3-phase to 2-phase Transformation

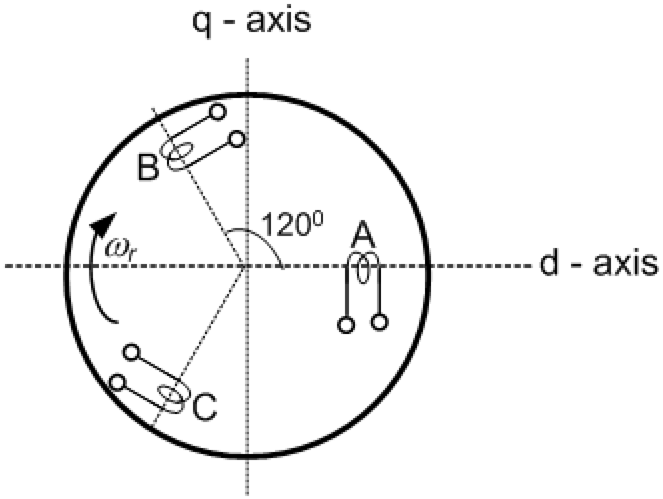
Day 7

ILOs – Day7

- Transform 3-phase machine quantities to equivalent 2-phase machine quantities
  - By changing current
  - By changing number of turns
  - By changing both number of turns and current

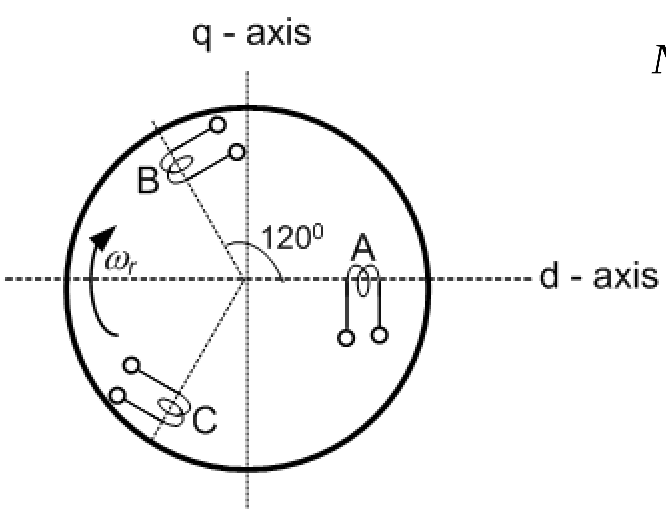
a, b, c to α, β transformation

- **Symmetrical 3-phase, 2-pole winding**
  - Rotor has 3 identical windings A, B, C 120^0 apart
  - Each coil has *N* number of turns

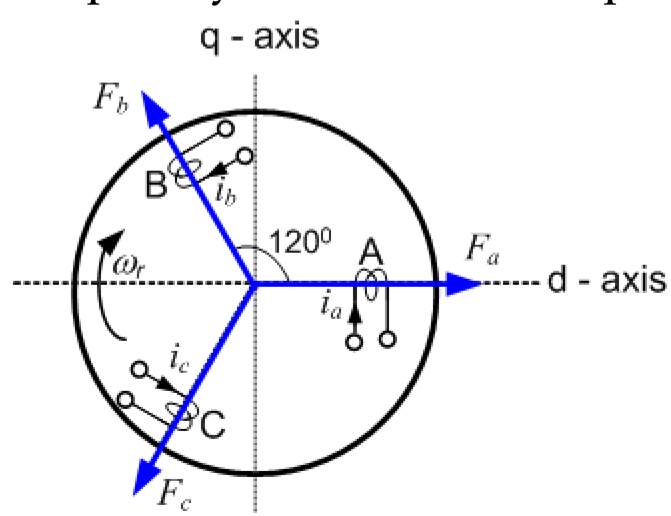


a, b, c to α, β transformation

- **Symmetrical 3-phase, 2-pole winding**
  - Coils carry currents i_a, i_b, and i_c
  - Maximum values of the MMFs F_a, F_b, F_c are along the respective coil axes
  - Combined effect of these three sinusoidally time-varying MMFs produce a rotating magnetic field in rotor that varies in space but is of constant magnitude (constant RMS value)
  - Speed of the RMF depends on supply frequency and number of poles



$$N_s = \frac{120\,f}{P}$$



a, b, c to α, β transformation

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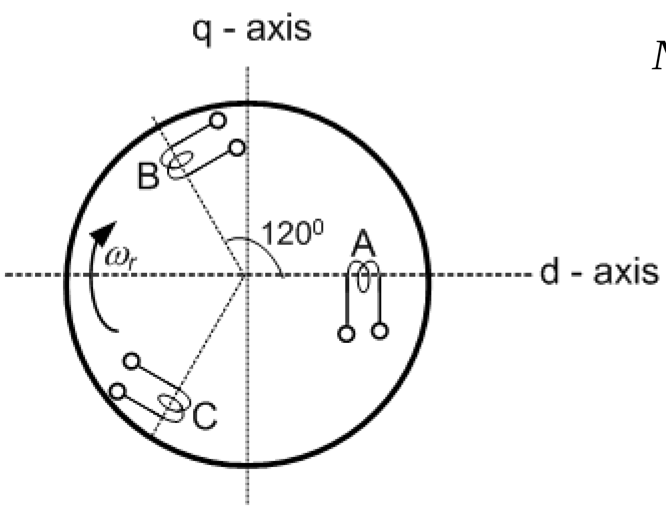

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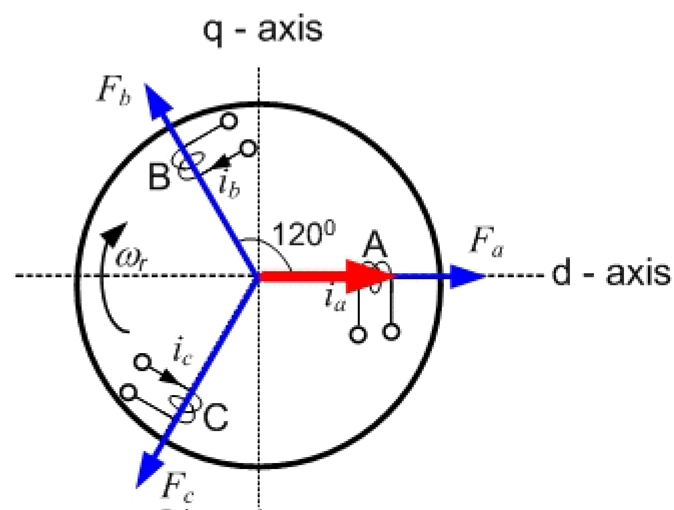


a, b, c to α, β transformation

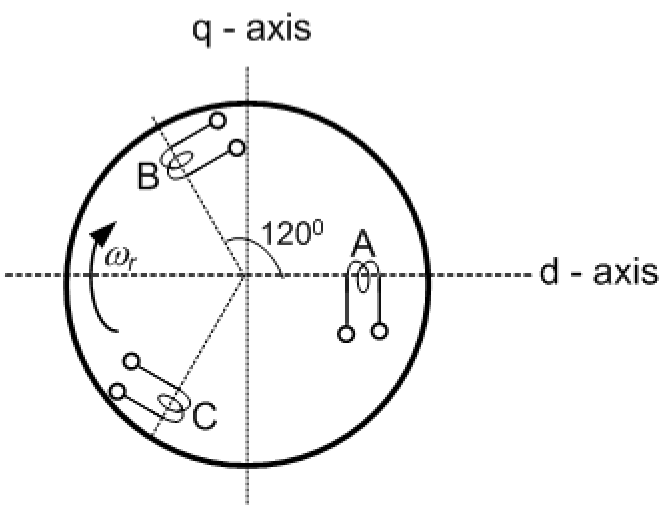
- **Symmetrical 3-phase, 2-pole winding**
  - Coils carry currents i_a, i_b, and i_c

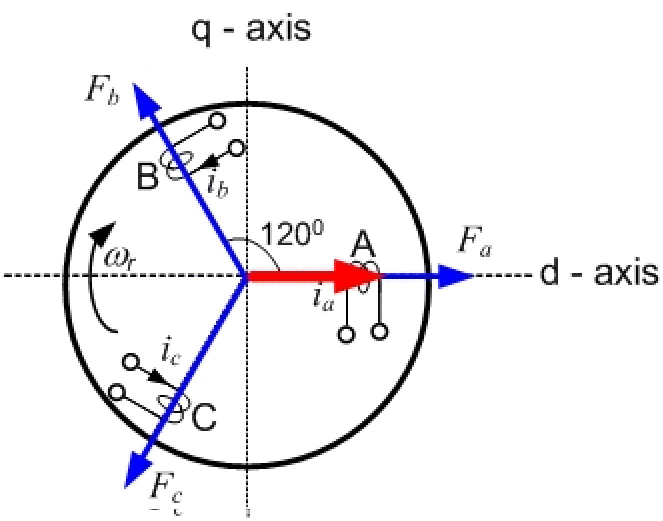
$$i_a = I_m \cos \omega t$$

$$i_b = I_m \left(\cos \omega t - \frac{2\pi}{3} \right)$$

$$i_c = I_m \left(\cos \omega t - \frac{4\pi}{3} \right)$$

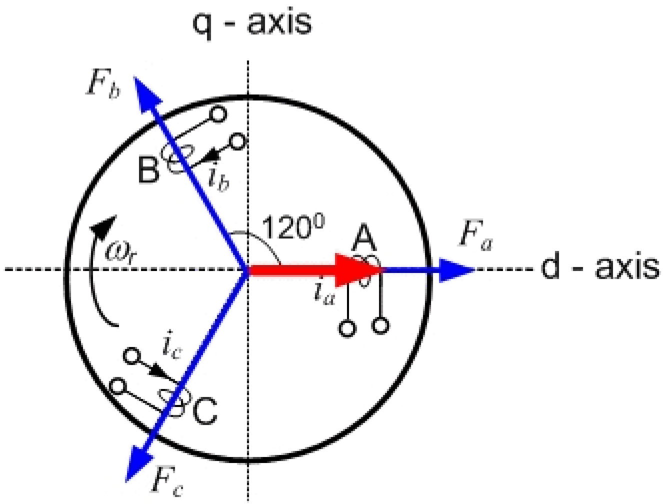
  - The resultant rotating MMF moves at a speed N_s rpm
  - It has constant magnitude (RMS) of $\frac{3NI_m}{2} = 1.5NI_m$

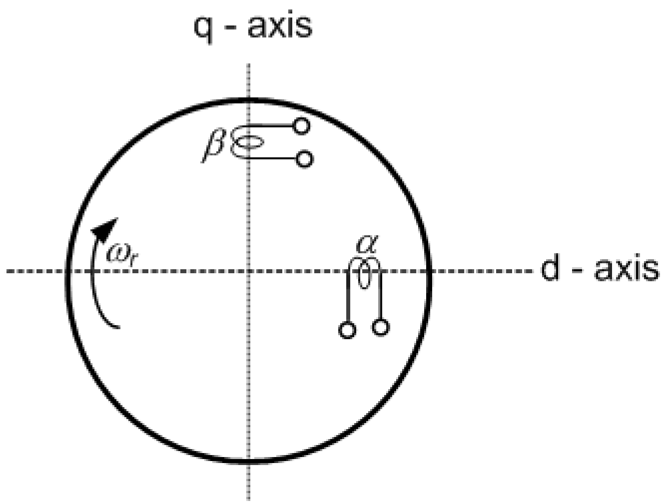




a, b, c to α, β transformation

- **Transformation from 3-phase to 2-phase**
  - 3-phase winding is equivalently represented by a balanced 2-phase winding
  - The two coils α and β are at 90^0
  - The coil α is taken along the same axis as coil A for convenience

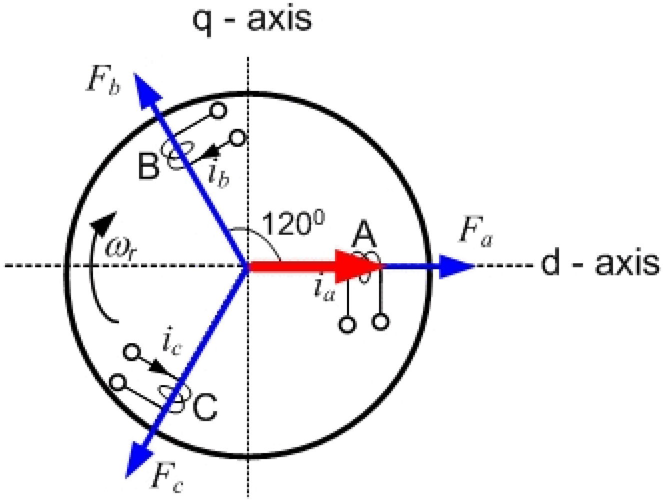


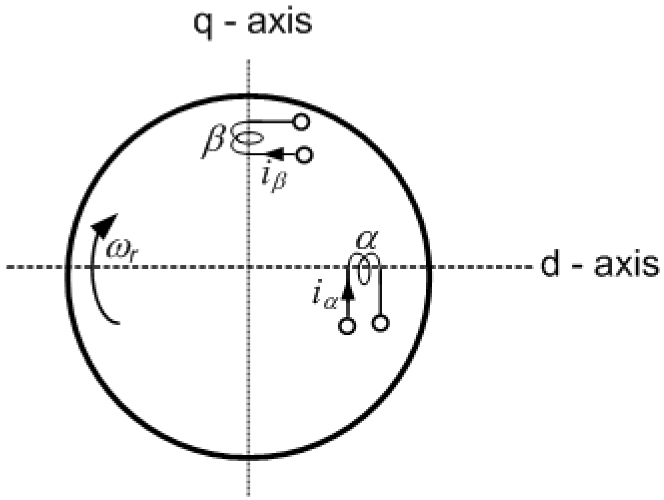


a, b, c to α, β transformation

- **Transformation from 3-phase to 2-phase**
  - The balanced 2-phase currents i_α and i_β are given by:

$$i_\alpha = I_m \cos \omega t$$
$$i_\beta = I_m \left(\cos \omega t - \frac{\pi}{2} \right) = I_m \sin \omega t$$



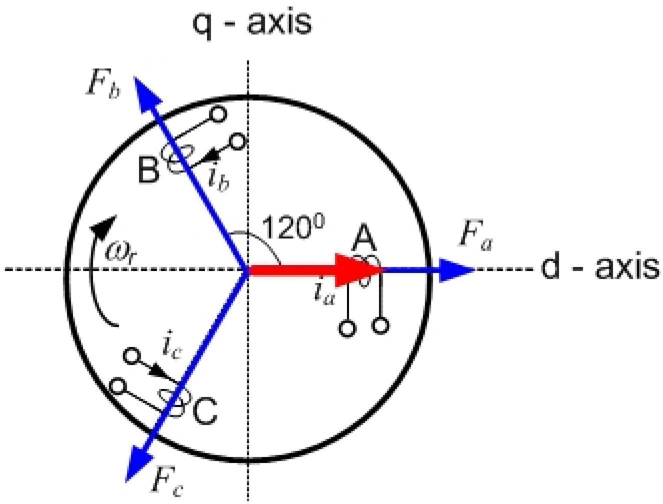


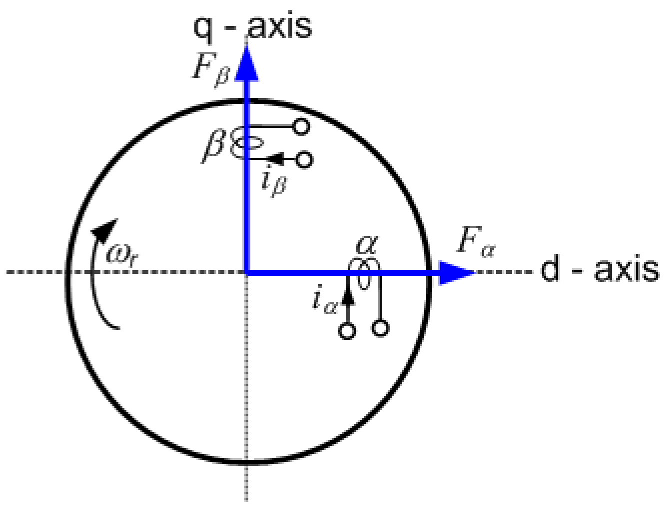
a, b, c to α, β transformation

- **Transformation from 3-phase to 2-phase**
  - The balanced 2-phase currents i_α and i_β are given by:

$$i_\alpha = I_m \cos \omega t$$
$$i_\beta = I_m \left(\cos \omega t - \frac{\pi}{2} \right) = I_m \sin \omega t$$

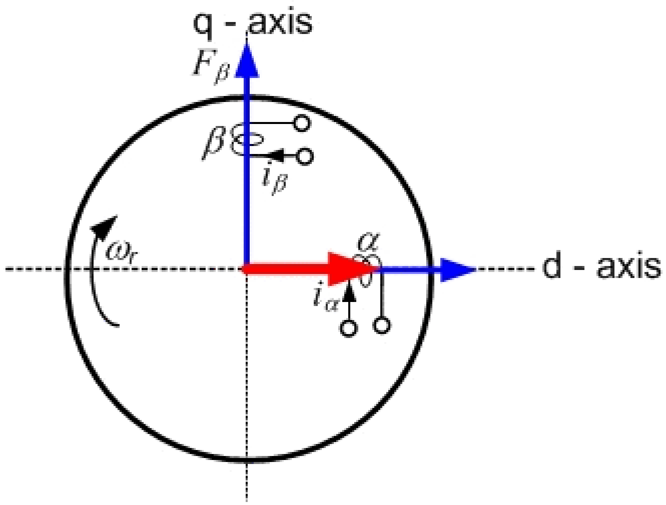
  - These two balanced 2-phase currents will produce MMFs F_α and F_β

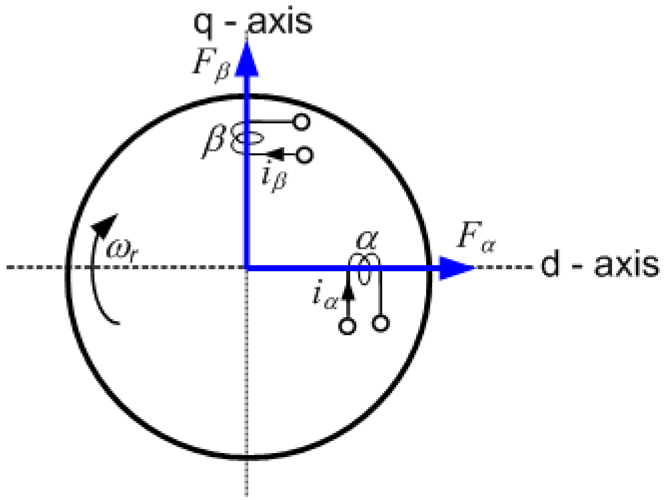




a, b, c to α, β transformation

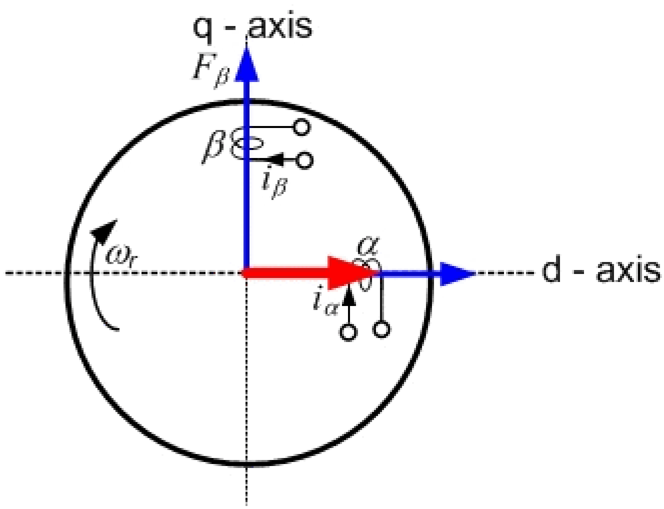
- **Transformation from 3-phase to 2-phase**
  - These two orthogonal (space) and sinusoidally (time) varying MMFs will give rise to a magnetic field that rotates at a constant speed N_s and has a constant magnitude (RMS) of $\boldsymbol{Ni_m}$

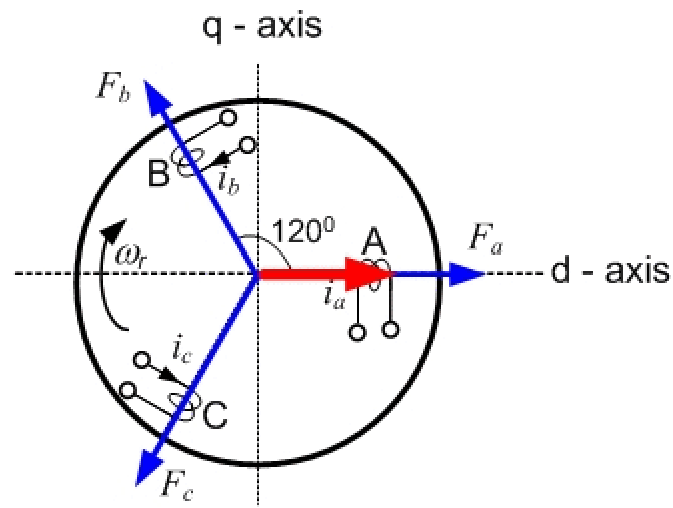




a, b, c to α, β transformation

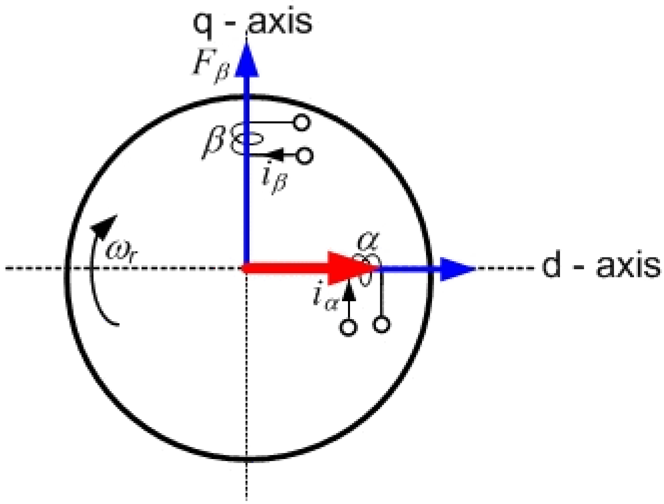
- **Transformation from 3-phase to 2-phase**
  - Thus, both the 3-phase system, as well as the 2-phase system can produce MMFs that rotate at a constant speed of N_s rpm
  - But, magnitude (RMS) of the 3-phase RMF is $1.5Ni_m$
  - Magnitude of the 2-phase RMF is Ni_m
  - These two systems can be equivalent if their RMF magnitudes can be made equal

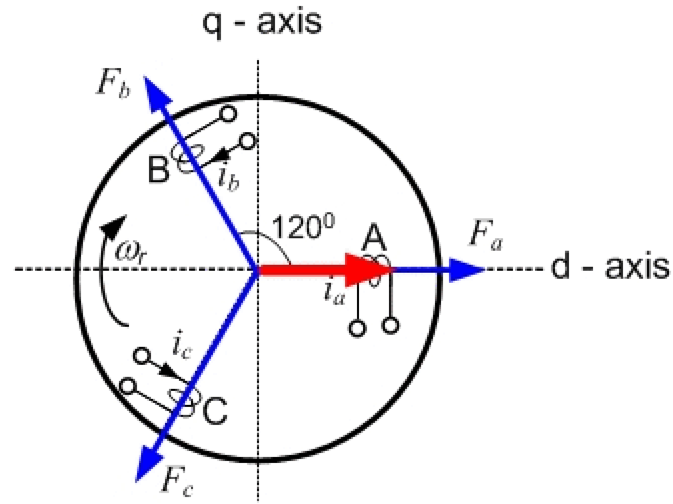




a, b, c to α, β transformation

- **Transformation from 3-phase to 2-phase**
  - RMF magnitudes of the 3-phase system and the 2-phase system can be made equal by:
    - Changing magnitude of the 2-phase currents
    - Changing number of turns in the 2-phase windings
    - Changing both current and number of turns in the 2-phase windings



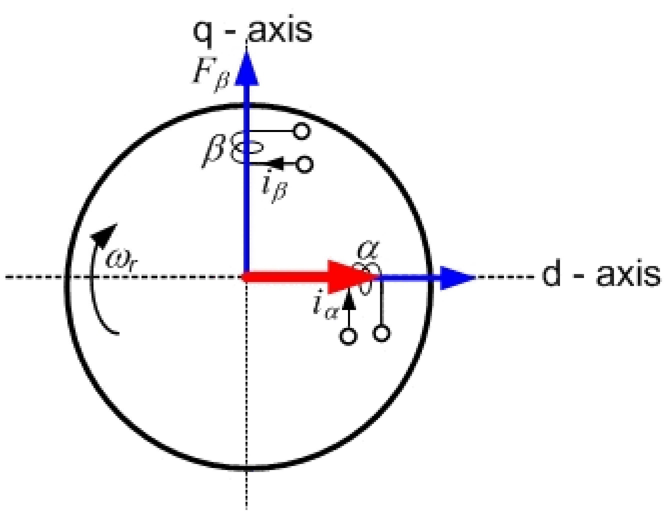


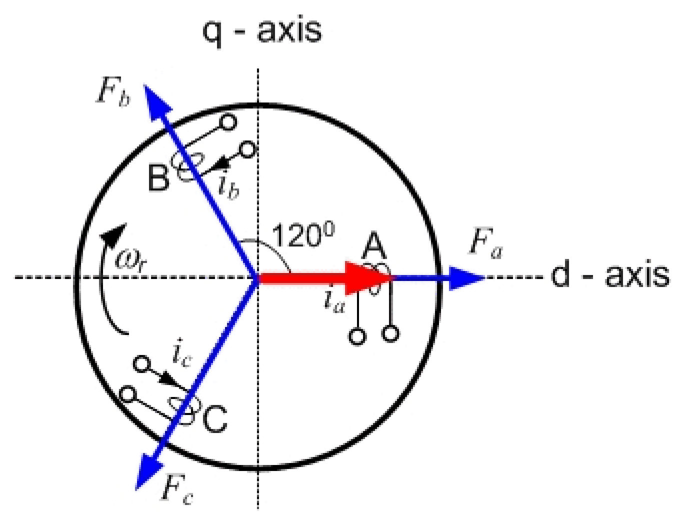
3-phase to 2-phase transformation

By changing the 2-phase currents

a, b, c to α, β transformation

- **Transformation from 3-phase to 2-phase**
  - Magnitude of the 3-phase RMF is $1.5Ni_m$
  - Magnitude of the 2-phase RMF is Ni_m
  - Thus, if the 3-phase and 2-phase system coils have same number of turns,
  - Then to make their MMFs equal, current in the 2-phase system must be 1.5 times higher than those in the equivalent 3-phase system





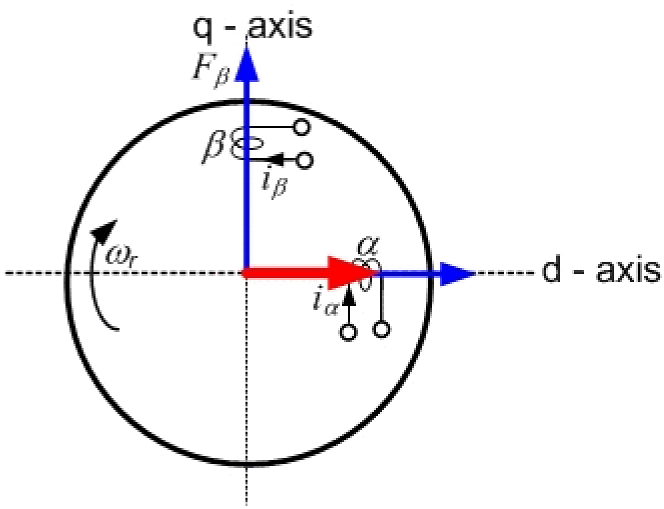
a, b, c to α, β transformation

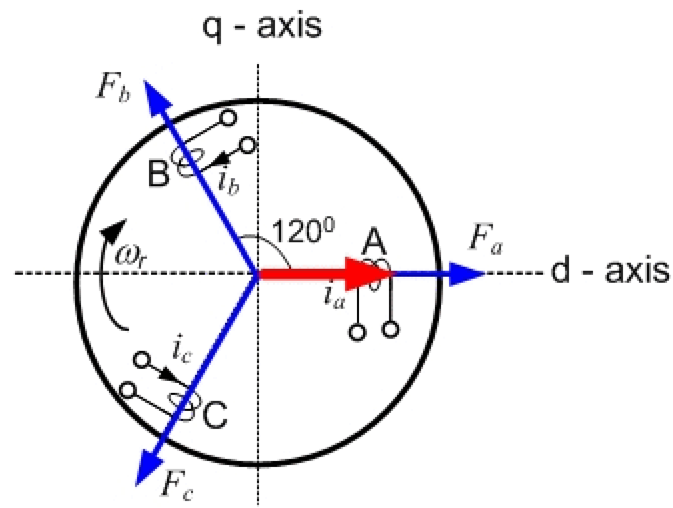
- **Transformation from 3-phase to 2-phase**
  - Resolving the instantaneous values of 3-phase MMFs along α-axis:

$$Ni_\alpha = N\left[i_a \cos 0^0 + i_b \cos 120^0 + i_c \cos 240^0\right]$$

or, $$Ni_\alpha = N\left[i_a + i_b\left(-\frac{1}{2}\right) + i_c\left(-\frac{1}{2}\right)\right]$$

or, $$i_\alpha = \left[i_a - \frac{1}{2}(i_b + i_c)\right]$$





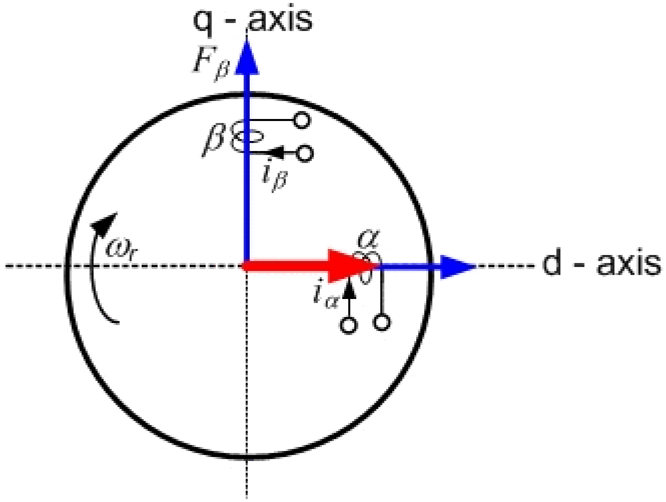
a, b, c to α, β transformation

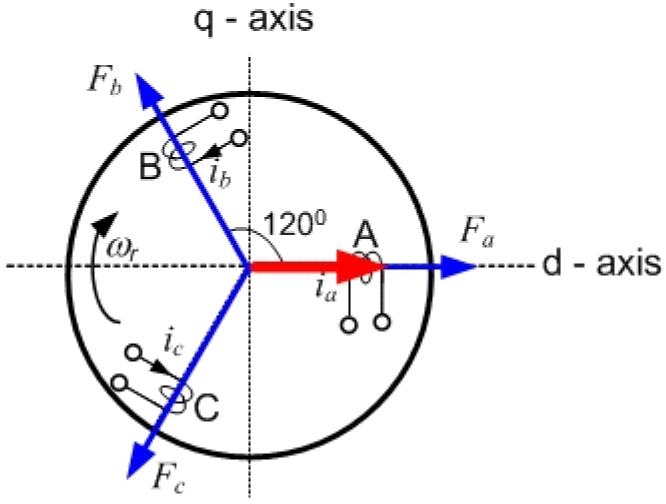
- **Transformation from 3-phase to 2-phase**
  - Similarly, along β-axis:

$$Ni_\beta = N\left[i_a \cos 270^0 + i_b \cos 30^0 + i_c \cos 150^0\right]$$

or, $$Ni_\beta = N\left[0 + i_b\left(\frac{\sqrt{3}}{2}\right) + i_c\left(-\frac{\sqrt{3}}{2}\right)\right]$$

or, $$i_\beta = \left[\frac{\sqrt{3}}{2}(i_b - i_c)\right]$$





a, b, c to α, β transformation

- **Transformation from 3-phase to 2-phase**
  - For a balanced 3-phase system:

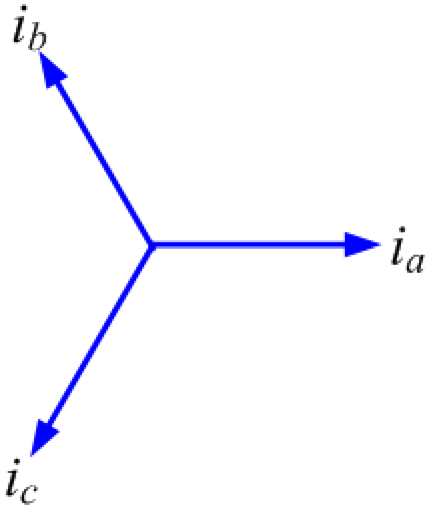
$$i_\alpha = \left[i_a - \frac{1}{2}(i_b + i_c) \right]$$

$$i_\beta = \left[\frac{\sqrt{3}}{2}(i_b - i_c) \right]$$

$$(i_a + i_b + i_c) = 0$$

Thus, $i_\alpha = \left[i_a - \frac{1}{2}(i_b + i_c) \right] = \left[i_a - \frac{1}{2}(-i_a) \right] = \frac{3}{2} i_a = 1.5 i_a$

Hence, $|i_\alpha| = 1.5 |i_a|$



a, b, c to α, β transformation

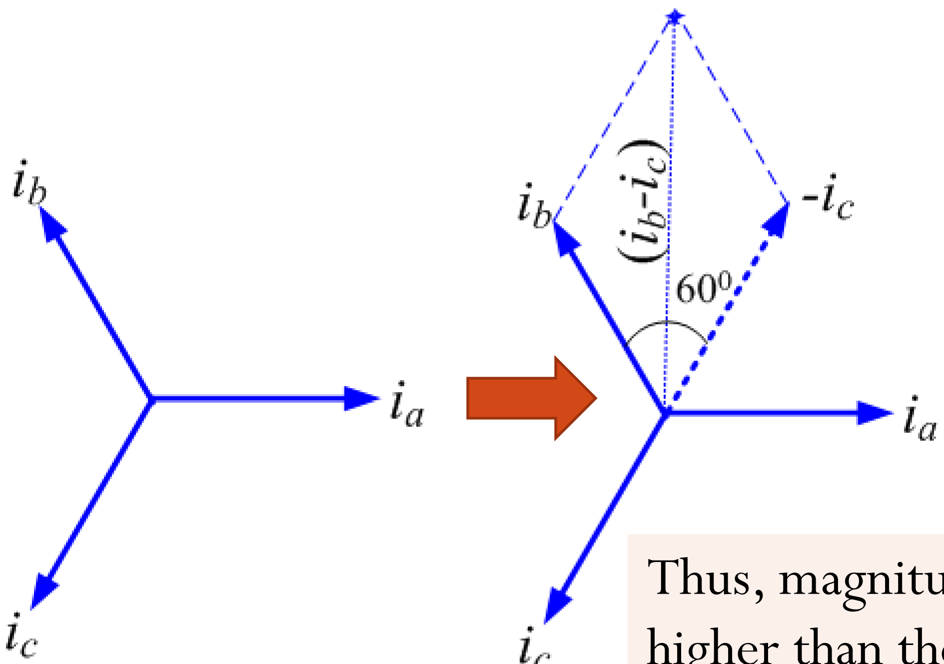
- **Transformation from 3-phase to 2-phase**

$$i_\alpha = \left[i_a - \frac{1}{2}(i_b + i_c) \right]$$

$$i_\beta = \left[\frac{\sqrt{3}}{2}(i_b - i_c) \right]$$

$$i_\beta = \left[\frac{\sqrt{3}}{2}(i_b - i_c) \right] \quad \text{Thus, } |i_\beta| = \left[\frac{\sqrt{3}}{2} |(i_b - i_c)| \right]$$

Drawing the phasor $-i_c$ and finding the phasor sum of $(i_b - i_c)$:,



$$|i_\beta| = \frac{\sqrt{3}}{2}\sqrt{\left(i_b^{\ 2} + i_c^{\ 2} + 2 i_b i_c \cos 60^0\right)}$$

Remember that: $|i_a| = |i_b| = |i_c|$

$$|i_\beta| = \frac{\sqrt{3}}{2}\sqrt{\left(i_a^{\ 2} + i_a^{\ 2} + 2 i_a i_a \frac{1}{2}\right)}$$

$$= \frac{\sqrt{3}}{2}\sqrt{3 i_a^{\ 2}} = \frac{3}{2} i_a$$

Hence, $|i_\beta| = 1.5|i_a|$ $\quad |i_\alpha| = 1.5|i_a|$

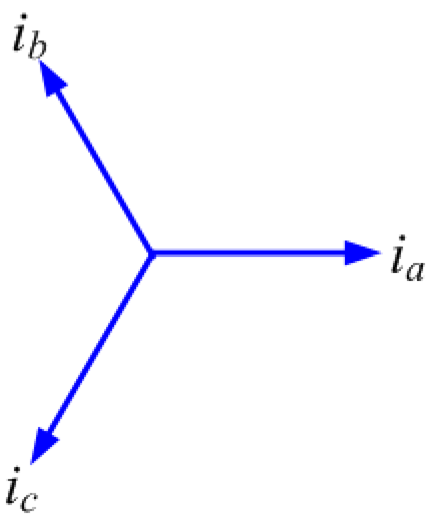
Thus, magnitude of the 2-phase currents are 1.5 times higher than those in the equivalent 3-phase system

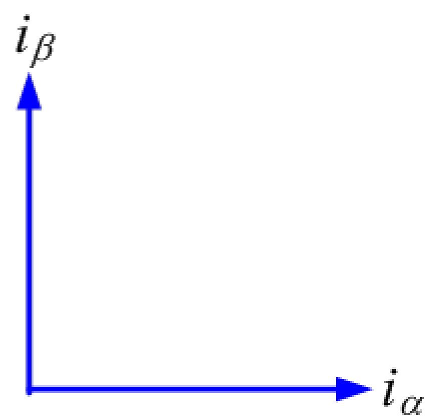
a, b, c to α, β transformation

- **Transformation from 3-phase to 2-phase**
  - Under this condition, the MMFs are equal in both 2-phase and 3-phase systems
  - With same MMF, the flux must also be equal in both 2-phase and 3-phase systems
  - With same of turns, the per phase induced EMF must also be equal in both 2-phase and 3-phase systems $E \approx V = 4.44\, f \phi_m N K_w$
  -

$$|i_\alpha| = 1.5|i_a|$$

$$|i_\beta| = 1.5|i_a|$$





a, b, c to α, β transformation

- **Under this condition:** $|i_\alpha| = 1.5|i_a|$ $|i_\beta| = 1.5|i_a|$

Parameter	3-phase system	2-phase system
Current per phase	$\lvert I_a \rvert$	$1.5\lvert I_a \rvert$
MMF (Equal)	$1.5\ NI_m$	
Flux (Equal)	MMF/Reluctance	
EMF (V) per phase (Equal)	$E \approx V = 4.44\, f\phi_m NK_w$	
Power per phase	$P_3 = V \times I = VI$	$P_2 = V \times 1.5I = 1.5VI$
Total power	$P = 3P_3 = 3VI$	$P = 2P_2 = 2 \times 1.5VI = 3VI$

Thus, the phenomena of **power invariance** is hence proved

3-phase to 2-phase transformation

By changing the number of turns in the 2-phase system

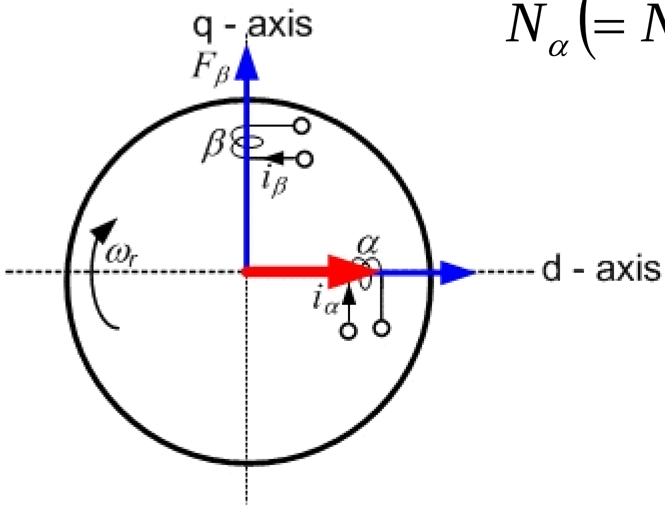
a, b, c to α, β transformation

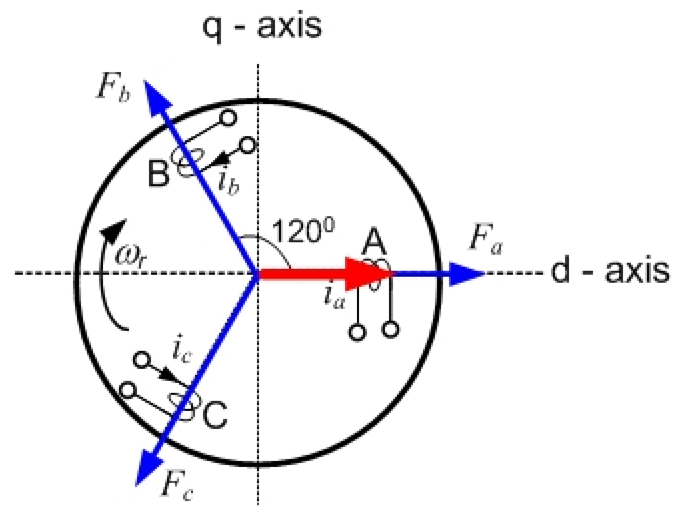
- **Transformation from 3-phase to 2-phase**
  - Magnitude of the 3-phase RMF is $1.5Ni_m$
  - Magnitude of the 2-phase RMF is Ni_m
  - Thus, if the 3-phase and 2-phase system have same per phase current,

$$|i_\alpha| = |i_\beta| = |i_a|$$

  - Then to make their MMFs equal, number of turns in the 2-phase system must be 1.5 times higher than those in the equivalent 3-phase system

$$N_\alpha \left(= N_\beta\right) = 1.5N$$





a, b, c to α, β transformation

- **Under this condition:** $|i_\alpha| = |i_\beta| = |i_a|$ $N_\alpha (= N_\beta) = 1.5N$

Parameter	3-phase system	2-phase system
Current (Equal)	$\|I_a\|$	$\|I_\alpha\| = \|I_a\|$
Number of turns	N	$1.5N$
MMF (Equal)	$1.5NI_m$	
Flux (Equal)	MMF/Reluctance	
EMF (V) per phase	$E_3 = 4.44 f\phi_m NK_w = V$	$E_2 = 4.44 f\phi_m (1.5N) K_w = 1.5V$
Power per phase	$P_3 = V \times I = VI$	$P_2 = 1.5V \times I = 1.5VI$
Total power	$P = 3P_3 = 3VI$	$P = 2P_2 = 2 \times 1.5VI = 3VI$

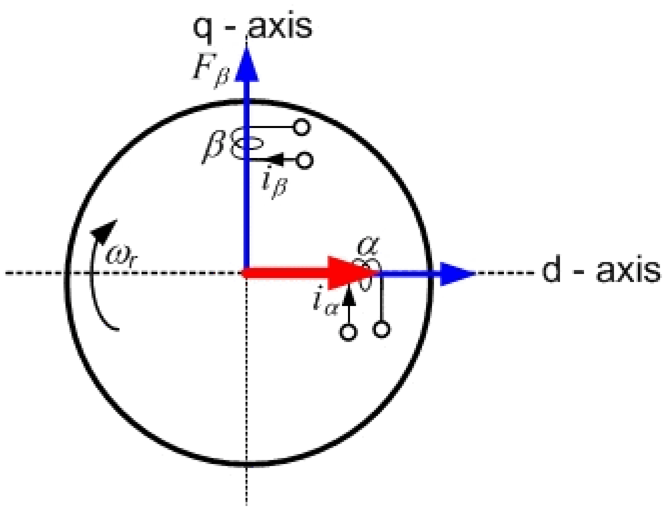
Thus, the phenomena of **power invariance** is once again proved

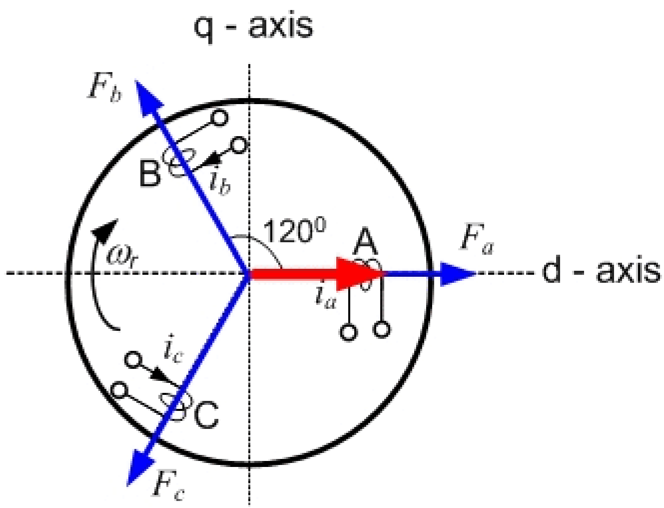
3-phase to 2-phase transformation

By changing the both the current and number of turns in the 2-phase system

a, b, c to α, β transformation

- **Transformation from 3-phase to 2-phase**
  - Magnitude of the 3-phase RMF is $1.5Ni_m$
  - Magnitude of the 2-phase RMF is Ni_m
  - Then to make their MMFs equal, both the current and number of turns in the 2-phase system can be suitably modified w.r.t those in the equivalent 3-phase system
  - This can give us identical transformations for voltage & current

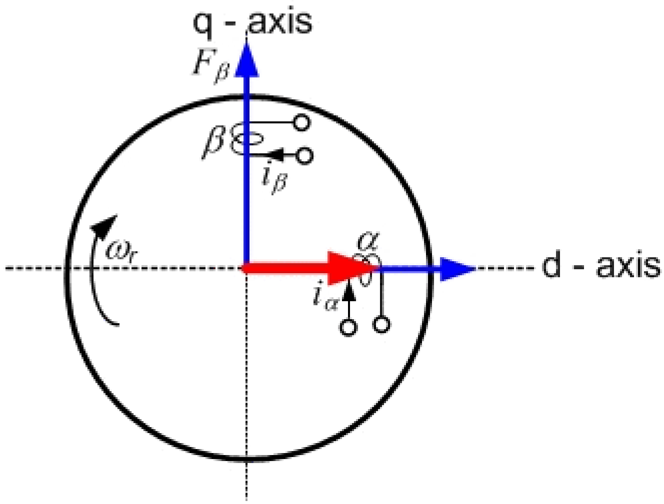


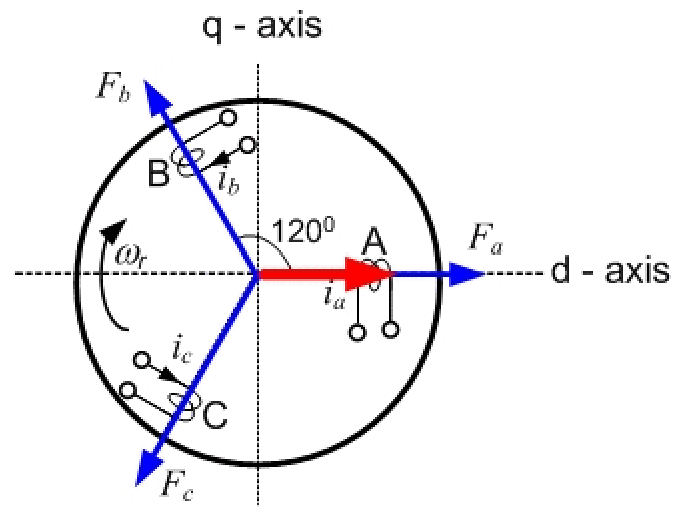


a, b, c to α, β transformation

- **Transformation from 3-phase to 2-phase**
  - Let the number of turns per phase in the equivalent 2-phase winding be made $\sqrt{\frac{3}{2}}$ times the per phase number of turns in 3-phase winding
  - Then, for equal MMFs in 3-phase and 2-phase systems, resolving the instantaneous values of 3-phase MMFs along α-axis:

$$\sqrt{\frac{3}{2}}Ni_\alpha = N\left[i_a \cos 0^0 + i_b \cos 120^0 + i_c \cos 240^0\right]$$





a, b, c to α, β transformation

- **Transformation from 3-phase to 2-phase**

$$\sqrt{\frac{3}{2}}Ni_\alpha = N\left[i_a \cos 0^0 + i_b \cos 120^0 + i_c \cos 240^0\right]$$

or, $$i_\alpha = \sqrt{\frac{2}{3}}\left[i_a + i_b\left(-\frac{1}{2}\right) + i_c\left(-\frac{1}{2}\right)\right]$$

or, $$i_\alpha = \sqrt{\frac{2}{3}}\left[i_a - \frac{1}{2}(i_b + i_c)\right]$$

or, $$i_\alpha = \sqrt{\frac{2}{3}}\left[i_a - \frac{1}{2}(-i_a)\right]$$

or, $$i_\alpha = \sqrt{\frac{2}{3}}\left[i_a + \frac{1}{2}i_a\right]$$

or, $$i_\alpha = \sqrt{\frac{2}{3}}\left[\frac{3}{2}i_a\right]$$ → $$i_\alpha = \sqrt{\frac{3}{2}}i_a$$ → $$|i_\alpha| = \sqrt{\frac{3}{2}}|i_a|$$

a, b, c to α, β transformation

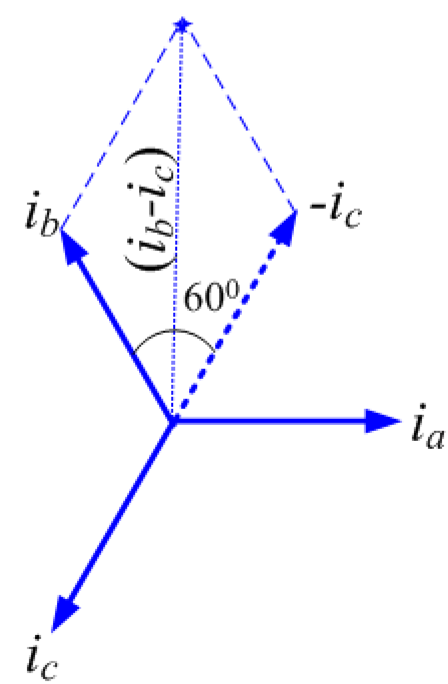
$$|i_\alpha| = \sqrt{\frac{3}{2}}|i_a|$$

- **Transformation from 3-phase to 2-phase**
  - Similarly, for equal MMFs in 3-phase and 2-phase systems, resolving the instantaneous values of 3-phase MMFs along β-axis:

$$\sqrt{\frac{3}{2}}Ni_\beta = N\left[i_a\cos 270^0 + i_b\cos 30^0 + i_c\cos 150^0\right]$$

or, $$i_\beta = \sqrt{\frac{2}{3}}\left[0 + i_b\left(\frac{\sqrt{3}}{2}\right) + i_c\left(-\frac{\sqrt{3}}{2}\right)\right]$$

or, $$i_\beta = \left[\frac{1}{\sqrt{2}}(i_b - i_c)\right]$$



Using vector algebra we get: $|i_\beta| = \frac{1}{\sqrt{2}}\sqrt{\left(i_b^{\ 2} + i_c^{\ 2} + 2i_b i_c\cos 60^0\right)}$

Remember that: $|i_a| = |i_b| = |i_c|$

$$|i_\beta| = \frac{1}{\sqrt{2}}\sqrt{\left(i_a^{\ 2} + i_a^{\ 2} + 2i_a i_a\frac{1}{2}\right)} = \frac{1}{\sqrt{2}}\sqrt{3i_a^{\ 2}} = \sqrt{\frac{3}{2}}i_a$$

$$|i_\beta| = \sqrt{\frac{3}{2}}|i_a|$$

a, b, c to α, β transformation

- **Under these conditions:** $N_\alpha (= N_\beta) = \sqrt{\frac{3}{2}} N$ $\qquad |I_\alpha| (= |I_\beta|) = \sqrt{\frac{3}{2}} |I_a|$

Parameter	3-phase system	2-phase system
Current	$\lvert I_a \rvert$	$\sqrt{\frac{3}{2}} \lvert I_a \rvert$
Number of turns	N	$\sqrt{\frac{3}{2}} N$
MMF (Equal)	$1.5NI_m$	
Flux (Equal)	MMF/Reluctance	
EMF (V) per phase	$E_3 = 4.44 f\phi_m N K_w = V$	$E_2 = 4.44 f\phi_m \left(\sqrt{\frac{3}{2}} N\right) K_w = \sqrt{\frac{3}{2}} V$

- Thus, both voltage and current are identically transformed, both are $\sqrt{\frac{3}{2}}$ times in the 2-phase system as compared to the 3-phase system
- Since *V* and *I* transformations are identical, impedance per phase are same for the 2- and 3-phase systems

a, b, c to α, β transformation

- **Under these conditions:** $N_\alpha \left(= N_\beta\right) = \sqrt{\frac{3}{2}} N$ $\quad |I_\alpha| \left(= |I_\beta|\right) = \sqrt{\frac{3}{2}} |I_a|$

Parameter	3-phase system	2-phase system
Current	$\|I_a\|$	$\sqrt{\frac{3}{2}} \|I_a\|$
EMF (V) per phase	$E_3 = 4.44 f \phi_m N K_w = V$	$E_2 = 4.44 f \phi_m \left(\sqrt{\frac{3}{2}} N\right) K_w = \sqrt{\frac{3}{2}} V$
Power per phase	$P_3 = V \times I = VI$	$P_2 = \sqrt{\frac{3}{2}} V \times \sqrt{\frac{3}{2}} I = \frac{3}{2} VI = 1.5VI$
Total power	$P = 3P_3 = 3VI$	$P = 2P_2 = 2 \times 1.5VI = 3VI$

Thus, the phenomena of **power invariance** is once again proved