DISSERTATION

ON

STUDY OF INVENTORY CLASSIFICATION UNDER A UTOPIAN ENVIRONMENT

THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR AWARD OF THE DEGREE OF

Master of Engineering

In

Production Engineering

(In the faculty of Engineering and Technology)

By

ANANDA RABI DHAR

(B.E. in Mechanical Engineering, 2000

From

Jalpaiguri Govt. Engg. College, University of North Bengal, West Bengal, India)

Examination Roll Number: M4PRD1602

DEPARTMENT OF PRODUCTION ENGINEERING

JADAVPUR UNIVERSITY, KOLKATA- 700032, INDIA

MAY, 2016

JADAVPUR UNIVERSITY

(IN THE FACULTY OF ENGINEERING AND TECHNOLOGY)

KOLKATA – 700032, INDIA

Certificate of Recommendation

I/ WE HERE BY RECOMMEND THAT THE THESIS ENTITLED **"STUDY OF INVENTORY CLASSIFICATION UNDER A UTOPIAN ENVIRONMENT"** CARRIED OUT UNDER MY / OUR SUPERVISION AND GUIDANCE, BY **MR. ANANDA RABI DHAR** BEARING EXAMINATION ROLL NO. **M4PRD1602** MAY BE ACCEPTED FOR THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF THE DEGREE OF MASTER OF ENGINEERING IN PRODUCTION ENGINEERING AT JADAVPUR UNIVERSITY, KOLKATA – 700032.

....

Professor Bijan Sarkar

Department of Production Engineering

Jadavpur University

Kolkata – 700032.

THESIS ADVISOR

DEPARTMENT OF PRODUCTION ENGINEERING

JADAVPUR UNIVERSITY

KOLKATA– 700032.

....

HEAD, Department of Production Engineering

Jadavpur University

....

DEAN

Faculty Council of Engineering & Technology, Jadavpur University

Kolkata – 700032.

JADAVPUR UNIVERSITY

(IN THE FACULTY OF ENGINEERING AND TECHNOLOGY)

KOLKATA – 700032, INDIA

Certificate of Approval*

The forgoing thesis is hereby approved as a creditable study of Master of Engineering in Production Engineering Department and presented in a manner satisfactory to warrant its acceptance as a prerequisite to the degree for which it has been submitted. It is understood that by this approval the undersigned do not necessarily endorse or approve any statement made, opinion expressed or conclusion therein but approve this thesis only for the purpose for which it is submitted.

COMMITTEE ON FINAL EXAMINATION FOR EVALUATION OF THE THESIS

……………………………………………………

……………………………………………………

……………………………………………………

Signatures of the Examiners

* Only in case the thesis is approved

JADAVPUR UNIVERSITY

(IN THE FACULTY OF ENGINEERING AND TECHNOLOGY)

KOLKATA – 700032, INDIA

Declaration of Originality and Compliance of Academic Ethics

I hereby declare that this thesis contains literature survey and original research work by the undersigned candidate, as part of his Master of Engineering in Production Engineering Department.

All information in this document has been obtained and presented in accordance with academic rules and ethical conduct.

I also declare that, as required by this rules and conduct, I have fully cited and given references of all material and results that are not original to this work.

Name (Block Letters):

Examination Roll No.: M4PRD1602

Thesis Title: "STUDY OF INVENTORY CLASSIFICATION UNDER A UTOPIAN ENVIRONMENT"

Signature with date:

ACKNOWLEDGEMENT

In the journey of this research work, I got had the privilege to receive help and support from many people. These sources of assistance contributed in many distinct ways to keep me on the right track and to make this academic challenge an enjoyable and unforgettable experience. In particular, I am grateful to the following persons.

I want to express my sincere gratitude to my supervisor, Prof. Bijan Sarkar, Department of the Production Engineering, Jadavpur University, Kolkata, for his relentless help and constant guidance else it would not have been possible for me to complete this thesis work. I had the chance to learn from his broad range of experience, and I particularly appreciated his enthusiasm and communicative interest in research. I also appreciated the trust and freedom he gave me. He never stuck to any particular topic himself from the very beginning and encouraged me to explore my own ideas, so that I could select my own topic of interest by means of travelling on the vast ocean of knowledge from each new corner of exposure of literature. He constantly encouraged me for building software systems from the known theory and ideas of production engineering. Finally, I will always remember the entertaining open discussions that were never restricted on the actual topic of this work. I enjoyed these greatly, and I am really grateful for all the time he dedicated to me. Prof. Bijan Sarkar is an icon of my inspiration and constant encouragement in fulfilling my task. He led me on my inward journey towards perfection. So, I really remain ever grateful to him.

I would also like to convey my deep regards to Prof. Debamalya Banerjee, Head, Production Engineering Department, Jadavpur University, Kolkata.

I greatly convey my unfeigned thanks to all my friends at Jadavpur University, my cousin brother Suman, and all those who have rendered me valuable support in the completion of my thesis work.

For direct and indirect assistance, I would like to thank the staff members at Jadavpur University for their support, without which it would have been difficult to carry out the project successfully. I want to thank also Jadavpur University as a whole for providing me such a wonderful facility including those for research and most importantly for giving me a place in the abode of learning and education – I really owe so much to you, JU.

Above all, I would like to convey my sincere regards to my ever caring and loving mother and elders in my family, without their blessings, moral support and enrichment I could not have landed with such consequence. I also thank my wife and my son for all the love and support they have shown during time of crisis. Whatever be the situation, they were always by my side giving me constant encouragement and enthusiasm. This thesis is dedicated to the entire trident in my life - MOTHER, WIFE, and SON.

Date: ……………………………….

……………………………………………………………

Ananda Rabi Dhar

Registration Number: 129412 of 2014-15

Examination Roll Number: M4PRD1602

Dedicated to my
MOTHER
WIFE &
SON

CONTENTS

Page No.

Title sheet …………………………………………………………………………….. I

Certificate of recommendation ……………………………………………………... II

Certificate of Approval ……………………………………………………………... III

Declaration ………………………………………………………………………… IV

Acknowledgement ………………………………………………………………… V

Contents ………………………………………………………………………… VIII

List of Figures ……………………………………………………………………… XII

List of Tables..XIII

Abbreviations used ……………………………………………………………….. XIV

Thesis outline …………………………………………………………………….. XV

Abstract …………………………………………………………………………… 01

1. Introduction 02

1.1 Inventory Management (IM) ……..…………………………………………..... 02

1.1.1 Definition of Inventory and Inventory Management …………………………… 02

1.1.2 Motivation for Holding Inventory …………………….…………….......................... 02

1.1.3 Importance of Inventory ……….……………….…………………………….. 03

1.1.4 Types of Inventory………………………….. 05

1.1.5 Inventory Costs…………………………….. 06

1.1.6 Basic Inventory Models….. …………………………………………………... 07

1.2 Inventory Classification …...………………. ………………………………...... 21

1.2.1 Selective Control Policies…..……………… …………………………... 21

1.2.2 Basic ABC Analysis …………………....……………………………... 22

1.2.3 Illustration …………..…………………..……………………………... 24

1.2.4 Multi-Criteria Inventory Classification - Background and Motivation……….. 27

2. Literature Review and Research Gap Analysis 29

2.1 Inventory Control……………… ……………….……………………………... 30

2.2 Multi-Criteria Inventory Classification Methods...……..…………………………... 36

2.2.1 Subjective Weighting and Rating…………… …………………………... 36

2.2.2 Linear Optimization………….……………. …………………………... 39

2.2.3 Clustering, Genetic Algorithm, and Neural Networks…...…………………... 40

2.2.4 Other Approaches……………………………………………………... 42

2.3 MOORA and MULTIMOORA…….…...………………………………………… 43

2.4 Research Gap Analysis…..…………………. ……...…………………………… 44

2.5 Aims, Scope of the Work, Objectives..………...……………………………………… 51

3. Proposed Methodology 56

3.1 Fuzzy Set Theory …………………………………………………….………... 56

3.2 The MULTIMOORA Method…………………..…………………….………... 59

3.2.1 The Ratio System……………… ……………………………………………... 60

3.2.2 The Reference Point Approach… ……………………………………………... 60

3.2.3 The Full Multiplicative Form..… ……………………………………………... 60

3.2.4 Final Ranking of the MULTIMOORA method……………………….…………...…... 61

3.3 The Target-based MULTIMOORA with Integrated Significant Coefficients ……... 61

3.3.1 The Target-based Normalization….………..………………………….………….. 61

3.3.2 The Significant Coefficient of Criteria……..………………………….………….. 62

3.3.2.1 Subjective Significant Coefficient……………………….………….... 62

3.3.2.2 Objective Significant Coefficient…………………….……...……... 62

3.3.2.3 Inter-attribute Correlation Effect Significant Coefficient……….. 63

3.3.2.4 Integration of the Significant Coefficients……………………… 64

3.3.3 Subordinate Parts of the Target-based MULTIMOORA Method……………..... 65

3.3.3.1 The Target-based Ratio System……………………………………….... 65

3.3.3.2 The Target-based Reference Point Approach.. 65

3.3.3.3 The Target-based Full Multiplicative Form…………………………... 66

3.3.4 Final Ranking of the Target-based MULTIMOORA Method………….……..... 66

4. Case Study **68**

4.1 Case Study – 1……………………..…………………………………………... 68

4.1.1 Data and Criteria Selection …………………………………………….…..... 68

4.1.2 Assigning of Weights………………………………..………………….…..... 69

4.1.3 Building Prototype……………………………………………….................... 71

4.1.4 Numerical Illustration……………………………………………................ 72

4.2 Case Study – 2……………………..…………………………………………... 78

4.2.1 Data and Criteria Selection ……………………………………………..... 78

4.2.2 Assigning Weights ……………………………………………................. 79

4.2.3 Using Prototype ……………………………………………..................... 79

4.2.4 Numerical Illustration ……………………………………………............... 80

5. Results and Discussions 83

5.1 Comparison with Previous Methods……..…………………………………………... 83

5.2 Validation………..……………………..…………………………………………... 86

6 Conclusions and Future Scopes 89

4. References 91

LIST OF FIGURES

Sl. No.	List of Figures	Page No.
Fig. 1.1.6.1	Annual Holding, Ordering, Total Cost and EOQ	9
Fig. 1.1.6.2	Inventory Pattern for EOQ Inventory Model	11
Fig. 1.1.6.3	Inventory Pattern for Production Lot Size Inventory Model	13
Fig. 1.1.6.4	Inventory Pattern for EOQ Inventory Model with Back Orders	15
Fig. 1.1.6.5	Inventory Pattern for Continuous Review Model with Probabilistic Demand	19
Fig. 1.1.6.6	Inventory Pattern for Periodic Review Model with Probabilistic Demand	20
Fig. 1.2.3.1	ABC Classification (Pareto Principle)	26
Fig. 2.1	Generic Research Flow Chart (GRFC)	30
Fig. 2.1.2	Decision making complexity and evolution	35
Fig. 2.2.1.1	Tip of ice berg resembling the metaphor of knowledge	37
Fig. 2.5.1	Generic Production Process and Inventory	52
Fig. 3.1.3.1	Trapezoidal fuzzy number $\tilde{A}$	57
Fig. 3.3	Flowchart of the target-based MULTIMOORA method	67
Fig. 4.1.2.1	Expert opinion capture snap-shot	70
Fig. 4.1.3.1	MULTIMOORA final ranking snapshot for Case Study – 1	71
Fig. 4.1.3.2	Extended MULTIMOORA final ranking snapshot (Case Study -1)	72
Fig. 4.2.2.1	Weight assignment from AHP	79
Fig. 4.2.3.1	Extended MULTIMOORA final ranking snapshot for Case Study – 2	79
Fig. 5.2.1	Spearman rank correlation coefficient for Case Study - 1	87
Fig. 5.2.2	Dendrogram showing 3 clusters in different colour	88

LIST OF TABLES

Sl. No.	List of Tables	Page No.
Table 1.2.1.1	Different Selective Control Policies	21
Table 1.2.2.1	ABC classification empirical rule (Pareto principle)	22
Table 1.2.3.1	ABC Classification Illustration (1)	25
Table 1.2.3.2	ABC Classification Illustration (2)	25
Table 2.4.1	Gap Analysis of MCIC Literature	45
Table 4.1.1	Illustration of Respiratory Therapeutic Unit Data REID (1987)	68
Table 4.1.2.1	Linguistic variable for rating the weights of criteria of inventory items	70
Table 4.1.3.1	Derivation of subjective significant coefficient	73
Table 4.1.3.2	Normalized ratings in MULTIMOORA and extended MULTIMOORA	74
Table 4.1.3.3	Information entropy and standard deviation measures in extended MULTIMOORA method	75
Table 4.1.3.4	Inter-attribute correlation effect measures in extended MULTIMOORA method	75
Table 4.1.3.5	Subjective weights, objective weights and inter-attribute correlation effect weights, and integrated weights for Case Study - 1	75
Table 4.1.3.6	Assessment values, subordinate rankings, and final rank in MULTIMOORA	76
Table 4.1.3.7	Assessment values, subordinate rankings, final rank in extended MULTIMOORA	77
Table 4.2.1.1	Decision matrix for Pharmaceutical industry data (Case Study – 2)	78
Table 4.2.4.1	Subjective weights, objective weights and inter-attribute correlation effect weights, and integrated weights for Case Study - 2	81
Table 4.2.4.2	Final ranking by MULTIMOORA method (Case Study – 2)	81
Table 4.2.4.3	Final ranking by Extended MULTIMOORA method (Case Study – 2)	82
Table 5.1.1	Classification result compared with other previous methods for REID (1987)	85
Table 5.1.2	Comparison of final ranks of items in Case Study – 2	86
Table 5.2.1	Spearman’s rank correlation coefficient for Case Study - 2	87

LIST OF ABBREVIATIONS

Abbreviated Form	Full Form
MCDM	Multi-Criteria Decision Making
AI	Artificial Intelligence
AHP	Analytical Hierarchy Process
FF-NN	Feed Forward Neural Network
BP	Back Propagation
ANP	Analytic. Network Process
FTOPSIS	Fuzzy Technique for Order of Preference by Similarity to Ideal Solution
RSM	Revised Silver Meal
GA	Genetic Algorithm
SKU	Stock Keeping Unit
MOORA	Multi-Objective Optimization on the basis of Ratio Analysis.
OCRA	Operational Competitiveness Ratings Analysis
COPRAS-G	COmplex PRoportional ASsessment of alternatives to Grey relations
EVAMIX	EVAluation of MIXed data
RLUC	Revised Least Unit Cost
VRSDP	Variance Retentive Stochastic Dynamic Programming
ILP	Integer Linear Programming
LTD	Lead Time Demand
DM	Decision Maker
DFE	Demand Forecast Error
BRB-IC	Belief-Rule-Based-Inventory Classification
EOBS	Extended Optimal Base Stock
FEMOGA	Fast and Elitist Multi Object Genetic Algorithm
BA	Bee Algorithm
RBES	Rule-Based Expert System
CBR	Case-Based Reasoning
GAMIC	Genetic Algorithm Multi-criteria Inventory Control
MCIC	Multi-Criteria Inventory Classification
IM	Inventory Management
PSO	Particle Swarm Optimization
MSE	Mean Square Error
ANN	Artificial Neural Network
DEA	Data Envelopment Analysis
FRV	Fuzzy Random Variable

THESIS OUTLINE

The central idea and contribution of this dissertation is the study of latest inventory control policies and out of this focus of selective inventory control by means of inventory classification methods considering multiple criteria under fuzzy environment using MULTIMOORA and extended MULTIMOORA.

Here, in this dissertation, I have also attempted to write and illustrate many concepts with flow charts, tables, coloured diagrams and graphs in such a manner that is hopefully make it accessible to a broad range of readers.

Here, there are brief illustrations of the Sections made in this thesis paper. They are as follows:

Section 1: In this section, Introduction of Inventory Control is illustrated. There are two sub-parts in this section, first is Inventory Management (IM) and second is Inventory Classification (IC). In the first sub-part, there are seven more sub-parts. They are (i) definition of inventory; (ii) motivation for holding inventory; (iii) importance of inventory; (iv) types of inventory; (v) costs of inventory; (vi) need for inventory management and (vii) basic inventory models. Under IC, there are four more sub parts. They are- (i) selective control policies; (ii) basic ABC analysis; (iii) illustration; (iv) Multi-Criteria Inventory Classification - background and motivation.

Section 2: This section illustrates the in-depth literature surveys and gap analysis with five important sub-sections, one is Inventory Management, which is very selective to acquaint with latest researches in inventory management especially under uncertainty, next one is Inventory Classification, then research gap analysis, the fourth one is MULTIMOORA, and last one is aims, scope of investigation and objectives of research study.

Section 3: This part of section describes the methodology used. This section contains three sub-sections i.e., i) fuzzy set theory; ii) MULTIMOORA; iii) extended MULTIMOORA. In each sub-sections of MULTIMOORA and Extended MULTIMOORA, ratio system, reference point approach and full multiplicative forms are discussed. Extended MULTIMOORA has additional sub-sections as target-based normalization and significant coefficients, derivation of each type of significant coefficients and eventually integration.

Section 4: It illustrates the application of proposed models to two case studies. Under this, there are five sub-sections for each case study. They are – (i) selection of data and criteria; ii) building prototype; (iii) assignment of weights; iv) numerical illustration..

Section 5: This section describes the results and discussions. This has two sub-sections. First one is the comparison with some previous results; second is validation of the results

Section 6: This section illustrates the conclusion part of this research study. This additionally states the limitations of the research work that can ride wave to make inroads into new aspects of future work in this related field.

Section 7: This section presents all the related References of this research work.

ABSTRACT

Inventory classification is an effective way to manage a large number of items. Thousands of inventory items in companies even with moderate size increase the risk of losing sight of the most important items and spending unnecessary resources in controlling less important ones. Therefore companies try to classify items and select appropriate control policy for each group. As a basic methodology, ABC analysis is widely used for classification. The traditional ABC classification is done by sorting the items in descending order based on single criteria i.e. the annual dollar value of each item. According to this approach, resources spent on inventory control should be related to the importance of each item. Therefore, class A contains few items (approx. 20%) but constitutes the largest amount of annual dollar value (approx. 80%), whilst class C holds a large number of items (approx. 50%) and forms a small amount of annual dollar value (approx. 5%). Items that fall in between these two classes are assigned to class B. However, it is generally recognized that multiple criteria should be used in practice. A considerable body of research in this direction over the last two decades and more already exists in the literature. A Multi 'Multi-Objective Optimization by Ratio Analysis' (MULTIMOORA) approach is proposed in this dissertation for multi-criteria inventory classification (MCIC) under a utopian environment. The proposed model assigns subjective significant coefficients or simply subjective weights for each criterion from five different sets of expert judgment under fuzzy environment. To get advantages of both exogenous and endogenous weight assignment the model is again extended in a paradigm of target-based decision making considering a combination of four - subjective weights, two sets of objective weights based on information entropy and standard deviation, and weights based on inter-attribute correlation effects. Comparisons of the proposed models (both MULTIMOORA and Extended MULTIMOORA) with some well-known previous methods are illustrated using a benchmark MCIC problem and an additional case study in pharmaceuticals domain. It is shown that our proposed model can provide more reasonable and comprehensive performance index.

KEYWORDS: Multiple Criteria Inventory Classification, MULTIMOORA, target-based normalization, integrated significant coefficients, fuzzy sets, and Spearman's rank correlation co-efficient.

Chapter 1

1. Introduction

"One of the great responsibilities that I have is to manage my assets wisely, so that they create value." - *Alice Walton of* Walmart

1.1 Inventory Management

1.1.1 Definition of Inventory and Inventory Management

The word "inventory" and "inventory management" have been defined in many ways as indicated in the literature. The following definitions are selected out of all as they are short and comprehensive:

"Inventory is a physical resource that a firm holds in stock with an intent of selling it or transforming into a more valuable state."

"Inventory Management is a set of policies and controls that monitors levels of inventory and determines what levels should be maintained, when stock should be replenished, and how large orders should be placed."

1.1.2 Motivation for Holding Inventory

There are several reasons that motivate companies to have stock. Five main reasons have been identified as motivation for holding stocks, namely

Economies of Sale:

The economies of scale in manufacturing, purchasing and transportation firm can be realized by holding inventory. A quantity discount is obtained if the business buys large amounts. Thus the transportation can move larger volumes and get economies of scale through better equipment utilization. If more material is inventoried, manufacturing can have longer production runs allowing per unit fixed cost reduction.

Balance in supply in demand:

Inventory helps in maintaining a balance between supply and demand. A Christmas tree manufacturer sees some demand year around but the demand increases by 60% or more in the Christmas season. By manufacturing to stock, production can be kept level throughout the year. The idle plant capacity is reduced while maintaining a relatively stable workforce and also keeping the cost down. In the production of canned fruits, where the demand is relatively constant but the input materials are seasonal finished inventory helps meet demand when the materials are no longer available.

Specialization:

The subsidized firms can specialize with the help of inventory. Instead of manufacturing a variety of products, each plant can manufacture a product and then ship the finished products directly to customers or to a ware house for storage. Thus by specializing, each plant can gain economies of scale through long production runs.

Protection from uncertainties:

In case of more demand and less supply of raw material stocks run out, the production line shuts down until more material is delivered. Likewise a shortage of work in process means the product cannot be finished. Finally if a customer orders outstrips finished good supply, the resulting stockout could lead to loss of customers. Therefore the primary reason to hold inventory is to have protection from uncertainties.

Buffer interface:

Inventory can buffer key interfaces, creating time and place utility. Key interfaces include 1) supplier and purchasing, 2) purchasing and production, 3) production and marketing, 4) marketing and distribution, 5) distribution and intermediary, and 6) intermediary and customer. Having inventory at these interfaces helps ensure that demand is met and stock outs are minimized.

1.1.3 Importance of Inventory

Inventory plays a major role in the growth and survival of an organization. Failure to an effective and efficient management of inventory means that the organization will lose customers and gradually sales will decline. Emphasizing on the importance of inventory on the balance sheet of companies, Coyle, Bardi, and Langley (2003) state that “inventory as an assets on the balance sheet of companies has taken an increased significance because of the strategy of many firms to reduce their investments in fixed assets, that is, plants, warehouses, office buildings, equipment and machinery and so on”.

As per the researches done in 1999, it was noted in the United States of America that about $700 million worth of inventory held by American businesses is financed by bank loans with the marketing relationship exists between inventory managers and commercial lending officers who write these inventory loans. Sufficient information must be provided by the inventory managers to their lenders to obtain financing at the lowest rate. Loan officers need to assess the degree of inventory risk in order to assign a proper interest rate. Issues of risk and return of inventory loans are matter of concern for both the inventory managers and the creditors.

For all types of businesses inventory management is an important concern. For companies such as JC Penny Limited, which operate on relatively low profit margins, poor inventory management can seriously undermine the business. The challenge is not to pare inventories to the bone to reduce costs or to have plenty around to satisfy all demands but to have the right amount to achieve the competitive priorities for business most efficiently. Finally according to the U.S Bureau of Census, inventories are found in such places as warehouses, yards, shop floors, transportation equipment and on retail store shelves. Having these inventories on hand can cost between 20 and 40 percent of their value per year. Therefore carefully managing inventory levels makes good economic sense. Even though many strides have been taken to reduce inventories through just in time, time compression, quick response and collaborative practices applied throughout the supply channel, the annual investment in inventories by manufacturers, retailers and merchant wholesalers, whose sales represent about 90 percent of GNP, is about 12 percent of the U.S gross domestic product.

1.1.4 Types of Inventory

According to latest study, inventories can be categorized in to six distinct forms that are:

Cycle stock:

Cycle stock is inventory that results from the replenishment process and is required in order to meet demand under the condition of certainty, that is when the item can predict demand and replenishment times (lead times) almost perfectly. For example if the rate of sales for a constant 20 units per day and the lead time is always 10 days, no inventory beyond the cycle stock would be required. Assumptions of constant demand and lead time remove the complexities involved in inventory management.

In-transit inventories:

In transit inventories are items that are en route from one location to another. They may be considered a part of cycle stock even though they are not available for sale and/ or shipment until after they arrive at the destination. For the calculation of inventory carrying costs, in-transit inventories should be considered as inventory at the place of shipment origin since the items are not available for the buyers, sale or subsequent reshipment.

Safety or buffer stock:

Safety of buffer stock is held in excess of cycle stock because of uncertainty in demand or lead time. The notion is that a portion of average inventory should be devoted to cover short range variations in demand and lead time. Average inventory at a stock keeping location that experience demand or lead time variability equal to half the order quantity plus the safety stock.

Anticipation stock:

Anticipation stock is inventory held for reasons other than satisfying current demand. For example, materials may be purchased in volumes larger than necessary in order to receive quantity discounts, because of a forecasted price increase or materials shortage, or to protect against the possibility of a strike.

Seasonal stocks:

Seasonal stocks is a form of anticipation stock that involves the accumulation of inventory before a season begins in order to maintain a stable labour force and stable production runs or, in the case of agricultural products, inventory accumulated as the result of a growing season that limits availability throughout the year.

Dead stock:

Dead stock is inventory that no one wants, at least immediately. The question is why an organization would incur the costs associated with holding these items rather than simply disposing of them. One reason might be that management expects demand to resume at some point in the future. Alternatively, it may cost more to get rid of an item that it does to keep it. But the most compelling reason for maintain these goods is customer service. Perhaps an important buyer has an occasional need for some of these items, so management keeps them to have a good will gesture.

1.1.5 Inventory Costs

There are four types of costs that must be considered in setting inventory levels.

Purchasing cost:

This is cost of purchasing or procuring inventory. Inventory is a liability from the point view of expenditure by means of locking working capital. To have inventory the firm mush have sufficient working capital. The unit cost of items purchased may fluctuate over the entire planning horizon. Hence, average unit price is considered for computing the purchasing cost of inventories.

Holding costs:

Holding cost or carrying costs are costs such as storage, handling, insurance, taxes, obsolescence, theft and interest on funds financing the goods. These charges increase as inventory levels rises. In order to minimise carrying costs management makes frequent orders of small quantities. Holding costs are commonly assessed as a percentage of unit value, that is 15 percent, 20 percent, rather than attempting to derive a monetary value for each of these costs individually. This practice is a reflection of the difficulty inherent in deriving a specific per unit costs for example theft or obsolescence.

Ordering costs:

Ordering costs are those costs associated with placing an order, including expenses related to personnel in a purchasing department, communications and the handling of the related paperwork. Lowering these costs would be accomplished by placing a small number of orders, each for a large quantity. Unlike carrying costs, ordering costs are generally expressed as a monetary value per order.

Stockout costs:

It included sales that are lost, both short and long term. These charges are probably the most difficult to compute, but arguably the most important because they represent the costs incurred by customers (internal or external) when inventory policies falter. Failure to understand these costs can lead management to maintain higher (or lower) inventory levels than customer requirement may justify.

1.1.6 Basic Inventory Models

The first mathematical inventory model is generally referred to as the Economic Order Quantity (EOQ) model which was developed by Harris [**45**] in 1915. There are several full length books attempted to explain how various extensions of EOQ can be used in practice [**82**, **15**, **21**]. Further research works showed that the EOQ model appears to be quite insensitive to errors in the specification of the appropriate cost parameters and the estimation of demand. The importance of the EOQ model is not only from the historical point of view but also because many other models designed to cope with different situations have been based on this very model.

The following models are discussed over here to get an overview of the general principles of inventory management.

Deterministic Models:

- The Economic Order Quantity (EOQ) Model

- The Economic Production Lot Size Model
- An Inventory Model with Planned Shortages

Probabilistic Models:

- Single-Period Inventory Models
- A Continuous Fixed Order Quantity Model
- A Fixed Time Period Model

The Economic Order Quantity (EOQ) Model

Overview:

The Economic Order Quantity (EOQ) applies only when demand for a product is constant over the year and each new order is delivered in full when inventory reaches zero. There is a fixed cost for each order placed, regardless of the number of units ordered. There is also a cost for each unit held in storage, commonly known as holding cost, sometimes expressed as a percentage of the purchase cost of the item.

The optimal number of units to order needs to be determined so that the total cost associated with the purchase, delivery and storage of the product can be minimized.

The required parameters to the solution are the total demand for the year, the purchase cost for each item, the fixed cost to place the order and the storage cost for each item per year. Note that the number of times an order is placed will also affect the total cost, though this number can be determined from the other parameters.

Assumptions:

- Demand for items from inventory is continuous and at a constant rate
- Orders to replenish inventory are made at regular intervals
- Ordering cost is fixed (independent of quantity produced)
- The lead time is fixed

- The purchase price of the item is constant, i.e. no discount is available
- The replenishment is made instantaneously

Total Minimum Cost Model:

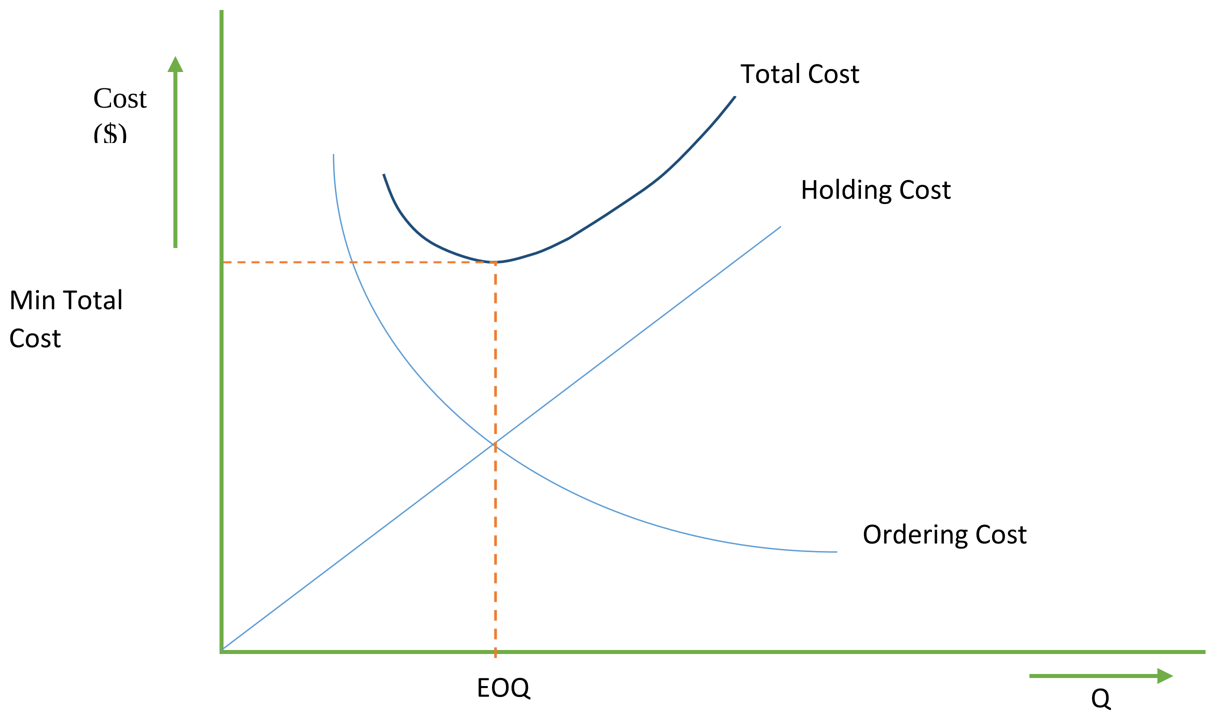


Fig. 1.1.6.1: Annual Holding, Ordering, Total Cost and EOQ

The single-item EOQ formula finds the minimum point of the following cost function:

Total Cost = purchase cost or production cost + ordering cost + holding cost

Where:

Purchase cost: This is the variable cost of goods: purchase unit price × annual demand quantity. This is P × D

Ordering cost: This is the cost of placing orders: each order has a fixed cost K, and it needs to order D/Q times per year. This is K * D/Q

Holding cost: the average quantity in stock (between fully replenished and empty) is Q/2, so this cost is h × Q/2

$$TC = PD + \frac{DK}{Q} + \frac{hQ}{2}.$$

To determine the minimum point of the total cost curve, calculate the derivative of the total cost with respect to Q (assume all other variables are constant) and set it equal to 0:

$$0 = -\frac{DK}{Q^2} + \frac{h}{2}$$

Solving for Q gives Q* (the optimal order quantity):

$$Q^{*2} = \frac{2DK}{h}$$

Formula:

D - Annual Demand, Q - Order Quantity, K - Cost of Placing Order, h - Annual per-unit Holding Cost, Ordering Cost = KD/Q, Holding Cost = HQ/2, Total Cost = KD/Q + hQ/2

$$Q^* = \sqrt{\frac{2DK}{h}}$$

Graphical Representation:

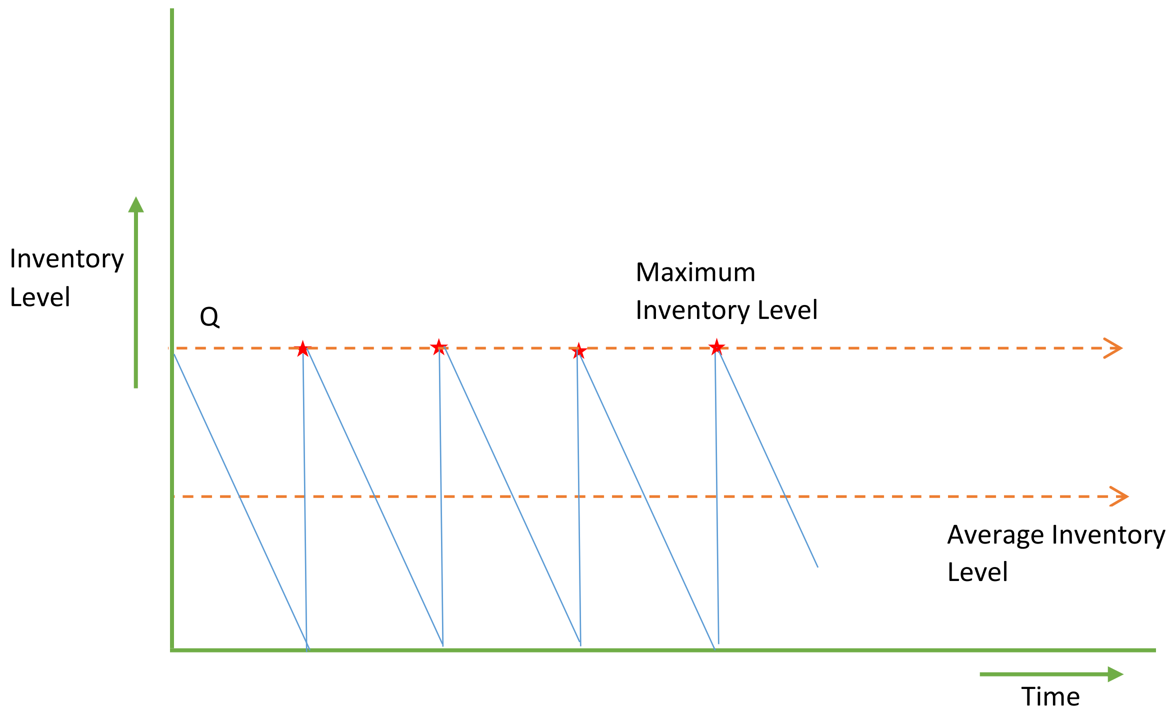


Fig. 1.1.6.2: Inventory Pattern for EOQ Inventory Model

The Economic Production Lot Size Model

Overview:

Economic Production Quantity (EPQ) or the Economic Production Lot Size Model only applies where the demand for a product is constant over the year and that each new order is delivered/produced incrementally when the inventory reaches zero. There is a fixed cost charged for each order placed, regardless of the number of units ordered. There is also a holding or storage cost for each unit held in storage (sometimes expressed as a percentage of the purchase cost of the item).

Here also, the optimal number of units of the product to order is determined so that the total cost associated with the purchase, delivery and storage of the product is the minimum.

The required parameters to the solution are the total demand for the year, the purchase cost for each item, the fixed cost to place the order and the storage cost for each item per year. The number of times an order is placed will also affect the total cost, however, this number can be determined from the other parameters

Assumptions:

- Demand for items from inventory is continuous and at a constant rate
- Production runs to replenish inventory are made at regular intervals
- During a production run, the production of items is continuous and at a constant rate
- Production set-up/ordering cost is fixed (independent of quantity produced)
- The lead time is fixed
- The purchase price of the item is constant, i.e. no discount is available
- The replenishment is made incrementally

Variables:

K = ordering/setup cost, D = demand rate, h = holding cost, T = cycle length,

P = production rate, $x = \frac{D}{P}$, Q = order quantity

Holding Cost per Year = $\frac{Q}{2} \times h\,(1 - x)$, where $\frac{Q}{2}$ is the average inventory level, and $h\,(1 - x)$ is the average holding cost. Therefore multiplying these two results in the holding cost per year.

Ordering Cost per Year = $\frac{D}{Q} \times K$

Where, $\frac{D}{Q}$ is the orders placed in a year, multiplied by K results in the ordering cost per year.

It can be noted from the equations above that the total ordering cost decreases as the production quantity increases. Inversely, the total holding cost increases as the production quantity increases. Therefore in order to get the optimal production quantity a holding cost needs to be set per year equal to ordering cost per year and solve for quantity (Q), which is the EPQ formula mentioned below. Ordering this quantity will result in the lowest total inventory cost per year.

EPQ Formula:

$$Q^* = \sqrt{\frac{2KD}{h(1-x)}}$$

Graphical Representation:

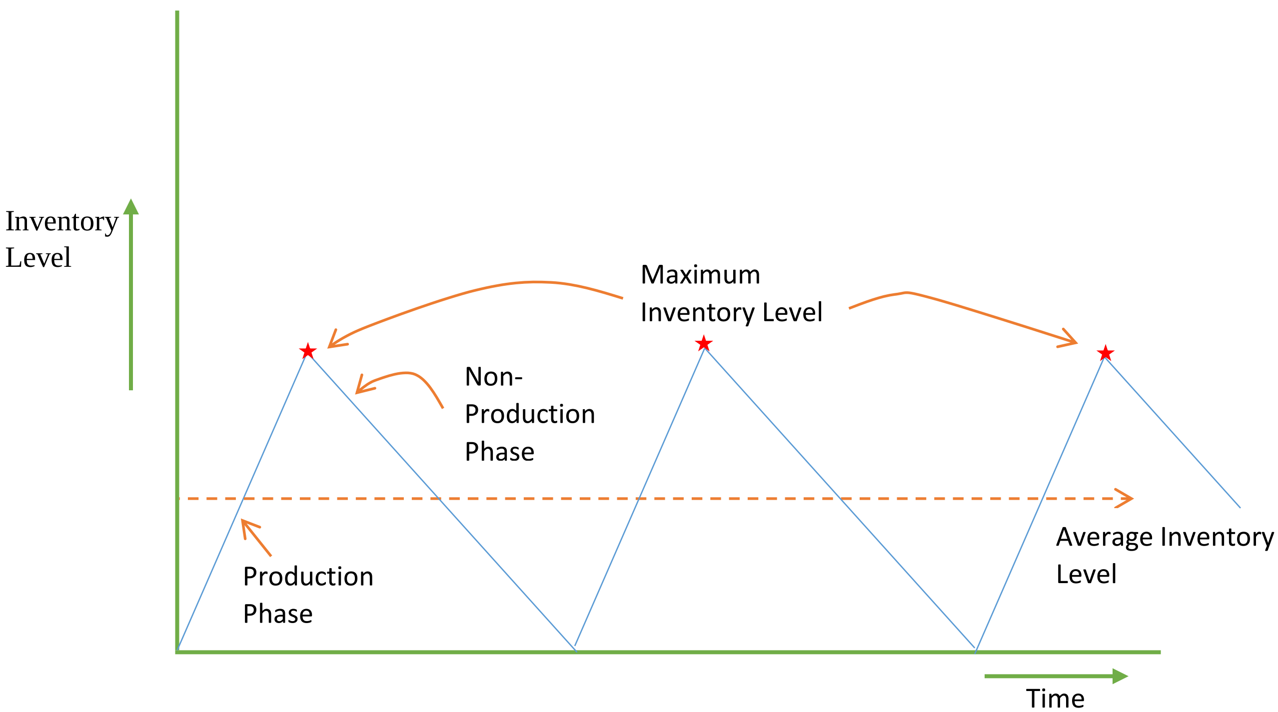


Fig. 1.1.6.3: Inventory Pattern for Production Lot Size Inventory Model

An Inventory Model with Planned Shortages

Overview:

One of the assumptions of our basic EOQ model is that shortages and back ordering are not allowed. The third model variation that will be described, the EOQ model with shortages, relaxes this assumption. However, it will be assumed that all demand not met because of inventory shortage can be back ordered and delivered to the customer later. Thus, all demand is eventually met. The EOQ model with shortages is illustrated in Fig. 1.1.6.4.

Assumptions:

- All assumptions made EOQ model with the following exception:
- Shortages are allowed as backorder assuming no lost sales.

Formula:

Bypassing the lengthy derivation of the individual cost components of the EOQ model with shortages, which requires the application of plane geometry to the graph in Fig. 1.1.6.4. The individual cost functions are provided as follows, where S equals the shortage level and C_S equals the annual per-unit cost of shortages, C_O equals the unit ordering cost, C_C equals the annual per-unit holding/carrying cost:

$$\text{total shortage costs} = C_s \frac{S^2}{2Q}$$

$$\text{total carrying cost} = C_c \frac{(Q-S)^2}{2Q}$$

$$\text{total ordering cost} = C_o \frac{D}{Q}$$

Combining these individual cost components results in the total inventory cost formula:

$$TC = C_s \frac{S^2}{2Q} + C_c \frac{(Q-S)^2}{2Q} + C_o \frac{D}{Q}$$

The only way to determine the optimal order size and the optimal shortage level , S , is to differentiate the total cost function with respect to Q and S , set the two resulting equations equal to zero, and solve them simultaneously . Doing so results in the following formulas for the optimal order quantity and shortage level:

$$Q_{opt} = \sqrt{\frac{2C_o D}{C_c}\left(\frac{C_s + C_c}{C_s}\right)}$$

$$S_{opt} = Q_{opt}\left(\frac{C_c}{C_c + C_s}\right)$$

Graphical Representation:

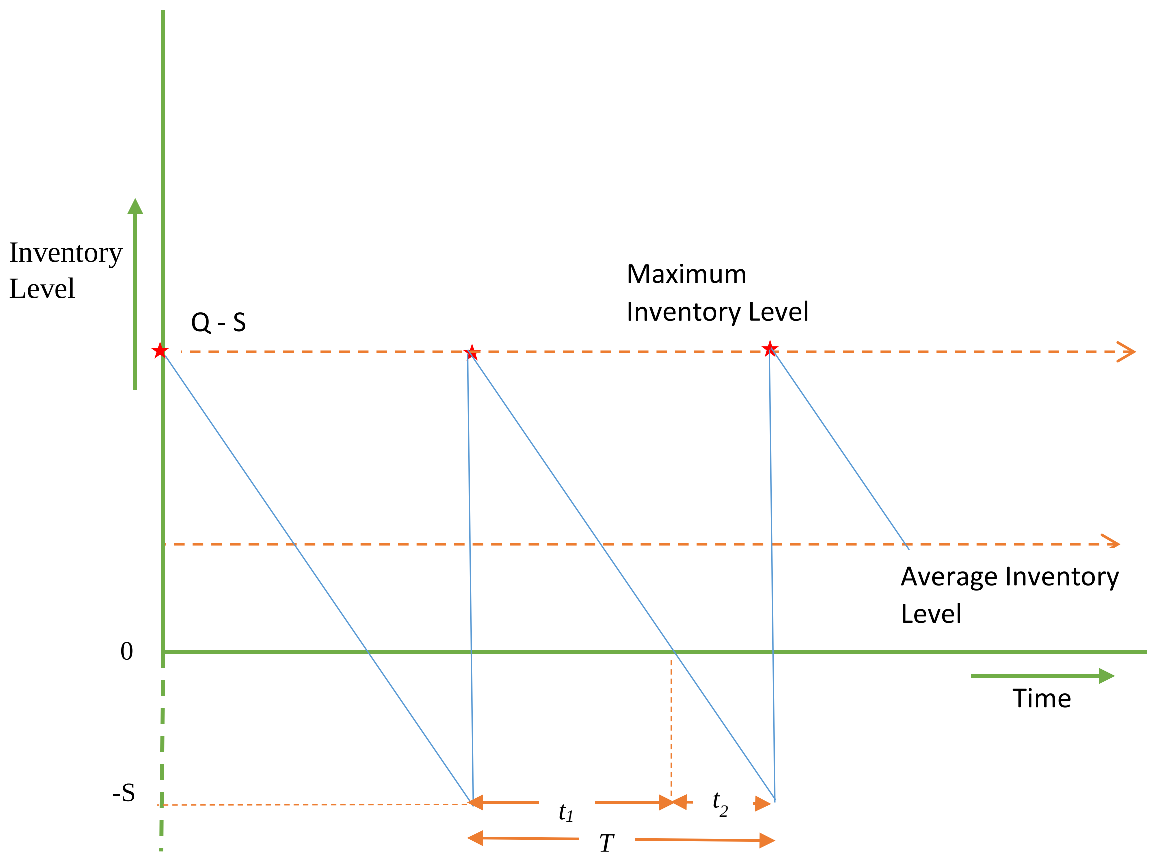


Fig. 1.1.6.4: Inventory Pattern for EOQ Inventory Model with Back Orders

Single-Period Inventory Models

Overview:

It is necessary to clarify the term single period. This term refers to the situation where the inventory is perishable and demand for that particular inventory exists only for the period at which it is ordered (or) procured. Newspaper selling is such an example. The newspaper ordered for today will not be sold at the same price tomorrow. Fashion selling is another example. Spring-summer designs will not sell during the autumn-winter season.

Derivation:

Increment analysis is used to determine the optimal order quantity for a single-period inventory model with probabilistic demand. The increment analysis addresses the how-much-to order question by comparing the cost or loss of ordering one additional unit with the cost or loss of not ordering one additional unit. Notation used in this model is listed below.

Co: Cost per unit of overestimating demand; represents the loss of ordering one additional unit that may not sell.

Cu : Cost per unit of underestimating demand; represents the loss of not ordering one additional unit for which demand existed otherwise.

Let the probability of the demand of inventory being more than a certain level y is $P(D > y)$, and the probability of the demand of inventory being less than or equal to this level y is $P(D <= y)$. Then, the expected loss (EL) is given by either of the two conditions below.

Overestimation: $EL(y + 1) = Co * P (D <= y)$

Underestimation: $EL(y) = Cu * P (D > y)$

Following which the optimal order quantity $(y *)$ can be found as follows: $EL(y * + 1) = EL(y *)$

Formula:

$Co * P (D <= y *) = Cu * P (D > y*)$; it is known that $P (D > y*) = 1 – P (D <= y *)$

Substituting above two equations, it becomes $Co * P (D <= y *) = Cu * [1 – P (D <= y *)]$

Solving for P (D <= y ∗), it is finally obtained that P (D <= y∗) = $\frac{C_u}{C_u + C_o}$

The above expression provides the general condition for the optimal order quantity y ∗ in the single-period inventory model. The determination of y ∗ depends on the probability distribution.

A Continuous Fixed Order Quantity Model

Overview:

In earlier periods, non-continuous, or periodic inventory systems were more prevalent. Starting in the 1970s digital computers made possible the ability to implement a perpetual inventory system. This has been facilitated by bar coding and lately radio frequency identification (RFID) labeling which allows computer systems to quickly read and process inventory information as part of transaction processing

The reorder point for replenishment of stock occurs when the level of inventory drops down to zero. In view of instantaneous replenishment of stock the level of inventory jumps to the original level from zero level.

In real life situations one never encounters a zero lead time. There is always a time lag from the date of placing an order for material and the date on which materials are received. As a result the reorder point is always higher than zero, and if the firm places the order when the inventory reaches the reorder point, the new goods will arrive before the firm runs out of goods to sell. The decision on how much stock to hold is generally referred to as the order point problem, that is, how low should the inventory be depleted before it is reordered.

The two factors that determine the appropriate order point are the delivery time stock which is the Inventory needed during the lead time (i.e., the difference between the order date and the receipt of the inventory ordered) and the safety stock which is the minimum level of inventory that is held as a protection against shortages due to fluctuations in demand.

Therefore: Reorder Point, ROP = Normal consumption during lead-time + Safety Stock

Several factors determine how much delivery time stock and safety stock should be held. In summary, the efficiency of a replenishment system affects how much delivery time is needed.

Since the delivery time stock is the expected inventory usage between ordering and receiving inventory, efficient replenishment of inventory would reduce the need for delivery time stock. And the determination of level of safety stock involves a basic trade-off between the risk of stockout, resulting in possible customer dissatisfaction and lost sales, and the increased costs associated with carrying additional inventory.

Another method of calculating reorder level involves the calculation of usage rate per day, lead time which is the amount of time between placing an order and receiving the goods and the safety stock level expressed in terms of several days' sales.

Reorder level = Average daily usage rate x lead-time in days = $D * L + Z * \sigma (L)$, where D is average demand during lead time, L = Lead Time, Z = standard normal variant according to the desired service level, $\sigma (L)$ = standard deviation of demand during lead time

Features:

- Order Quantity (Q) - Fixed, the same amount is ordered each time, less error-prone and paper work is reduced.
- When to place an order- When inventory position drops to the reorder point (ROP).
- Recordkeeping*- Each time a withdrawal or addition is made. With the advent of modern computer systems and inventory records policy, this effort is much reduced
- Size of inventory- Less than Fixed-Time Period Model because it provides safety stock only for the lead time.
- Time to maintain - Minimal because the point-of-sale scanners update inventory each time a sale is made.
- Type of items- Higher priced, critical, or important items (e.g. diamonds, computer chips, etc.) because average inventory

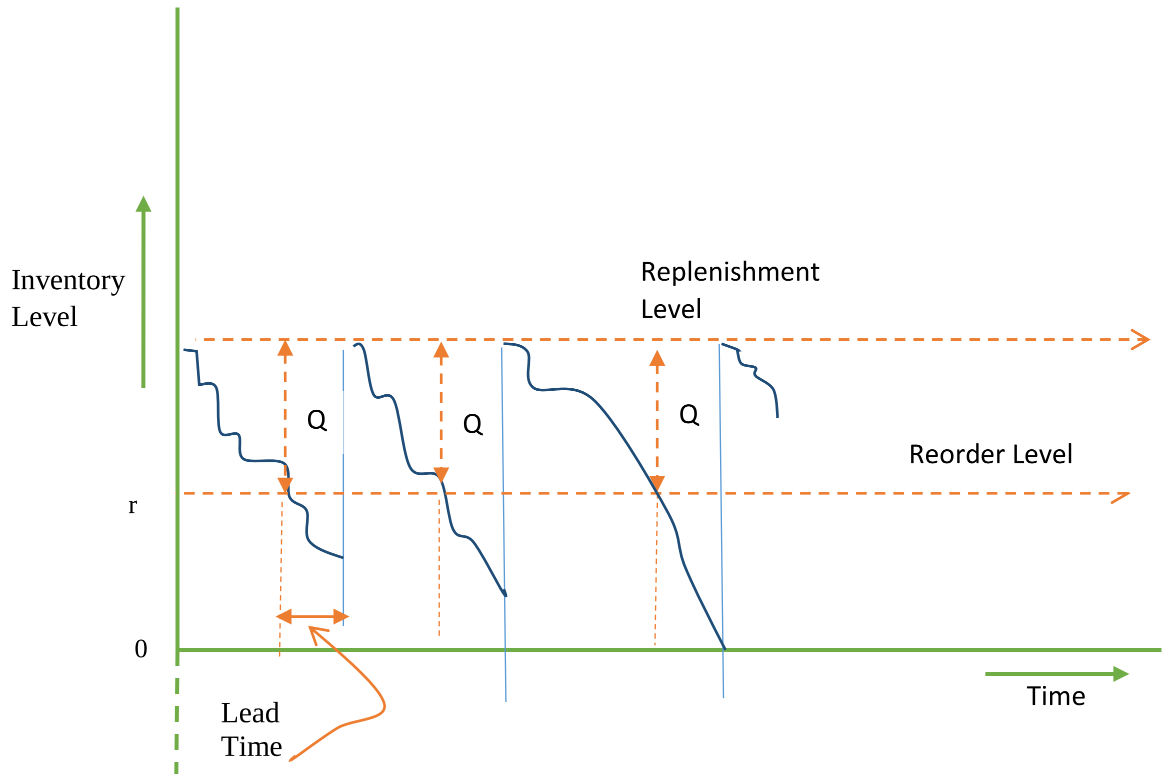


Fig. 1.1.6.5: Inventory Pattern for Continuous Review Model with Probabilistic Demand

A Fixed Time Period Model

Overview:

In fixed time-period models (also known as P-models), orders are placed at fixed periods, irrespective of the demand or usage pattern. Inventory is not reviewed continuously as in Q-model; but in periodic intervals. An order quantity to replenish available inventory to a maximum level is placed. Because of the uncertain demand pattern, a safety stock is usually maintained; safety stock is the minimum stock levels maintained, which is not accounted for in evaluating the order quantity. Reiterating, order quantity is determined based on demand forecast, and the actual order placed will be over and above the safety stock level. The model is illustrated in Fig. 1.1.6.6. The order quantity in this model is dependent on demands, safety stock and current inventory. Q = D $*$ (T + L) + Z $*$ σ (T + L) - I where, D is the average demand T is the periodicity of review, L is the lead time, Z is the number of standard deviations for a specified service probability, σ (T + L) is the standard deviation of demand over review and lead time, I is the current inventory (including those being processed in order), σ(T + L) = $\sqrt{\sum_{i=1}^{T+L} \sigma_{(D_i)}{}^2}$

Now, $\sigma_{(D_i)}$ can be assumed to be constant, as demands are considered independent over any period. Therefore, SS = Z * σ (T + L) = $Z \times \sqrt{(\mathrm{T} + \mathrm{L}) * \sigma_{\mathrm{L}}{}^2}$

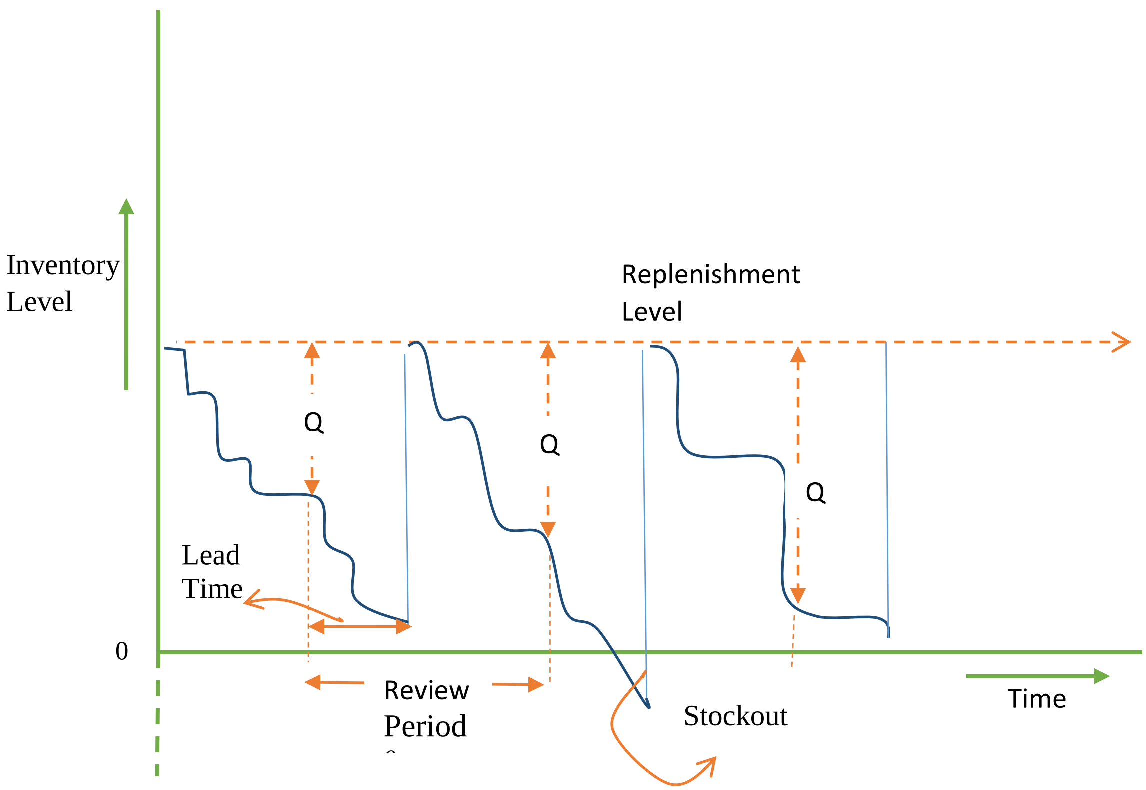


Fig. 1.1.6.6: Inventory Pattern for Periodic Review Model with Probabilistic Demand

The safety stock to be maintained is SS = $Z \times \sqrt{(\mathrm{T} + \mathrm{L}) * \sigma_{\mathrm{L}}{}^2}$

1.2 Inventory Classification

1.2.1 Selective Control Policies

Inventory in a company consists of thousands of different items in stock. The control of all these items creates a serious problem to the management if the same amount of control is exercised on each of these items.

Therefore, in order to execute proper control, it is necessary to take selective approach and find the attention required for each item according to its importance. This is essential for achieving maximum benefits with minimum efforts and costs.

Depending upon the alternatives and purposes, different analysis have been developed to help in bringing practical solution to the problem of inventory control. The commonly used systems can be classified as:

Table 1.2.1.1: Different Selective Control Policies

Technique	Explanation	Characteristics
ABC Analysis	Always Better Control	Value (Volume X Unit Price) of Annual Demand
VED Analysis	Vital, Essential, Desired	Criticality of Items/Parts
SDE Analysis	Scarce, Difficult, Easily available	Procurement Process of Items
HML Analysis	High, Medium, Low	Unit Price of Items
FSN Analysis	Fast moving, Slow moving, Non-moving	Demand Volume of Items

1.2.2 Basic ABC Analysis

ABC analysis divides into three categories in terms of percentage of number of items and percentage of total value. It is based on Pareto analysis. The great Italian scantiest Pareto in 1990 observed that 80% of the population in Milan city own only 20% land, and 20% of the population hold 80% of it. It is known as Pareto principle or 80/20 rule.

In ABC analysis important items (high usage valued items) are grouped in A', while trivial items (low usage valued items) are grouped in C', and the remaining middle level items are considered B' items. The inventory control is exercised on the principle of "management by exception", i.e. rigorous controls are exercised on A' items and routine loose controls on C' items and moderate controls on B' items.

The items classified by virtue of usage is shown in Table 1.2.2.1.

Table 1.2.2.1: ABC classification empirical rule (Pareto principle)

Category	% of items (approx.)	% of Usage Value (approx.)
High Value Items	20%	80%
Medium Value Items	30%	15%
Low Value Items	50%	5%

A' Items:

In the total inventory items A' items are few in number and represent a small percentage of total items. However, due to high cost and huge consumption, they represent a large percentage of company's total expenditure. It is common that approx. 20% of the total quantity of the items represent approx. 80% of the amount spent on the all the inventory items. These items require accurate records and careful handling and storage under tight control. Minimum and maximum limits and reorder pint is set for each of such items. Such items are thought of in advance and purchased well in time. A detailed records of receipts and proper handling and storage facilities are provided for them. Such items being costly are purchased in smaller quantities often and just before their use. This, of course, increases the procurement cost and involves little risk of non-availability.

However, inventory holding costs decrease and the problem of storage and caretaking are minimized.

B' Items:

These are middle level items which do not require as detailed and close control as A' items but they need more attention than C' items. These items usually represent approx. 30% of the total quantity of all the items and represent 15-20% approx. of the total expenditure in inventory stock for the company.

Control Policy for A' items:

- A' items are high valued items. Hence, they should be ordered more and in small quantity in order to reduce capital locked up at any time.
- The future requirement must be planned in advance so that the required quantity arrive a little before they are required for consumption.
- Purchases and control of A' items should be looked into by the top management executive in purchase department.
- Maximum efforts should be made to expedite the delivery. The safety stock should be as less as possible.
- Ordering quantities, reorder point, minimum and maximum stock level should be revised more frequently.

Control Policy for B' items:

- The policies are in between A' and C' items.
- Orders for these items should be placed less frequently.
- The safely stock should be medium (3 months' consumption stock in general).
- B' items are subjected to moderate control

Control Policy for C' items:

- C' items are the low valued items. Therefore, the safety stocks of such items should be liberal. (more than 3 months)
- Annual or Half yearly orders should be placed to reduce paper work and ordering cost.
- In case of these items only routine checks are required

Steps in ABC Analysis:

- Calculate the annual usage of each item
- Calculate the annual usage value in terms of Rupees (₹) / Dollar ($) / Euro (€) / or any other currency
- Rank the items from highest annual usage to lowest value.
- Find the cumulative annual value of items in the ranked order
- Compute total value
- Find the % of cumulative value for each item
- A graph can be plotted between % of items on X axis and % of annual cumulative value on Y axis

1.2.3 Illustration

Following example illustrates the ABC analysis.

Table 1.2.3.1: ABC Classification Illustration (1)

Item	Annual Demand	Unit Cost ($)	Annual Usage ($)	Rank
A	30000	0.01	300	6
B	2800	1.50	4200	1
C	300	0.10	30	9
D	1100	0.05	550	4
E	400	0.05	20	10
F	2200	1.00	2200	2
G	1500	0.05	75	8
H	8000	0.05	400	5
I	3000	0.30	900	3
J	800	0.10	80	7

Table 1.2.3.2: ABC Classification Illustration (2)

Item	Annual Usage	Cumulative Usage	Cumulative %	Category
B	4200	4200	47.97	A
F	2200	6400	73.10	A
I	900	7300	83.38	B
D	550	7850	89.66	B
H	400	8250	94.23	B
A	300	8550	97.66	C
J	80	8630	98.57	C
G	75	8705	99.43	C
C	30	8735	99.77	C
E	20	8755	100	C

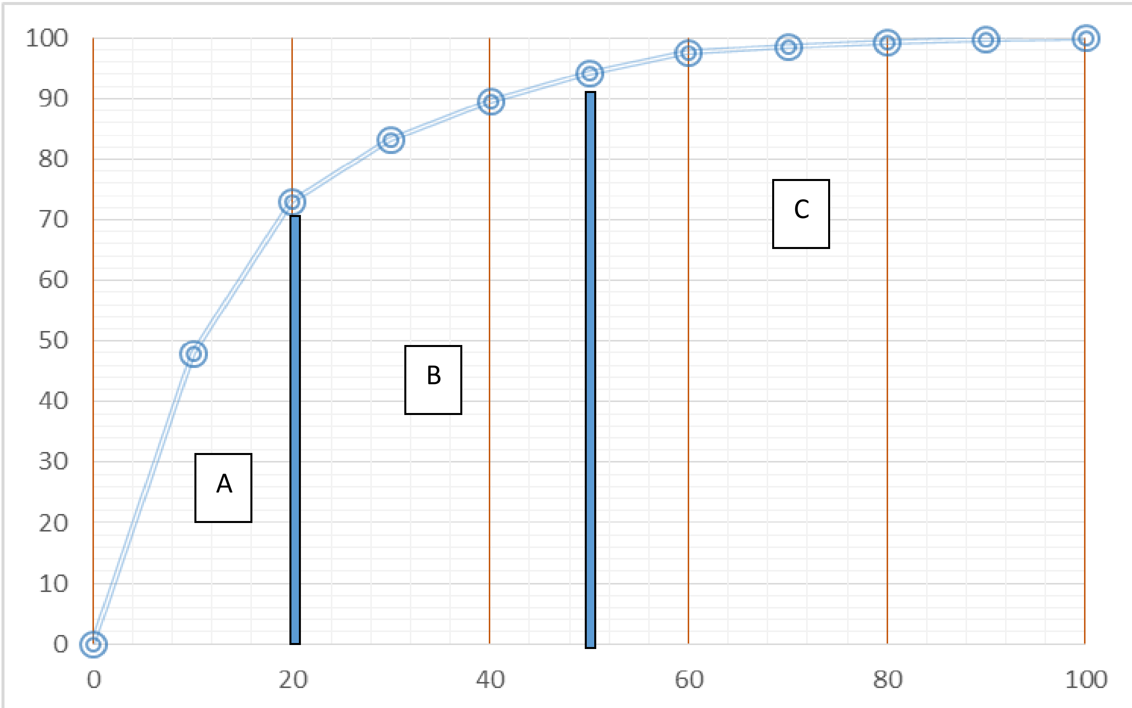


Fig. 1.2.3.1: ABC Classification (Pareto Principle)

The control policies for A, B, C items are based on the following principles:

- To keep capital tied up in inventories as low as possible.
- To ensure that all the materials would be available when required.
- ABC analysis can be effectively used in material management. The various stages where it can be used are as follows:
- Information of items which require higher degree of control
- To evolve useful recording strategy
- Stock records
- Priority treatment to different items
- Determination of safety stock items
- Stores layout
- Value analysis

1.2.4 Multi-Criteria Inventory Classification - Background and Motivation

Because of its easy-to-implement nature, applicability to numerous situations, empirically observed benefits, and remarkable effectiveness in many inventory systems, this approach is still popularly used in practice. However, the method has a serious drawback that may inhibit the effectiveness of the procedure in some situations. The criterion used in the conventional ABC classification is the annual dollar usage, so using one criterion may create problems of significant financial loss. Hence, only one criterion is not always very efficient measure for decision making. Therefore, multiple criteria decision making (MCDM) methods are used (Flores, Whybark, 1986, 1987) [**35, 36**]. Apart from annual dollar value, other criteria like lead time of supply, part criticality, availability, stock-out penalty cost, ordering cost, scarcity, durability, substitutability, reparability, risk of obsolescence, etc., have been taken into consideration (Flores, Whybark, 1986, 1987; Zhou and Fan, 2007) [**35, 36, 105**]. More studies have been carried-out on the field of MCIC in the past 20-25 years. So many different methods for classifying inventory and taking into consideration multiple criteria have been used and developed. The considerable body of research (as discussed in literature review section) shows many feasible ways of implementing multiple criteria ABC analysis in practice – some extremely simple while others quite sophisticated. The earlier researches based on Analytic Hierarchy Process (AHP) in this direction suffer by subjectivities associated with assignment of weights from expert judgment (Flores, Partovi, Kabir) [**37, 75, 51**]. The unavoidable biasness and familiarity of different experts with each of the criterion may lead to inconsistencies and unsatisfactory results. While optimization techniques and soft computing based approaches (rough set theory, artificial intelligence, clustering, etc.) by large were motivated to get away from subjective weights, it is realized that subjectivity in this context is a good thing to reflect the management priorities. In some of the approaches even though management decisions were additionally captured in the form of exogenously ranking the weights by the Decision Maker (DM), the interpretation of what did not seem to be simple and comprehensive enough in the perspective of further training requirements with them.

The second point of concern is that the several previous methods in the literature are fully compensatory in multiple criteria aggregation (Liu, 2016) [**63]**. This means that an item scoring badly on one or more key criteria may still be placed in the best class because these bad performances could be compensated by other criteria**.** Thus, it is necessary to consider the non-compensation in the multiple criteria ABC analysis. To the best of our knowledge the ABC classification problem with non-compensation among criteria has not been studied sufficiently. Some exceptions include the studies developed by Zhou and Fan [**105**], Hadi- Vencheh [**41**], and Lolli et al. [**60**].

The third known issue is the limitation associated with imposing fixed cardinality on each classification. The limit of cardinality should rest solely on manager's decision taking into consideration the company's vision, service level requirements for customers etc.

The need of a combinatorial but transparent way of dealing with weights of criteria is addressed in this paper. This is accomplished by integrating subjective weights considering fuzzy environment, two types of objective weights based on the concept of information entropy and standard deviation, and inter-attribute correlation effect based weights.

In the proposed approach, taking into account the requirement of the ABC analysis, the cardinality limitation of items in each class is specified in advance (A-20%, B- 30%, C- 50%). Hence, the previous approaches which do not have the limitation in place are not compared with the proposed method.

The Multi Objective Optimization on the basis of Ratio Analysis (MOORA) [**12**] and its updated form (MULTIMOORA) [**14**] methods are effective and simple MCDM techniques. In this dissertation, the inventory classification is based on both MULTIMOORA and a comprehensive form of the same method which is referred to as EXTENDED MULTIMOORA (XMULTIMOORA) method. The proposed methodology in solving the MCIC problem was developed through considering target-based normalization technique and integrated significant coefficients.

As one can note, the Reference Point prevents the MULTIMOORA from becoming a fully compensatory technique. Whereas the Ratio System and the Full Multiplicative Form are fully compensatory methods, the Reference Point is not of that kind. MULTIMOORA combines all these three subordinate methods and arrives at final ranking of the alternatives.

Chapter 2

2. Literature Review and Research Gap Analysis

Inventory classification has no aim in itself. The objective of the classification is to reduce the inventory value which in turn aims at simplifying the task of inventory management as a whole. Inventory control strategies have been rigorously explored over the last few decades. With the advancement of computer technologies and theoretical study of novel algorithms and use of cutting edge artificial intelligence (AI) techniques, answering the inventory management questions like what to order, how much to order, at what time to order, and at what stage to order has become the attractive and interesting field of research. There has been a huge body of literature in the field of inventory classification too over the last two decades or so.

The literature survey will be restricted to an esoteric selection of latest researches in inventory control and instead, will be focused on an exhaustive search in the area of multi criteria inventory classification techniques and methodologies.

2.1 Inventory Control

Researches in the field of inventory control are by and large motivated to apply different permutations and combinations of demand and lead time in type, and consider additionally the effects of different constraints like simple product or multiple products constraints, single or multiple periods, space constraint, perishability and lost sales constraints, requirement for backorders to name a few. The inventory management techniques evolved from simple Economic Order/Production Quantity (EOQ/EPQ) to fuzzy belief-based systems which are highly probabilistic and enormously conflicting in nature, and systems using AI techniques like Clustering, Rough Set, Genetic Algorithm (GA), Artificial Neural Networks (ANN), Rule-based Expert Systems (RBES), Case-based Reasoning Systems (CBR) etc.

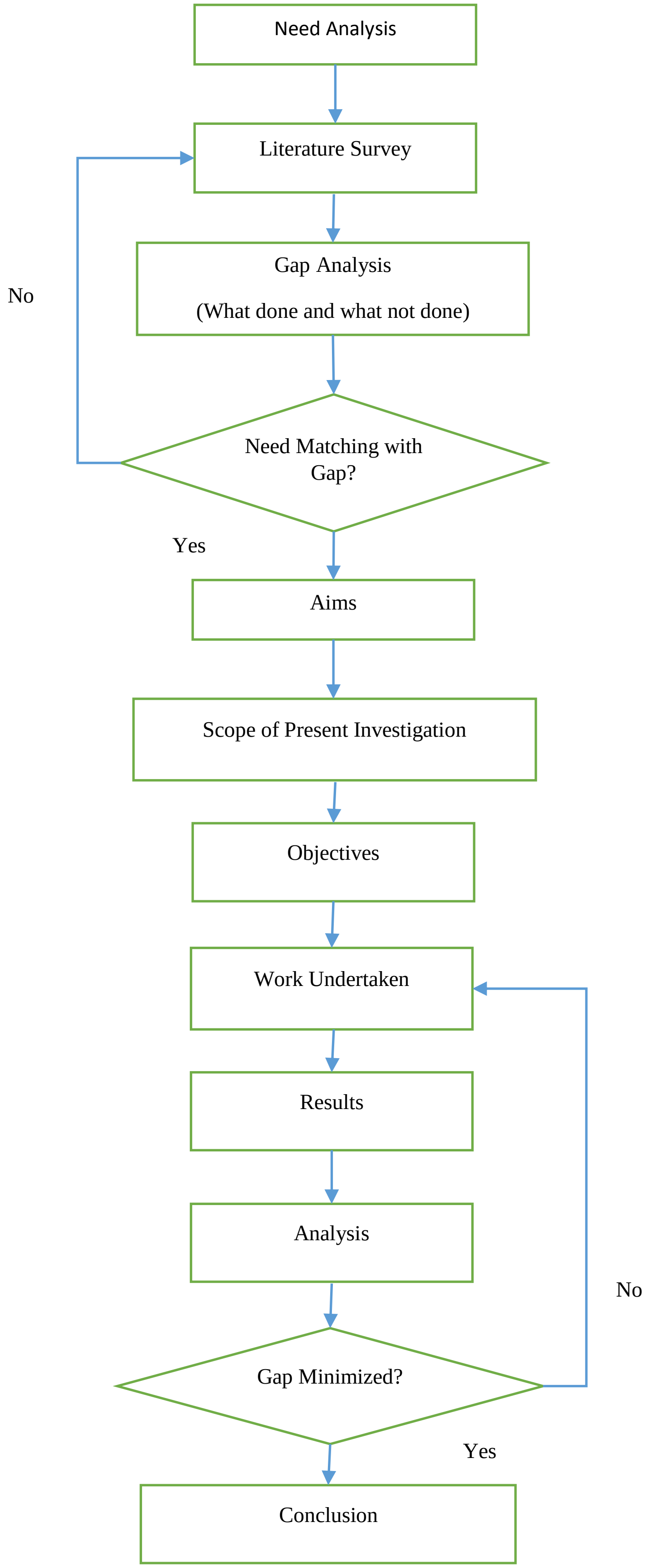


Fig. 2.1: Generic Research Flow Chart (GRFC)

Li et al. [**61**] devoted their research work to investigating inventory control problems under non-stationary and uncertain demand. A belief-rule-based inventory control (BRB-IC) method was developed, which can be applied in situations where demand and demand-forecast-error (DFE) do not follow certain stochastic distribution and forecasting demand is given in single-point or interval styles. The method could assist decision-making through a belief-rule structure that can be constructed, initialized and adjusted using both manager’s knowledge and operational data. An extended optimal base stock (EOBS) policy was proved for initializing the belief-rule-base (BRB), and a BRB-IC inference approach with interval inputs is proposed. A numerical example and a case study were examined to demonstrate potential applications of the BRB-IC method. These studies show that the belief-rule-based expert system is flexible and valid for inventory control. The case study also showed that the BRB-IC method can compensate DFE by training BRB using historical demand data for generating reliable ordering policy.

Mohammaditabar et al. [**68**] worked on inventory classification by simulated annealing method based on optimization of cost with respect to selection of control policies. According to them, some researchers have studied the appropriate inventory policy for each group and some worked on the inventory classification itself. Since both the actions of categorization and policy selection are sub-optimal solutions for the original problem of efficient inventory control policy, the authors proposed an integrated model to categorize the items and find the best policy simultaneously. As it is difficult to find a global solution, simulated annealing was used to find appropriate solutions. The model results were compared with the findings of other methods both for dissimilarity and total inventory values.

Bera et al. [**6**] worked on the real-world inventory control problems which are normally imprecisely defined and human interventions are often required in solving these decision-making problems. In their work, a realistic inventory problem with an infinite rate of replenishment over a prescribed finite but imprecise time horizon was formulated considering time dependent ramp type demand, which increases with time. Lead time was also assumed as fuzzy in nature. Shortages were allowed and backlogged partially. Two models were considered depending upon the ordering policies of the decision maker (DM). The imprecise parameters were first transformed to corresponding nearest interval numbers depending upon some distance metric on fuzzy numbers and then following the interval mathematics, the objective function for total profit from the planning horizon was obtained (which is an interval function). Then interval objective decision making problem was reduced to multi-objective problems using different approaches. Finally a fast and elitist multi-objective genetic algorithm (FEMOGA) was used for solving these multi-objective models to find Pareto-optimal decisions for the DM.

Şenyiğit et al. [**85**] considered multi-period single-item lot sizing problem under stochastic environment which has been tackled by few researchers and still remains in need of further studies. It is mathematically intractable due to its complex structure. In their work, an optimum lot-sizing policy based on minimum total relevant cost under price and demand uncertainties was studied by using various artificial neural networks trained with heuristic-based learning approaches; genetic algorithm (GA) and bee algorithm (BA). These combined approaches have been examined with three domain-specific costing heuristics comprising revised silver meal (RSM), revised least unit cost (RLUC), cost benefit (CB). It was concluded that the feed-forward neural network (FF-NN) model trained with BA outperforms the other models with better prediction results. In addition, RLUC was found the best operating domain-specific heuristic to calculate the total cost incurring of the lot-sizing problem. Hence, the best paired heuristics to help decision makers were suggested as RLUC and FF-NN trained with BA.

Tsai and Liu [**98**] presented a simulation-based decision support system for solving the multi-echelon constrained inventory problem. The goal was to determine the optimal setting of stocking levels to minimize the total inventory investment costs while satisfying the expected response time targets for each field depot. The authors derived new decision support algorithms to be applied in different scenarios, including small-sample and large-sample cases. The first case required that the set of alternative solutions was known at the beginning of the experiment, and the number of evaluated solutions might depend on the simulation budget (i.e., the time available to solve the problem). In the second case, the alternative solutions were generated sequentially during the searching process, and the algorithm could be terminated when the specified sampling budget was exhausted. Empirical studies were conducted to compare the performance of the proposed algorithms with other conventional optimization approaches.

Basu and Nair [**4**] applied mean-variance analysis for multi-period inventory control. Traditional inventory management focuses on risk-neutral decision making with the objective of maximizing the expected rewards or minimizing costs over a specified time horizon. However, for items marked by high demand volatility such as fashion goods and technology products, this objective needs to be balanced against the risk associated with the decision. Stochastic dynamic programming models have been extensively used for sequential decision making in the context of multi-period inventory management, but in the traditional way where one either minimizes costs or maximizes profits. Risk is implicitly considered by accounting for stock-out costs. Considering risk and reward simultaneously and explicitly in a stochastic dynamic setting is a cumbersome task and often difficult to implement for practical purposes, since dynamic programming is designed to optimize on one variable, not two. The authors developed an algorithm, Variance-Retentive Stochastic Dynamic Programming that tracks variance as well as expected reward in a stochastic dynamic programming model for inventory control.

Saracoglu et al. [**84**] formulated an approach for multi-product multi-period (Q, r) inventory models that calculated the optimal order quantity and optimal reorder point under the constraints of shelf life, budget, storage capacity, and "extra number of products" promotions according to the ordered quantity. Detailed literature reviews conducted in both fields uncovered no other study proposing such a multi-product (Q, r) policy that also had a multi-period aspect and which took all the aforementioned constraints into consideration. A real case study of a pharmaceutical distributor in Turkey dealing with large quantities of perishable products, for whom the demand structure varies from product to product and shows deterministic and variable characteristics, was presented and an easily-applicable (Q, r) model for distributors operating in this manner proposed. First, the problem was modeled as an integer linear programming (ILP) model. Next, a genetic algorithm (GA) [**38**] solution approach with an embedded local search was proposed to solve larger scale problems. The results indicated that the proposed approach yields high-quality solutions within reasonable computation times.

Noblesse et al. [**72**] studied that lot sizing decisions in inventory management trade-off the cost of placing orders against the cost of holding inventory. However, when these lot sizes are to be produced in a finite capacity production/inventory system, the lot size has an important impact on the lead times, which in turn determine inventory levels (and costs). The authors further studied the lot sizing decision in a production/inventory setting, where lead times are determined by a queuing model that is linked endogenously to the orders placed by the inventory model. Assuming a continuous review (s, S) inventory policy, they developed a procedure to obtain the distribution of lead times and the distribution of inventory levels, when lead times are endogenously determined by the inventory model. This procedure allows to determine the optimal inventory parameters within the class of (s, S) policies that minimize the expected ordering and inventory related costs over time. The authors also numerically showed that ignoring the endogeneity of lead times may lead to inappropriate lot sizing decisions and significantly higher costs. This cost discrepancy is very outspoken if the lot size based on the economic order quantity deviates significantly from desirable production lot sizes. In these cases, the endogenous treatment of lead times is of particular importance.

Çelebi [**19**] presented a case study to determine the optimal inventory levels in a spare parts distribution system. The authors developed a solution based on a Genetic Algorithm (GA) for an effective management of the distribution network of a Turkish automotive manufacturer under centralized control. They provided a specific approach to address the two-echelon inventory control problem in its combinatorial and sequential behavior, dealing with a large number of specific properties that are considered in practice. Findings of the case study revealed that the use of the proposed inventory control system may provide substantial cost savings to the case company.

Kumar and Goswami [**57**] worked on EPQ model in stochastic framework. As it is clear, when both lead-time and demand rate are deterministic and constant, then demand during the lead-time is constant, and is referred to as zero lead-time. Moreover, when either or both of them are random variables, then lead-time demand (LTD) is a random variable. In such a case, a crucial question is: “when the order should be placed?” On the other hand, the distributional information on demand may not always be available or there may be many distribution functions in the practice, which have same mean and variance, but their frequencies are different. In this study, the authors developed an EPQ model in stochastic framework, wherein the distribution function of demand is unknown, but the mean and variance are known. The inventory level is continuously reviewed, and an order is placed when it reaches the reorder level. To address the contingency of imprecise and non-stationary nature of variables in real world, the authors further extended the model in the fuzzy random environment by considering demand rate as a fuzzy random variable (FRV). Furthermore, they mathematically analyzed the cost function and proposed a heuristic procedure to find the global optimum.

Kazemi et al. [**53**] concentrated on modelling the decision maker's characteristics and their effect on his/her decisions and consequently on the planning outcome had not attracted much attention in the literature. In order to fill this research gap and model reality more accurately, the authors developed a new fuzzy EOQ inventory model with backorders that considered human learning over the planning horizon. The paper was an extension of an existing EOQ inventory model with backorders in which both demand and lead times are fuzzified. Here, the assumption of constant fuzziness was relaxed by incorporating the concept of learning in fuzziness into the model considering that the degree of fuzziness reduces over the planning horizon. The proposed fuzzy EOQ inventory model with backorders and learning in fuzziness had a good performance in efficiency. Finally, it is worth mentioning that learning in fuzziness decreases the total inventory cost.

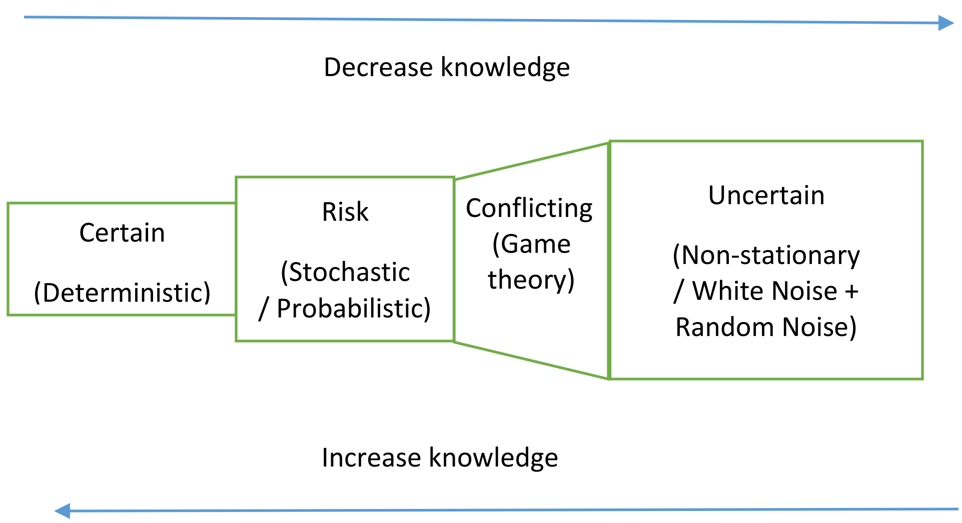


Fig. 2.1.2: Decision making complexity and evolution

2.2 Multi Criteria Inventory Classification Methods

Since Flores and Whybark (1987) [**36**] first proposed looking at more than one criterion, this has been an area of active research. There has been broad agreement that ABC analysis should consider more than one criterion. The methodology involves three main steps once the relevant criteria have been identified. The first is to determine what weights to assign to the different criteria and the second is to score each item on each criterion. If the criteria are measured on a variety of scales, this second step might involve rescaling the scores onto a 0-1 or 0-100 scale. The final step is to combine weights and scores to produce the weighted score. Over the years, three broad approaches have emerged to perform the weighting. It has been assumed that the different criteria permit unambiguous scoring of the items and that this is not an issue.

2.2.1 Subjective Weighting and Rating

"*Human being is a biological rarity*." The tacit knowledge hidden in an expert can never be all understood from all commensurable attributes. Thought it is hard to bring-out those unseen knowledge and experience in order to do improved and enriched decision making, enormous efforts are being put into it to make it happen. Analytic Hierarchy Process (AHP) is one such mechanism developed by SAATY [**83**], which has found applications in many fields.

This approach scores each type of inventory item on each criterion and then combines the different scores using a subjective weighting scheme. Many researchers have used the framework provided by the AHP to accomplish this (Flores, Olsen, & Dorai, 1992; Partovi & Burton, 1993; Gajpal, Ganesh, & Rajendran, 1994; Kabir, Hasin, & Khondokar, 2011; Braglia, Grassi, & Montanari, 2004) [**37**, **75**, **39**, **51**, **8**]. AHP relies on pairwise comparisons of criteria with respect to an overall objective to derive the weights for the criteria. Alternatives too can be compared pairwise with respect to each criterion. In this case, the alternatives are the various inventory items. Pairwise comparison of thousands of items with respect to each criterion is clearly a mammoth task. Instead, the alternatives are rated along each criterion and the weights are applied to these ratings. This is AHP in its ratings mode. The result is a weighted rating that can be used to rank the items prior to classifying them into different categories. The pairwise comparisons needed to determine the weights are performed by managers who are knowledgeable about the inventory items and the trade-offs among the different criteria to deal with. This is a one-time task as long as the set of criteria or the management preferences among them do not alter.

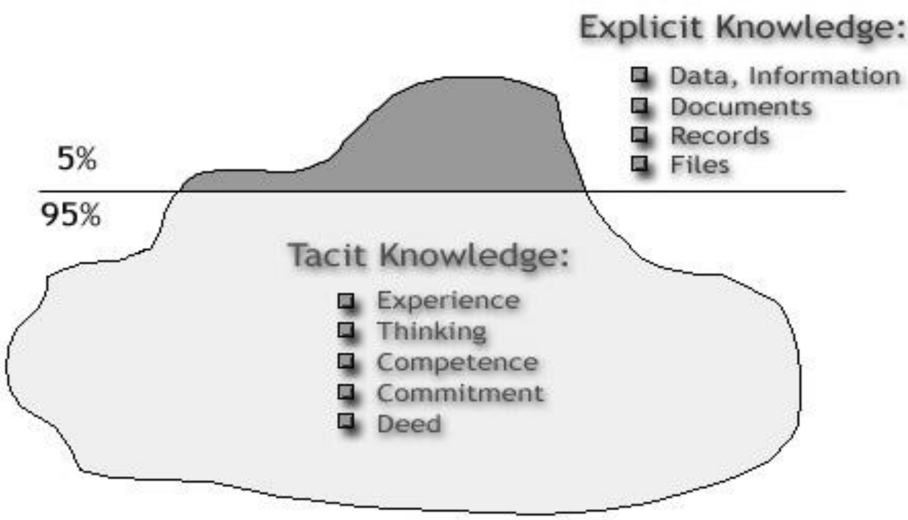


Fig. 2.2.1.1: Tip of ice berg resembling the metaphor of knowledge

AHP has found place in a variety of business decision-making phases and decision-makers also have found it intuitive and easy to use (Saaty, 1995; Zahedi, 1986; Vargas, 1990) [**83**, **104**, **99**]. Its theoretical underpinnings are strong and it has been incorporated into software (Expert Choice) that makes the implementation of the decision making process easy.

There are alternative ways of implementing rating and weighting schemes. Researchers might not have indicated them in the light of ABC analysis. For example, Multi-Attribute Utility Theory provides theory and methodology for assessing weights, rating alternatives, and combining weights and ratings to arrive at a final rating (or utility) for an alternative. The most robust and easy to use model is an additive model that is very similar to the AHP in its ratings mode. Software, as for example, SMART (Edwards & Barron, 1994) [**31**] also exists by which this process can be easily implemented.

Ketkar and Vaidya [**54**] developed ordering policy based on multiple inventory classification schemes. They considered ABC (traditional), VED, FSN, and SDE ratings for each product based on AHP and employed SAW (Simple Additive Weighting) method to arrive at final rankings

Balaji and Kumar (2014) [**3**] presented multi-criteria inventory classification technique for the classification of inventory of an automobile rubber component manufacturing industry. For estimating the value of inventory system an analytic hierarchy process had been used for dividing complex problems into sub problems based upon criteria and attributes. For classification of inventory items, Soylu and Akyol [**90**] incorporated the preference of the decision-maker into the decision making process. They applied two utility-function-based sorting methods to solve the MCIC problem. Bacchetti et al. [**2**] proposed a hierarchical multi-criteria classification method for inventory management purposes and applied it in a case study of the spare parts business of a household appliance manufacturer.

A novel fuzzy linear assignment method is developed [BAYKASUGLU] for multi-attribute group decision making problems. Since uncertain nature of many decision problems, the proposed method incorporates various concepts from fuzzy set theory such as fuzzy arithmetic and aggregation, fuzzy ranking and fuzzy mathematical programming into a fuzzy concordance based group decision making process. Fuzziness in the group hierarchy and quantitative type criteria are also taken into account. In order to present the validity and practicality of the proposed method, it is applied to a real life multi-criteria spare part inventory classification problem.

A web-based inventory classification system is proposed by Cakir and Conbolat [**16**] using fuzzy analytic hierarchy process (FAHP) technique.

Whichever method is opted for use, once the weights are obtained, the weighting and rating can be simply performed on a spreadsheet.

2.2.2 Linear Optimization

Other researchers (Ramanathan, 2006; Ng, 2007; Zhou & Fan, 2007; Hadi-Vencheh, 2010) [**78**, **71**, **105**, **41**] utilized a linear optimization approach to determining the weights. Their view is that the subjective inputs needed in the weighting and rating approach are cumbersome to obtain and undesirable because of possible inconsistencies. Instead, they would rather let the data itself decide weights that minimize some reasonable criteria or objective functions.

Ramanathan (2006) [**78**] solved a linear programming problem for each item in inventory to determine weights that maximize the weighted score for that item subject to constraints that the weighted sum for every item using this same set of weights is less than or equal to one. Thus, one immediate criticism of this model is that with more than a handful of items, the process will not only become more cumbersome but would also consume more time.

Ng (2007) [**71**] addressed this issue by proposing a DEA-type model similar to Ramanathan's. By this technique, the original optimization model is then transformed into another set of problems, the structure of which makes it easy to recognize the optimal solution without the use of a linear optimizer. Apart from endogenous derivation of final criteria weights by the optimal solver, the decision makers are also allowed to exogenously specify the exact values of weights in the form of a ranking of the weights associated with the criteria for each item, but this ranking is not critical to the mechanics of the method which can be implemented on a spreadsheet. At the end of the process, each item in inventory is given a rating which could then be used to perform the ABC analysis. Hadi-Vencheh (2010) [**41**] proposed a nonlinear extension to the Ng model.

A second criticism of Ramanathan's model is that the method can provide high scores to items that score highly on an unimportant criterion. Zhou & Fan (2007) [**105**] proposed a refinement which avoids this problem.

Liu & Huang (2006) [**64**] and Torabi, Hatefi, & Pay (2012) [**96**] presented modified versions of a DEA model to take both quantitative and qualitative criteria into account in ABC analysis

Park et al. [**73**] proposed a cross-evaluation-based weighted linear optimization model for the MCIC problem. Their proposed method incorporates a cross-efficiency evaluation approach into a weighted linear optimization model for finer classification of inventory items.

Mitchell et al. [**67**] proposed a model which simultaneously optimizes the number of inventory groups, their corresponding service levels and assignment of SKUs to groups, under limited inventory spending budget. The methodology adopted provides inventory and purchasing managers with a decision-support tool to optimally exploit the tradeoff among service level, inventory cost and net profit.

2.1.3 Clustering, Genetic Algorithms, and Neural Networks

A third approach to categorization for the purpose of ABC analysis relies on the methods of artificial intelligence and data-mining. All these methods start with a training set – a set of inventory items that have already been classified on the basis of multiple criteria as A, B, or C, by managers who are familiar with them - to learn the appropriate transformations necessary to combine criteria values and determine cut-offs.

Guvenir and Erel (1998) [**1**] proposed an approach called GAMIC which starts with the framework of AHP to deal with multi-criteria ABC analysis. GAMIC uses genetic algorithms [**38**] to learn from the training set the weights to be assigned to each criterion and, further, to determine the cut-offs between the three categories. Unknown weights and cutoffs are encoded as chromosome vectors that result in a particular classification. Given this encoding scheme, the method applies standard genetic operators (reproduction, crossover, and mutation) to create new generations of solutions. Each chromosome (solution) is tested using a fitness function and the best solutions become members of the next generation. This process continues iteratively until the algorithm converges on the training set; i.e., provides weights and cut-offs that reproduce (for the training set) the decision-maker's categorizations. These weights and cut-offs can then be used for other inventory categorization tasks. In their comparisons, their algorithm performed better than AHP – in the sense of having fewer misclassifications when compared with the decision-maker's classifications of the items. One limitation of this approach is that criteria can only be quantitative.

Partovi and Anandarajan (2001) [**74**] followed a similar process but using Artificial Neural Networks (ANN) to solve an inventory classification problem with four criteria - unit price, ordering cost, demand range, and lead time. The inputs to the network are values of these criteria for different inventory items. The output of the network is a categorization of a set of criteria values as A, or B, or C. Thus, their network consists of four input neurons (one for each input criterion), 16 hidden neurons, and 3 output neurons (one for each inventory category). Two kinds of learning algorithms were used - back propagation and genetic algorithms. Once the network was trained, it was used on stored data (validation) as well as on test sample. Results (% misclassification compared with decision-maker categorization) were encouraging and proved ANN to be viable of performing multi-criteria ABC analysis. Yu, 2011 [**102**] also used ANN for solving MCIC problem.

Gulsen and Ozkan (2013) [**40**] treated ABC analysis as a clustering problem in which the inventory items that have to be categorized are partitioned into 3 "fuzzy" clusters by minimizing some appropriate clustering function. Fuzzy clustering is the appropriate technique to use given that it is possible for some inventory items to belong to more than one cluster. The center of a cluster is described by an n-dimensional vector, where n is the number of criteria to be used for the ABC analysis. Each inventory item is similarly an n-dimensional vector. Membership of the clusters is indicated by a membership value that is between 0 and 1. The objective to be minimized is the distance between the current centers of each cluster and each inventory item weighted by the membership value modified by a "fuzzifier". The algorithm starts with initial values for the cluster centers, followed by calculating a membership value for each inventory item. This allows recalculation of the cluster centers. If the new cluster centers are within some ε of the current cluster centers, the algorithm stops; otherwise, the next iteration begins with the new cluster centers. Once the stopping rule has been met, the output of the algorithm is the membership value for each item for each cluster. An item is assigned to a cluster based upon the highest of its membership values. Thus, at the end of the process, three (for three categories) clusters will have been identified. The next step is to label the clusters appropriately. Labeling is done on the basis of the average criterion value within a cluster. This is calculated by adding all the criterion values for all items within a cluster and dividing by the number of items in the cluster. The cluster with the highest average criterion value is labeled A, the next highest as B, and the last one as C. In actual application of the method, it is suggested that item ratings on each criterion be rescaled to a 0-1 scale using a simple linear transform.

In concept, each of the above three approaches will produce an ABC categorization with high reliability; in other words, there is a high degree of overlap with the categorizations of human decision-makers.

Tsai and Yeh [**97**] proposed a particle swarm optimization approach for the multi-criteria ABC problem.

Lolli et al. (2012) [**60**] used a K-Means clustering for solving the MCIC problem, which is very similar to the clustering method which works on the principle of minimizing the overall summation of centroid distances, then it solves the fully compensatory issue by utilizing a veto rule on top of the proposed algorithm.

Liu et al. [**63**] employed a classification approach based on the outranking model. They used a combination of clustering and simulated annealing to find the optimal classification for the inventories.

2.1.5 Other Approaches

Other approaches have been proposed to the ABC categorization problem. Rough set theory (Pawlak, 1991) [**76**] has been used by Gomes and Ferreira (1995) [**34**] and Chen, Li, Levy, Hipel, and Kilgour (2008) [**25**] to perform the ABC categorization with the use of training sets. Bhattacharya, Sarkar, and Mukherjee (2007) [**7**] presented a distance-based consensus method using the concepts of ideal and negative ideal solutions from the TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) approach to ranking. They demonstrate the practicality of their approach by applying it to the inventory items of a pharmaceutical company. An application of the case-based distance model to solve the MCIC problem can be seen in Chen et al. [CHEN]. A novel approach based on loss profit is proposed to deal with ABC analysis [**101**]. Similarly, Rezaei and Dowlatshahi [**80**] present a rule-based method for classifying inventory items in a multi-criteria setting. However, these methods are too complicated to be applied in practice so that they may not be easily understood by inventory managers. Rezaei and Salimi **[81**] developed an interval programming model for ABC inventory classification. Their proposed model provides optimal results instead of an expert-based classification and it does not require precise values of item parameters.

Other statistical techniques use more than one characteristic. When considering a pair of characteristics, researchers used tables, matrices or graphical techniques to illustrate their classification. For instance, D'Alessandro and Baveja (2000) [**27**] plotted all products on a graph with mean weekly demand volume along one axis, and the associated coefficient of variance on the other. For each quadrant in this graph, a production strategy is determined. Syntetos *et al.* (2005) [**94**] distinguished four quadrants based on the mean inter-demand interval and the squared coefficient of variation of the demand sizes (when demand occurs). The cut-off values for their quadrants are based on a comparison of theoretical MSEs (mean squared errors) of different forecasting methods

Another interesting technique is the decision tree. Here, the classification was performed in a stepwise fashion, one characteristic at a time. For instance, Porras and Dekker (2008) [**77**] first looked at the criticality of the product, then at the demand volume, and finally at price. For each combination, a specific inventory management procedure was developed. Kobbacy and Liang (1999) [**55**] included statistical tests for each step in a decision tree to determine, for example, whether there is a trend (e.g. seasonality) in the demand pattern.

In the successive works of Kaabi et al. [**49, 50**], two classification models are proposed - one based on TOPSIS and Continuous Variable Neighborhood Search (CVNS), .and the other based on Genetic Algorithm (GA) [**38**] to infer the criteria weights. The two classifications of items obtained by these models optimize respectively the two following objective functions: The Total Relevant Cost (TRC to be minimized) and the Inventory Turnover Ratio (ITR to be maximized).

Chu et al. [**26]** showed a technique of controlling inventory by combining ABC analysis and fuzzy classification.

2.3 MOORA and MULTIMOORA

In one way or the other, MCIC is typically a multi-criteria decision making problem. This involves several criteria or factors on which decision maker's knowledge is usually vague and imprecise. There are several MCDM methods available in literature. Our work was focused on

MOORA and MULTIMOORA as it was recently gaining immense interests among the researchers in various domains and neither MOORA nor MULTIMOORA was ever used considering the MCIC problem.

The MULTIMOORA method (Brauers and Zavaskas 2010) [**14**] is a recently introduced new MCDM method based on multi-objective optimization by ratio analysis. (MOORA) (Brauers and Zavaskas, 2006) [**12**]. Due to its characteristics and capabilities, the use of MULTIMOORA has been increasing in the literature. Brauerrs et al. (2013) [**11**] employed the MULTIMOORA to analyze the construction sectors of European countries from a macroeconomic point of view by comparing construction market variations appeared during recession. Streimikiene and Balezenitis (2013) [**93**] proposed a MCDM methodology by using the MULTIMOORA for assessing mitigation strategies for climate changes and applied it for determining the optimal mitigation policies in Lithuania. Brauers et al. (2012) [**10**] used the MULTIMOORA to estimate economic worth of European Union (EU) member states towards 2020. Karande and Chakraborty (2012) [**52**] used the ratio system, the reference point approach, and the full multiplicative form to solve some of the common material selection problems. Mandal and Sarkar (2012) [**65**] used the fuzzy MULTIMORRA for comparing the intelligent systems of conflicting nature. Lie et al. (2014) [LIE] used the MULTIMOORA method under fuzzy environment to evaluate the failure modes to address the problem of infant abduction from hospital. Mishra et al. (2015) [**66**] applied fuzzy integrated MULTIMOORA method towards supplier/partner selection in agile supply chain. Hafezalkotob and Hafezalkotob (2015) [**43**] successfully used the target-based MULTIMOORA with integrated significant coefficients is solving material selection in biomedical engineering. Dey et al. [**29**] presented a supplier selection strategy selection methodology based on fuzzy MULTIMOORA.

2.4 Research Gap Analysis

A research gap analysis should be used to analyze gaps in research processes and the gulf between the existing outcome and the desired outcome. This step process can be illustrated by the example below:

Identify the existing process: fishing by using fishing rods

– Identify the existing outcome: someone can manage to catch 20 fish per day

– Identify the desired outcome: she/he wants to catch 100 fish per day

– Identify the process to achieve the desired outcome: she/he can use an alternative method such as using a fishing net

– Identify and document the gap: it is a difference of 80 fish

– Develop the means to fill the gap: she/he acquires and uses a fishing net

– Develop and prioritize requirements to bridge the gap

A gap analysis can also be used to compare one process to others performed elsewhere, which are often identified through benchmarking. In this usage, one compares each process side-by-side and step-by-step and then notes the differences. One then analyzes each deviation to determine if there is any benefit to changing to the alternate process. The results of this analysis (in the context of the benefits and detriments of changing processes) may support the maintenance of the current process, the wholesale adoption of an alternate process, or a fusion of different aspects of each process.

The gap analysis as performed in multiple criteria ABC inventory classification, which is scope of our research is presented in the Table 2.4.1.

Table 2.4.1: Gap Analysis of MCIC Literature

Technique	Authors(Year)	Work Done	Limitations
AHP	Flores et al. (1992) [**37**]	Used AHP to reduce multiple criteria to single and consistent measure considering multi objectives	Qualitative criteria not considered. May suffer from subjective biasness and imprecise specifications.
	Gajpal et al. (1994) [**39**]	Various modes of criteria considered in AHP and absolute measurement of part criticality achieved	Did not consider other classification techniques. May suffer from subjective biasness and imprecise specifications.

Technique	Authors(Year)	Work Done	Limitations
AHP	Partovi and Burton (1993) [75]	Saaty's AHP to classify inventory items	No fuzziness considered. May suffer from subjective biasness and imprecise specifications.
FAHP	Kabir et al. (2011) [**51**]	FAHP used to classify materials	No integration with heuristic MCDM methods
SAW	Ketkar and Vaidya (2014) [**54**]	Simple Additive Weighting method to classify items from different classification schemes like ABC, VED, FSN, and HML	Fuzziness not considered. Too much expert effort required if other classification scheme results are not obtained beforehand. May suffer from subjective biasness and imprecise specifications.
TOPSIS	Bhattacharya et al. (2007) [**7**]	Used TOPSIS model to classify items as A, B, and C	Fuzzy classifier not considered. May suffer from subjective biasness and imprecise specifications.
Distance modeling	Chen et al. (2008) [**24**]	Used case-based distance function to evaluate the rankings of items	Did not consider fuzziness. May suffer from subjective biasness and imprecise specifications.
ABC Pareto	Reid (1987) **[79]**	Used ABC analysis for items in a Respiratory Therapeutic Unit	Only single criteria like annual dollar consumption value considered. Does not comply with managerial judgment of giving high importance to the slow moving but critical moderate value items

Technique	Authors(Year)	Work Done	Limitations
Bi-Criteria ABC	Flores and Whybark (1986) [**35**]	Bi-criteria approach of ABC analysis. One criterion at a time and then combined	A simultaneous approach missing. Fuzziness not considered.
	Flores and Whybark (1986) [**35**]	Bi-criteria approach of ABC analysis. One criterion at a time and then combined	A simultaneous approach missing. Fuzziness not considered.
	Flores and Whybark (1987) [**36**]	Bi-Criteria ABC analysis. One criterion at a time and then combined	Multiple criteria not considered. No fuzziness taken into account.
	Harhalakis et al. (1989) [**44**]	A dynamic planning and control system for inventories and raw materials	Multiple criteria not considered. No fuzziness taken into account.
Graphical /2x2 matrix	D'Alessandro and Baveja (2000) [**27**]	Used a graphical plotting technique based on demand and coefficient of covariance. For each quadrant on the graph, a production strategy is determined for inventory control.	Computationally expensive procedures. Did not consider multiple criteria.

Technique	Authors(Year)	Work Done	Limitations
Graphical /2x2 matrix	Syntetos et al. (2005) [**94**]	Used graphical plotting technique considering mean inter-demand interval and the squared coefficient of variation of the demand sizes	Computationally expensive procedures. Did not consider multiple criteria. The cut-off values for quadrants are not easy to compute.
Decision tree	Kobbacy and Liang (1999) [**55**]	Classification performed in a stepwise fashion. Included statistical tests for each step in a decision tree to determine, for example, whether there is a trend (e.g. seasonality) in the demand pattern	The cut-off/threshold values for each tree node deciding the subsequent branches is the key, which is not so straight forward. Managerial preference cannot be combined.
	Porras and Dekker (2008) [**77**]	Used decision tree of multiple criteria like criticality, demand, and price and took appropriate inventory management strategy based on item's position the tree structure.	The cut-off/threshold values for each tree node deciding the subsequent branches is the key, which is not so straight forward. Managerial preference cannot be combined.

Technique	Authors(Year)	Work Done	Limitations
Cluster analysis	Canetta et al. (2005) [**17**]	Used self-organizing map (SOM) -based clustering approach	Computationally expensive. Introduction of new items leads to re-clustering. Managerial preference cannot be combined.
	Ernst and Cohen (1990) [**33**]	Used the concept of operation related groups (ORG) by means of statistical clustering utilizing full range of significant attributes	Computationally expensive. Introduction of new items leads to re-clustering. Managerial preference cannot be combined.
Optimization	Chakravarty (1981) [**20**]	Sorted items according to annual usage value(price), and classified by dynamic programming	Qualitative attributes not considered
	Ng (2007) [**69**]	Used DEA type model	Considered only linear optimization.
	Ramanathan (2006) [**78**]	Used Weighted Linear optimizer scheme	Meta-heuristic not used. Cumbersome and also time-consuming.
	Stanford and Martin (2007) [**92**]	Used Integration of control rules and ABC classification	A simultaneous approach is missing.

Technique	Authors(Year)	Work Done	Limitations
Optimization	Zhou and Fan (2007) [**105**]	Extended Ramanathan's by most favourable and least favourable	AHP not used
	Hadi-Vencheh (2010) [**41**]	Non-linear extension to Ng's model of optimization.	Qualitative criteria not considered
	Hadi-Vencheh and Mohamadghasemi (2011) [**42**]	Integrated fuzzy analytic hierarchy process-data envelopment analysis for multiple criteria ABC classification	The method does not consider integration with modern AI techniques
	Torabi et al. (2012) [**96**]	Modified version DEA model using linear optimization for both quantitative and qualitative criteria	The method does not consider integration with modern AI techniques. Fuzziness not considered
Neural networks	Huiskonen et al. (2005) [**46**]	Used ANN for evaluating importance of C items for customer specific factors	ABC analysis in general was missing as it assumed A and B class items correctly classified.
	Partovi and Anandarajan (2002) [**74**]	Classified inventory items based on artificial neural networks modelling integrated with GA	ANN is not hybridized with any MCDM technique. Fuzziness not considered. Prerequisite of training data already classified

Technique	Authors(Year)	Work Done	Limitations
Artificial Intelligence	M. C. Yu (2011) [**102**]	Compared results based on AI techniques	Prerequisite of training data already classified. AI techniques not so comprehensive for training purpose.
Genetic Algorithm	Güvenir and Erel (1998)	Used GA for Multi-criteria classification	GA is not hybridized with AHP. Prerequisite of classified training data.
Rough Set	Gomes and Ferreira (1995) [**34**]	Used rough set theory in determining the ABC classification	Prerequisite of training data already classified.
Simulated Annealing	Mohammaditabar et al (2012) [**68**]	Integrated inventory classification with inventory control strategy for fulfilling multiple objectives	Did not consider fuzziness. Highly cumbersome in nature
Rule-based system	Rezaei and Dowlatshahi (2010) [**80**]	Used inventory manger's set of rules for ranking inventory items	Did not consider fuzziness. Did not compare with other MCDM methods.
Outranking Model	Liu et al (2016) [**63**]	Used peer outranking model to classify inventory items	
Veto K-Means	Lolli et al. (2012) [**60**]	Used K-Means algorithm along with a Veto rule	Cardinality limitation not imposed on ABC analysis by the use of Veto rule
PSO (Particle Swarm Optimization)	Tsai and Yeh (2008) [**97**]	Particle Swarm Optimization method for inventory classification	Computationally expensive. Results not compared with GA or AHP methods.

2.5 Aims, Scope of Work, and Objectives

Aims:

The final aim of any work in operations management is smooth, sustainable, and profitable operations. It has to take the comparative economic advantages by virtue of prudent management decisions in a highly competitive environment with an additional inclination towards better social, political, and environmental wellbeing. The inventory classification, which in broader sense often coined as Stock Keeping Unit (SKU) classification, is no exception to that.

SKU classification is a matter of importance for any business of any size from very large to very small. The huge body of research directed towards achieving a desired classification speaks out in this favour. For running a production process smoothly, there must be proper planning, hassle-free on-demand procurement, efficient and effective manufacturing, prompt delivery, and monitoring and controlling of the entire process for getting the desired customer satisfaction.

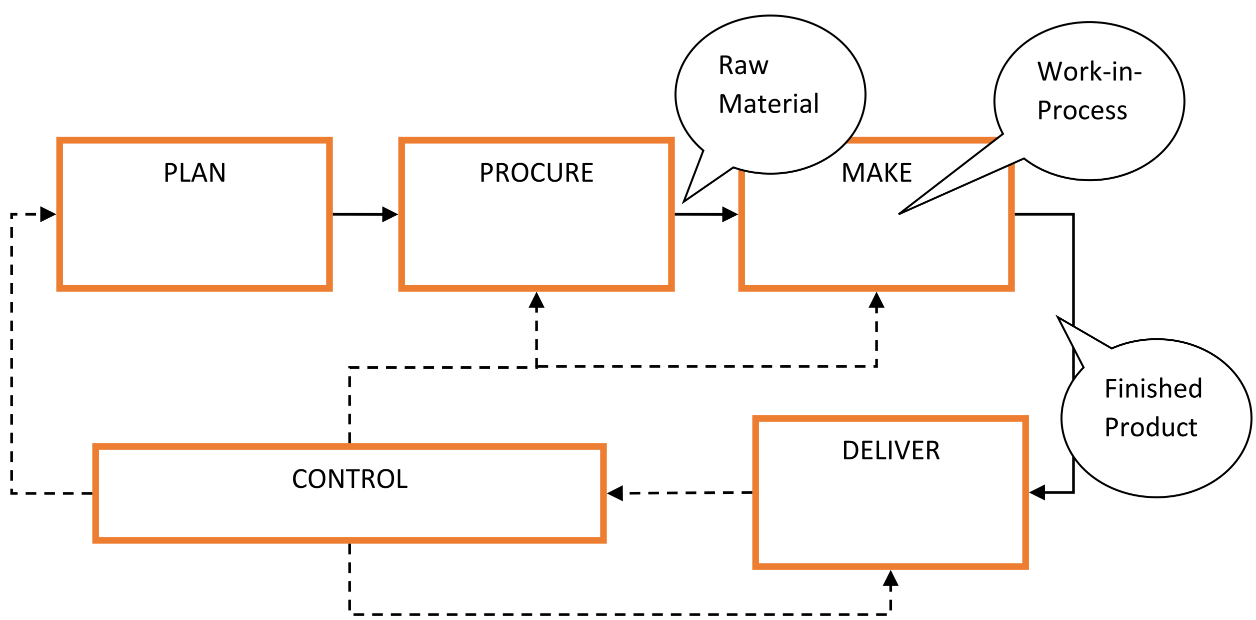


Fig. 2.5.1: Generic Production Process and Inventory

The aim of the SKU classification has three major facets.

Firstly, this enables the forecasting to be accomplished without much complications and with minimum costs. The most important items need to be forecasted with greater attention. High order statistical computation and managerial judgment would be highly solicited in the forecasting process. The use of computer technologies along with AI algorithms may be used for this purpose. The least important classification needs much simpler forecasting methods like moving average (MA).

Secondly, this strengthens inventory management. The need for inventory control policy does not hold equally good for all the items in the store. A category products need to be regularly monitored for availability of stock and ordering should be placed well in advance so that stock-out does not occur during replenishment period. C type items do not need any perpetual review of stock keeping. B type items need intermediate attention considering their moderate importance level.

Thirdly, this aims to define production strategy in a more convincing and simplified manner. The classification helps to decide optimum levels of production for different items and product mix, the need for inspection and control for different group of items, the requirement of quality adherence for different items and delivery schedule in the context of customer satisfaction.

Scopes of Present Work:

The aim of the classification can be any combination of the three aims discussed above. The primary focus of the classification is inventory management which defines the scope of the present work and has been explained in some detail in earlier chapters.

Hence, the considerations as regards to the optimum level of stock (S), the reorder point (r), any requirement of safety stock (ss), and/or the ordering schedules/intervals (T/n), and/or ordering quantity (Q), and most importantly the service level target agreement within the organization for dependent demand and between the organization and the customer for independent demand are the scopes of the present work undertaken and discussed in the dissertation.

The advantages of the classification will be compared in the context of the inventory management considering the different models and levels of control policies and costs associated with it.

Objectives:

The broad objective of the proposed classification model is development of an efficient and cost effective algorithm and scheme which serves four major and four minor goals as follows:

Major Goals

Cost Effective:

The classification should lead to considerable savings of inventory and related management costs. The simple EOQ model can be used with considerations of shortage-costs and safety stock considering risk in demand and lead time.

Accurate Classifier:

The classification should comply with the clustering algorithm within a certain degree of deviations. It must be validated with some known techniques for comparisons or it can be directly validated with sample trained data before implementation.

Minor Goals

Easy to Implement:

The classification method should be easily implemented on an average computer system with reasonable speed of computation and generation of results. The complexity of the algorithm can be expressed in terms of number of criteria *(m)* and number of items *(n)* and cross-checked so that it does not exceed $O\ (m \times n)$ in all cases.

Easy to Understand:

The classification method, when implemented, should be easily understandable and can be handled by any store manager or shop-floor executive.

Dynamic:

It should be dynamic to accommodate new items in store or delete old (not to be used) items from the store.

Scalable:

It should be ideally scalable and extensible considering addition of new parameters or changes in the existing setup.

Chapter 3

3. Proposed Methodology

3.1 Fuzzy Set Theory

Definition 3.1.1 A fuzzy set Ã in a universe of discourse X is characterized by a membership function $\mu_{\tilde{A}}(x)$, which maps each element $x \in X$ to a real number in the interval [0, 1]. The function value $\mu_{\tilde{A}}(x)$ is termed as grade of membership x in $\tilde{A}$ (Zadeh, 1965, 1975) [**103**].

Definition 3.1.2 A fuzzy number is a fuzzy subset in a universe of discourse X that is both convex and normal (Liu, 2014) [**62**]. A fuzzy set Ã of the universe of discourse X is convex if and only if for all x_1, x_2 in X, $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda) x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$, where $\lambda \in [0,1]$. A fuzzy set Ã of the universe of discourse X is called a normal fuzzy set implying that $\exists x_i \in X$, $\mu_{\tilde{A}}(x_i) = 1$.

Triangular and trapezoidal fuzzy numbers are the most common used fuzzy numbers both in theory and practice. In fact a triangular fuzzy number is a special case of trapezoidal fuzzy number. When the two most promising values are the same, trapezoidal fuzzy number reduces to a triangular fuzzy number. Thus, trapezoidal fuzzy numbers are adopted for representing the linguistic variables in this study for simplifying the discussion and without the loss of generality.

Definition 3.1.3 A positive trapezoidal fuzzy number $\tilde{A}$ can be denoted as (a_1, a_2, a_3, a_4), shown in Fig. 3.1.3.1. The membership function $\mu_{\tilde{A}}(x)$, is defined as (Liu, 2014) [**62**]

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a1 \\ \dfrac{x - a1}{a2 - a1}, & a1 \leq x \geq a2 \\ 1, & a2 \leq x \geq a3 \\ \dfrac{x - a4}{a3 - a4}, & a3 \leq x \geq a4 \\ 0, & x > a4 \end{cases} \tag{1}$$

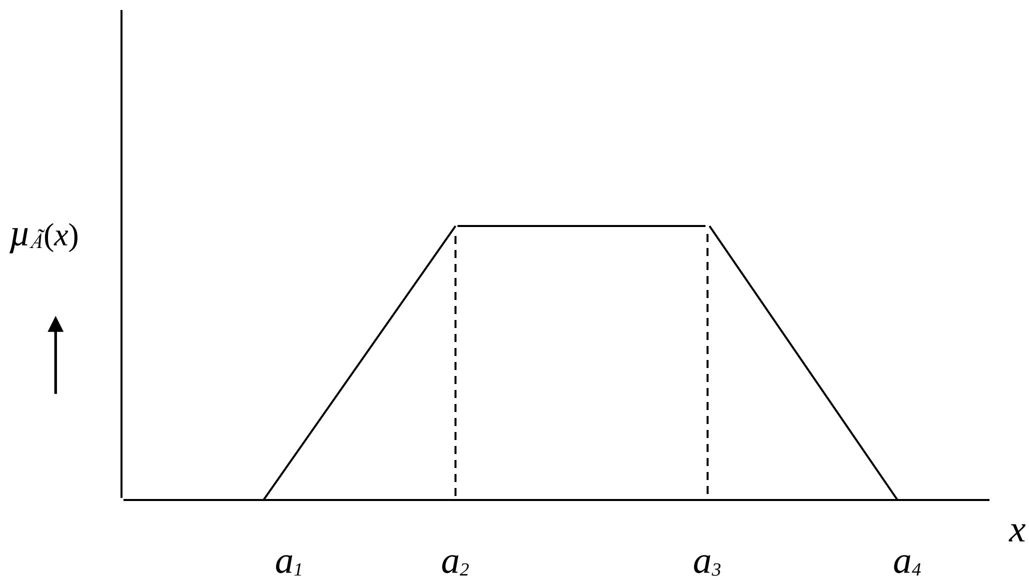


Fig. 3.1.3.1: Trapezoidal fuzzy number $\tilde{A}$

For a trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$, if $a_2 = a_3$ then $\tilde{A}$ is called a triangular fuzzy number. Give any two positive trapezoidal fuzzy numbers $\tilde{A} = (a_1, a_2, a_3, a_4)$, $\tilde{B} = (b_1, b_2, b_3, b_4)$, and a positive real number r, the basic operations of these two fuzzy numbers can be expressed as follows:

$$\tilde{A} \oplus \tilde{B} = [a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4] \quad (2)$$

$$\tilde{A} \ominus \tilde{B} = [a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4] \quad (3)$$

$$\tilde{A} \otimes \tilde{B} = [a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4] \quad (4)$$

$$\tilde{A} \oslash \tilde{B} = [a_1 / b_4, a_2 / b_3, a_3 / b_2, a_4 / b_1] \quad (5)$$

$$\tilde{A} \otimes r = [a_1 r, a_2 r, a_3 r, a_4 r] \quad (6)$$

Definition 3.1.4 Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers, then the distance between them can be calculated by using the vertex method as

$$d\left(\tilde{A}, \tilde{B}\right) = \sqrt{\frac{1}{6}\left[(a_1 - b_1)^2 + 2(a_2 - b_2)^2 + 2(a_3 - b_3)^2 + (a_4 - b_4)^2\right]} \quad (7)$$

Definition 3.1.5 A linguistic variable is a variable whose values are linguistic terms (Zadeh, 1975) [**103**]. It is very useful in dealing with situations which are too complex or too ill-defined to be reasonably described in conventional quantitative expressions.

Definition 3.1.6 An important step regarding the use of the fuzzy members is the defuzzification task which transforms a fuzzy number into a crisp value (essentially from a linguistic term to a quantifiable term). The centroid method is widely used for defuzzification, which can be presented as follows: (Liu et al., 2014) [**62**]

$$\bar{x}(\tilde{A}) = \frac{\int x\mu\tilde{A}(x)}{\int \mu\tilde{A}(x)} \tag{8}$$

In Eq. (8) $\bar{x}$ ($\tilde{A}$) is the defuzzified value. For a trapezoidal fuzzy number $\tilde{A}$ = (a1, a2, a3, a4), the centroid based defuzzified value turns-out to be

$$\bar{x}(\tilde{A}) = \frac{1}{3}\left[(a_1 + a_2 + a_3 + a_4) - \frac{(a_4a_3 - a_1a_2)}{(a_4 + a_3) - (a_1 + a_2)} \right] \tag{9}$$

Suppose, there are l cross-functional members TM_k (k = 1, 2…l) in an organization concerned with inventory management problem and responsible for prioritizing criteria for inventory classification problem.

Each team member TM_k is given a weight $\lambda_k > 0$ (k = 1, 2… l) satisfying $\sum_{k=1}^{l} \lambda_k = 1$ to reflect his/her relative importance in the process computed based on individual qualification, experience, and other related parameters.

Let, $\tilde{w}_j^k = (w_{j1}^k, w_{j2}^k, w_{j3}^k, w_{j4}^k)$ is fuzzy weight of j^{th} criterion given by TM_k.

The aggregated fuzzy weight of the criterion is as follows:

$$\tilde{w}j = (w_{j1}, w_{j2}, w_{j3}, w_{j4})$$
$$= \left(\sum_{k=1}^{l} \lambda_k\left(w_{j1}{}^k\right), \sum_{k=1}^{l} \lambda_k\left(w_{j2}{}^k\right), \sum_{k=1}^{l} \lambda_k\left(w_{j3}{}^k\right), \sum_{k=1}^{l} \lambda_k\left(w_{j4}{}^k\right)\right) \tag{10}$$

3.2 The MULTIMOORA Method

The MULTIMOORA method consists of two components that are the ratio system and the reference point approach. However, the updated MOORA method, named MULTIMOORA, consists of three parts i.e. the ratio system, the reference point approach, and the full multiplicative form. Similar to all other MCDM methods, a decision matrix X is considered for the MULTIMOORA method. The arrays of the decision matrix, i.e.x_{ij}, denote the responses of alternative $A_i$to attribute $a_i$called alternative ratings, i = 1, 2… m and j = 1, 2… n:

$$X = \quad [x_{ij}]_{m \times n} \tag{11}$$

The decision matrix is normalized to obtain comparable and dimensionless values named as normalized alternative ratings x_{ij}^*. Normalization typically is a comparison between an alternative rating on a certain criterion or attribute, as a numerator, and a denominator that is representative of for all alternative ratings on that criterion. Brauers and Zavadskas [**13**, **14**] recommended the following normalization ratio for the MULTIMOORA method.

$$x_{ij}^* = \frac{x_{ij}}{\sqrt{[\sum_{i=1}^{m}(x_{ij})^2]}} \tag{12}$$

Relative significant coefficients of criteria, i.e., w_j, can be considered in formulation of the MULTIMOORA method. Significant coefficients of criteria satisfy $\sum_{j=1}^{n} w_j = 1$

3.2.1 The Ratio System

The normalized ratings are multiplied by significant coefficients. The resultants are added for beneficial criteria and subtracted for non-beneficial criteria to obtain the assessment value of the ratio system Y_i [9]

$$Y_i = \sum_{j=1}^{g}(w_j x_{ij}{}^*) - \sum_{j=g+1}^{n}(w_j x_{ij}{}^*) \tag{13}$$

In Eq. (13), g indicates the number of beneficial criteria and (n-g) shows the number of non-beneficial criteria. The optimal alternative can be specified by listing the assessment values in descending order [9, 33].

$$A_{RS}{}^{*} = \{A_i \mid max_i Y_i\} \tag{14}$$

3.2.2 The Reference Point Approach

This approach is established on the concept of maximal objective reference point (MORP) and Tchebycheff min-max metric. The coordinate j of MORP is calculated as follows [8, 9]:

$$r_j = \begin{cases} max_i x_{ij}{}^{*}, & j \leq g \\ min_i x_{ij}{}^{*}, & j > g \end{cases} \tag{15}$$

In Eq. (15), g is the number of beneficial criteria. The derivation of normalized rating $x_{ij}{}^{*}$from reference pint r_j can be obtained as:

$$d_{ij} = \left| r_j - x_{ij}{}^{*} \right| \tag{16}$$

The assessment value of reference point approach can be determined as [8, 9]:

$$Z_i = max_j (w_j d_{ij}) \tag{17}$$

The assessment values are listed in ascending order to produce the optimal alternative of the reference point approach [BRAUER, BRAUER2, **62**].

$$A_{RP}{}^{*} = \{A_i \mid min_i Z_i\} \tag{18}$$

3.2.3 The Full Multiplicative Form

The third part of the MULTIMOORA approach is the full multiplicative form. In this technique, the allocation of significant coefficients as multiplier is regarded as meaningless. Instead, the significant coefficients should be considered as exponents [8, 9].The assessment values of the full multiplicative form can be obtained as follows:

$$U_i = \frac{\prod_{j=1}^{g} (x_{ij}{}^{*(w_j)})}{\prod_{g+1}^{n} (x_{ij}{}^{*(w_j)})} \tag{19}$$

In Eq. (19), g denotes the number of beneficial criteria. Similar to the ratio system, the optimal alternative in this technique is calculated by finding the maximum assessment value:

$$A_{MF}{}^{*} = \{A_i \mid max_i U_i\} \tag{20}$$

3.2.4 Final Ranking of the MULTIMOORA Method

The subordinate ranks obtained in sections 3.2.1, 3.2.2, and 3.2.3 can be summarized into a final ranking, called the MULTIMOORA rank, by employing the theory of dominance. This theory is structured based on propositions such as dominance, being dominated, transitivity, and equability. Translating of ranks into a final ranking using the concept of dominance theory may lead to circular reasoning. Hence, a simplified method is used by the authors taking the mean of the subordinate ranks and listing the alternatives in ascending order of the mean ranks.

3.3 The Target-based MULTIMOORA with Integrated Significant Coefficients

3.3.1 Target-based Normalization

In traditional MCDM methods, normalization is accomplished with ratings of alternatives on beneficial and non-beneficial criteria to obtain comparable values. However, the goal of achieving a certain target value of a criterion in practice highlights the necessity of target-based normalization techniques.

The target (the goal or the most favorable values for all criteria, i.e. T_j , $j = 1, 2, \dots n$ can constitute the following set:

$$T = \{ \ T_1, \ T_2, \dots\dots \ T_j, \dots., \qquad T_n \ \} \tag{21}$$

Based on the norm of comprehensive VIKOR [**47**] a normalization technique was employed to consider target-based criteria for the MULTIMOORA method as follows:

$$f_{ij} = \quad e^{-\frac{|\ T_j - x_{ij}\ |}{[\max\left(max_i x_{ij}\ ,\ T_j\right) - \min\left(min_i x_{ij}\ ,\ T_j\right)]}} \tag{22}$$

In this exponential target-based normalization technique is employed in derivation of the assessment indices and significant coefficients of criteria.

In section 3.2.1, Eq. (13) can also be utilized in traditional MULTIMOORA method in which only beneficial and non-beneficial criteria exits without the existence of target-based criteria.

3.3.2 Significant Coefficient of Criteria

3.3.2.1. Subjective Significant Coefficient

Relative importance of criteria obtained straight from decision maker's opinion is called subjective significant coefficient. This is obtained by defuzzification of the trapezoidal fuzzy variables as collectively obtained from different levels of experts who also have their relative importance or weights. The relative importance is also calculated based on their qualification, experience, age, etc. The subjective significant coefficients are labeled as w_j^S.

3.3.2.2 Objective Significant Coefficient

The concepts including entropy and standard deviation are considered in this paper to assign objective significant coefficients or objective weight to each criterion.

Entropy concept has been largely utilized in many fields in social and physical sciences such as economics, language modeling, and spectral analysis to name a few. Shannon [**87**] developed the concept into information entropy to measure uncertainty in data. Information entropy can be effectively utilized in the process of decision making because of its ability to evaluate existent contrast in between different sets of data.

To calculate objective significant coefficients, the technique as used by Jahan et al. [**47**] is adopted, which computes the significant coefficients based on information entropy.

The following procedures are used to determine the significant coefficients. First, $f_{ij}{}'$is created from $f_{ij}$to avoid the insignificance associated with $l_n(f_{ij}{}')$ in later Eq. (24).

$$f_{ij}{}' = \frac{(1 + f_{ij})}{\sum_{i=1}^{m}(1 + f_{ij})} \tag{23}$$

The entropy is calculated as:

$$H_j = -\sum_{i=1}^{m} f_{ij}{}' \, l_n\left(f_{ij}{}'\right) \tag{24}$$

Derivation degree $G_j$is defined as:

$$G_j = 1 - H_j \tag{25}$$

Derivation degree $G_j$is higher if the value of information entropyH_j is smaller. The information entropy based significant coefficient $w_j{}^S$ is generated as:

$$w_j{}^H = \frac{G_j}{\sum_{j=1}^{n} G_j} \tag{26}$$

Based on entropy concept, distributions with higher entropy represent more disorder, are smoother, are more probable, are less predictable, or assume less [**47**]. Thus the set of ratings on a given criterion that is smoother shows higher information entropyH_j, lower deviation degreeG_j, and lower information entropy based significant coefficient $w_j{}^H$.

In statistics, standard deviation is measure utilized to find the amount of variation of a data set. Generally, a standard deviation that is close to 0 shows that the set of data is near to the mean point. On the other hand, a high standard deviation denotes a great spread of values. The standard deviation σ_j for application in the target-based MCDM methods can be defined as follows [**47**]:

$$\sigma_j = \sqrt{\frac{\sum_{i=1}^{m} (f_{ij} - \bar{f}_j)^2}{m}} \tag{27}$$

In Eq. (27), $\bar{f}_j = \frac{1}{m} \sum_{i=1}^{m} f_{ij}$ (28)

The objective significant coefficient based on standard deviation is formulated as:

$$w_j{}^\sigma = \frac{\sigma_j}{\sum_{j=1}^{n} \sigma_j} \tag{29}$$

Based on Eq. (29), the set of ratings on a given criterion with higher variance leads to higher standard deviation based significant coefficient $w_j{}^\sigma$.

3.3.2.3 Inter-attribute Correlation Effect Significant Coefficient

This significant coefficient is established on correlation effect of attributes. Here, only difference is that instead of attributes, the objectives are specified in terms of several criteria. Hence, the terms criteria and attributes can interchangeably be used without any impact. The idea behind this type of significant coefficient is when correlation effect of one specific criterion over another is higher, lesser importance should be considered on the higher correlated criterion [**30**]. Diakoulaki introduced the original inter-attribute correlation effect significant coefficient based on standard deviation approach. Jahan and Edwards [**48**] updated the significant coefficient for application in the target-based MCDM methods. Inter-attribute correlation effect measure can be obtained as follows:

$$R_{jk} = \frac{\sum_{i=1}^{m}\left(f_{ij} - \bar{f}_j\right)(f_{ik} - \bar{f}_k)}{\sqrt{\sum_{i=1}^{m}(f_{ij} - \bar{f}_j)^2 \sum_{i=1}^{m}(f_{ik} - \bar{f}_k)^2}} \tag{30}$$

In Eq. (30), $k = 1, 2, \ldots n$; moreover, f_{ij} or f_{ik} is the target-based normalization ratings obtained using Eq. (22) and $\bar{f}_j$ or $\bar{f}_k$ can be calculated through Eq. (28). Inter-attribute correlation effect significant coefficient is determined from the correlation effect measure obtained using Eq. (30) [**48**].

$$w_j{}^C = \frac{\sum_{k=1}^{n}(1 - R_{jk})}{\sum_{j=1}^{n}\sum_{k=1}^{n}(1 - R_{jk})} \tag{31}$$

Based on Eq. (31), inter-attribute correlation effect significant coefficient $w_j{}^C$ increases as inter-attribute correlation effect measure R_{jk} decreases.

3.3.2.4 Integration of the Significant Coefficients

Subjective, information entropy based and standard deviation based objective, and inter-attribute correlation effect significant coefficients can be combined together to generate the integrated form in a very similar way done in [**47**] by Jahan et al. The selection of specific objective significant coefficient based on mode is not considered here. Instead, the two objective coefficients together are taken into account. The resultant significant coefficient can be obtained as follows:

$$w_j{}^R = \frac{(w_j{}^S w_j{}^H w_j{}^\sigma w_j{}^C)^{\frac{1}{4}}}{\sum_{j=1}^{n}(w_j{}^S w_j{}^H w_j{}^\sigma w_j{}^C)^{\frac{1}{4}}} \tag{32}$$

3.3.3 Subordinate Parts of the Target-Based MULTIMOORA Method

3.3.3.1 The Target-Based Ratio System

By considering Eq. (13), the assessment value of the target-based ratio system can be computed as follows:

$$Y_i = \sum_{j=1}^{n} w_j{}^R f_{ij} \tag{33}$$

Unlike Eq. (13), that has two terms – one for beneficial criteria and the other one for non-beneficial criteria, Eq. (33) has been simplified to only one term as beneficial, non-beneficial, and target-based criteria are taken into account in the normalized rating f_{ij}. $w_j{}^R$ In Eq. (33), is the resultant significant coefficient of criteria as discussed in section 3.3.2.

The resultant optimal alternative of the target-based ratio system is formulated as follows:

$$A_{TRS}{}^* = \{A_i \mid max_i Y_i\} \tag{34}$$

3.3.3.2 The Target-Based Reference Point Approach

The reference point r_j of the original MULTIMOORA method, i.e., Eq. (16), is translated to one for the target-based MULTIMOORA method. The corresponding form of the reference point can be conceived as normalized target value that is equal to 1 for all criteria.

Thus, the deviation of the normalized rating f_{ij} from normalized target value of 1 is obtained as:

$$D_{ij} = 1 - f_{ij} \tag{35}$$

Eq. (35) shows that greater value of f_{ij} leads to lower deviation D_{ij}. By considering this deviation and Eq. (17), the assessment value target-based reference point approach can be specified as follows:

$$Z_i = \sum_{j=1}^{n} w_j{}^R D_{ij} \tag{36}$$

Then, alternatives can be listed in ascending order of the assessment values to find the optimal of the target-based reference point approach as follows:

$$A_{TRP}{}^{*} = \{A_i \mid min_i Z_i\} \tag{37}$$

3.3.3.3 The Target-Based Full Multiplicative Form

Similar to the target-based ratio system, the target-based full multiplicative form has only one term. The assessment values of the full multiplicative form is transformed into Eq. (38) when target-based criteria are considered as formulated below:

$$U_i = \prod_{j=1}^{n} f_{ij}{}^{w_j{}^R} \tag{38}$$

The alternatives can be ranked by listing them in in ascending order of the assessment values:

$$A_{TMF}{}^{*} = \{A_i \mid max_i U_i\} \tag{39}$$

3.3.4 Final Ranking of the Target-Based MULTIMOORA Method

By employing the aggregation logic instead of dominance theory the ranks obtained in section 3.3.3.1, 3.3.3.2, and 3.3.3.3 can be integrated into a final ranking.

$$R_i{}^{mean} = \frac{1}{3}\left[R_i{}^{RS} + R_i{}^{RP} + R_i{}^{MF}\right] \tag{40}$$

$$A_{TMOORA}{}^{*} = \{A_i \mid min_i R_i{}^{mean}\} \tag{41}$$

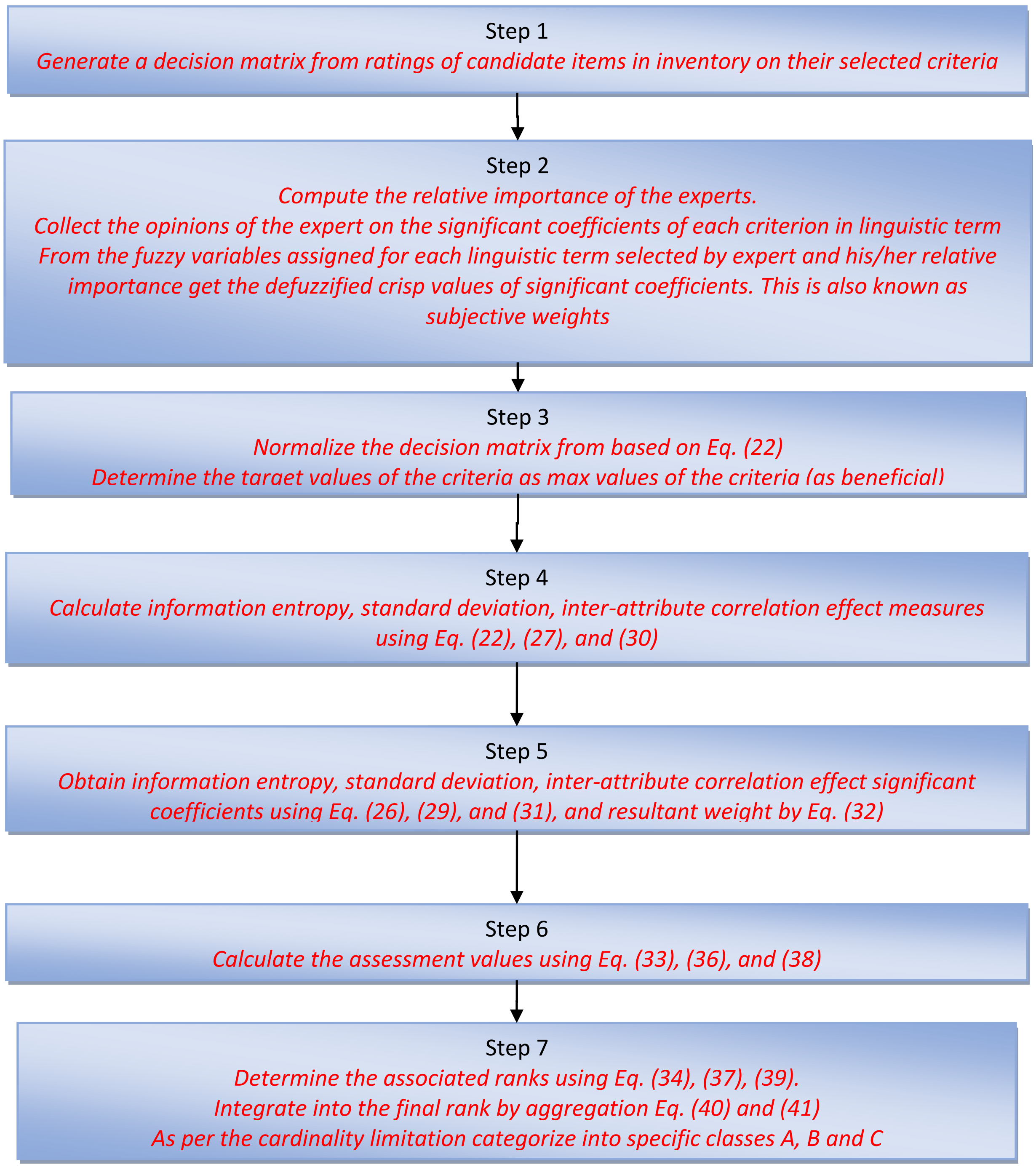


Fig. 3.3 Flowchart of the target-based MULTIMOORA method

Chapter 4

4. Case Study

4.1 Case Study – 1

4.1.1 Data and Criteria Selection

The data set as provided by Flores et al [**37**] has 47 items with four criteria: average unit cost (AUC), annual dollar value/usage (ADU), lead time (LT), and criticality factor (CF). All these criteria are positively related to importance of inventory items. In other words all them are beneficial criteria,. Unlike previous optimization methods based on works by Zhou and Fan (2007), Ng (2007), Hadi and Vencheh (2010), and Chen (2011) [**105, 69, 41, 24**] that omitted the CF criteria because of its discontinuity and non-linearity nature, this dissertation also accounts for it as did Flores et al. (1992), Ramanathan (2006), Lolli et al.(2012), Mohammaditabar et al. (2012) [**37, 78, 60, 78**], and many others. As assigned by Ramanathan, CF criteria may have three discrete values: 1 - for very critical items, 0.5 – for moderately critical items, and 0.1 – for non-critical items.

Table 4.1.1: Illustration of Respiratory Therapeutic Unit Data REID (1987)

Product Code	Unit Price ($)	Annual Usage	Lead Time (Weeks)	Criticality
S01	49.92	117	2	1
S02	210	27	5	1
S03	23.76	212	4	1
S04	27.73	172	1	0.01
S05	57.98	60	3	0.5
S06	31.24	94	3	0.5
S07	28.2	100	3	0.5
S08	55	48	4	0.01
S09	73.44	33	6	1
S10	160.5	15	4	0.5
S11	5.12	210	2	1
S12	20.87	50	5	0.5
S13	86.5	12	7	1
S14	110.4	8	5	0.5
S15	71.2	12	3	1

Product Code	Unit Price ($)	Annual Usage	Lead Time (Weeks)	Criticality
S16	45	18	3	0.5
S17	14.66	48	4	0.5
S18	49.5	12	6	0.5
S19	47.5	12	5	0.5
S20	58.45	8	4	0.5
S21	24.4	19	4	1
S22	65	7	4	0.5
S23	86.5	5	4	1
S24	33.2	12	3	1
S25	37.05	10	1	0.01
S26	33.84	10	3	0.01
S27	84.03	4	1	0.01
S28	78.4	4	6	0.01
S29	134.34	2	7	0.01
S30	56	4	1	0.01
S31	72	3	5	0.5
S32	53.02	4	2	1
S33	49.48	4	5	0.01
S34	7.07	27	7	0.01
S35	60.6	3	3	0.01
S36	40.82	4	3	1
S37	30	5	5	0.01
S38	67.4	2	3	0.5
S39	59.6	2	5	0.01
S40	51.68	2	6	0.01
S41	19.8	4	2	0.01
S42	37.7	2	2	0.01
S43	29.89	2	5	0.01
S44	48.3	1	3	0.01
S45	34.4	1	7	0.01
S46	28.8	1	3	0.01
S47	8.46	3	5	0.01

4.1.2 Assigning Weights

A team of five different experts from different functional and hierarchical levels with a fair amount of knowledge on inventory items and the relative importance of each criteria are

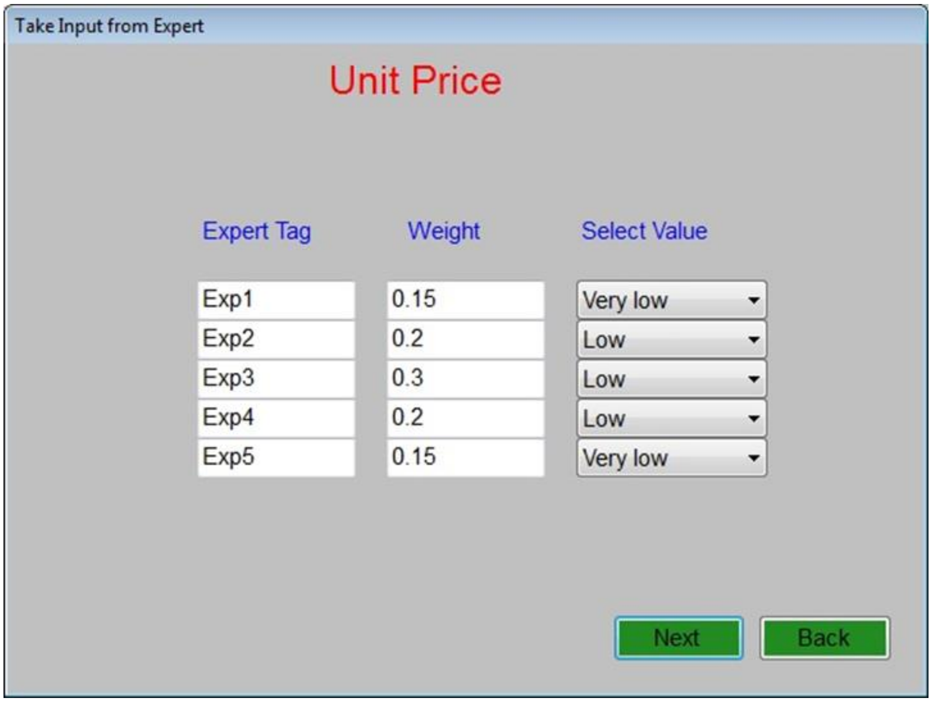


Fig. 4.1.2.1 Expert opinion capture snap-shot

Table 4.1.2.1 Linguistic variable for rating the weights of criteria of inventory items

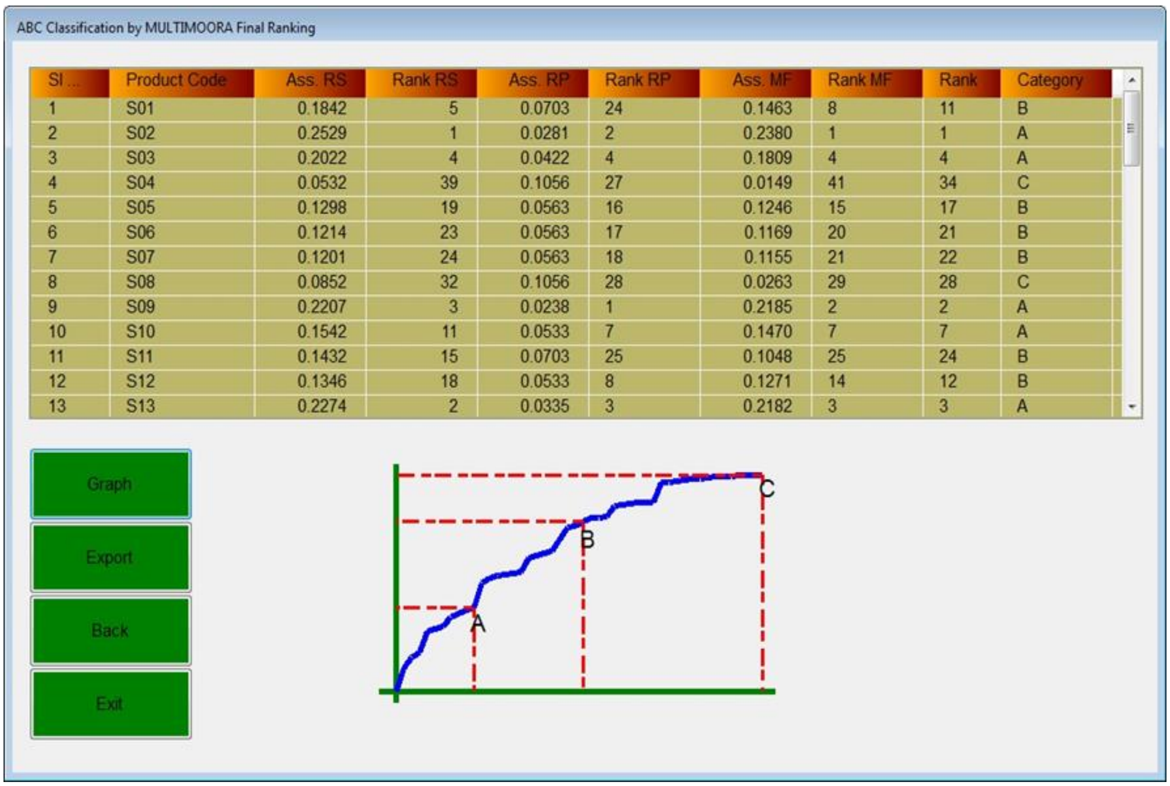
Linguistic variables	fuzzy Score1	fuzzy Score2	fuzzy Score3	fuzzy Score4
Very low (VL)	0	0	0.1	0.2
Low (L)	0.1	0.2	0.2	0.3
Medium low (ML)	0.2	0.3	0.4	0.5
Medium (M)	0.4	0.5	0.5	0.6
Medium high (MH)	0.5	0.6	0.7	0.8
High (H)	0.7	0.8	0.8	0.9
Very high (VH)	0.8	0.9	1	1

presented the four selected criteria one after another and the options are given as per Table 4.1.2.1. A supporting prototype is built to capture the imprecise expert's opinion in linguistic terms, the screen-shot of which is presented here for the purpose of better clarity. However, a predefined set of weights can also be set in the settings table stored in database

The five team members are assigned the following relative weights 0.15, 0.20, 0.30, 0.20, 0.15 in the criteria prioritizing process based on their different domain knowledge and expertise.

4.1.3 Building Prototype

A working prototype of the MULTIMOORA and target-based MULTIMOORA methods including the auxiliary fuzzy weight capturing module from experts are built to solve the MCIC problem in particular using VB.NET and MS Access database. The screenshots showing the subordinate ranks, assessment values, final ranks and categories in the extended MULTIMOORA method are presented in Fig. 4.1.3.1.

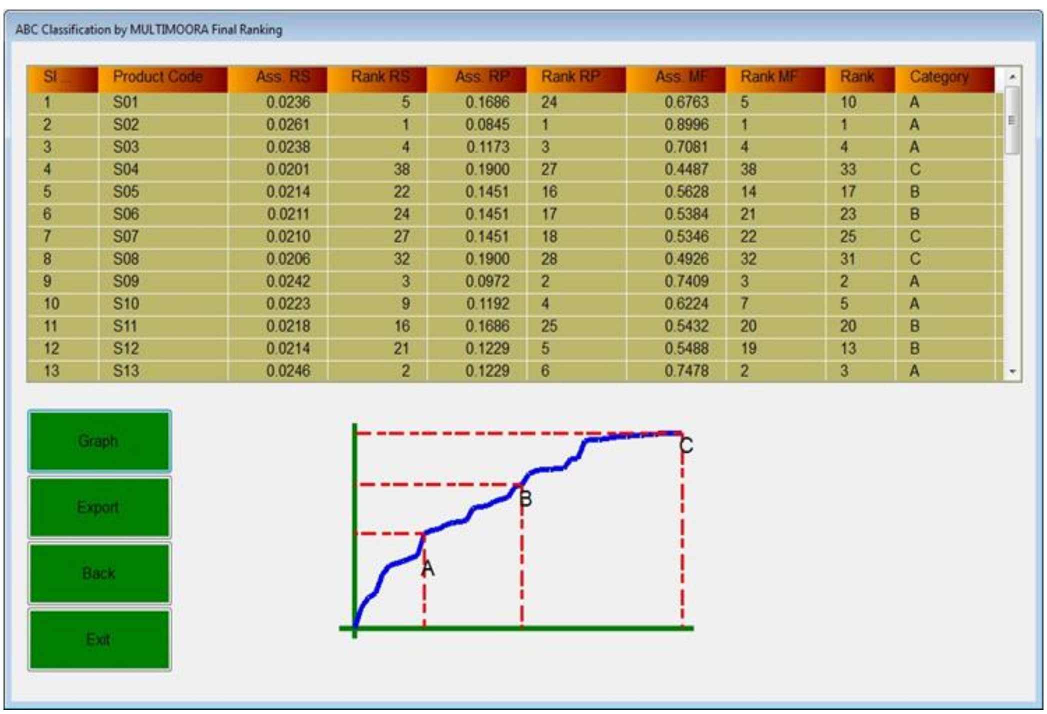


Sl ...	Product Code	Ass. RS	Rank RS	Ass. RP	Rank RP	Ass. MF	Rank MF	Rank	Category
1	S01	0.1842	5	0.0703	24	0.1463	8	11	B
2	S02	0.2529	1	0.0281	2	0.2380	1	1	A
3	S03	0.2022	4	0.0422	4	0.1809	4	4	A
4	S04	0.0532	39	0.1056	27	0.0149	41	34	C
5	S05	0.1298	19	0.0563	16	0.1246	15	17	B
6	S06	0.1214	23	0.0563	17	0.1169	20	21	B
7	S07	0.1201	24	0.0563	18	0.1155	21	22	B
8	S08	0.0852	32	0.1056	28	0.0263	29	28	C
9	S09	0.2207	3	0.0238	1	0.2185	2	2	A
10	S10	0.1542	11	0.0533	7	0.1470	7	7	A
11	S11	0.1432	15	0.0703	25	0.1048	25	24	B
12	S12	0.1346	18	0.0533	8	0.1271	14	12	B
13	S13	0.2274	2	0.0335	3	0.2182	3	3	A

Fig. 4.1.3.1 MULTIMOORA final ranking snapshot for Case Study - 1

4.1.4 Numerical Illustration

After translating into corresponding fuzzy numbers the experts' evaluations are aggregated using Eq. (10) to get the aggregated fuzzy weights of criteria. The fuzzy weights are defuzzified using Eq. (9), the crisp subjective significant coefficients are obtained as shown in Table 4.1.4.1. Say, for an example, the criterion weights for Unit Price (AUC) in terms of linguistic terms selected by the 5 experts are VL, L, L, L, and VL respectively. The relative importance among the experts are assigned as 0.15, 0.2, 0.3, 0.2, and 0.15 respectively based on their experiences and other factors as stated previously. Then the first fuzzy variable of the trapezoidal fuzzy set (fuzzy weights1) for the Unit Price criterion can be computed by Eq. (10) as $0.15 \times 0 + 0.2 \times .1 + 0.3 \times 0.1 + 0.2 \times 0.1 + 0.15 \times 0 = 0.07$. For the sake of fair comparison with the AHP-based classification methods proposed by Flores et al. (1992) [**37**], the subjective weights given to the selected criteria must be the same ($W_{AUC} = 0.079$, $W_{ADU} = 0.091$, $W_{LT} = 0.410, W_{CF} = 0.420$), i.e., step 2 is skipped in the flow diagram given in Fig. 3.3.


ABC Classification by MULTIMOORA Final Ranking

Sl	Product Code	Ass. RS	Rank RS	Ass. RP	Rank RP	Ass. MF	Rank MF	Rank	Category
1	S01	0.0236	5	0.1686	24	0.6763	5	10	A
2	S02	0.0261	1	0.0845	1	0.8996	1	1	A
3	S03	0.0238	4	0.1173	3	0.7081	4	4	A
4	S04	0.0201	38	0.1900	27	0.4487	38	33	C
5	S05	0.0214	22	0.1451	16	0.5628	14	17	B
6	S06	0.0211	24	0.1451	17	0.5384	21	23	B
7	S07	0.0210	27	0.1451	18	0.5346	22	25	C
8	S08	0.0206	32	0.1900	28	0.4926	32	31	C
9	S09	0.0242	3	0.0972	2	0.7409	3	2	A
10	S10	0.0223	9	0.1192	4	0.6224	7	5	A
11	S11	0.0218	16	0.1686	25	0.5432	20	20	B
12	S12	0.0214	21	0.1229	5	0.5488	19	13	B
13	S13	0.0246	2	0.1229	6	0.7478	2	3	A

Fig. 4.1.3.2 Extended MULTIMOORA final ranking snapshot (Case Study – 1)

The product ratings on each criterion are normalized using Eq. (22) as shown in Table 4.1.4.2. The measures of information entropy and standard deviations for the MCIC problem tabulated in Table 4.1.4.3 are computed using Eq. (24) and (27) respectively. Table 4.1.4.4 exhibits the arrays of inter-attribute correlation effect measures obtained through Eq. (30). The

Table 4.1.3.1 Derivation of subjective significant coefficients

Name	Expert Selection (.15, 0.2, 0.3, 0.2, 0.15)	Fuzzy Weight1	Fuzzy Weight2	Fuzzy Weight3	Fuzzy Weight4	Subjective Weight
Unit Price	[VL, L, L, L, VL]	0.07	0.14	0.17	0.27	0.1079
Dollar Value	[VL, ML, ML, ML, VL]	0.14	0.21	0.31	0.41	0.1764
Lead Time	[M, MH, ML, ML, MH]	0.335	0.435	0.52	0.62	0.3135
Criticality Factor	[MH, H, M, MH, M]	0.495	0.595	0.63	0.73	0.4022

subjective weights (as assigned by [**37**], two objective weights based on information entropy and standard deviation, and inter-attribute correlation effect based weights, and finally the resultant weight by combining all as computed using Eq. (26) , (29), (31), and (32) are shown in Table 4.1.4.5. The assessment values related to three subordinate parts of MULTIMOORA and extended MULTIMOORA are obtained using Eq. (13), (17), and (19) and Eq. (33), (36), and (38) and illustrated along with subordinate rankings and the final aggregated ranks considering Eq. (14), (18), and (20) and Eq. (34), (37), (40), and Eq. (41) respectively in the Table 4.1.4.6 and Table 4.1.4.7 respectively.

Table 4.1.3.2 Normalized ratings in MULTIMOORA and extended MULTIMOORA

Product Code	MULTIMOORA				Extended MULTIMOORA			
	Unit Price	Dollar Value	Lead Time	Crit. Factor	Unit Price	Dollar Value	Lead Time	Crit. Factor
S01	0.1095	0.4476	0.0686	0.254	0.4578	1	0.4346	1
S02	0.4607	0.4346	0.1715	0.254	1	0.9711	0.7165	1
S03	0.0521	0.386	0.1372	0.254	0.4029	0.8709	0.6065	1
S04	0.0608	0.3655	0.0343	0.0025	0.4108	0.8318	0.3679	0.3679
S05	0.1272	0.2666	0.1029	0.127	0.4762	0.6662	0.5134	0.6035
S06	0.0685	0.2251	0.1029	0.127	0.4179	0.6069	0.5134	0.6035
S07	0.0619	0.2161	0.1029	0.127	0.4117	0.5949	0.5134	0.6035
S08	0.1207	0.2023	0.1372	0.0025	0.4693	0.5767	0.6065	0.3679
S09	0.1611	0.1857	0.2058	0.254	0.5135	0.5557	0.8465	1
S10	0.3521	0.1845	0.1372	0.127	0.7854	0.5541	0.6065	0.6035
S11	0.0112	0.0824	0.0686	0.254	0.3679	0.4407	0.4346	1
S12	0.0458	0.08	0.1715	0.127	0.3973	0.4383	0.7165	0.6035
S13	0.1898	0.0796	0.2401	0.254	0.5473	0.4379	1	1
S14	0.2422	0.0677	0.1715	0.127	0.615	0.4264	0.7165	0.6035
S15	0.1562	0.0655	0.1029	0.254	0.5079	0.4242	0.5134	1
S16	0.0987	0.0621	0.1029	0.127	0.4469	0.421	0.5134	0.6035
S17	0.0322	0.0539	0.1372	0.127	0.3854	0.4134	0.6065	0.6035
S18	0.1086	0.0455	0.2058	0.127	0.4569	0.4057	0.8465	0.6035
S19	0.1042	0.0437	0.1715	0.127	0.4524	0.404	0.7165	0.6035
S20	0.1282	0.0358	0.1372	0.127	0.4773	0.3969	0.6065	0.6035
S21	0.0535	0.0355	0.1372	0.254	0.4042	0.3967	0.6065	1
S22	0.1426	0.0349	0.1372	0.127	0.4928	0.3961	0.6065	0.6035
S23	0.1898	0.0331	0.1372	0.254	0.5473	0.3946	0.6065	1
S24	0.0728	0.0305	0.1029	0.254	0.4219	0.3923	0.5134	1
S25	0.0813	0.0284	0.0343	0.0025	0.4299	0.3904	0.3679	0.3679
S26	0.0742	0.0259	0.1029	0.0025	0.4232	0.3882	0.5134	0.3679
S27	0.1844	0.0258	0.0343	0.0025	0.5407	0.3881	0.3679	0.3679
S28	0.172	0.024	0.2058	0.0025	0.5261	0.3866	0.8465	0.3679
S29	0.2947	0.0206	0.2401	0.0025	0.6912	0.3836	1	0.3679
S30	0.1229	0.0172	0.0343	0.0025	0.4716	0.3807	0.3679	0.3679
S31	0.158	0.0166	0.1715	0.127	0.5099	0.3801	0.7165	0.6035
S32	0.1163	0.0163	0.0686	0.254	0.4648	0.3799	0.4346	1
S33	0.1086	0.0152	0.1715	0.0025	0.4568	0.379	0.7165	0.3679
S34	0.0155	0.0146	0.2401	0.0025	0.3714	0.3785	1	0.3679
S35	0.133	0.0139	0.1029	0.0025	0.4823	0.3779	0.5134	0.3679
S36	0.0896	0.0125	0.1029	0.254	0.4379	0.3767	0.5134	1
S37	0.0658	0.0115	0.1715	0.0025	0.4154	0.3758	0.7165	0.3679
S38	0.1479	0.0103	0.1029	0.127	0.4986	0.3749	0.5134	0.6035
S39	0.1308	0.0091	0.1715	0.0025	0.4799	0.3739	0.7165	0.3679
S40	0.1134	0.0079	0.2058	0.0025	0.4617	0.3728	0.8465	0.3679
S41	0.0434	0.0061	0.0686	0.0025	0.3952	0.3713	0.4346	0.3679
S42	0.0827	0.0058	0.0686	0.0025	0.4313	0.3711	0.4346	0.3679
S43	0.0656	0.0046	0.1715	0.0025	0.4152	0.3701	0.7165	0.3679
S44	0.106	0.0037	0.1029	0.0025	0.4542	0.3693	0.5134	0.3679
S45	0.0755	0.0026	0.2401	0.0025	0.4244	0.3685	1	0.3679
S46	0.0632	0.0022	0.1029	0.0025	0.413	0.3681	0.5134	0.3679
S47	0.0186	0.0019	0.1715	0.0025	0.3739	0.3679	0.7165	0.3679

Table 4.1.3.3: Information entropy and standard deviation measures in extended MULTIMOORA method

	Unit Price	Dollar Value	Lead Time	Critical Factor
Information Entropy	3.847575773	3.844732172	3.844425511	3.837809052
Standard Deviation	0.456053537	0.440207265	0.600195873	0.578182694

Table 4.1.3.4: Inter-attribute correlation effect measures in extended MULTIMOORA method

	Unit Price (AUC)	Dollar Value (ADU)	Lead Time (LT)	Critical Factor (CF)
Unit Price (AUC)	1	0.322219389	0.210874999	0.192074906
Dollar Value (ADU)	0.322219389	1	-0.16613782	0.371605967
Lead Time (LT)	0.210874999	-0.16613782	1	-0.069061112
Critical Factor (CF)	0.192074906	0.371605967	-0.069061112	1

Table 4.1.3.5: Subjective weights, objective weights and inter-attribute correlation effect weights, and integrated weights for Case Study - 1

Subjective Weights w_j^S	Information Entropy Objective Weights w_j^H	Standard Deviation Objective Weights w_j^σ	Inter-Attribute Correlation Effect Weights w_j^C	Integrated Weights w_j^R
0.079	0.2503	0.1569	0.2214	0.1825
0.091	0.2501	0.2262	0.2406	0.2187
0.410	0.2501	0.2512	0.2943	0.2982
0.420	0.2495	0.3657	0.2438	0.3006

Table 4.1.3.6 Assessment values, subordinate rankings, and final rank in MULTIMOORA

Sl No.	Product Code	Ass. RS	Rank RS	Ass. RP	Rank RP	Ass. MF	Rank MF	Rank	Category
1	S01	0.1842	5	0.0703	24	0.1463	8	11	B
2	S02	0.2529	1	0.0281	2	0.238	1	1	A
3	S03	0.2022	4	0.0422	4	0.1809	4	4	A
4	S04	0.0532	39	0.1056	27	0.0149	41	34	C
5	S05	0.1298	19	0.0563	16	0.1246	15	17	B
6	S06	0.1214	23	0.0563	17	0.1169	20	21	B
7	S07	0.1201	24	0.0563	18	0.1155	21	22	B
8	S08	0.0852	32	0.1056	28	0.0263	29	28	C
9	S09	0.2207	3	0.0238	1	0.2185	2	2	A
10	S10	0.1542	11	0.0533	7	0.147	7	7	A
11	S11	0.1432	15	0.0703	25	0.1048	25	24	B
12	S12	0.1346	18	0.0533	8	0.1271	14	12	B
13	S13	0.2274	2	0.0335	3	0.2182	3	3	A
14	S14	0.1489	13	0.0533	9	0.1427	10	8	A
15	S15	0.1672	8	0.0563	19	0.1492	6	9	A
16	S16	0.109	26	0.0563	20	0.107	24	25	C
17	S17	0.117	25	0.0533	10	0.1088	22	20	B
18	S18	0.1504	12	0.0533	11	0.1393	11	10	A
19	S19	0.1359	17	0.0533	12	0.1283	13	13	B
20	S20	0.123	22	0.0533	13	0.1169	19	19	B
21	S21	0.1704	7	0.0422	5	0.1459	9	6	A
22	S22	0.124	21	0.0533	14	0.1176	18	18	B
23	S23	0.1809	6	0.0422	6	0.1602	5	5	A
24	S24	0.1574	9	0.0563	21	0.131	12	14	B
25	S25	0.0241	47	0.1056	29	0.0121	46	41	C
26	S26	0.0515	41	0.1056	30	0.0187	37	36	C
27	S27	0.032	45	0.1056	31	0.0128	45	40	C
28	S28	0.1012	30	0.1056	32	0.0264	28	29	C
29	S29	0.1246	20	0.1056	33	0.0289	27	27	C
30	S30	0.0264	46	0.1056	34	0.0119	47	44	C
31	S31	0.1376	16	0.0533	15	0.1214	17	15	B
32	S32	0.1455	14	0.0703	26	0.1087	23	23	B
33	S33	0.0813	34	0.1056	35	0.0226	31	31	C
34	S34	0.102	29	0.1056	36	0.0222	32	30	C
35	S35	0.055	38	0.1056	37	0.0185	38	38	C
36	S36	0.1571	10	0.0563	22	0.1228	16	16	B
37	S37	0.0776	35	0.1056	38	0.0212	35	37	C
38	S38	0.1082	27	0.0563	23	0.0938	26	26	C
39	S39	0.0825	33	0.1056	39	0.0219	33	33	C
40	S40	0.0951	31	0.1056	40	0.023	30	32	C
41	S41	0.0332	44	0.1056	41	0.0133	44	46	C
42	S42	0.0362	43	0.1056	42	0.0139	43	45	C
43	S43	0.077	36	0.1056	43	0.0195	36	39	C
44	S44	0.0519	40	0.1056	44	0.0161	40	43	C
45	S45	0.1057	28	0.1056	45	0.0215	34	35	C
46	S46	0.0484	42	0.1056	46	0.0147	42	47	C
47	S47	0.073	37	0.1056	47	0.0163	39	42	C

Table 4.1.3.7 Assessment values, subordinate rankings, final rank in extended MULTIMOORA

Sl. No.	Product Code	Ass. RS	Rank RS	Ass. RP	Rank RP	Ass. MF	Rank MF	Rank	Category
1	S01	0.0236	5	0.1686	24	0.6763	5	10	A
2	S02	0.0261	1	0.0845	1	0.8996	1	1	A
3	S03	0.0238	4	0.1173	3	0.7081	4	4	A
4	S04	0.0201	38	0.19	27	0.4487	38	33	C
5	S05	0.0214	22	0.1451	16	0.5628	14	17	B
6	S06	0.0211	24	0.1451	17	0.5384	21	23	B
7	S07	0.021	27	0.1451	18	0.5346	22	25	C
8	S08	0.0206	32	0.19	28	0.4926	32	31	C
9	S09	0.0242	3	0.0972	2	0.7409	3	2	A
10	S10	0.0223	9	0.1192	4	0.6224	7	5	A
11	S11	0.0218	16	0.1686	25	0.5432	20	20	B
12	S12	0.0214	21	0.1229	5	0.5488	19	13	B
13	S13	0.0246	2	0.1229	6	0.7478	2	3	A
14	S14	0.022	14	0.1255	7	0.5907	10	8	A
15	S15	0.0224	8	0.1451	19	0.6005	8	11	B
16	S16	0.0206	31	0.1451	20	0.5032	28	26	C
17	S17	0.0209	29	0.1283	8	0.5127	25	24	B
18	S18	0.022	13	0.13	9	0.5817	11	9	A
19	S19	0.0215	19	0.1303	10	0.552	17	14	B
20	S20	0.021	26	0.1319	11	0.5284	24	21	B
21	S21	0.0225	7	0.1319	12	0.5965	9	7	A
22	S22	0.0211	25	0.1321	13	0.5312	23	22	B
23	S23	0.0228	6	0.1324	14	0.6297	6	6	A
24	S24	0.0221	11	0.1451	21	0.5707	12	12	B
25	S25	0.019	47	0.19	29	0.3834	47	42	C
26	S26	0.0195	41	0.19	30	0.4218	41	37	C
27	S27	0.0192	43	0.19	31	0.3993	43	40	C
28	S28	0.0212	23	0.19	32	0.509	27	27	C
29	S29	0.0223	10	0.19	33	0.5613	15	18	B
30	S30	0.019	46	0.19	34	0.3878	46	44	C
31	S31	0.0215	17	0.1356	15	0.5567	16	16	B
32	S32	0.0218	15	0.1686	26	0.5488	18	19	B
33	S33	0.0205	34	0.19	35	0.4699	34	34	C
34	S34	0.0214	20	0.19	36	0.4997	30	29	C
35	S35	0.0197	39	0.19	37	0.4294	39	38	C
36	S36	0.0221	12	0.1451	22	0.5695	13	15	B
37	S37	0.0203	35	0.19	38	0.461	35	36	C
38	S38	0.0207	30	0.1451	23	0.5005	29	28	C
39	S39	0.0205	33	0.19	39	0.4728	33	35	C
40	S40	0.021	28	0.19	40	0.4931	31	32	C
41	S41	0.0191	45	0.19	41	0.3925	45	47	C
42	S42	0.0192	44	0.19	42	0.3988	44	45	C
43	S43	0.0203	36	0.19	43	0.4594	36	39	C
44	S44	0.0196	40	0.19	44	0.4226	40	43	C
45	S45	0.0215	18	0.19	45	0.509	26	30	C
46	S46	0.0195	42	0.19	46	0.415	42	46	C
47	S47	0.0202	37	0.19	47	0.4501	37	41	C

4.2 Case Study – 2

4.2.1 Data and Criteria Selection

The proposed methodology for classifying inventory items was tested on a second case study data from a pharmaceutical industry [**70**].

A sample of 20 items are selected for experimentation using the proposed model. Five pieces of information was recorded for each sample, viz. unit cost (INR) per kg, or g, or units, consumption rate (kg, or g, or units/day), lead time (weeks), perishability of items (year), and cost of storing (INR/unit/day).

A decision matrix is computed using attribute values and shown in Table 4.2.1.1

Table 4.2.1.1: Decision matrix for Pharmaceutical industry data (Case Study – 2)

Product Code	Unit Cost	Lead Time	Consumption Rate	Perishability	Storage Cost
C01	1108.29	1.5	216	0.5	3.5
C02	1144	3.5	249.4	1	12
C03	318.4	4.5	874.55	1	3
C04	25	4.5	1000	1	2.1
C05	2413.72	4	4935.78	1	12
C06	186.04	3.5	25.8	1	3
C07	761.08	4.5	73	0.83	12
C08	45.3	3.5	11.7	1	11.5
C09	316	3.5	7.9	1	3
C10	800	3.5	2.1	1	11.5
C11	181.3	3.5	55.9	1	11.5
C12	360	3.5	11.7	1	3
C13	55	3.5	232.3	1	11.5
C14	233.68	4.5	4511.3	1	11.5
C15	340.2	4.5	121.5	1	3
C16	2006.13	4.5	6930	1	12.8
C17	274.92	4.5	3820.4	1	11.5
C18	988.88	4.5	2.5	1	2.1
C19	80.25	4.5	14116.9	1	12.8
C20	66.95	7	10623.6	1	12.8

4.2.2 Assigning Weights

Criteria weights are calculated as per SAATY's pair-wise comparison which is illustrated in the Table 4.2.2.1. This is subjective weights for the criteria. A weight capturing mechanism from different experts under fuzzy environment is also implemented here, and that is an alternative provision to assigning weights for the criteria. The screen-shot for weights in lead time is shown in Fig. 4.2.2.1. The weights from the AHP framework is considered for comparison purpose.

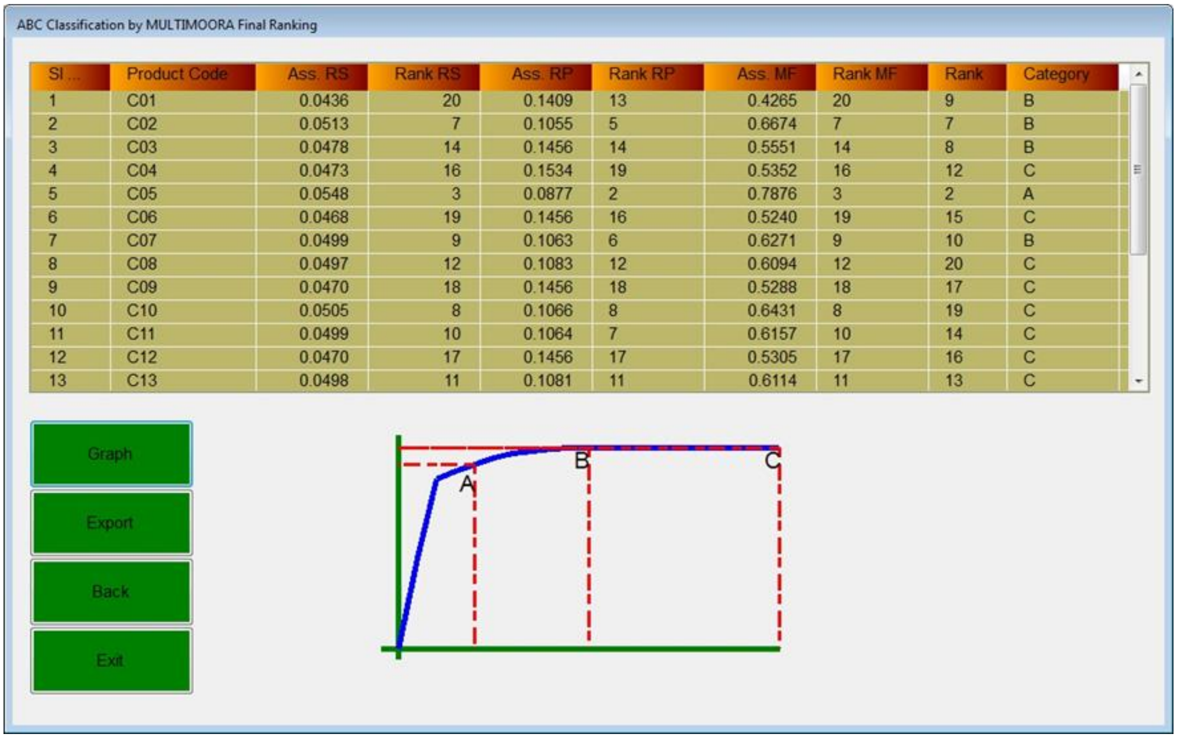
$$P = \begin{bmatrix} 1 & 2/5 & 3/4 & 4/5 & 2/7 \\ 5/2 & 1 & 3/2 & 2/1 & 5/7 \\ 4/3 & 2/3 & 1 & 5/4 & 2/5 \\ 5/4 & 1/2 & 4/5 & 1 & 1/3 \\ 7/2 & 7/5 & 5/2 & 3/1 & 1 \end{bmatrix}$$

Criteria	Unit Cost	Lead Time	Consumption Rate	Perishability	Storage Cost
Weight	0.105	0.25	0.152	0.125	0.368

Fig. 4.2.2.1: Weight assignment from AHP

4.2.3 Using Prototype

The same prototype which was used for case study – 1 was extended so that it works with a completely different dataset with a new set of parameters. The MULTIMOORA and extended MULTIMOORA methods were applied and the results of classification was obtained. Fig. 4.2.3.1 shows the snap-shot of Extended MULTIMOORA method.



Sl ...	Product Code	Ass. RS	Rank RS	Ass. RP	Rank RP	Ass. MF	Rank MF	Rank	Category
1	C01	0.0436	20	0.1409	13	0.4265	20	9	B
2	C02	0.0513	7	0.1055	5	0.6674	7	7	B
3	C03	0.0478	14	0.1456	14	0.5551	14	8	B
4	C04	0.0473	16	0.1534	19	0.5352	16	12	C
5	C05	0.0548	3	0.0877	2	0.7876	3	2	A
6	C06	0.0468	19	0.1456	16	0.5240	19	15	C
7	C07	0.0499	9	0.1063	6	0.6271	9	10	B
8	C08	0.0497	12	0.1083	12	0.6094	12	20	C
9	C09	0.0470	18	0.1456	18	0.5288	18	17	C
10	C10	0.0505	8	0.1066	8	0.6431	8	19	C
11	C11	0.0499	10	0.1064	7	0.6157	10	14	C
12	C12	0.0470	17	0.1456	17	0.5305	17	16	C
13	C13	0.0498	11	0.1081	11	0.6114	11	13	C

Fig. 4.2.3.1 Extended MULTIMOORA final ranking snapshot for Case Study - 2

4.2.4 Numerical Illustration

Applying the three subordinate methods, the final ranking of MUTIMOORA methods are obtained. The extended XMLTIMOORA can be applied after all the different significant coefficients values are obtained. The different significant coefficients obtained are tabulated in Table 4.2.4.1 and the final result is shown in Table 4.2.4.2 and Table 4.2.4.3 for MULTIMOORA and XMULTIMOORA methods.

Table 4.2.4.1: Subjective weights, objective weights and inter-attribute correlation effect weights, and integrated weights for Case Study - 2

Subjective Weights w_j^S	Information Entropy Objective Weights w_j^H	Standard Deviation Objective Weights w_j^σ	Inter-Attribute Correlation Effect Weights w_j^C	Integrated Weights w_j^R
0.105	0.2	0.1466	0.2511	0.1722
0.250	0.2003	0.1823	0.1826	0.2087
0.152	0.1999	0.1358	0.1723	0.1687
0.125	0.2002	0.3025	0.2158	0.2077
0.368	0.1996	0.2328	0.1782	0.2427

Table 4.2.4.2: Final ranking by MULTIMOORA method (Case Study – 2)

Product Code	Ass. RS	Rank RS	Ass. RP	Rank RP	Ass. MF	Rank MF	Rank	Category
C01	0.0964	20	0.1029	11	0.0713	14	9	B
C02	0.2129	7	0.1027	9	0.1549	7	7	B
C03	0.1302	14	0.0981	8	0.1048	10	8	B
C04	0.1154	16	0.0971	7	0.0718	13	12	C
C05	0.2884	4	0.068	4	0.2729	2	2	A
C06	0.107	19	0.1043	15	0.0547	17	15	C
C07	0.2098	8	0.104	13	0.1284	8	10	B
C08	0.1772	12	0.1044	17	0.0688	16	20	C
C09	0.1104	18	0.1045	18	0.0484	19	17	C
C10	0.1974	9	0.1045	20	0.0709	15	19	C
C11	0.1812	10	0.1041	14	0.1001	11	14	C
C12	0.1116	17	0.1044	16	0.0522	18	16	C
C13	0.1791	11	0.1028	10	0.1098	9	13	C
C14	0.229	5	0.0711	5	0.2135	5	4	A
C15	0.1252	15	0.1036	12	0.0782	12	11	C
C16	0.3059	3	0.0532	1	0.2972	1	1	A
C17	0.225	6	0.0762	6	0.2118	6	5	B
C18	0.134	13	0.1045	19	0.0413	20	18	C
C19	0.3073	2	0.0627	2	0.2362	4	3	A
C20	0.3146	1	0.0631	3	0.2478	3	6	B

Table 4.2.4.3: Final ranking by Extended MULTIMOORA method (Case Study – 2)

Product Code	Ass. RS	Rank RS	Ass. RP	Rank RP	Ass. MF	Rank MF	Rank	Category
C01	0.0436	20	0.1409	13	0.4265	20	9	B
C02	0.0513	7	0.1055	5	0.6674	7	7	B
C03	0.0478	14	0.1456	14	0.5551	14	8	B
C04	0.0473	16	0.1534	19	0.5352	16	12	C
C05	0.0548	3	0.0877	2	0.7876	3	2	A
C06	0.0468	19	0.1456	16	0.524	19	15	C
C07	0.0499	9	0.1063	6	0.6271	9	10	B
C08	0.0497	12	0.1083	12	0.6094	12	20	C
C09	0.047	18	0.1456	18	0.5288	18	17	C
C10	0.0505	8	0.1066	8	0.6431	8	19	C
C11	0.0499	10	0.1064	7	0.6157	10	14	C
C12	0.047	17	0.1456	17	0.5305	17	16	C
C13	0.0498	11	0.1081	11	0.6114	11	13	C
C14	0.0514	5	0.1031	4	0.677	5	4	A
C15	0.0477	15	0.1456	15	0.551	15	11	C
C16	0.0552	2	0.0762	1	0.8107	2	1	A
C17	0.0513	6	0.1019	3	0.6734	6	5	B
C18	0.0482	13	0.1534	20	0.5665	13	18	C
C19	0.0548	4	0.1074	9	0.7698	4	3	A
C20	0.0559	1	0.1077	10	0.8111	1	6	B

Chapter 5

5. Results and Discussions

5.1 Comparison with Previous Methods

The final classification results obtained in the MULTIMOORA and extended MULTIMOORA methods in Case Study - 1 (REID, 1987) [**79**] are compared with the classification results of some previous methods like Flores et al. (1992), Ramanathan (2006), Ng (2007), S. A. Torabi et al. (2012), D. Mohammaditabar et al. (2012) [**37, 78, 71, 96, 68**] as all these methods employed the same benchmark dataset in the Hospital Respiratory Therapy Unit and all of them considered 4 criteria (ADU, AUC, LT, CF). The comparison is presented in Table 5.1.1.

To evaluate each inventory classification, the inventory cost function proposed by Mohammaditabar et al., [**68**] has been modified. It is OK to consider group-based optimal interval T_g for class C type items while computing the Total Relevant Cost (TRC) of inventory. But for class A and B items, initially every individual item's optimal interval time T_i must be considered using the same formula and based on that, individual item's inventory cost (TRC_i) can be computed in that group. In short, the ordering policy becomes comparatively tighter in this case. Then all the individual item's costs are added to find the Total Relevant Costs of that group. Apart from that, in our new computational model for comparison, the stock-out cost is incorporated for all critical items classified in class C. Similarly, for items which have higher/moderate annual usage, high/moderate lead time, and high/moderate criticality factor, and still wrongly classified as B items may incur some shortage costs in times because of lighter control assigned to it. And on the contrary, items classified as A items may not necessarily have all moderate/high criteria values, which means extra unnecessary stock needs to be piled up for it, which is also a cost addition. To make it a bit simpler, only C class stockout cost for critical items is considered in our computational model. The important functions which compute Total Relevant Costs (TRC) of the obtained ABC classification as follows:

$TRC = [\sum_{g \in group(C)} \left\{ \left(\frac{\sum_{i \in group(g)} (S_i + SO_i)}{T_g} \right) + \left(\frac{1}{2} T_g . \sum_{i \in group(g)} D_i h_i \right) \right\} +$

$\sum_{g \in group(A,B)} \{ \left(\sum_{i \in group(g)} \left(\frac{S_i}{T_i} + \frac{1}{2} . T_i . D_i h_i \right) \right) \}]$, where $T_g = \sqrt{\frac{2 . \sum_{i \in group(g)} S_i}{\sum_{i \in group(g)} D_i h_i}}$ is the optimal joint replenishment cycle for item of category C (g = C) which means all the items in that group share a single optimal reorder interval, hence the ordering policy and management becomes easier and simpler, and $T_i = \sqrt{\frac{2.(S_i)}{(D_i h_i)}}$ is the optimal replenishment cycle for item of category A or B (g = A,B), S_i is the set up cost of item i, D_i is the demand per unit time of item i and h_i is the holding cost per unit of time of item i, and SO_i is the stock-out cost of item i, which is critical (very critical or moderately critical). The stock-out cost can be obtained by computing the Average Quantity of Stock-outs (AQSO), when optimal interval for an individual item T_i is smaller than the group interval T_g, which is approximately calculated as $AQSO_i = \frac{1}{2} D_i (T_g - T_i)$

In order to calculate the total inventory cost it is essential to know setup costs and since setup cost was not included in Flores et al. (1992) [**37**] **,** it is considered equivalent to the lead time multiplied by a fixed coefficient for all items (*1.078*). Inventory holding cost is assumed to be 10% of the item average cost. The inventory stock-out cost is also assumed to be 30% of the item average cost multiplied by the critical factor of the individual item. The inventory control policy is integrated with the classification by calculating order intervals for each group. The TRC of each MCIC method is also computed based on the above logic and presented in the Table 5.1.1.

It is shown that the TRC computed in case of MULTIMOORA is the minimum and the extended MUTIMOORA is the second lowest in the table. By raising the stock-out cost above 30% the savings in terms of cost is further dominantly visible.in case of both MULTIMOORA and XMULTIMOORA. By lowering the stock-out-cost to 10% and then finally to 0%, the XMULTIMOORA method substantially improves over MULTIMOORA and so do the other previously established methods.

Table 5.1.1 Classification result compared with other previous methods for REID (1987)

Product Code	Traditional	Flores et al.	Ramanathan	Ng	S. A. Torabi et al.	D. Mohammaditabar et al.	MULTIMOORA	XMULTIMOORA
S01	**A**	**A**	**A**	**A**	**A**	**A**	**B**	**A**
S02	**A**	**A**	**A**	**A**	**A**	**A**	**A**	**A**
S03	**A**	**A**	**A**	**A**	**B**	**A**	**A**	**A**
S04	**A**	**A**	**B**	**A**	**C**	**A**	**C**	**C**
S05	**A**	**B**	**B**	**A**	**B**	**A**	**B**	**B**
S06	**A**	**B**	**C**	**A**	**C**	**A**	**B**	**B**
S07	**A**	**C**	**C**	**B**	**C**	**A**	**B**	**C**
S08	**A**	**C**	**B**	**B**	**C**	**A**	**C**	**C**
S09	**A**	**C**	**A**	**A**	**A**	**A**	**A**	**A**
S10	**A**	**C**	**B**	**A**	**A**	**B**	**A**	**A**
S11	**B**	**A**	**C**	**C**	**B**	**C**	**B**	**B**
S12	**B**	**A**	**B**	**B**	**C**	**B**	**B**	**B**
S13	**B**	**A**	**A**	**A**	**A**	**A**	**A**	**A**
S14	**B**	**A**	**B**	**B**	**A**	**B**	**A**	**A**
S15	**B**	**A**	**C**	**C**	**A**	**C**	**A**	**B**
S16	**B**	**A**	**C**	**C**	**C**	**C**	**C**	**C**
S17	**B**	**B**	**C**	**C**	**C**	**B**	**B**	**B**
S18	**B**	**B**	**A**	**B**	**B**	**B**	**A**	**A**
S19	**B**	**B**	**B**	**B**	**B**	**B**	**B**	**B**
S20	**B**	**B**	**C**	**C**	**B**	**C**	**B**	**B**
S21	**B**	**B**	**C**	**C**	**B**	**C**	**A**	**A**
S22	**B**	**B**	**C**	**C**	**B**	**C**	**B**	**B**
S23	**B**	**B**	**C**	**B**	**A**	**C**	**A**	**A**
S24	**B**	**C**	**C**	**C**	**B**	**C**	**B**	**B**
S25	**C**	**B**	**C**	**C**	**C**	**C**	**C**	**C**
S26	**C**	**B**	**C**	**C**	**C**	**C**	**C**	**C**
S27	**C**	**B**	**C**	**C**	**C**	**C**	**C**	**C**
S28	**C**	**B**	**A**	**B**	**B**	**B**	**C**	**C**
S29	**C**	**B**	**A**	**A**	**A**	**C**	**C**	**B**
S30	**C**	**C**	**C**	**C**	**C**	**C**	**C**	**C**
S31	**C**	**C**	**B**	**B**	**B**	**B**	**B**	**B**
S32	**C**	**C**	**C**	**C**	**A**	**C**	**B**	**B**
S33	**C**	**C**	**B**	**B**	**C**	**B**	**C**	**C**
S34	**C**	**C**	**A**	**B**	**C**	**B**	**C**	**C**
S35	**C**	**C**	**C**	**C**	**C**	**C**	**C**	**C**
S36	**C**	**C**	**C**	**C**	**B**	**C**	**B**	**B**
S37	**C**	**C**	**B**	**C**	**C**	**B**	**C**	**C**
S38	**C**	**C**	**C**	**C**	**B**	**C**	**C**	**C**
S39	**C**	**C**	**B**	**B**	**B**	**C**	**C**	**C**
S40	**C**	**C**	**B**	**B**	**C**	**B**	**C**	**C**
S41	**C**	**C**	**C**	**C**	**C**	**C**	**C**	**C**
S42	**C**	**C**	**C**	**C**	**C**	**C**	**C**	**C**
S43	**C**	**C**	**B**	**C**	**C**	**C**	**C**	**C**
S44	**C**	**C**	**C**	**C**	**C**	**C**	**C**	**C**
S45	**C**	**C**	**A**	**B**	**C**	**B**	**C**	**C**
S46	**C**	**C**	**C**	**C**	**C**	**C**	**C**	**C**
S47	**C**	**C**	**B**	**C**	**C**	**C**	**C**	**C**
TRC	**4661.17**	**4974.20**	**5147.86**	**5006.45**	**4763.79**	**4697.92**	**4501.64**	**4626.80**

The Case Study – 2 results are compared with other methods like TOPSIS, COPRAS-G, OCRA, Desirability Function, and EVAMIX. From the Table 5.1.2 the high degree of overlap and correlation can be realized.

Table 5.1.2: Comparison of final ranks of items in Case Study - 2

Product Code	TOPSIS	COPRAS-G	OCRA	Desirability Function	EVAMIX	MULTIMOORA	XMULTIMOORA
C01	20	20	10	20	20	9	9
C02	8	7	9	5	9	7	7
C03	14	13	8	13	8	8	8
C04	16	16	7	16	11	12	12
C05	4	4	4	4	3	2	2
C06	19	19	17	19	17	15	15
C07	7	15	13	6	7	10	10
C08	12	11	18	12	18	20	20
C09	18	18	20	18	19	17	17
C10	9	8	15	9	15	19	19
C11	10	9	14	10	14	14	14
C12	17	17	19	17	16	16	16
C13	11	10	11	11	13	13	13
C14	5	5	5	7	5	4	4
C15	15	14	12	15	10	11	11
C16	3	2	3	3	1	1	1
C17	6	6	6	8	6	5	5
C18	13	12	16	14	12	18	18
C19	2	3	1	2	4	3	3
C20	1	1	2	1	2	6	6

5.2 Validation

The Spearman's [**91**] rank correlation coefficient, $R = 1 - \frac{6\sum d^2}{n(n^2-1)}$,where d is the distance between the final ranks of the two methods, n is the number of alternatives i.e. the number of items (which is 47), is calculated for both the methods, which happens to be 0.9742 , suggesting a strong correlation (mutual dependencies) of the two MOORA-based methods. The rank correlation coefficients of MULTIMOORA and XMULIMORRA computed against the traditional method are obtained as 0.7014 and 0.6719 respectively. Fig. 5.2.1 clearly indicates the strong correlation between the employed methods and also the same with the traditional method. Some of the other performance indices namely clustering validation ratio, etc. can be also computed in this regard to judge the efficacy of the employed methods. Table 5.2.1 shows the high values of spearman's rank correlation coefficient for Case Study -2 when compared with other MCDM methods. The final results of classifications obtained by MULTIMOORA and extended MULTIMOORA is just identical for Case Study – 2.

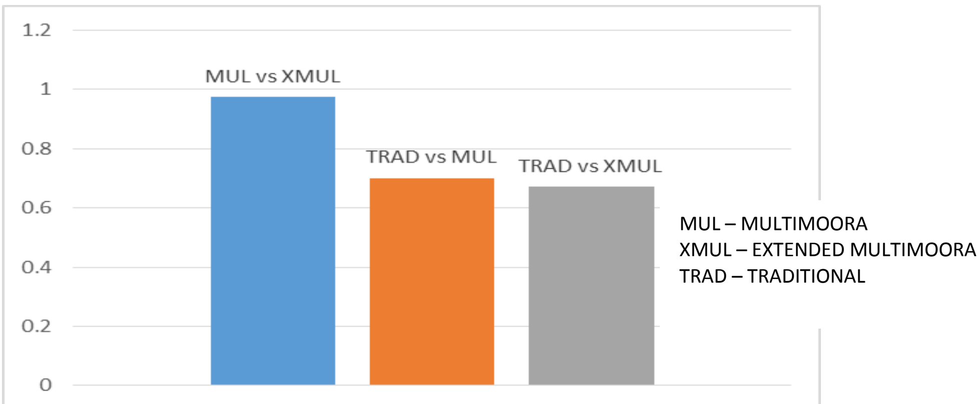


Fig. 5.2.1: Spearman rank correlation coefficient for Case Study - 1

Table 5.2.1: Spearman's rank correlation coefficient for Case Study - 2

Methods	TOPSIS	COPRAS-G	OCRA	Desirability Function	EVAMIX
MULTIMOORA	0.653	0.611	0.91	0.648	0.835
XMULTIMOORA	0.653	0.611	0.91	0.648	0.835

In order to further validate the proposed model, the weighted distance matrix for Case Study - 1 (47×47) was computed for the normalized scores obtained from MULTIMOORA, and a dendrogram having predefined number of clusters as 3 (for, A, B, C classes) was drawn using MINITAB, which is shown is Fig. 5.2.2. The percentage of overlap between the items in the same cluster is 70% for class A items, 64% for class B items, and 86% for class C items.

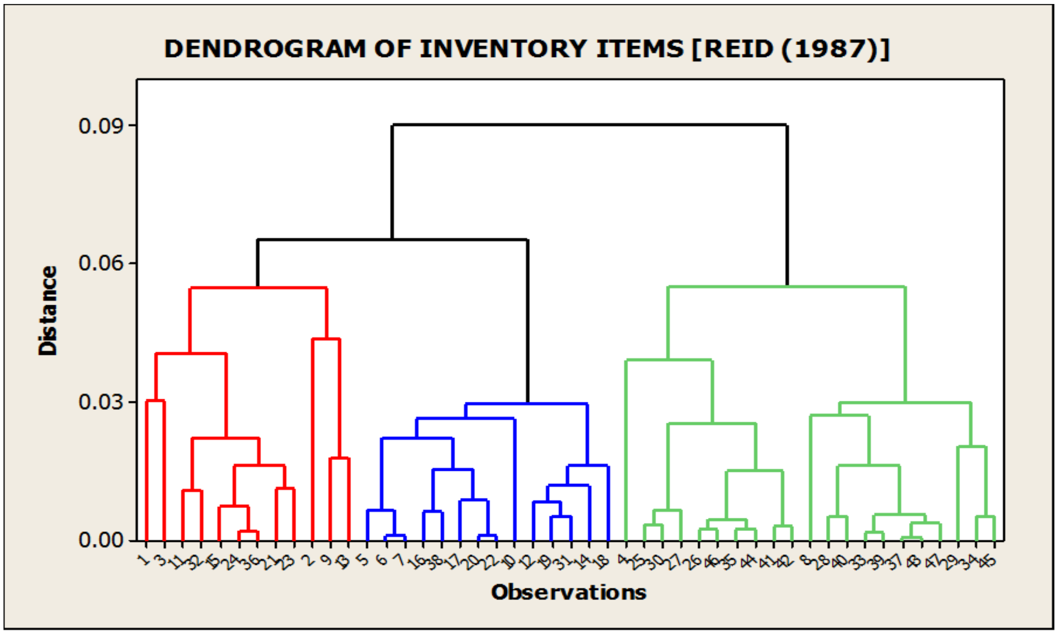


Fig. 5.2.2: Dendrogram showing 3 clusters in different colours

Chapter 6

6. Conclusions and Future Scopes

In this dissertation the multi-criteria ABC inventory classification is addressed using the MULTIMOORA method and extending the same by target-based normalization and integrated significant coefficients. The subjective, objective, and inter-attribute correlation effect significant coefficients are combined together to formulate the extended method. Information entropy and standard deviation concepts are used to derive the two sets of objective significant coefficients, while standard deviation based correlation effect measures are also obtained for figuring out the inter-attribute correlation effect significant coefficients. The exogenously decided weights i.e. subjective significant coefficient as obtained from fuzzy linguistic variables assigned by different experts are aggregated based on their relative importance, whereas the endogenous weights are derived from the data itself in the form of the other three significant coefficients. Hence, both the advantages of subjectivity and objectivity are taken into account by the combinatorial approach. The three subordinate parts of the MULTIMOORA method find the corresponding rankings. This is worth highlighting that the use of reference point approach prevents the proposed model from becoming fully compensatory technique. The aggregation mechanism applied while doing final ranking avoid the possible circular reasoning trap in dominance theory. The cardinality limitation for each class items is also adhered from the desired service level management perspective. Two case studies were performed. First one was on the benchmark dataset of Reid, 1987 [**79**], used by many previous researchers. The second case study was on a pharmaceutical industry dataset. The results of the classification in case study - 1 made by both MULTIMOORA and XMULTIMOORA methods are compared with some previous methods using a new computational model. It is shown that both these methods are superior in terms of cost effectiveness. The results obtained in case study -2 also reveals strong correlation with other MCDM approaches.

There are certain limitations in the research work carried-out and methodology applied. These can be considered as scopes for future research directions.

The other criteria like ordering cost, holding cost, shortage cost or cost due to lost sales, durability, perishability, risk of obsolescence, etc. can be additionally considered for the Case

Study -1 to classify the items more accurately. A sensitivity analysis for the changes in weights using the same MULTIMOORA approaches should be carried out. A DOE analysis for the subjective weight assignment under fuzzy environment can also attract interesting research work.

The fuzzy environment can be extended to support qualitative attributes/ criteria so that crisp values are obtained for decision matrix requirement.

The cost computational model can be again enhanced for the shortage cost and extra safety stock cost with respect to the wrongly assigned control policies for class A and B items.

The assessment values obtained in each subordinate method of MULTIMOORA can be expressed as mass probability function, and subsequently an evidential combination approach employing Dempster-Shafer Theory (DST) [**28, 86**] can be used to compute the final assessment and subsequently the final score.

There is a need for general framework for comparing any classification dataset by MULTIMOORA and the allied methodology, and preferably for all other MCDM approaches.

Both the MULTIMOORA and extended MULTIMOORA methods can be applied in other virgin production management paradigms.

Chapter 7

7. References

[1] Altay Guvenir, H., & Erel, E. (1998). Multi-criteria inventory classification using a genetic algorithm. *European Journal of Operational Research, 105*(1), 29-37.

[2] Bacchetti, A., Plebani, F., Saccani, N., & Syntetos, A. A. (2013). Empirically-driven hierarchical classification of stock keeping units. *International Journal of Production Economics, 143, 2,* 263–274.

[3] Balaji, K. and Kumar, V. S. S. (2014). Multicriteria inventory ABC classification in an automobile rubber components manufacturing industry. Procedia CIRP, 17, 43 - 468.

[4] Basu, P. and Nair, S.K. (2014). A decision support system for mean–variance analysis in multi-period inventory control, *Decision Support Systems, 57*, 285-295.

[5] Baykasoğlu, A., Subulan, K., & Karaslan, F. S. (2016). A new fuzzy linear assignment method for multi-attribute decision making with an application to spare parts inventory classification. *Applied Soft Computing, 42,* 1-17.

[6] Bera, U.K., Maiti, M.K. and Maiti, M. (2012). Inventory model with fuzzy lead-time and dynamic demand over finite time horizon using a multi-objective genetic algorithm, *Computers and Mathematics with Applications, 64(6),* 1822-1838.

[7] Bhattacharya, A., Sarkar, B., & Mukherjee, S. K. (2007). Distance-based consensus method for ABC analysis. *International Journal of Production Research, 45*(15), 3405-3420.

[8] Braglia, M., Grassy, A., & Montana, R. (2004). Multi-attribute classification method for spare parts inventory management. *Journal of Quality in Maintenance Engineering, 10*(1), 55-65.

[9] Brauers, W.K.M., Baležentis, A., Baležentis, T. (2011). MULTIMOORA for the EU member states updated with fuzzy number theory. *Technology and Economic Development*, 17, 259–290.

[10] Brauers, W.K.M., Baležentis, A., Baležentis, T. (2012). EUROPEAN Union member states preparing for EUROPE 2020. An application of the MULTIMOORA method. *Technology and Economic Development*, *18*, 567–587.

[11] Brauers, W.K.M., Kildienė, S., Zavadskas, E.K., & Kaklauskas, A. (2013). The construction sector in twenty European countries during the recession 2008–2009– country ranking by MULTIMOORA. *International Journal of Strategic Property Management*, *17*, 58–78.

[12] Brauers, W.K.M., & Zavadskas, E.K. (2006). The MOORA method and its application to privatization in a transition economy. *Control and Cybernetic Journal*, *35*, 445–469.

[13] Brauers, W.K.M., & Zavadskas, E.K. (2010). Project management by MULTIMOORA as an instrument for transition economies. *Technology and Economic Development*, *16*, 5–24.

[14] Brauers, W.K.M., & Zavadskas, E.K. (2012). Robustness of MULTIMOORA: a method for multi-objective optimization. *Informatica* 23, 1–25

[15] Buffa, E.S. and Sarin, K. (2007). Modern Production/Operation Management, Edn. 8, John Wiley & Sons.

[16] Cakir, O., & Canbolat, M. S. (2008). A web-based decision support system for multi-criteria inventory classification using fuzzy AHP methodology. *Expert Systems with Applications, 35(3),* 1367-1378.

[17] Canetta, L., Cheikhrouhou, N., & Glardon, R. (2005). Applying two-stage SOM-based clustering approaches to industrial data analysis, *Production Planning & Control*, *16(8*), 774-784.

[18] Cavalieri, S., Garetti, M., Macchi, M., & Pinto, R. (2008). A decision-making framework for managing maintenance spare parts, *Production Planning & Control, 19(4)*, 379-396.

[19] Çelebi, D. (2015). Inventory control in a centralized distribution network using genetic algorithms: A case study, Computers & Industrial Engineering, 87, 532-539.

[20] Chakravarty, A. K. (1981). Multi-item inventory aggregation into groups, *Journal of the Operational Research Society, 32(1)*, 19-26.

[21] Chary, S.N. (2009). Production & Operations Management, Edn. 4, Tata McGraw-Hill Education

[22] Chen, J. X. (2011). Peer-estimation for multiple criteria ABC inventory classification. *Computers & Operations Research, 38(12*), 1784–91.

[23] Chen, J. X. (2012). Multiple criteria ABC inventory classification using two virtual items. *International Journal of Production Research, 50(6),* 1702–13.

[24] Chen, Y., Li, K. W.,Kilgour, D. M.,& Hipel, K. W. (2008). A case-based distance model for multiple criteria ABC analysis. *Computers & Operations Research*, *35 (3)*, 776–96.

[25] Chen, Y., Li, K. W., Levy, J., Hype, K. W., & Kilgore, D. M. (2008). A rough set approach to multiple criteria ABC analysis. In *Transactions on rough sets VIII* (pp. 35-52). Springer Berlin Heidelberg.

[26] Chu, C. W., Liang, G. S., & Liao, C. T. (2008). Controlling inventory by combining ABC analysis and fuzzy classification. *Computers & Industrial Engineering.*

[27] D'Alessandro, A. J. & Baveja, A. (2000). Divide and conquer: Rohm and Haas' response to a changing specialty chemicals market, *Interfaces*, *30(6)*, 1-16.

[28] Dempster, A.P. (1968). A generalization of Bayesian inference (with discussion). *Journal of the Royal Statistical Society Series B, 30(2)*, 205–247

[29] Dey, B., Bairagi, B., Sarkar, B., & Sanyal, S. (2012), A MOORA based fuzzy multi-criteria decision making approach for supply chain strategy selection. *International Journal of Industrial Engineering Computations, 3,* 649–662

[30] Diakoulaki, D., Mavrotas, G., & Papayannakis, I. (1995) Determining objective weights in multiple criteria problems: the CRITIC method. *Computers &. Operations Research, 22*, 763–770.

[31] Edwards, W., & Barron, F. H. (1994). SMARTS and SMARTER: Improved simple methods for multi-attribute utility measurement. *Organizational Behavior and Human Decision Processes, 60*(3), 306-325.

[32] Elgazzar, S.H., Nicoleta, N.S., Hubbard, N.J. and Leach, D.L. (2012). Linking supply chain processes' performance to a company's financial strategic objectives, *European Journal of Operational Research, 223(1),* 276-289.

[33] Ernst, R., &Cohen, M. A. (1990). Operations related groups (ORGs): a clustering procedure for production/inventory systems. *Journal of Operations Management (4),* 574–598.

[34] Gomes, L. F. A. M., & Ferreira, A. C. S. (1995). The multi-criteria ABC analysis – An application of rough set theory. *Foundations of Computing and Decision Sciences*, *20*(3).

[35] Flores, B.E., & Whybark, D.C. (1986). Multiple Criteria ABC Analysis. *International Journal of Operations and Production Management*, 6(3), 38-46.

[36] Flores, B. E., & Whybark, D. C. (1987). Implementing multiple criteria ABC analysis. *Journal of Operations Management, 7*(1), 79-85.

[37] Flores, B. E., Olson, D. L., & Dorai, V. K. (1992). Management of multi-criteria inventory classification. *Mathematical and Computer Modelling, 16*(12), 71-82.

[38] Goldberg, D. (1989). Genetic algorithms in search, optimization & machine learning, Harlow, England: Addison Wesley.

[39] Gajpal, P. P., Ganesh, L. S., & Rajendran, C. (1994). Criticality analysis of spare parts using the analytic hierarchy process. *International Journal of Production Economics, 35*(1), 293-297.

[40] Gulsen, A. K., & Coskun, O. (2013). Multiple Criteria ABC Analysis with FCM Clustering. *Journal of Industrial Engineering, 87274.*

[41] Hadi-Vencheh, A. (2010). An improvement to multiple criteria ABC inventory classification. *European Journal of Operational Research, 201*(3), 962-965.

[42] Hadi-Vencheh, A., & Mohamadghasemi, A. (2011). A AHP-DEA approach for multiple criteria ABC inventory classification. *Expert Systems with Applications, 38*, 3346-3352.

[43] Hafezalkotob, A., & Hafezalkotob, A. (2015). Comprehensive MULTIMOORA method with target-based attributes and integrated significant coefficients for materials selection in biomedical applications, *Materials & Design (87)*, 949-959.

[44] Harhalakis, G., Sharma, P., & Zachmann, W. S. (1989). A dynamic planning and control system for inventories of raw materials, *Production and Inventory Management Journal, 30(2)*, 12-17.

[45] Harris, F.W. (1913). How many parts to make at once, *Factory Magazine. Management, 10(2*), 135–136.

[46] Huiskonen, J., Niemi, P., & Pirttila, T. (2005). The role of C-products in providing customer service -refining the inventory policy according to customer-specific factors, *International Journal of Production Economics*, *93-94(1)*, 139-149.

[47] Jahan, A., Mustapha, F., Ismail, M.Y.,Sapuan, S., & Bahraminasab, M. (2011) A comprehensive VIKOR method for material selection. *Material Design, 32,* 1215–122.

[48] Jahan, A., Bahraminasab, M., & Edwards, K. (2012). A target-based normalization technique for materials selection. Material Design, 35, 647–654

[49] Kaabi, H.; Jabeur, K. & Enneifar, L. (2015), Learning criteria weights with TOPSIS method and continuous VNS for multi-criteria inventory classification. *Electronic Notes in Discrete Mathematics, 47*, 197-204.

[50] Kaabi, H., Jabeur, K. & Ladhari, T. (2014). Genetic Algorithm to infer criteria weights for Multicriteria Inventory Classification. *In 2014 International Conference on Control, Decision and Information Technologies (CoDIT)*, 276–281.

[51] Kabir, G., Hasin, M. A. A., & Khondokar, M. A. H. (2011). Fuzzy analytical hierarchical process for multi-criteria inventory classification. *Proceedings of the International Conference on Mechanical Engineering*, 18-20 December, Dhaka, Bangladesh.

[52] Karande, P., & Chakraborty, S. (2012). Application of multi-objective optimization on the basis of ratio analysis (MOORA) method for materials selection. *Material Design, 37*, 317–324.

[53] Kazemi, N., Shekarian, E., Eduardo, L., Cárdenas-Barrón, E. and Olugu, U. (2015). Incorporating human learning into a fuzzy EOQ inventory model with backorders, *Computers & Industrial Engineering*, *87*, 540-542.

[54] Ketkar, M. and Vaidya, O. S. (2014). Developing ordering policy based on multiple inventory classification schemes, *Procedia - Social and Behavioral Sciences, 133*, 180–188.

[55] Kobbacy, K. A. H. & Liang, Y. (1999). Towards the development of an intelligent inventory management system", *Integrated Manufacturing Systems*, *10(6*), 354-366.

[56] Kracka, M., Zavadskas, E.K., 2013. Panel building refurbishment elements effective selection by applying multiple-criteria methods. *International Journal. of Strategic Property Management*, *17*, 210–219.

[57] Kumar, R.S. and Goswami, A. (2015). A continuous review production–inventory system in fuzzy random environment: Minmax distribution free procedure, *Computers & Industrial Engineering*, *79*, 65-75

[58] Lajili, I., Babai, M. Z., & Ladhari, T. (2012) Inventory performance of multicriteria inventory classification methods: An empirical investigation. *In proceedings of 9th International Conference of Modeling, Optimization and Simulation (MOSIM).*

[59] Lei, Q.S., Chen, J., & Zhou, Q. (2005). Multiple criteria inventory classification based on principal components analysis and neural network. *Proceedings of Advances in neural networks*, Berlin, 1058-1063.

[60] Lolli, F.,Ishizaka, A.,& Gamberini, R. (2014). New AHP-based approaches for multi-criteria inventory classification. *International Journal of Production Economics*, *156*, 62–74.

[61] Li, B., Wang, H.W., Yang, J. B., Guo, M. and Qi, C. (2011). A belief-rule-based inventory control method under nonstationary and uncertain demand, *Expert Systems with Applications, 38(12),* 14997-15008.

[62] Liu, H. C., Fan, X. J., Li, P., & Chen, Y. Z. (2014). Evaluating the risk of failure modes with extended MULTIMOORA method under fuzzy environment, *Engineering Applications of Artificial Intelligence, 34*, 168-177.

[63] Liu, J., Liao, X., Zhao, W., & Yang, N. (2016). A classification approach based on the outranking model for multiple criteria ABC analysis, Omega.

[64] Liu, Q., & Huang, D. (2006). Classifying ABC inventory with multi-criteria using a data envelopment analysis approach. In *Intelligent systems design and applications, 2006.* ISDA'06. Sixth International Conference on IEEE (Vol. 1, pp. 1185-1190).

[65] Mandal, U. K., & Sarkar, B. (2012). Selection of best intelligent manufacturing system (IMS) under fuzzy MOORA conflicting MCDM environment, International Journal of Emerging Technology and Advanced Engineering, 2 (2012) 2250–2459.

[66] Mishra, S., Sahu,A. K., Datta,S., & Mahapatra, S. S. (2015). Application of fuzzy integrated MULTIMOORA method towards supplier/partner selection in agile supply chain. *International Journal of Operational Research, 22,* 466–514.

[67] Mitchell, A., Millstein, L. Y., & Haitao, L. (2014). Optimizing ABC inventory grouping decisions. *International Journal of Production Economics, 148*, 71-80

[68] Mohammaditabar, D., Ghodsypour, S. H., & O' Brien, C. (2012). Inventory control system design by integrating inventory classification and policy selection, *International Journal Production Economics, 40*, 655–659.

[69] Nahmias, S. (2004). *Production and Operations Analysis*. 5th Edition, Irwin/McGraw Hill, Burr Ridge, IL, USA, 213-215.

[70] Nandi, S. K. (2014). De NOVO Approach in ABC Analysis, *UG Thesis, Jadavpur University, Kolkata, India*

[71] Ng, W. L. (2007). A simple classifier for multiple criteria ABC analysis. *European Journal of Operational Research, 177*(1), 344-353.

[72] Noblesse, A.M., Boute, R.N., Lambrecht, M.R. and Van Houdt, B. (2014). Lot sizing and lead time decisions in production/inventory systems, *International Journal of Production Economics, 155,* 351–360.

[73] Park, J., Bae, H., & Bae, J. (2014). Cross-evaluation-based weighted linear optimization for multi-criteria ABC inventory classification. *Computers & Industrial Engineering, 76*, 40–48.

[74] Partovi, F. Y., & Anandarajan, M. (2002). Classifying inventory using an artificial neural network approach. *Computers & Industrial Engineering, 41*(4), 389-404.

[75] Partovi, F. Y., & Burton, J. (1993). Using the analytic hierarchy process for ABC analysis. *International Journal of Operations & Production Management, 13*(9), 29-44.

[76] Pawlak, Z. (1991). *Rough sets: Theoretical aspects of reasoning about data*. Dordrecht: Kluwer Academic Publishing. ISBN 0-7923-1472-7.

[77] Porras, E. & Dekker, R. (2008). An inventory control system for spare parts at a refinery: An empirical comparison of different re-order point methods, *European Journal of Operational Research, 184(1),* 101-132.

[78] Ramanathan, R. (2006). ABC inventory classification with multiple-criteria using weighted linear optimization. *Computers & Operations Research, 33*(3), 695-700.

[79] Reid, R. A. (1987). The ABC method in hospital inventory management: A practical approach. *Production and Inventory Management, 28 (4),* 67–70

[80] Rezaei, J., & Dowlatshahi, S. (2010). A rule-based multi-criteria approach to inventory classification. *International Journal of Production Research, 48 (23*), 7107–26.

[81] Rezaei, J. & Salimi, N. (2013). Optimal ABC inventory classification using interval programming. *International Journal of Systems Science*.

[82] Riggs, J.L (1981). Production Systems: Planning, Analysis & Control, Edn.4, John Wiley & Sons.

[83] Saaty, T. L. (1995). Transport planning with multiple criteria: The analytic hierarchy process applications and progress review. *Journal of advanced transportation*, *29*(I), 81-126.

[84] Saracoglu, I., Topaloglu, S. and Keskinturk, T. (2014). A genetic algorithm approach for multi-product multi-period continuous review inventory models, *Expert Systems with Applications, 41(18),* 8189–8202.

[85] Şenyiğit, E., Düğenci, M., Aydin, M. E. and Zeydan, M. (2013). Heuristic-based neural networks for stochastic dynamic lot sizing problem, *Applied Soft Computing, 13(3),* 1332–1339.

[86] Shafer, G. (1976). A Mathematical Theory of Evidence, Princeton University Press, Princeton.

[87] Shannon, C. E. (1948). A mathematical theory of communication. *Bell System Technical Journal, 27*, 379–423.

[88] Silver, E.A. and Meal, H.C. (1973). A heuristic for selecting lot size quantities for the case of a deterministic time varying demand rate and discrete opportunities for replenishment, *Production Inventory Management ,. 14(2),* 64–74.

[89] Šimunović, K., Šimunović, G., & Šarić, T. (2009). Application of Artificial Neural Networks to Multiple Criteria Inventory Classification. *Strojarstvo*, *51(4),* 313-321.

[90] Soylu, B., & Akyol, B. (2014). Multi-criteria inventory classification with reference items. *Computers and Industrial Engineering*, *69*, *1*, 12–20.

[91] Spearman, C. (1904) General intelligence, objectively determined and measured, *Am.J. Psychol, 15 (2),* 201–292.

[92] Stanford, R. E. & Martin, W. (2007). Towards a normative model for inventory cost management in a generalized ABC classification system, *Journal of the Operational Research Society, 58(7),* 922-928.

[93] Streimikiene, D., & Balezentis, T. (2013). Multi-objective ranking of climate change mitigation policies and measures in Lithuania. *Renewable and Sustainable Energy Review 18*, 144–153.

[94] Syntetos, A. A., Boylan, J. E., & Croston, J. D. (2005. On the categorization of demand patterns, *Journal of the Operational Research Society, 56(5*), 495-503.

[95] Teunter, R. H., Babai, M. Z., & Syntetos, A. A. (2010). ABC classification: Service levels and Inventory costs. *Production and Operations Management*.

[96] Torabi, S. A.,Hatefi, S. M., & Saleck, PayB. (2012) ABC inventory classification in the presence of both quantitative and qualitative criteria. *Computers & Industrial Engineering, 63(3)*, 530–537.

[97] Tsai, C.Y., & Yeh, S. W. (2008). A multiple objective particle swarm optimization approach for inventory classification. *International Journal of Production Economics, 114(2)*, 656-666

[98] Tsai, S.C. and Liu, C.H. (2014). A simulation-based decision support system for a multi-echelon inventory problem with service level constraints, *Computers & Operations Research, 53*, 118–127.

[99] Vargas, L. G. (1990). An overview of the analytic hierarchy process and its applications. *European Journal of Operational Research*, North-Holland.

[100] Wagner, H.M. and Whitin, T.M. (1958). Dynamic version of the economic lot size model, *Management Science, 5*, 89–96

[101] Xiao, Y.Y., Zhang, R.Q., & Kaku, I. (2011). A new approach of inventory classification based on loss profit. *Expert Systems with Applications, 38(8)*, 9382–9391.

[102] Yu, M. C. (2011). Multi-criteria ABC analysis using artificial-intelligence-based classification techniques. *Expert Systems with Applications, 38*(4), 3416-3421.

[103] Zadeh, L.A. (1975). The concept of a linguistic variable and its application to approximate reasoning. *Part I, Information Sciences, 8*, 199–249

[104] Zahedi, F. (1986).The analytic hierarchy process-a survey of the method and its applications. *Interfaces*, *16*(4).

[105] Zhou, P., & Fan, L. (2007). A note on multi-criteria ABC inventory classification using weighted linear optimization. *European Journal of Operational Research*, *182*(3), 1488-1491.