Linear Transformations in Machines

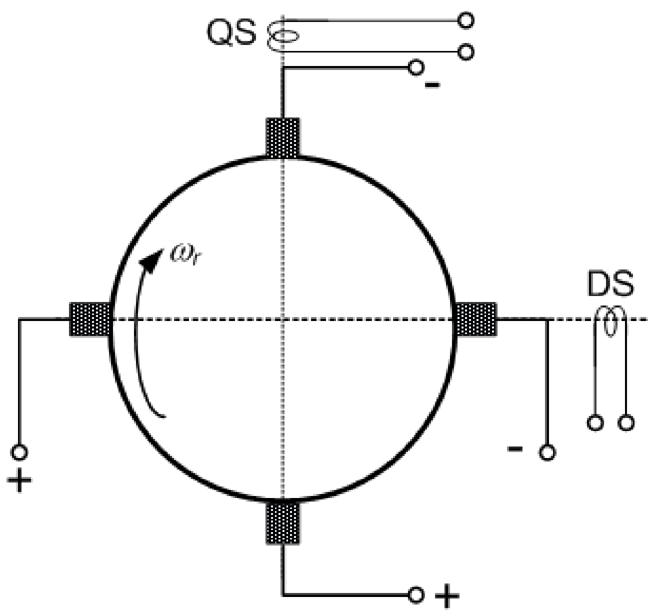
Day 6

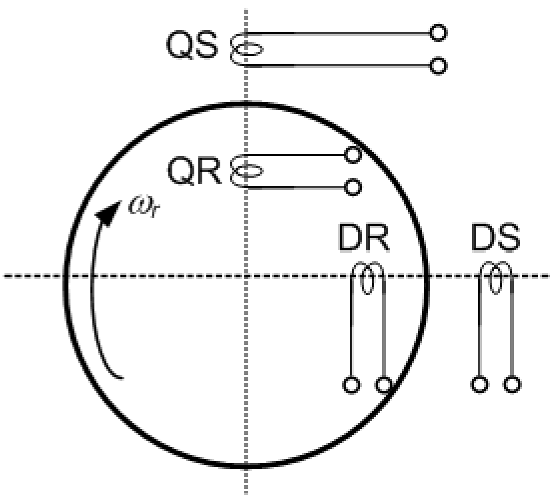
ILOs – Day6

- Understand the need for linear transformations in electric machines
- List the pre-requisites of such transformations
- Perform one such sample transformation (displaced brush axis)

Why Transformation?

- **Primitive (basic) 2-pole machine**
  - Can be shown to be equivalent to any rotating machine
  - The rotor coils are fitted with brush and commutator
  - The rotor magnetic field is stationary (both in time and space) – due to the presence of brush and commutator





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  - Can be shown to be equivalent to any rotating machine
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- **These conditions are most closely met by:**
  - DC machine
    - They have brush and commutator
    - Hence their rotor magnetic field is stationary both in time and space

- **DC machines are most closely equivalent to the primitive machine with appropriate number of coils in each of the fixed axes depending on type of the DC machine**

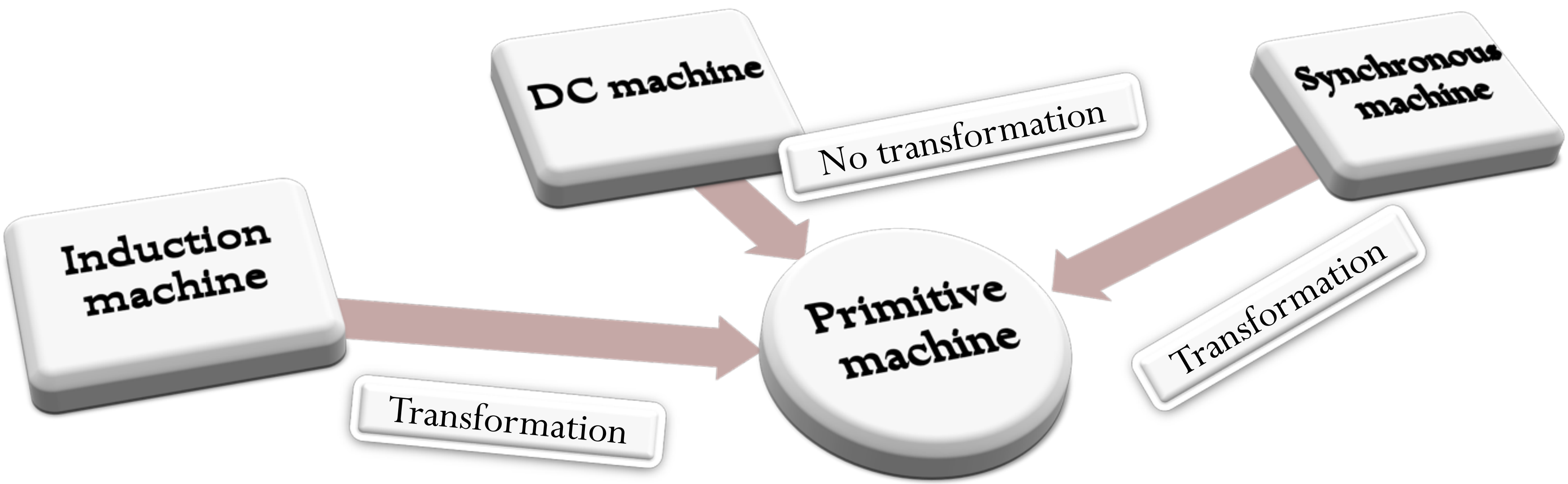
Why Transformation?

- Many rotating machines are poly-phase AC machines (induction, synchronous)
- Their construction is different from that of the primitive machine
  - They do not have brush and commutator
  - They have polyphase rotating winding in rotor
  - They have polyphase stationary winding in stator
  - Their rotor magnetic field is both time and space varying
- So they can not be directly made equivalent to the primitive machine

- **However, in those cases also, primitive machine concept can be used for analysis, provided:**
  - The rotating polyphase rotor coils and stationary polyphase stator coils can be suitably represented by the *d*–*q* axes coils of the primitive machine
  - Such an equivalent representation is termed as "***Transformation***"

What is Transformation?

- The process of replacing one set of variables by another set of variables for equivalent representation electrical machines is called ***winding transformation*** or simply ***transformation***.
  - Since DC machines resemble the primitive machine structure directly, no transformation is necessary
  - Polyphase machines, however, need such transformations so that they can be fit into the primitive machine model and analyzed



Linear Transformation

- The process of transformation between old and new set of variables and *vice versa* is described by linear equations
  - Transformations are generally done to:
    - Simply the solution process
    - Reduce number of variables to be dealt with
    - Reduce number of equations to be solved
- *Transformation matrix* is used to show the relationship between old and new variables in *transformation equations:*

[Old variables] = [Transformation matrix] [New variables]

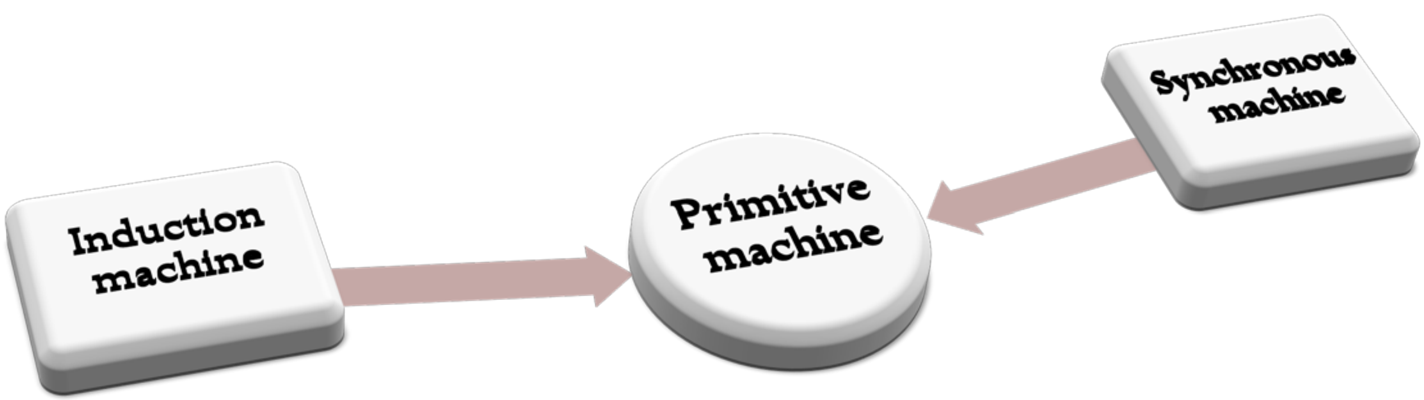
or, [New variables] = [Transformation matrix] [Old variables]

Linear Transformation

- **Examples of linear transformation:**
  - Logarithmic & anti-logarithmic transformations
  - Laplace & inverse Laplace transformations (time to *s* domain)
  - Symmetrical components in power system fault analysis
  - Referring primary quantities to secondary or secondary quantities to primary in a transformer

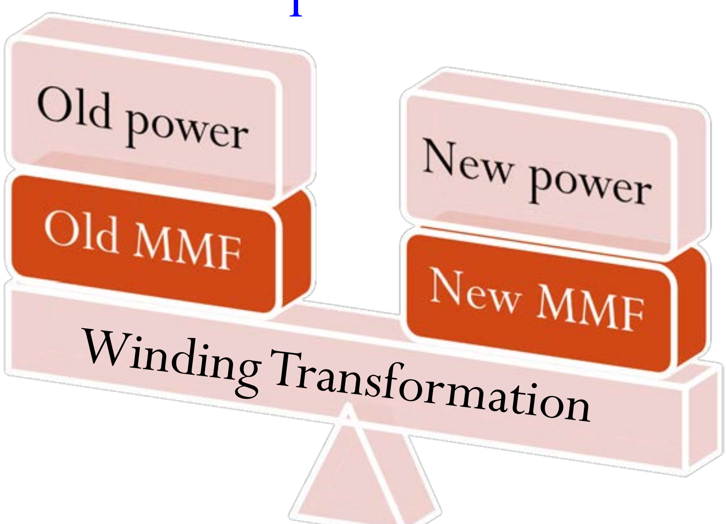
Why Transformation in machines?

- Fitting polyphase machine models to generalized machine model
- Reducing number of equations to be solved
- 3-phase machine require three voltage equations to be solved
- But its generalized model has only two coils, so only two equations to be solved
- Magnetic coupling between the three windings of a 3-phase machine makes the circuit equations much more complicated
- In the generalized 2-axis model, the two coils along *d* and *q* axes are magnetically independent since they are at 90^0



Pre-requisites of Transformation

- Voltage and current transformation matrices can be different
- But they should be so related that the total; power remain invariant between the old and the new (transformed) system [**power invariance**]
- Current transformations should be done in such a way that MMF must remain same (magnitude and direction) in the new set as compared to the old set.
- In other words, MMF produced in the two cases (pre and post transformation) must be time and space invariant



Different Transformations in machines

Transformation from a displaced brush axis

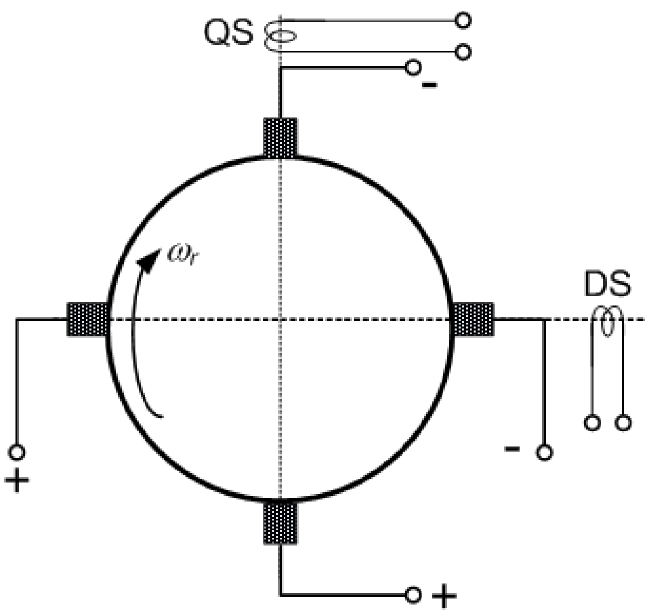
Transformation from 3-phase to 2-phase (a-b-c to α-β)

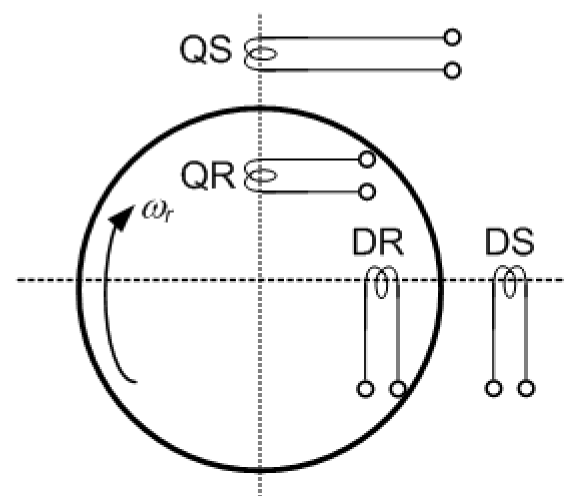
Transformation from rotating axes to stationary axes (α-β-0 to d-q-0)

Transformations from a displaced brush axis

Transformations from a displaced brush axis

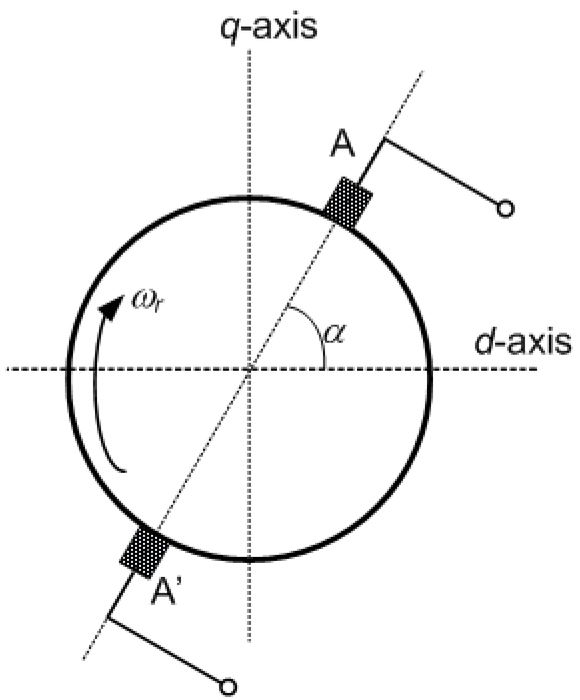
- The generalized 2-pole structure, the brush axes are assumed to coincide with the *d*- and *q*- axes





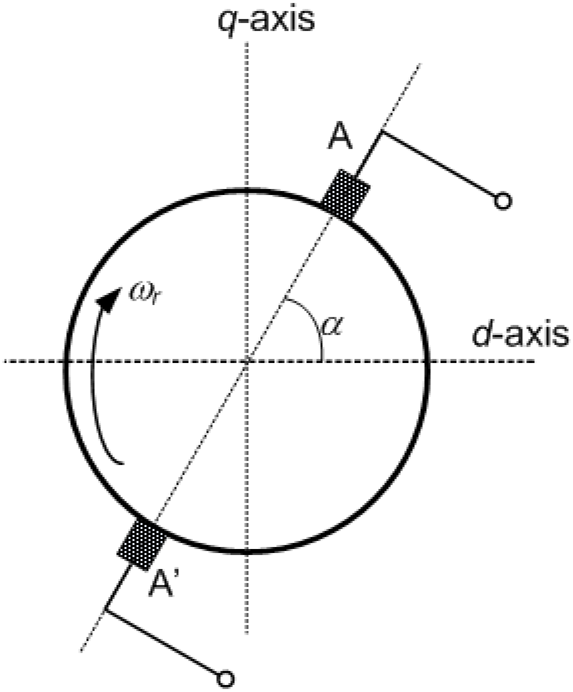
Transformations from a displaced brush axis

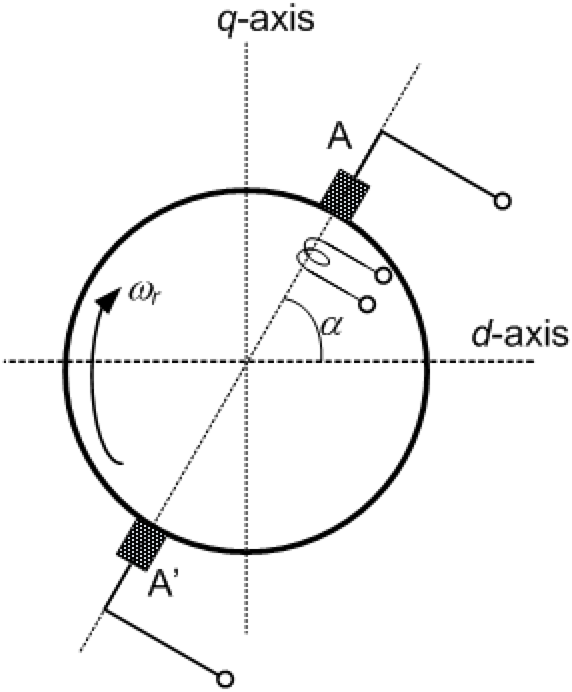
- The generalized 2-pole structure, the brush axes are assumed to coincide with the *d*- and *q*- axes
- However, a DC machine may have its brushes displaced from *d*- or *q*- axes
- In such a case, a transformation is necessary from brush axes to *d*-*q* axes



Transformations from a displaced brush axis

- Let, one such set of brush AA' is shifted by an angle α w.r.t. the *d*-axis
- The armature coil thus can be assumed to be located along the AA' axis





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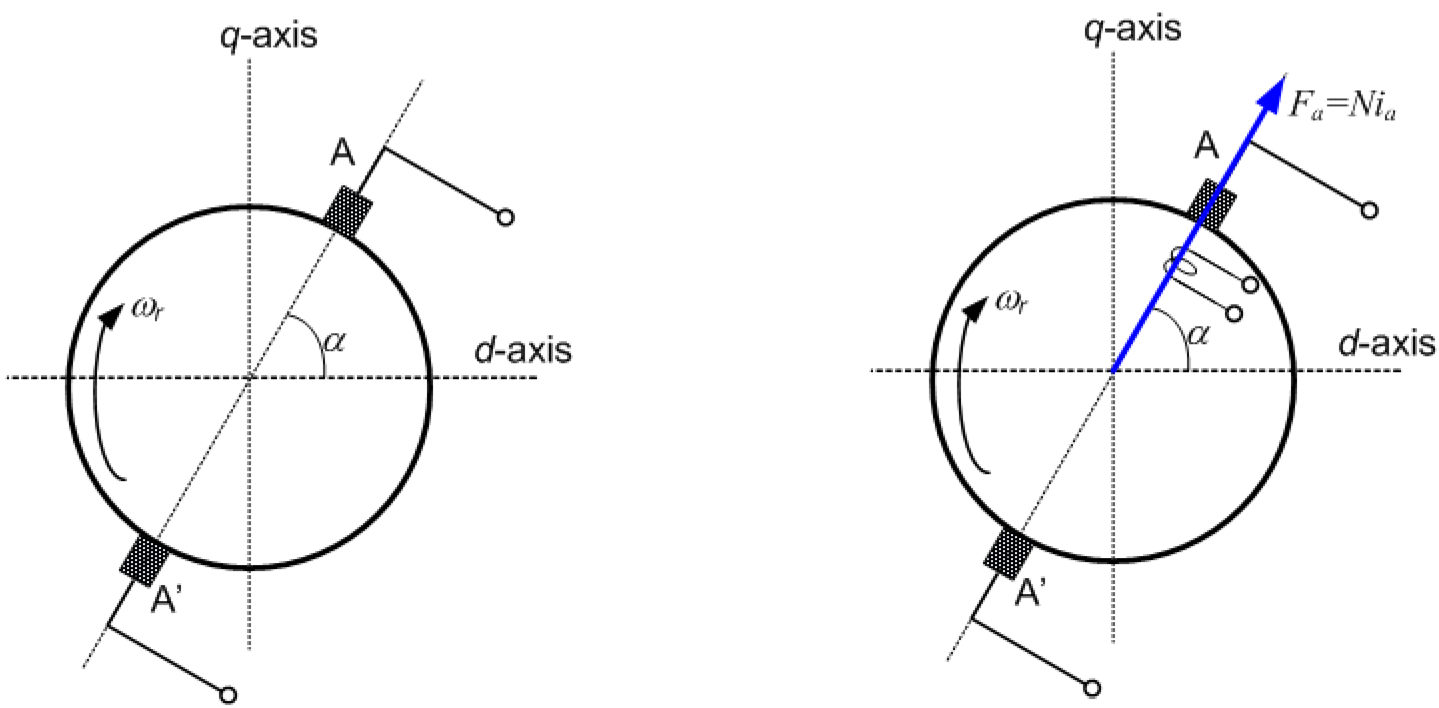
- The armature coil develops an MMF F_a along the brush axis



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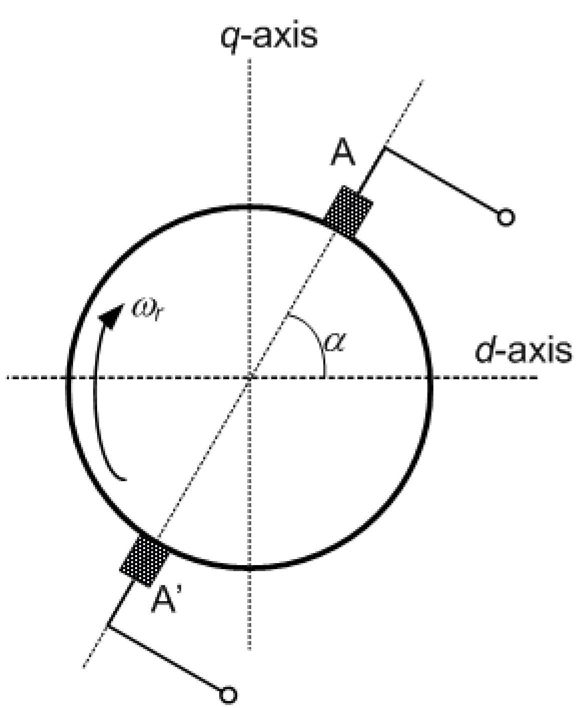
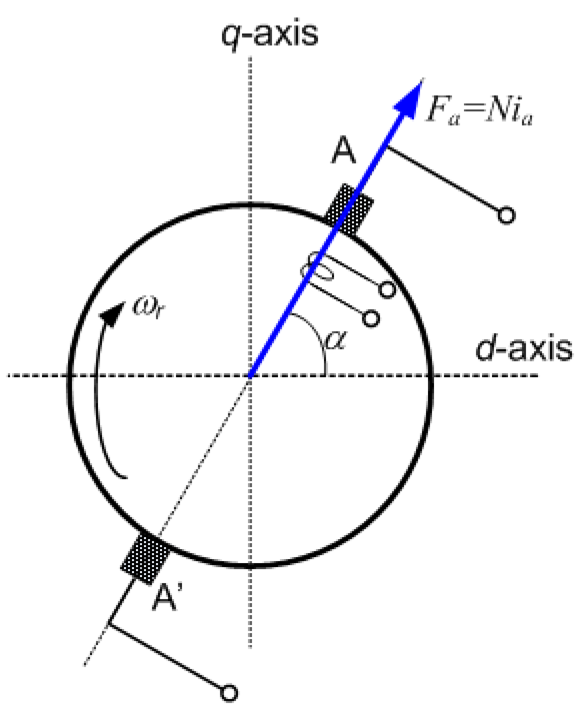
- The armature coil develops an MMF F_a along the brush axis
- This MMF F_a can be resolved in two orthogonal components

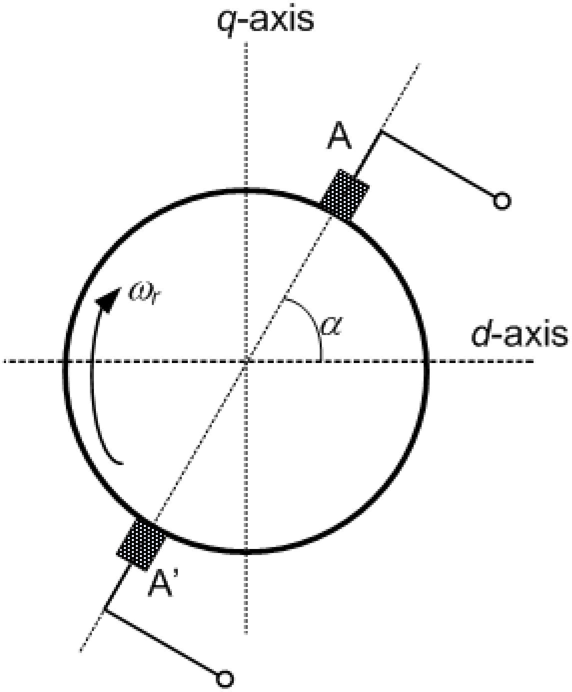


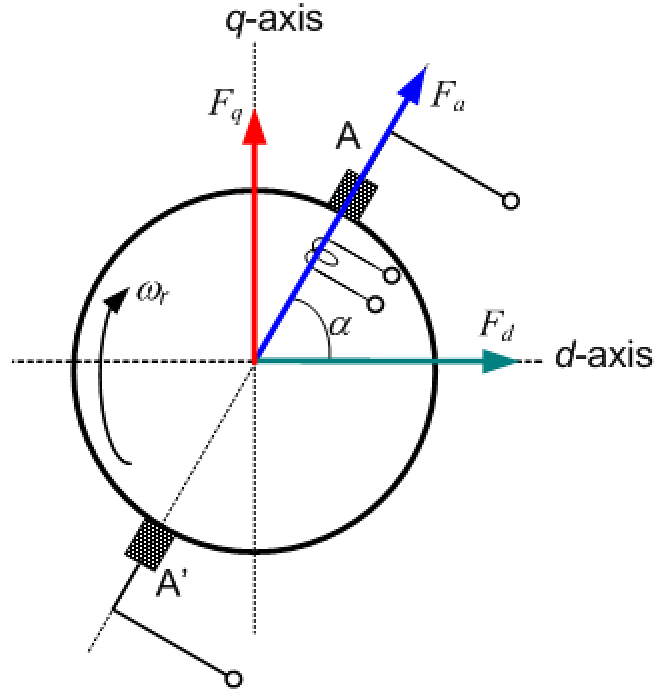


Transformations from a displaced brush axis

- F_d along d-axis and F_q along q-axis

$$F_d = F_a \cos\alpha \qquad F_q = F_a \sin\alpha$$





Transformations from a displaced brush axis

- F_d along d-axis and F_q along q-axis

$$F_d = F_a \cos\alpha \qquad F_q = F_a \sin\alpha$$

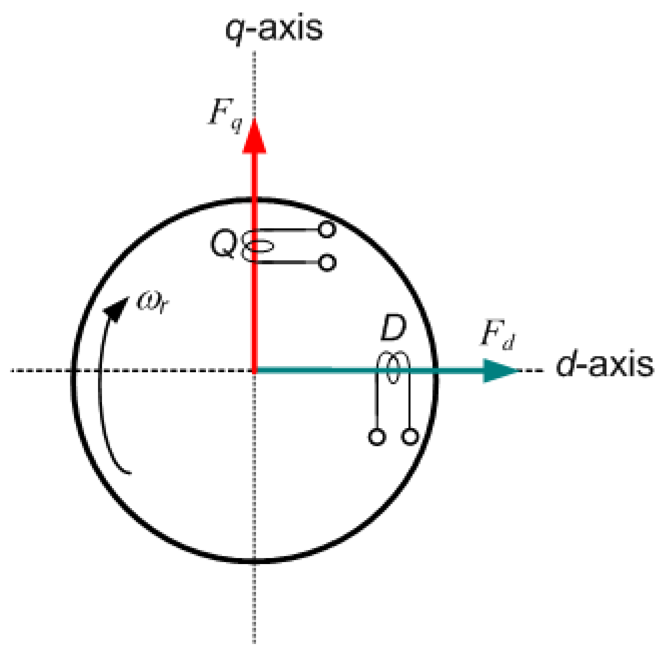
- These two component MMFs (F_d and F_q) can be assumed to be developed by two equivalent coils along *d*-, and *q*-axes
- Each having same number of turns *N* as the armature coil
- Thus, we have the current relations:

$$Ni_d = Ni_a \cos\alpha$$

or, $\boxed{i_d = i_a \cos\alpha}$

$$Ni_q = Ni_a \sin\alpha$$

or, $\boxed{i_q = i_a \sin\alpha}$



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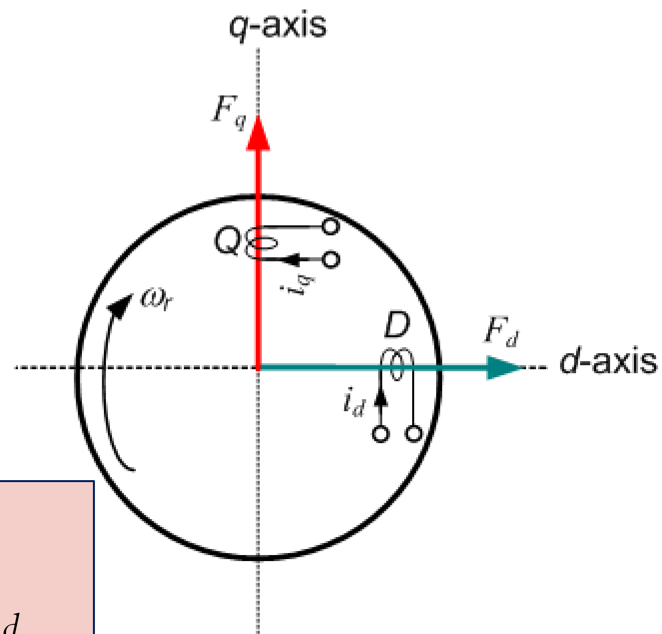
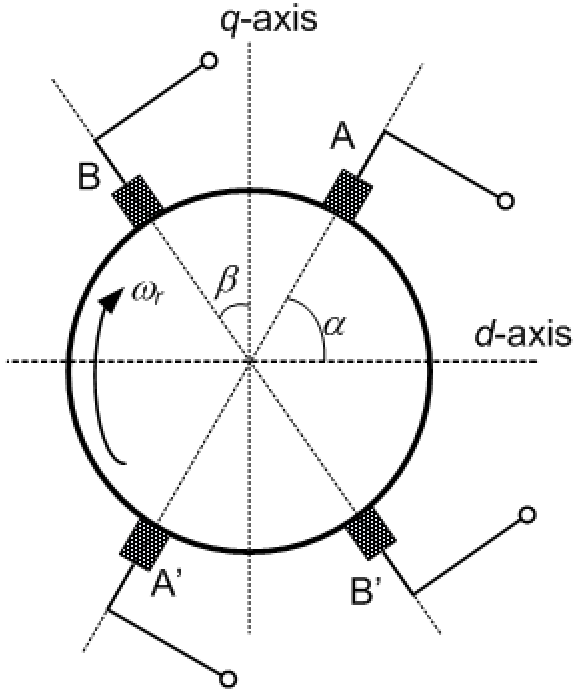
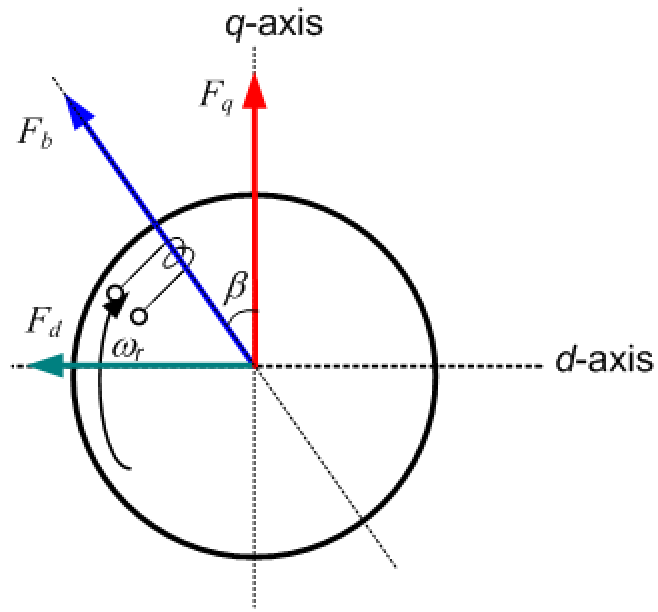


Thus, we have the equivalent **(transformed)** component currents i_d and i_q in d- and q-axis coils respectively

Transformations from a displaced brush axis

- In addition to the brush pair AA', let us assume another pair of brushes BB' that makes an angle β w.r.t. the q-axis
- Armature coil along BB' produces an MMF F_b that can also be resolved in two orthogonal components along d- and q-axes





Transformations from a displaced brush axis

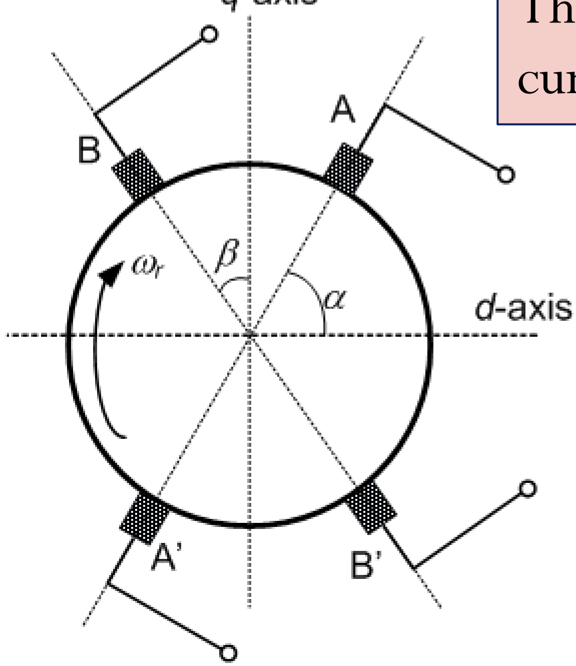
- Let us combine the effects of components of F_a and F_b along *d*- and *q*-axes

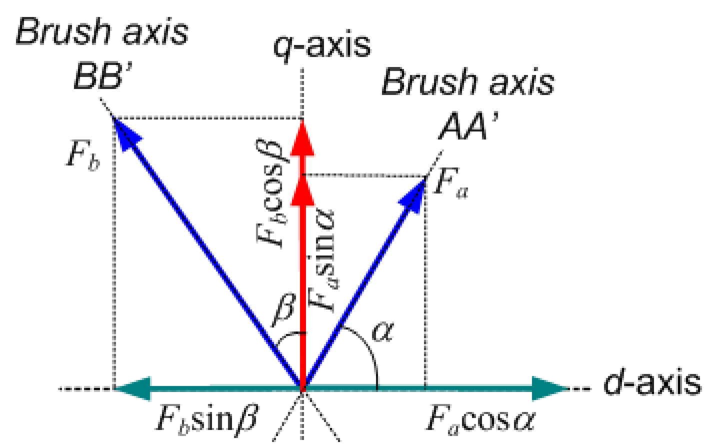
$$F_d = F_a \cos\alpha - F_b \sin\beta \qquad F_q = F_a \sin\alpha + F_b \cos\beta$$

- Assuming equal number turns in the main armature coils and also in the equivalent *d*- and *q*- axes coils, we have the transformed current relations:

$$i_d = i_a \cos\alpha - i_b \sin\beta \qquad i_q = i_a \sin\alpha + i_b \cos\beta$$

Thus, we have the equivalent **(transformed)** component currents i_d and i_q in d- and q-axis coils respectively





Transformations from a displaced brush axis

- Transformation equations $i_d = i_a \cos\alpha - i_b \sin\beta$ $i_q = i_a \sin\alpha + i_b \cos\beta$
- In matrix form:

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} cos\alpha & -sin\beta \\ sin\alpha & cos\beta \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix}$$

New variables

Transformation matrix

Old variables