Tutorial 2

Day 21

ILOs – Day21

- Solve numerical problems related to heating & cooling of electrical machines

Recapitulation

Matching

Steady state temperature rise after heating

Heating curve

Temperature rise starting from cold condition

Heating time constant

$$\theta = \theta_m\left(1 - e^{-\frac{t}{\tau_h}}\right)$$

$$\tau_h = \frac{Gh}{\lambda S}$$

$$\theta = \theta_m\left(1 - e^{-\frac{t}{\tau_h}}\right) + \theta_i e^{-\frac{t}{\tau_h}}$$

$$\theta_m = \frac{Q}{\lambda S} = \frac{Qc}{S}$$

- Q = Power loss or heat developed, W
- G = Mass of the machine, kg
- h = specific heat, J/kg-^{0}C
- S = cooling surface area, m^2
- λ = specific heat dissipation, W/m^2-^{0}C
- $c = 1/\lambda$ = cooling coefficient

- θ = temperature rise at any time t, ^{0}C
- θ_m = final steady temperature rise while heating, ^{0}C
- θ_n = final steady temperature rise while cooling, ^{0}C
- θ_i = initial temperature rise over ambient medium, ^{0}C
- τ_h = heating time constant, s
- τ_c = cooling time constant, s

Recapitulation

Matching

Steady state temperature rise after cooling

$$\theta = \theta_n\left(1 - e^{-\frac{t}{\tau_c}}\right) + \theta_i e^{-\frac{t}{\tau_c}}$$

Cooling curve

$$\theta_n = \frac{Q}{\lambda S}$$

Temperature rise when cooled to ambient temperature

$$\tau_c = \frac{Gh}{\lambda S}$$

Cooling time constant

$$\theta = \theta_i e^{-\frac{t}{\tau_c}}$$

- Q= Power loss or heat developed, W
- G = Mass of the machine, kg
- h = specific heat, J/kg-^{0}C
- S = cooling surface area, m^2
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- θ_i = initial temperature rise over ambient medium, ^{0}C
- τ_h = heating time constant, s
- τ_c = cooling time constant, s

#1) *The initial temperature of a machine is 40^0C. Calculate the temperature of the machine after 1 hour if its final steady state temperature rise is 80^0C and the heating time constant is 2 hours. The ambient temperature is 30^0C.*

Given,

- θ_m = final steady temperature rise while heating = 80^0C
- τ_h = heating time constant = 2 hr
- t = time = 1 hr

Initial temperature rise: $\theta_i = 40^0 C - 30^0 C = 10^0 C$

Heating equation: $$\theta = \theta_m \left(1 - e^{-\frac{t}{\tau_h}}\right) + \theta_i e^{-\frac{t}{\tau_h}}$$

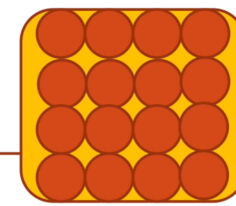
$\therefore$ Temperature rise after 1 hr: $\theta = 80\left(1 - e^{-\frac{1}{2}}\right) + 10e^{-\frac{1}{2}} = 37.54^0 C$

$\therefore$ Temperature of the machine after 1 hr $= 37.54^0 + 30^0 = 67.54^0 C$

#2) *A field coil has a heat dissipating surface of 0.15 m^2 and a length of mean turn of 1 m. It dissipates loss of 150 W, the emissivity being 34 $W/m^2\text{-}^0C$. Estimate the final steady state temperature rise of the coil and its time constant if the cross section of the coil is 100x50 mm^2. Specific heat of copper is 390 $J/kg\text{-}^0C$. The space factor is 0.56. Copper weighs 8900 kg/m^3.*

Given

- Q = Power loss = 150 W
- h = specific heat = 390 $J/kg\text{-}^0C$
- S = cooling surface area = 0.15 m^2
- λ = specific heat dissipation = 34 $W/m^2\text{-}^0C$



$$\text{Volume of copper} = \text{cross section} \times \text{length of coil} \times \text{space factor}$$
$$= 100 \times 50 \times 10^{-6} \times 1 \times 0.56$$
$$= 2.8 \times 10^{-3} \quad m^3$$

$$\text{Weight of copper } G = \text{volume} \times \text{density} = 2.8 \times 10^{-3} \times 8900 = 24.92 \quad \text{kg}$$

$\therefore$ Final steady state temperature rise:

$$\theta_m = \frac{Q}{\lambda S} = \frac{150}{34 \times 0.15} = 29.4^0 C$$

Heating time constant:

$$\tau_h = \frac{Gh}{\lambda S} = \frac{24.92 \times 390}{34 \times 0.15} = 1906 \ s$$

#3) The initial temperature rise of a transformer is 25^0C after one hour and 37.5^0C after two hours of starting from cold condition. Calculate its final steady state temperature rise and the heating time constant. If its temperature falls from the final steady state value to 40^0C in 2.5 hours when disconnected, calculate its cooling time constant. The ambient temperature is 30^0C.

When heating:

The transformer starts from cold condition, so heating equation is:

$$\theta = \theta_m \left(1 - e^{-\frac{t}{\tau_h}} \right)$$

Given: $\theta = 25^0$C at $t = 1$ hr and $\theta = 37.5^0$C at $t = 2$ hr

$$\therefore\ 25 = \theta_m \left(1 - e^{-\frac{1}{\tau_h}} \right) \quad \text{and} \quad 37.5 = \theta_m \left(1 - e^{-\frac{2}{\tau_h}} \right)$$

$$\Rightarrow \frac{\left(1 - e^{-\frac{2}{\tau_h}} \right)}{\left(1 - e^{-\frac{1}{\tau_h}} \right)} = \frac{37.5}{25} = 1.5 \quad \Rightarrow \left(1 + e^{-\frac{1}{\tau_h}} \right) = 1.5 \quad \Rightarrow e^{-\frac{1}{\tau_h}} = 0.5 \quad \Rightarrow \tau_h = 1.44 \text{ hr}$$

Final steady state temperature rise: $25 = \theta_m \left(1 - e^{-\frac{t}{\tau_h}} \right) \Rightarrow 25 = \theta_m (1 - 0.5) \Rightarrow \theta_m = 50^0 C$

#3) *The initial temperature rise of a transformer is 25⁰C after one hour and 37.5⁰C after two hours of starting from cold condition. Calculate its final steady state temperature rise and the heating time constant. If its temperature falls from the final steady state value to 40⁰C in 2.5 hours when disconnected, calculate its cooling time constant. The ambient temperature is 30⁰C.*

When cooling:

After 2.5 hours, the temperature is 40^0C. Thus, temperature **rise** after 2.5 hr:

$$\theta = 40^0 C - 30^0 C = 10^0 C$$

Since the transformer was disconnected, its final steady state temperature while cooling is equal to the ambient temperature. Thus $\theta_n = 0^0$C

Initial temperature rise during cooling = Final steady state temperature rise during heating

$$\Rightarrow \theta_i = \theta_m = 50^0 C$$

$\therefore$ From cooling equation: $\theta = \theta_i e^{-\frac{t}{\tau_c}}$

$$\Rightarrow \theta = \theta_i e^{-\frac{t}{\tau_c}} \Rightarrow 10 = 50 e^{-\frac{2.5}{\tau_c}}$$

$\therefore$ Cooling time constant $\tau_c = 1.55$ hr

- θ = temperature rise at any time t, ^{0}C
- θ_n = final steady temperature rise while cooling, ^{0}C
- θ_i = initial temperature rise over ambient medium, ^{0}C
- τ_c = cooling time constant, s
- t = time, s