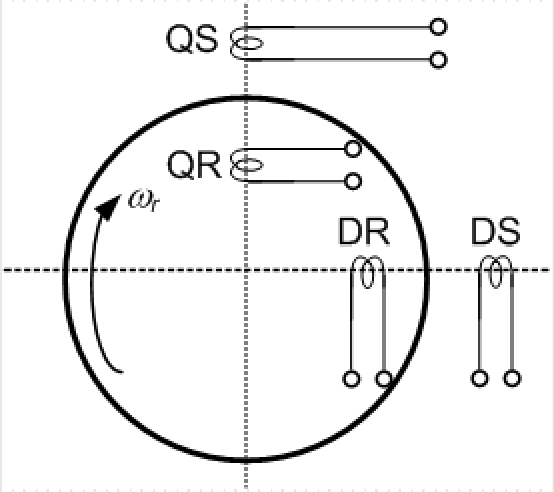
# Voltage equations in Kron's machine

Day 4

# ILOs – Day4

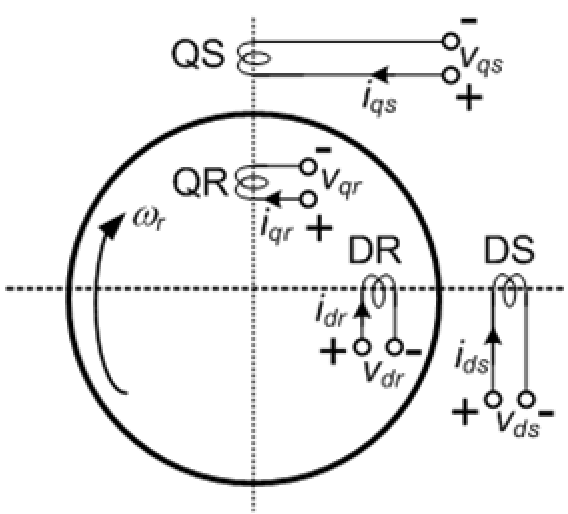
- Derive expressions for voltages in stator and armature coils in Kron's primitive machine
- Build impedance matrix of Kron's primitive machine

# Voltage in stator field coil DS



# Voltage in stator field coil DS

- **Stator coil DS will have voltage drops due to:**
  - Its own resistance $r_{ds}$
  - Its own leakage inductance $l_{ds}$
  - Its mutual inductance $M_d$ with other d-axis coils (transformer EMF)
  - Q axis coils can not have any transformer coupling effect on DS since they are mutually perpendicular
  - Since coil DS is in stator, there will be no rotational EMF




Total inductance of stator coil DS = $L_{ds} = (M_d + l_{ds})$

Thus applied voltage $v_{ds}$ is balanced by:

$$v_{ds} = i_{ds} r_{ds} + L_{ds} p i_{ds} + M_d p i_{dr}$$

Drop in resistance

Drop in self inductance

Drop in mutual inductance with rotor coil DR

# Voltage in stator field coil DS

- **Stator coil DS will have voltage drops due to:**

Thus applied voltage $v_{ds}$ is balanced by:

$$v_{ds} = i_{ds} r_{ds} + L_{ds} p i_{ds} + M_d p i_{dr}$$

Drop in resistance

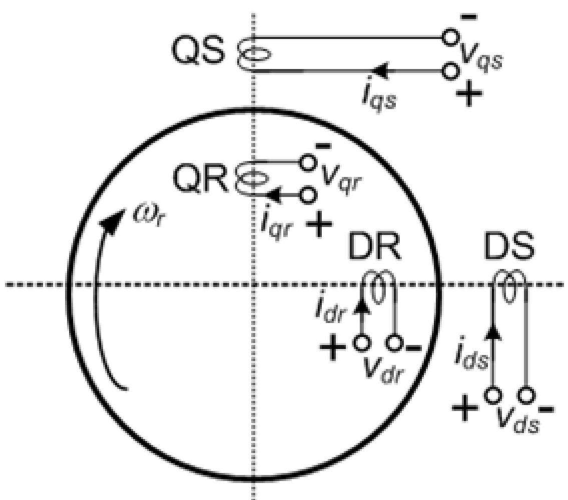
Drop in self inductance

Drop in mutual inductance with rotor coil DR

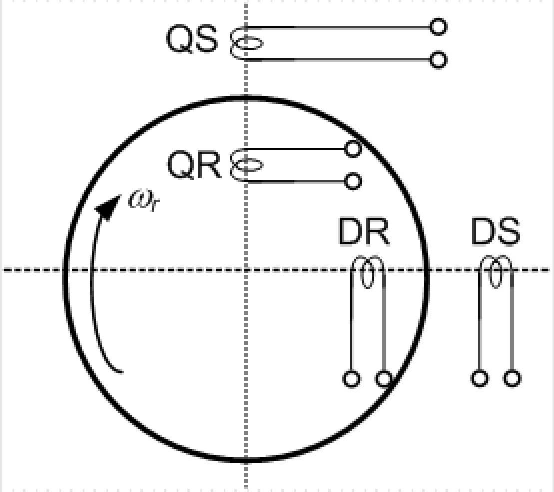
Stator resistance, stator current

Stator inductance, stator current

Mutual inductance, **rotor** current



# Voltage in stator field coil QS



# Voltage in stator field coil QS

- **Stator coil QS:**
  - Similar to stator coil DS, $v_{ds} = i_{ds} r_{ds} + L_{ds} p i_{ds} + M_d p i_{dr}$

Applied voltage $v_{qs}$ in stator coil QS is balanced by:

$$v_{qs} = i_{qs} r_{qs} + L_{qs} p i_{qs} + M_q p i_{qr}$$

Drop in resistance

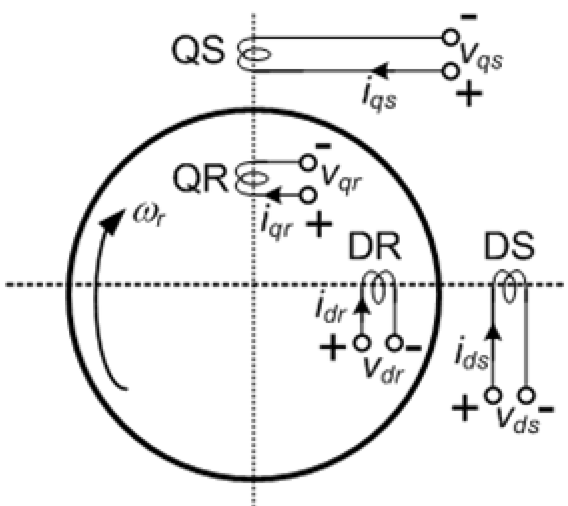
Drop in self inductance

Drop in mutual inductance with rotor coil QR

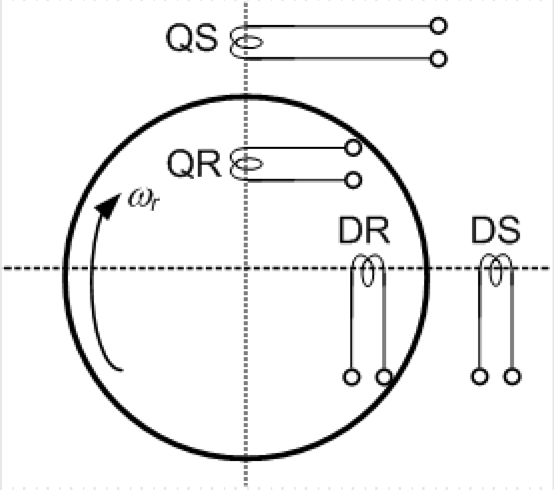
Stator resistance, stator current

Stator inductance, stator current

Mutual inductance, **rotor** current



# Voltage in rotor armature coil DR



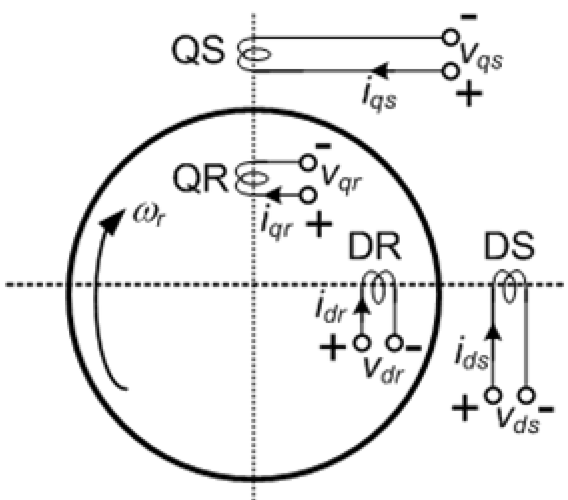
# Voltage in armature coil DR

- **Rotor armature coil DR:**
  - Since the armature coil is a pseudo-stationary coil placed in the rotating element (rotor), there will be rotational EMF (back EMF) induced in it
  - This rotational EMF in DR due to flux along the q-axis is: (remember that rotational EMF is maximum when the coil and flux are orthogonal)

$$e_{dr} = \omega_r \psi_q \sin\theta = \omega_r \psi_q \sin 90^0 = \omega_r \psi_q$$

Thus applied voltage $v_{dr}$ is balanced by:

$$v_{dr} = i_{dr} r_{dr} + L_{dr} p i_{dr} + M_d p i_{ds} - e_{dr}$$




Drop in resistance

Drop in self inductance

Drop in mutual inductance with stator coil DS

Induced rotational back EMF

The negative sign before the induced rotational EMF indicates that this **back EMF** opposes the supply voltage

# Voltage in armature coil DR

- **Rotor armature coil DR:**

$$e_{dr} = \omega_r \psi_q$$

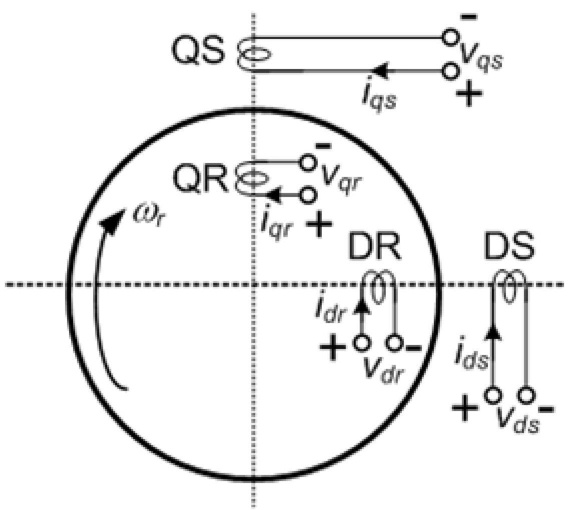
$$v_{dr} = i_{dr} r_{dr} + L_{dr} p i_{dr} + M_d p i_{ds} - e_{dr}$$

Back EMF (rotational EMF)

Rotor resistance, rotor current

Mutual inductance, **stator** current

Rotor inductance, rotor current



# Voltage in armature coil DR

- **Rotor armature coil DR:**

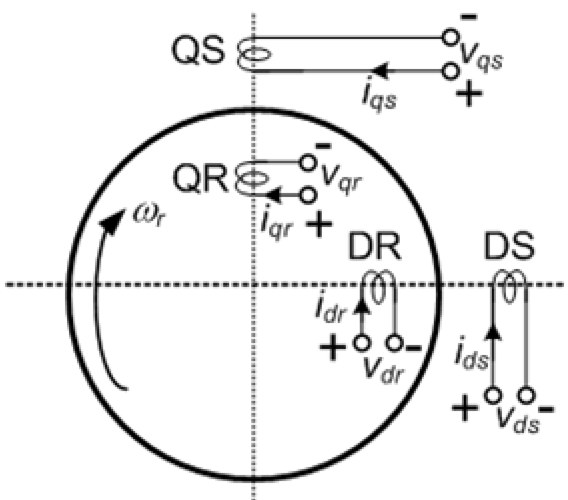
$$e_{dr} = \omega_r \psi_q \qquad v_{dr} = i_{dr} r_{dr} + L_{dr} p i_{dr} + M_d p i_{ds} - e_{dr}$$

The total flux linkage $\psi_q$ with the armature (rotor) in q-axis is given by:

$$\psi_q = M_q (i_{qs} + i_{qr}) + l_{qr} i_{qr}$$

Mutual inductance along Q axis coils in stator and rotor

Leakage inductance of the Q axis coil in rotor

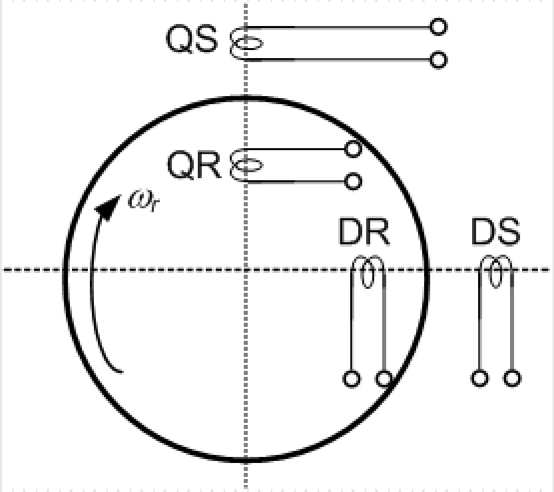



Thus: $v_{dr} = i_{dr} r_{dr} + L_{dr} p i_{dr} + M_d p i_{ds} - \omega_r M_q (i_{qs} + i_{qr}) - \omega_r l_{qr} i_{qr}$

$$v_{dr} = i_{dr} r_{dr} + L_{dr} p i_{dr} + M_d p i_{ds} - \omega_r \left[ (M_q + l_{qr}) i_{qr} \right] - \omega_r M_q i_{qs}$$

$$\boxed{v_{dr} = i_{dr} r_{dr} + L_{dr} p i_{dr} + M_d p i_{ds} - \omega_r L_{qr} i_{qr} - \omega_r M_q i_{qs}}$$

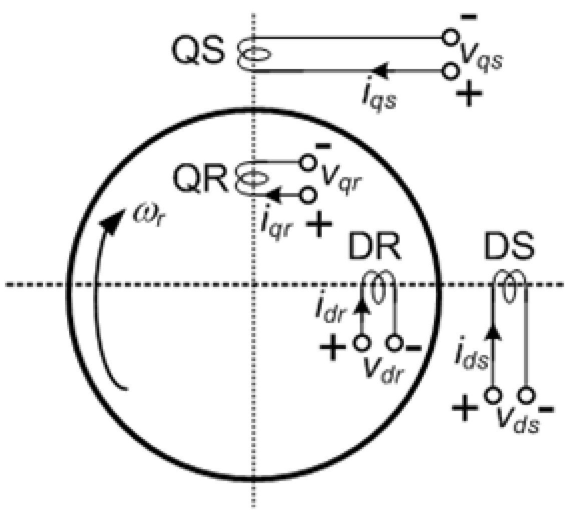
# Voltage in rotor armature coil QR



# Voltage in armature coil QR

- **Rotor armature coil DR:** $v_{dr} = i_{dr} r_{dr} + L_{dr} p i_{dr} + M_d p i_{ds} - \omega_r L_{qr} i_{qr} - \omega_r M_q i_{qs}$
- **Rotor armature coil QR:** $v_{qr} = i_{qr} r_{qr} + L_{qr} p i_{qr} + M_q p i_{qs} + \omega_r L_{dr} i_{dr} + \omega_r M_d i_{ds}$
  - Since this armature coil QR is also a pseudo-stationary coil placed in the rotating element (rotor), there will be rotational EMF (back EMF) induced in it
  - This rotational EMF in QR due to flux along the d-axis is:

$e_{qr} = \omega_r \psi_d \sin\theta = \omega_r \psi_d \sin 270^0 = -\omega_r \psi_d$   Where, $\psi_d = M_d (i_{ds} + i_{dr}) + l_{dr} i_{dr}$




Thus applied voltage $v_{qr}$ is balanced by:

$$v_{qr} = i_{qr} r_{qr} + L_{qr} p i_{qr} + M_q p i_{qs} - e_{qr}$$

Thus, $v_{qr} = i_{qr} r_{qr} + L_{qr} p i_{qr} + M_q p i_{qs} + \omega_r M_d (i_{ds} + i_{dr}) + \omega_r l_{dr} i_{dr}$

$$v_{qr} = i_{qr} r_{qr} + L_{qr} p i_{qr} + M_q p i_{qs} + \omega_r [(M_d + l_{dr}) i_{dr}] + \omega_r M_d i_{ds}$$

$$v_{qr} = i_{qr} r_{qr} + L_{qr} p i_{qr} + M_q p i_{qs} + \omega_r L_{dr} i_{dr} + \omega_r M_d i_{ds}$$

# Voltage equations in matrix form

# Voltage equations in matrix form

- **Stator coil DS:** $v_{ds} = i_{ds}r_{ds} + L_{ds}pi_{ds} + M_d pi_{dr}$
- **Stator coil QS:** $v_{qs} = i_{qs}r_{qs} + L_{qs}pi_{qs} + M_q pi_{qr}$
- **Rotor armature coil DR:** $v_{dr} = i_{dr}r_{dr} + L_{dr}pi_{dr} + M_d pi_{ds} - \omega_r L_{qr}i_{qr} - \omega_r M_q i_{qs}$
- **Rotor armature coil QR:** $v_{qr} = i_{qr}r_{qr} + L_{qr}pi_{qr} + M_q pi_{qs} + \omega_r L_{dr}i_{dr} + \omega_r M_d i_{ds}$

The above four equations can be conveniently written in matrix form:

$$
\begin{bmatrix} v_{ds} \\ v_{qs} \\ v_{dr} \\ v_{qr} \end{bmatrix}
=
\begin{array}{c} ds \\ qs \\ dr \\ qr \end{array}
\overset{\begin{array}{cccc} ds & qs & dr & qr \end{array}}{
\begin{bmatrix}
r_{ds} + L_{ds}p & & M_d p & \\
 & r_{qs} + L_{qs}p & & M_q p \\
M_d p & -M_q\omega_r & r_{dr} + L_{dr}p & -\omega_r L_{qr} \\
M_d\omega_r & M_q p & \omega_r L_{dr} & r_{qr} + L_{qr}p
\end{bmatrix}}
\begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}
$$

Voltage matrix — Impedance matrix — Current matrix

# Observations

- Main diagonal boxes of the impedance matrix contain only self-impedances (resistance & self-inductance)
- The remaining boxes contain either mutual impedances (terms with M), or rotational EMFs (terms with ω), if they exist

$$
\begin{bmatrix} v_{ds} \\ v_{qs} \\ v_{dr} \\ v_{qr} \end{bmatrix}
=
\begin{array}{c|cccc}
 & ds & qs & dr & qr \\
\hline
ds & r_{ds}+L_{ds}p & & M_d p & \\
qs & & r_{qs}+L_{qs}p & & M_q p \\
dr & M_d p & -M_q\omega_r & r_{dr}+L_{dr}p & -\omega_r L_{qr} \\
qr & M_d\omega_r & M_q p & \omega_r L_{dr} & r_{qr}+L_{qr}p
\end{array}
\begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}
$$

Voltage matrix

Impedance matrix

Current matrix

# Observations

- Main diagonal boxes of the impedance matrix contain only self-impedances (resistance & self-inductance)
- The remaining boxes contain either mutual impedances (terms with M), or rotational EMFs (terms with ω), if they exist
- The rotational EMF terms appear only in the last two rows of the impedance matrix

| | | | $ds$ | $qs$ | $dr$ | $qr$ | |
|---|---|---|---|---|---|---|---|
| $v_{ds}$ | | $ds$ | $r_{ds} + L_{ds} p$ | | $M_d p$ | | $i_{ds}$ |
| $v_{qs}$ | | $qs$ | | $r_{qs} + L_{qs} p$ | | $M_q p$ | $i_{qs}$ |
| $v_{dr}$ | $=$ | $dr$ | $M_d p$ | $-M_q \omega_r$ | $r_{dr} + L_{dr} p$ | $-\omega_r L_{qr}$ | $i_{dr}$ |
| $v_{qr}$ | | $qr$ | $M_d \omega_r$ | $M_q p$ | $\omega_r L_{dr}$ | $r_{qr} + L_{qr} p$ | $i_{qr}$ |

Voltage matrix

Impedance matrix

Current matrix

# Observations

- Main diagonal boxes of the impedance matrix contain only self-impedances (resistance & self-inductance)
- The remaining boxes contain either mutual impedances (terms with M), or rotational EMFs (terms with ω), if they exist
- The rotational EMF terms appear only in the last two rows of the impedance matrix
- $p$ and $\omega_r$ (or $-\omega_r$) are interchanged between column elements in last two rows

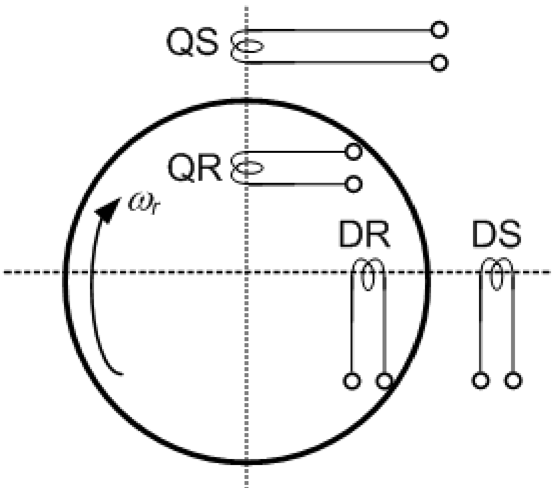
$$\begin{bmatrix} v_{ds} \\ v_{qs} \\ v_{dr} \\ v_{qr} \end{bmatrix} = \begin{array}{c} \\ ds \\ qs \\ dr \\ qr \end{array} \begin{array}{cccc} ds & qs & dr & qr \\ \hline r_{ds}+L_{ds}p & & M_d p & \\ & r_{qs}+L_{qs}p & & M_q p \\ M_d p & -M_q\omega_r & r_{dr}+L_{dr}p & -\omega_r L_{qr} \\ M_d\omega_r & M_q p & \omega_r L_{dr} & r_{qr}+L_{qr}p \end{array} \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}$$

Voltage matrix

Impedance matrix

Current matrix

# Observations




- $M_d$ in the term $M_d p$ is the mutual inductance between coils DS and DR
- These two coils are on the same magnetic axis
- Since this mutual coupling is due to transformer action, $M_d$ may be called ***transformer mutual inductance***

$$\begin{bmatrix} v_{ds} \\ v_{qs} \\ v_{dr} \\ v_{qr} \end{bmatrix} = \begin{array}{c|cccc} & ds & qs & dr & qr \\ \hline ds & r_{ds}+L_{ds}p & & M_d p & \\ qs & & r_{qs}+L_{qs}p & & M_q p \\ dr & M_d p & -M_q\omega_r & r_{dr}+L_{dr}p & -\omega_r L_{qr} \\ qr & M_d\omega_r & M_q p & \omega_r L_{dr} & r_{qr}+L_{qr}p \end{array} \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}$$

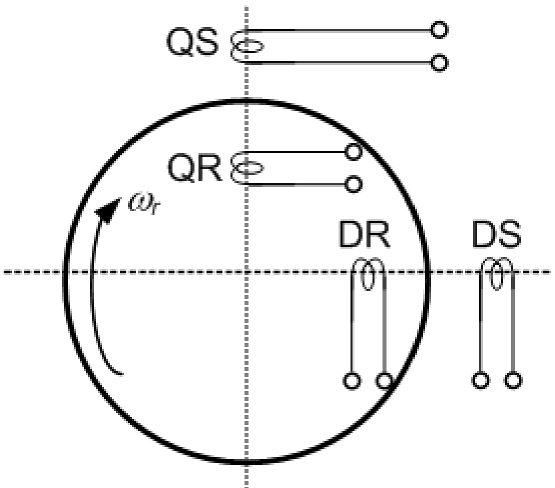
Voltage matrix

Impedance matrix

Current matrix

# Observations

- Similarly, $M_q$ in the term $M_q p$ is the mutual inductance between coils QS and QR
- These two coils are on the same magnetic axis
- Since this mutual coupling is due to transformer action, $M_q$ may also be called ***transformer mutual inductance***



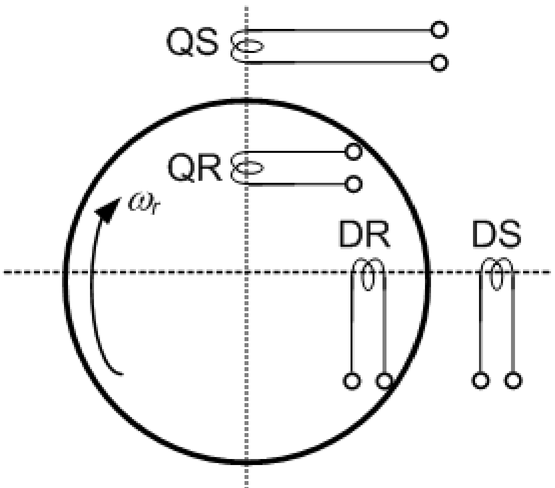

| Voltage matrix | = | | ds | qs | dr | qr | Current matrix |
|---|---|---|---|---|---|---|---|
| $v_{ds}$ | | ds | $r_{ds} + L_{ds} p$ | | $M_d p$ | | $i_{ds}$ |
| $v_{qs}$ | = | qs | | $r_{qs} + L_{qs} p$ | | $M_q p$ | $i_{qs}$ |
| $v_{dr}$ | | dr | $M_d p$ | $-M_q \omega_r$ | $r_{dr} + L_{dr} p$ | $-\omega_r L_{qr}$ | $i_{dr}$ |
| $v_{qr}$ | | qr | $M_d \omega_r$ | $M_q p$ | $\omega_r L_{dr}$ | $r_{qr} + L_{qr} p$ | $i_{qr}$ |

Voltage matrix

Impedance matrix

Current matrix

# Observations




- $M_d$ in the term $M_d\omega_r$ is the mutual inductance between coils DS and QR
- These two coils magnetically in quadrature ($90^0$)
- Since this mutual coupling is due to rotational action, $M_d$ in this case may be called ***rotational mutual inductance*** or ***motional inductance***

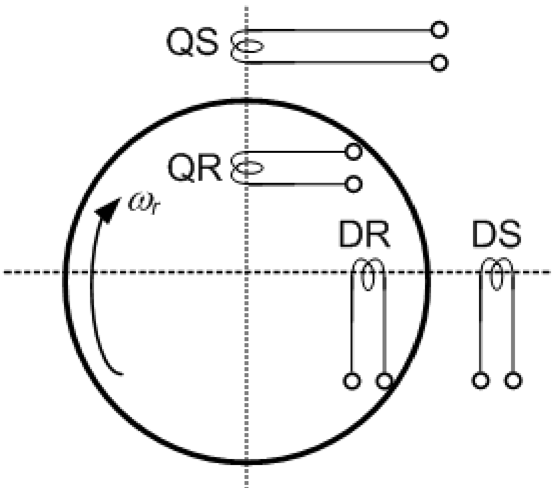
$$\begin{bmatrix} v_{ds} \\ v_{qs} \\ v_{dr} \\ v_{qr} \end{bmatrix} = \begin{array}{c|cccc} & ds & qs & dr & qr \\ \hline ds & r_{ds}+L_{ds}p & & M_d p & \\ qs & & r_{qs}+L_{qs}p & & M_q p \\ dr & M_d p & -M_q\omega_r & r_{dr}+L_{dr}p & -\omega_r L_{qr} \\ qr & M_d\omega_r & M_q p & \omega_r L_{dr} & r_{qr}+L_{qr}p \end{array} \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}$$

Voltage matrix

Impedance matrix

Current matrix

# Observations




- Similarly, $M_q$ in the term $M_q\omega_r$ is the mutual inductance between coils QS and DR
- These two coils magnetically in quadrature ($90^0$)
- Since this mutual coupling is due to rotational action, $M_q$ in this case may be called ***rotational mutual inductance*** or ***motional inductance***

| Voltage matrix | | | ds | qs | dr | qr | Current matrix |
|---|---|---|---|---|---|---|---|
| $v_{ds}$ | | ds | $r_{ds}+L_{ds}p$ | | $M_d p$ | | $i_{ds}$ |
| $v_{qs}$ | = | qs | | $r_{qs}+L_{qs}p$ | | $M_q p$ | $i_{qs}$ |
| $v_{dr}$ | | dr | $M_d p$ | $-M_q\omega_r$ | $r_{dr}+L_{dr}p$ | $-\omega_r L_{qr}$ | $i_{dr}$ |
| $v_{qr}$ | | qr | $M_d\omega_r$ | $M_q p$ | $\omega_r L_{dr}$ | $r_{qr}+L_{qr}p$ | $i_{qr}$ |

Voltage matrix

Impedance matrix

Current matrix

# Abbreviated forms

$$[\text{Voltage column matrix}] = [\text{Impedance matrix}][\text{Current column matrix}]$$

$$[v] = [Z][i]$$

$$[v] = [[R] + [L]p + [G]\omega_r][i]$$

$$\begin{array}{c} \\ \\ \\ \\ \end{array}
\begin{bmatrix} v_{ds} \\ v_{qs} \\ v_{dr} \\ v_{qr} \end{bmatrix}
=
\begin{array}{c|c|c|c|c}
 & ds & qs & dr & qr \\
\hline
ds & r_{ds} + L_{ds}p & & M_d p & \\
qs & & r_{qs} + L_{qs}p & & M_q p \\
dr & M_d p & -M_q \omega_r & r_{dr} + L_{dr}p & -\omega_r L_{qr} \\
qr & M_d \omega_r & M_q p & \omega_r L_{dr} & r_{qr} + L_{qr}p
\end{array}
\begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}$$

Voltage matrix

Impedance matrix

Current matrix

# Abbreviated forms

$$[v] = [[R] + [L]p + [G]\omega_r][i]$$

$$[R] = \begin{bmatrix} r_{ds} & & & \\ & r_{qs} & & \\ & & r_{dr} & \\ & & & r_{qr} \end{bmatrix}$$

Resistance matrix

$$[G] = \begin{bmatrix} & & & \\ & & & \\ & -M_q & & -L_{qr} \\ M_d & & L_{dr} & \end{bmatrix}$$

Motional Inductance matrix or torque matrix (terms involving $\omega_r$)

$$[L] = \begin{bmatrix} L_{ds} & & M_d & \\ & L_{qs} & & M_q \\ M_d & & L_{dr} & \\ & M_q & & L_{qr} \end{bmatrix}$$

Static Inductance matrix (terms without $\omega_r$)

$$\begin{bmatrix} v_{ds} \\ v_{qs} \\ v_{dr} \\ v_{qr} \end{bmatrix} = \begin{array}{c|cccc} & ds & qs & dr & qr \\ \hline ds & r_{ds} + L_{ds}p & & M_d p & \\ qs & & r_{qs} + L_{qs}p & & M_q p \\ dr & M_d p & -M_q\omega_r & r_{dr} + L_{dr}p & -\omega_r L_{qr} \\ qr & M_d\omega_r & M_q p & \omega_r L_{dr} & r_{qr} + L_{qr}p \end{array} \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}$$

# Abbreviated forms

$$[v] = \big[[R] + [L]p + [G]\omega_r\big][i]$$

$$[R] = \begin{bmatrix} r_{ds} & & & \\ & r_{qs} & & \\ & & r_{dr} & \\ & & & r_{qr} \end{bmatrix}$$

$$[L] = \begin{bmatrix} L_{ds} & & M_d & \\ & L_{qs} & & M_q \\ M_d & & L_{dr} & \\ & M_q & & L_{qr} \end{bmatrix}$$

$$[G] = \begin{bmatrix} & & & \\ & & & \\ & -M_q & & -L_{qr} \\ M_d & & L_{dr} & \end{bmatrix}$$

$$\begin{bmatrix} v_{ds} \\ v_{qs} \\ v_{dr} \\ v_{qr} \end{bmatrix} = \begin{array}{c} \\ ds \\ qs \\ dr \\ qr \end{array} \begin{array}{c} \begin{array}{cccc} ds & qs & dr & qr \end{array} \\ \begin{bmatrix} r_{ds} + L_{ds}p & & M_d p & \\ & r_{qs} + L_{qs}p & & M_q p \\ M_d p & -M_q\omega_r & r_{dr} + L_{dr}p & -\omega_r L_{qr} \\ M_d\omega_r & M_q p & \omega_r L_{dr} & r_{qr} + L_{qr}p \end{bmatrix} \end{array} \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}$$