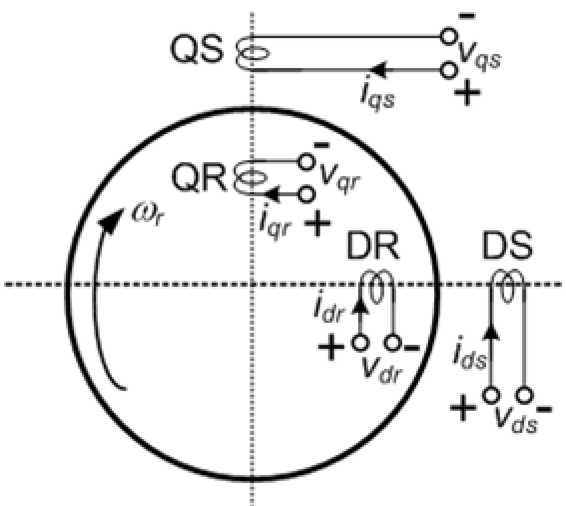
Power & Torque expressions in Kron's machine

Day 5

ILOs – Day5

- Derive expressions for power and torque from impedance matrix of Kron's primitive machine



$$
\begin{array}{c}
\end{array}
\begin{bmatrix} v_{ds} \\ v_{qs} \\ v_{dr} \\ v_{qr} \end{bmatrix}
=
\begin{array}{c|c|c|c|c}
 & ds & qs & dr & qr \\
\hline
ds & r_{ds}+L_{ds}p & & M_d p & \\
qs & & r_{qs}+L_{qs}p & & M_q p \\
dr & M_d p & -M_q\omega_r & r_{dr}+L_{dr}p & -\omega_r L_{qr} \\
qr & M_d\omega_r & M_q p & \omega_r L_{dr} & r_{qr}+L_{qr}p
\end{array}
\begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}
$$

Power Expressions

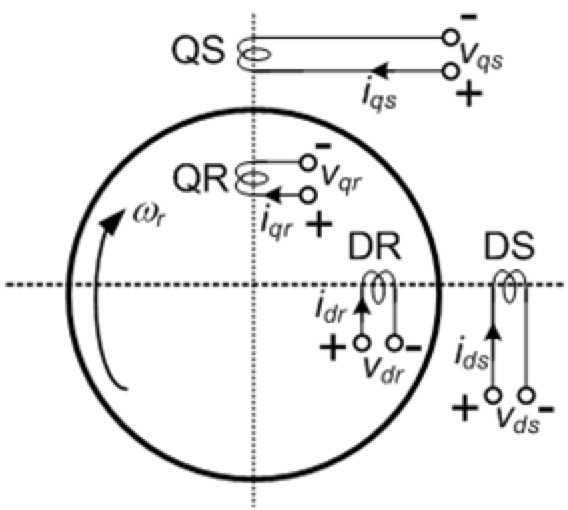
$$
\begin{array}{c}
\begin{bmatrix} v_{ds} \\ v_{qs} \\ v_{dr} \\ v_{qr} \end{bmatrix}
=
\begin{array}{c|c|c|c|c|}
 & ds & qs & dr & qr \\
\hline
ds & r_{ds} + L_{ds}p & & M_d p & \\
\hline
qs & & r_{qs} + L_{qs}p & & M_q p \\
\hline
dr & M_d p & -M_q \omega_r & r_{dr} + L_{dr}p & -\omega_r L_{qr} \\
\hline
qr & M_d \omega_r & M_q p & \omega_r L_{dr} & r_{qr} + L_{qr}p \\
\hline
\end{array}
\begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}
\end{array}
$$

Total electrical power input

- **Voltage supply is given to two stator coils and two rotor coils**
- Assuming the machine to be operating in motor mode
- The total input electrical power is:

$$P_i = v_{ds} i_{ds} + v_{qs} i_{qs} + v_{dr} i_{dr} + v_{qr} i_{qr}$$

In matrix form:



$$P_i = \begin{bmatrix} i_{ds} & i_{qs} & i_{dr} & i_{qr} \end{bmatrix} \begin{bmatrix} v_{ds} \\ v_{qs} \\ v_{dr} \\ v_{qr} \end{bmatrix} = [i_t][v]$$

Here $[i_t]$ is the transpose of current column matrix $[i]$

Total electrical power input

$$P_i = \begin{bmatrix} i_{ds} & i_{qs} & i_{dr} & i_{qr} \end{bmatrix} \begin{bmatrix} v_{ds} \\ v_{qs} \\ v_{dr} \\ v_{qr} \end{bmatrix} = [i_t][v]$$

- Remember the expression: $[v] = [Z][i] = \big[[R] + [L]p + [G]\omega_r\big][i]$

Thus: $[P_i] = [i_t]\big[[R] + [L]p + [G]\omega_r\big][i]$

$$= [i_t][R][i] + [i_t][L]p[i] + [i_t][G]\omega_r[i]$$

Total electrical power input

$$[P_i]=[i_t][R][i]+[i_t][L]p[i]+[i_t][G]\omega_r[i]$$

- The first term:

$$[i_t][R][i]=\begin{bmatrix} i_{ds} & i_{qs} & i_{dr} & i_{qr}\end{bmatrix}\begin{bmatrix} r_{ds} & & & \\ & r_{qs} & & \\ & & r_{dr} & \\ & & & r_{qr}\end{bmatrix}\begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr}\end{bmatrix}$$

$$\begin{bmatrix} A & B & C & D\end{bmatrix}\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p\end{bmatrix}\begin{bmatrix} Q \\ R \\ S \\ T\end{bmatrix}=\begin{bmatrix} AaQ+AbR+AcS+AdT+ \\ BeQ+BfR+BgS+BhT+ \\ CiQ+CjR+CkS+ClT+ \\ DmQ+DnR+DoS+DpT\end{bmatrix}$$

Total electrical power input

$$[P_i] = [i_t][R][i] + [i_t][L]p[i] + [i_t][G]\omega_r[i]$$

- The first term:

$$[i_t][R][i] = \begin{bmatrix} i_{ds} & i_{qs} & i_{dr} & i_{qr} \end{bmatrix} \begin{bmatrix} r_{ds} & & & \\ & r_{qs} & & \\ & & r_{dr} & \\ & & & r_{qr} \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}$$

$$= i_{ds}{}^2 r_{ds} + i_{qs}{}^2 r_{qs} + i_{dr}{}^2 r_{dr} + i_{qr}{}^2 r_{qr}$$

$$\begin{bmatrix} A & B & C & D \end{bmatrix} \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix} \begin{bmatrix} Q \\ R \\ S \\ T \end{bmatrix} = \begin{bmatrix} AaQ + AbR + AcS + AdT + \\ BeQ + BfR + BgS + BhT + \\ CiQ + CjR + CkS + ClT + \\ DmQ + DnR + DoS + DpT \end{bmatrix}$$

Total electrical power input

$$[P_i] = [i_t][R][i] + [i_t][L]p[i] + [i_t][G]\omega_r[i]$$

- The first term:

$$[i_t][R][i] = \begin{bmatrix} i_{ds} & i_{qs} & i_{dr} & i_{qr} \end{bmatrix} \begin{bmatrix} r_{ds} & & & \\ & r_{qs} & & \\ & & r_{dr} & \\ & & & r_{qr} \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}$$

$$= i_{ds}{}^2 r_{ds} + i_{qs}{}^2 r_{qs} + i_{dr}{}^2 r_{dr} + i_{qr}{}^2 r_{qr}$$

Thus, the first term is the sum of ohmic losses (copper losses) in all the four windings

Total electrical power input

$$[P_i] = [i_t][R][i] + [i_t][L]p[i] + [i_t][G]\omega_r[i]$$

- The second term:

$$[i_t][L]p[i] = \begin{bmatrix} i_{ds} & i_{qs} & i_{dr} & i_{qr} \end{bmatrix} \begin{bmatrix} L_{ds}p & & M_d p & \\ & L_{qs}p & & M_q p \\ M_d p & & L_{dr}p & \\ & M_q p & & L_{qr}p \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}$$

$$= i_{ds}L_{ds}pi_{ds} + i_{ds}M_d pi_{dr} + i_{qs}L_{qs}pi_{qs} + i_{qs}M_q pi_{qr}$$
$$+ i_{dr}M_d pi_{ds} + i_{dr}L_{dr}pi_{dr} + i_{qr}M_q pi_{qs} + i_{qr}L_{qr}pi_{qr}$$

Separately writing the self and mutual inductance terms:

$$[i_t][L]p[i] = \left[i_{ds}L_{ds}pi_{ds} + i_{qs}L_{qs}pi_{qs} + i_{dr}L_{dr}pi_{dr} + i_{qr}L_{qr}pi_{qr}\right]$$
$$+ \left[i_{ds}M_d pi_{dr} + i_{qs}M_q pi_{qr} + i_{dr}M_d pi_{ds} + i_{qr}M_q pi_{qs}\right]$$

Total electrical power input

$$[i_t][L]p[i] = [i_{ds}L_{ds}pi_{ds} + i_{qs}L_{qs}pi_{qs} + i_{dr}L_{dr}pi_{dr} + i_{qr}L_{qr}pi_{qr}] + [i_{ds}M_d pi_{dr} + i_{qs}M_q pi_{qr} + i_{dr}M_d pi_{ds} + i_{qr}M_q pi_{qs}]$$

- Expression for energy stored in a magnetic field due to its self inductance:

$$W_1 = \frac{1}{2}Li^2$$

$\therefore$ Rate at which this stored energy changes:

$$\frac{dW_1}{dt} = \frac{d}{dt}\left(\frac{1}{2}Li^2\right) = \frac{1}{2}L2i\frac{di}{dt} = iLpi$$

- This is nothing but the power being stored in the self-field

Total electrical power input

$$[i_t][L]p[i] = [i_{ds}L_{ds}pi_{ds} + i_{qs}L_{qs}pi_{qs} + i_{dr}L_{dr}pi_{dr} + i_{qr}L_{qr}pi_{qr}]$$
$$+ [i_{ds}M_d pi_{dr} + i_{qs}M_q pi_{qr} + i_{dr}M_d pi_{ds} + i_{qr}M_q pi_{qs}]$$

- Expression for energy stored in a magnetic field due to mutual inductance:

$$W_2 = Mi_1i_2$$

$\therefore$ Rate at which this stored energy in mutual magnetic field changes:

$$\frac{dW_2}{dt} = \frac{d}{dt}(Mi_1i_2) = Mi_1\frac{di_2}{dt} + Mi_2\frac{di_1}{dt} = i_1Mpi_2 + i_2Mpi_1$$

- This is nothing but the power being stored in mutual fields

Total electrical power input

$$[i_t][L]p[i] = [i_{ds} L_{ds} pi_{ds} + i_{qs} L_{qs} pi_{qs} + i_{dr} L_{dr} pi_{dr} + i_{qr} L_{qr} pi_{qr}] + [i_{ds} M_d pi_{dr} + i_{qs} M_q pi_{qr} + i_{dr} M_d pi_{ds} + i_{qr} M_q pi_{qs}]$$

- Power stored in self-fields $= iLpi$
- Power stored in mutual fields $= i_1 M pi_2 + i_2 M pi_1$

- Thus, the first component $[i_{ds} L_{ds} pi_{ds} + i_{qs} L_{qs} pi_{qs} + i_{dr} L_{dr} pi_{dr} + i_{qr} L_{qr} pi_{qr}]$
- It represents the total power being stored in self-fields

- And, the second component $[i_{ds} M_d pi_{dr} + i_{qs} M_q pi_{qr} + i_{dr} M_d pi_{ds} + i_{qr} M_q pi_{qs}]$
- It represents the total power being stored in mutual-fields

- Thus, the second term $[i_t][L]p[i]$ correspond to the total power being stored in the magnetic field

Total electrical power input

- Now going back to the original input power expression:

$$[P_i] = [i_t][R][i] + [i_t][L]p[i] + [i_t][G]\omega_r[i]$$

Power input | Ohmic power loss | Power stored in magnetic field | Power output

- According to the principle of conservation of energy:

  Power input = Power lost + Power stored + Power output

- Thus, the third term $[i_t][G]\omega_r[i]$ must correspond to the power output

Power output & torque Expressions

$$[P_i] = [i_t][R][i] + [i_t][L]p[i] + [i_t][G]\omega_r[i]$$

Power input

Ohmic power loss

Power stored in magnetic field

Power output

Power & torque output

$$[P_i]=[i_t][R][i]+[i_t][L]p[i]+[i_t][G]\omega_r[i]$$

Power input | Ohmic power loss | Power stored in magnetic field | Power output, *P*

- Since the system is a rotating one, the power output should mean the amount of electrical power that is being converted to mechanical power
- Thus, P = Power converted from electrical to mechanical

$$=\omega_r[i_t][G][i]$$

- As we know: Electrical torque $T_e=\dfrac{P}{\omega_r}=[i_t][G][i]$

Power & torque output

$$T_e = [i_t][G][i]$$

$$= \begin{bmatrix} i_{ds} & i_{qs} & i_{dr} & i_{qr} \end{bmatrix} \begin{bmatrix} & & & \\ & & & \\ & -M_q & & -L_{qr} \\ M_d & & L_{dr} & \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}$$

$$T_e = [i_t][G][i] = -i_{dr} M_q i_{qs} - i_{dr} L_{qr} i_{qr} + i_{qr} M_d i_{ds} + i_{qr} L_{dr} i_{dr}$$

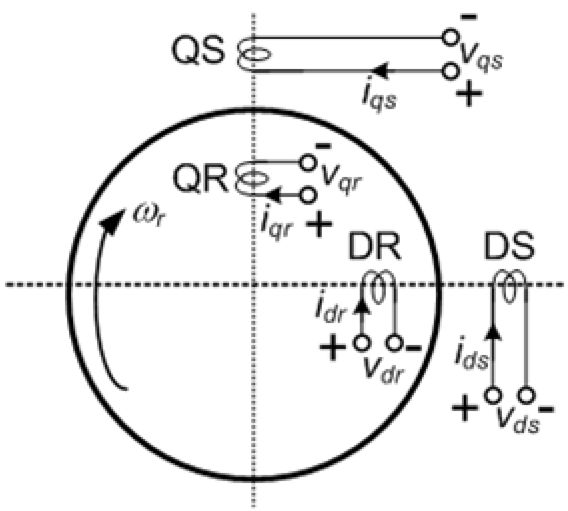
$$= M_d i_{qr} i_{ds} - M_q i_{dr} i_{qs} + (L_{dr} - L_{qr}) i_{dr} i_{qr}$$

$$\boxed{T_e = M_d i_{qr} i_{ds} - M_q i_{dr} i_{qs} + (L_{dr} - L_{qr}) i_{dr} i_{qr}}$$

Power & torque output

$$T_e = M_d i_{qr} i_{ds} - M_q i_{dr} i_{qs} + \left(L_{dr} - L_{qr}\right) i_{dr} i_{qr}$$

- Note that all the terms in torque expression contain two currents that are at 90^0 to each other in space
- Thus, for development of electrical torque, two currents that are at quadrature need to interact with each other
- Currents along the same axis cannot produce torque



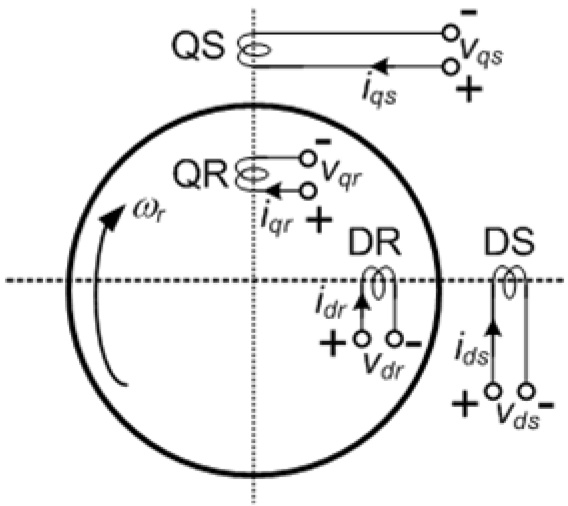
Torque components

$$T_e = M_d i_{qr} i_{ds} - M_q i_{dr} i_{qs} + \left(L_{dr} - L_{qr}\right) i_{dr} i_{qr}$$

Electromagnetic torque

Reluctance torque

- The electromagnetic torque $\left(M_d i_{qr} i_{ds} - M_q i_{dr} i_{qs}\right)$ requires two currents at space quadrature, one current on stator, and the other on rotor
- The reluctance torque $\left(L_{dr} - L_{qr}\right) i_{dr} i_{qr}$ also requires two currents at space quadrature, but both are on stator (or both on rotor)



- The reluctance torque is present only in salient pole machines where $L_{dr} > L_{qr}$
- In uniform air-gap machines, where $L_{dr} = L_{qr}$, the reluctance torque is absent

Torque in terms of flux linkage

$$T_e = M_d i_{qr} i_{ds} - M_q i_{dr} i_{qs} + \left(L_{dr} - L_{qr}\right) i_{dr} i_{qr}$$

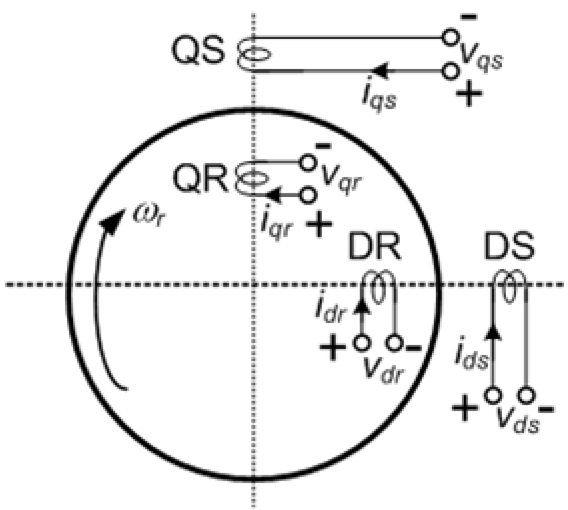
- The torque expression can be written in terms of d- and q-axes armature flux linkages with $L_{dr} = M_d + l_{dr}$ and $L_{qr} = M_q + l_{qr}$

$\psi_d = M_d\left(i_{ds} + i_{dr}\right) + l_{dr} i_{dr}$ and $\psi_q = M_q\left(i_{qs} + i_{qr}\right) + l_{qr} i_{qr}$

$$T_e = M_d i_{qr} i_{ds} - M_q i_{dr} i_{qs} + \left(M_d + l_{dr}\right) i_{dr} i_{qr} - \left(M_q + l_{qr}\right) i_{dr} i_{qr}$$

$$= i_{qr}\left[M_d\left(i_{ds} + i_{dr}\right) + l_{dr} i_{dr}\right] - i_{dr}\left[M_q\left(i_{qs} + i_{qr}\right) + l_{qr} i_{qr}\right]$$

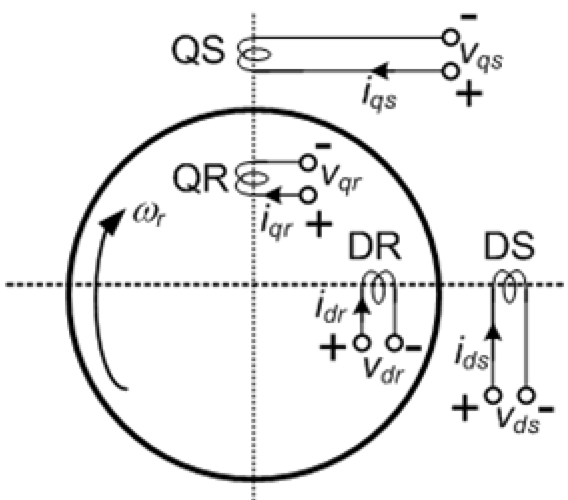
$$= i_{qr}\psi_d - i_{dr}\psi_q$$



Torque in terms of flux linkage

$$T_e = i_{qr}\psi_d - i_{dr}\psi_q$$

- The first term is due to interaction between q-axis armature current and d-axis flux
- It has positive sign indicating that it is a motoring term
- The second term is due to interaction between d-axis armature current and q-axis flux
- It has negative sign indicating that it is a generating term



- No torque is produced by interaction between flux and current that are on the same axis