 16, no. 3, Short paper, pp. 462-463, 1980.

[Pappas 1980] T. Pappas, T. A.J. Laub, and N. R. Sandell, "On the numerical solution of the discrete-time algebraic Riccati equation," *IEEE Trans. Automat. Contr.* Vol. AC-25, no. 4 ,pp 631-641, 1980.

[Vaughn 1970] D. R. Vaughan, "A nonrecursive algebraic solution for the discrete Riccati Equation," *IEEE Trans. Automat.Contr,* vol. AC-15, no. 5, pp 597-599, 1970.

[Gould 1969] L.A.Gould, *Chemical Process Control: Theory and Applications,* Reading, MA-Addison-Wesley, 1969.

[Bialkowski 1978] W. L. Bialkowski, "Application of steady state Kalman filters-Theory with field results," In *Joint Automatic Control Conference*, no. 15, pp. 361-374, 1978.

[Moler 1973] C. B. Moler and G. W. Stewart, "An algorithm for generalized matrix eigenvalue problems," *SIAM Journal on Numerical Analysis* vol.10, pp. 241-256, 1973.

[Laub 1979] A. J. Laub, "A Schur method for solving algebraic Riccati Equations," *IEEE TransactionsAutomate.Contr,* vol. 24, no. 6, pp. 913-921, 1979

[Dooren 1981] P. Van Dooren, "A generalized eigenvalue approach for solving Riccati equations." *SIAM Journal on Scientific and Statistical Computing* 2, vol. no. 2, pp. 121-135,1981.

[Kennedy 1995] R. C. Eberhart and J. Kennedy. "A new optimizer using particle swarm theory." In *Proceedings of the sixth international symposium on micro machine and human science*, vol. 1, pp. 39-43. 1995.

[Kennedy 1997] J. Kennedy, "The particle swarm: social adaptation of knowledge." In *IEEE International Conference on Evolutionary Computation*, pp. 303-308, 1997.

[Shi 1998] Y. Shi and Russell Eberhart, "A modified particle swarm optimizer," In *IEEE World Congress on Computational IntelligenceEvolutionary*

Computation Proceedings,The 1998 IEEE International Conference on, pp. 69-73. IEEE, 1998.

[Peer 2003] E. S. Peer, F .V. D Bergh and A. P. Engelbrecht, "Using neighbourhoods with the guaranteed convergence PSO," In *Swarm Intelligence Symposium, 2003.SIS'03. Proceedings of the 2003 IEEE*, pp. 235-242. IEEE, 2003.

[Gao 2004] L. Gao, H.Gao and C. Zhou, "Particle swarm optimization based algorithm for machining parameter optimization," In *Fifth World Congress onIntelligent Control and Automation*, vol. 4, pp. 2867-2871. IEEE, 2004.

[Gaing 2004] Z. L. Gaing. "A particle swarm optimization approach for optimum design of PID controller in AVR system." *IEEE Transactions onEnergy Conversion,* vol. 19, no. 2, pp. 384-391, 2004.

[Naidu 2002] D. S. Naidu, *Optimal control systems*. CRC press, 2002.

[Freudenberg 1985] J. S. Freudenberg and D. P. Looze, "Right half plane poles and zeros and design trade-offs in feedback systems," *IEEE Transactions on Automatic Control* 30, no. 6 (1985): 555-565.

[Kalman 1964] R. E. Kalman, "When is a linear control system optimal?" *Journal of Basic Engineering* 86, no. 1 (1964): 51-60.

[Soroka 1984] E. Soroka and U. Shaked, "On the robustness of LQ regulators." *IEEE transactions on automatic control* 29, no. 7 (1984): 664-665.

[Subudhi 2002] B. Subudhi and A. S. Morris, "Dynamic modelling, simulation and control of a manipulator with flexible links and joints," *Robotics and Autonomous Systems* 41, vol. no. 4, pp. 257-270, 2002.

[Guirrez 1998] L. B. Gutierrez, F. L. Lewis, and J. Andy Lowe. "Implementation of a neural network tracking controller for a single flexible link: comparison with PD and PID controllers." *Industrial Electronics, IEEE Transactions on* 45, no. 2 (1998): 307-318.

[Quanser Manuals] Quanser Manuals: Rotary Flexible Link, http://www.quanser.com

[Solihin 2009] M. I. Solihinand RiniAkmeliawati, “PSO-based optimization of state feedback tracking controller for a flexible link manipulator,” In *International Conference ofSoft Computing and Pattern Recognition*, pp. 72-76. IEEE, 2009.