3-phase to 2-phase Transformation Matrix

Day 8

ILOs – Day8

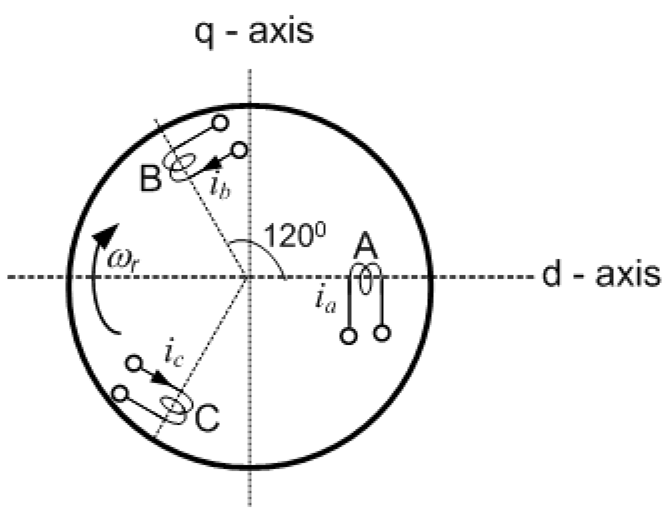
- Derive the transformation matrix (**a, b, c to α, β, 0**)
- Derive the inverse transformation matrix (**α, β, 0 to a, b, c**)

Transformation Matrix

- **a, b, c to α, β transformation**
  - The transformation equations are:

$$i_\alpha = \sqrt{\frac{2}{3}}\left[i_a + i_b\left(-\frac{1}{2}\right) + i_c\left(-\frac{1}{2}\right)\right]$$

$$i_\beta = \sqrt{\frac{2}{3}}\left[0 + i_b\left(\frac{\sqrt{3}}{2}\right) + i_c\left(-\frac{\sqrt{3}}{2}\right)\right]$$



Transformation Matrix

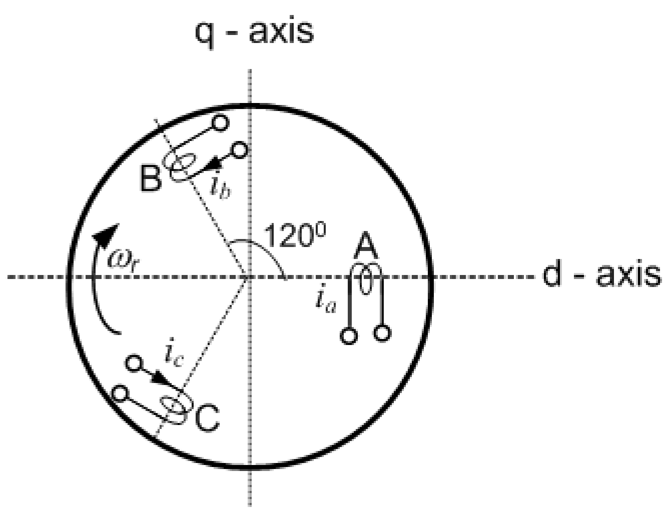
- **a, b, c to α, β transformation**
  - In matrix form:

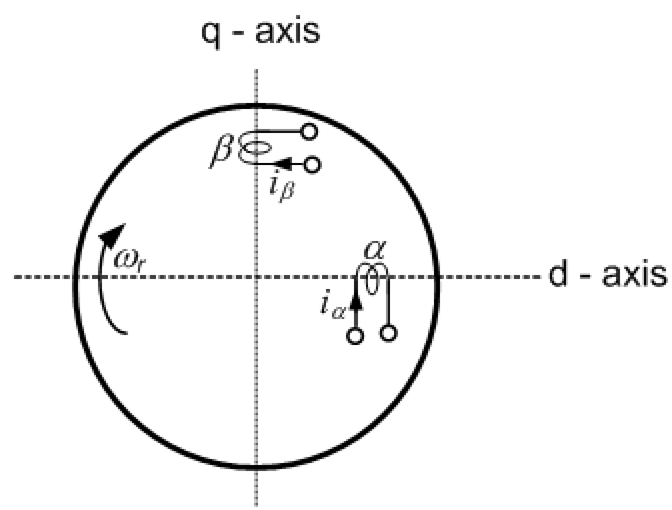
$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

Transformation matrix

$$i_\alpha = \sqrt{\frac{2}{3}}\left[i_a + i_b\left(-\frac{1}{2}\right) + i_c\left(-\frac{1}{2}\right)\right]$$

$$i_\beta = \sqrt{\frac{2}{3}}\left[0 + i_b\left(\frac{\sqrt{3}}{2}\right) + i_c\left(-\frac{\sqrt{3}}{2}\right)\right]$$





Transformation Matrix

- **a, b, c to α, β transformation**

$$i_\alpha = \sqrt{\frac{2}{3}}\left[i_a + i_b\left(-\frac{1}{2}\right) + i_c\left(-\frac{1}{2}\right)\right]$$

$$i_\beta = \sqrt{\frac{2}{3}}\left[0 + i_b\left(\frac{\sqrt{3}}{2}\right) + i_c\left(-\frac{\sqrt{3}}{2}\right)\right]$$

  - In matrix form:

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

  - The transformation matrix is a singular one (No determinant since it is not a square matrix)
  - Thus, i_a, i_b, i_c can not be obtained from i_α, i_β since inverse of a singular matrix doe not exist
  - This situation can be overcome by making the matrix a square one
  - i.e. we need a third equation containing i_a, i_b, and i_c

Transformation Matrix

- **a, b, c to α, β transformation**
  - In matrix form:

$$i_\alpha = \sqrt{\frac{2}{3}}\left[i_a + i_b\left(-\frac{1}{2}\right) + i_c\left(-\frac{1}{2}\right)\right]$$

$$i_\beta = \sqrt{\frac{2}{3}}\left[0 + i_b\left(\frac{\sqrt{3}}{2}\right) + i_c\left(-\frac{\sqrt{3}}{2}\right)\right]$$

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

  - The third equation required to make the transformation matrix a square one should not disturb the MMF
  - One obvious choice is the zero sequence current:

$$i_0 = \left[i_a + i_b + i_c\right]$$

  - Note that in a balanced system $(i_a + i_b + i_c) = 0$, and thus the main operation is not disturbed if we introduce the zero sequence current i_0 in the equation

Transformation Matrix

- **a, b, c to α, β transformation**
  - In matrix form:

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$$i_\alpha = \sqrt{\frac{2}{3}}\left[i_a + i_b\left(-\frac{1}{2}\right) + i_c\left(-\frac{1}{2}\right)\right]$$

$$i_\beta = \sqrt{\frac{2}{3}}\left[0 + i_b\left(\frac{\sqrt{3}}{2}\right) + i_c\left(-\frac{\sqrt{3}}{2}\right)\right]$$

$$i_0 = \left[i_a + i_b + i_c\right]$$

  - The zero sequence current does not produce any RMF; and hence simply to suit the transformations we choose an arbitrary multiplying factor $\frac{1}{\sqrt{3}}$:

$$i_0 = \frac{1}{\sqrt{3}}\left[i_a + i_b + i_c\right]$$

  - The 3-phase currents i_a, i_b, i_c are now replaced by 2-phase currents i_α, i_β and the zero sequence current i_0

Transformation Matrix

- **a, b, c to α, β, 0 transformation**
  - In matrix form:

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$$i_\alpha = \sqrt{\frac{2}{3}}\left[i_a + i_b\left(-\frac{1}{2}\right) + i_c\left(-\frac{1}{2}\right)\right]$$

$$i_\beta = \sqrt{\frac{2}{3}}\left[0 + i_b\left(\frac{\sqrt{3}}{2}\right) + i_c\left(-\frac{\sqrt{3}}{2}\right)\right]$$

$$i_0 = [i_a + i_b + i_c]$$

  - The zero sequence current does not produce any RMF; and hence simply to suit the transformations we choose an arbitrary multiplying factor $\frac{1}{\sqrt{3}}$:

$$i_0 = \frac{1}{\sqrt{3}}[i_a + i_b + i_c]$$

  - The 3-phase currents i_a, i_b, i_c are now replaced by 2-phase currents i_α, i_β and the zero sequence current i_0 is expressed as :

$$i_0 = \frac{1}{\sqrt{3}}[i_a + i_b + i_c] = \sqrt{\frac{2}{3}}\left[\frac{1}{\sqrt{2}} i_a + \frac{1}{\sqrt{2}} i_b + \frac{1}{\sqrt{2}} i_c\right]$$

Transformation Matrix

- **a, b, c to α, β, 0 transformation**
  - In matrix form:

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{array}{c|ccc} & a & b & c \\ \hline \alpha & 1 & -\frac{1}{2} & -\frac{1}{2} \\ \beta & 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{array} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$$i_\alpha = \sqrt{\frac{2}{3}}\left[i_a + i_b\left(-\frac{1}{2}\right) + i_c\left(-\frac{1}{2}\right)\right]$$

$$i_\beta = \sqrt{\frac{2}{3}}\left[0 + i_b\left(\frac{\sqrt{3}}{2}\right) + i_c\left(-\frac{\sqrt{3}}{2}\right)\right]$$

$$i_0 = \sqrt{\frac{2}{3}}\left[\frac{1}{\sqrt{2}} i_a + \frac{1}{\sqrt{2}} i_b + \frac{1}{\sqrt{2}} i_c\right]$$

  - The transformation matrix is now non-singular and its inverse can be obtained

Transformation Matrix

- **a, b, c to α, β, 0 transformation and vice versa**
  - Transformation equation

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{array}{c} \\ \alpha \\ \beta \\ 0 \end{array} \begin{array}{ccc} a & b & c \\ \hline 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{array} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

  - In alternate form

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{array}{c} \\ \alpha \\ \beta \\ 0 \end{array} \begin{array}{ccc} a & b & c \\ \hline \cos 0^0 & \cos 120^0 & \cos 240^0 \\ \sin 0^0 & \sin 120^0 & \sin 240^0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{array} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

Inverse Transformation Matrix

- **α, β, 0 to a, b, c transformation**
  - Inverse transformation matrix

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{ccc} \alpha & \beta & 0 \\ \hline 1 & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \end{array} \begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix}$$

  - In alternate form

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{ccc} \alpha & \beta & 0 \\ \hline \cos 0^0 & \sin 0^0 & \frac{1}{\sqrt{2}} \\ \cos 120^0 & \sin 120^0 & \frac{1}{\sqrt{2}} \\ \cos 240^0 & \sin 240^0 & \frac{1}{\sqrt{2}} \end{array} \begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix}$$

  - If the zero sequence current is not present, then i_a, i_b, i_c can be obtained in terms of i_α, i_β only simply by omitting the third column (marked 0) of the inverse transformation matrix