Heating & Cooling of Electric Machines

Day 20

ILOs – Day20

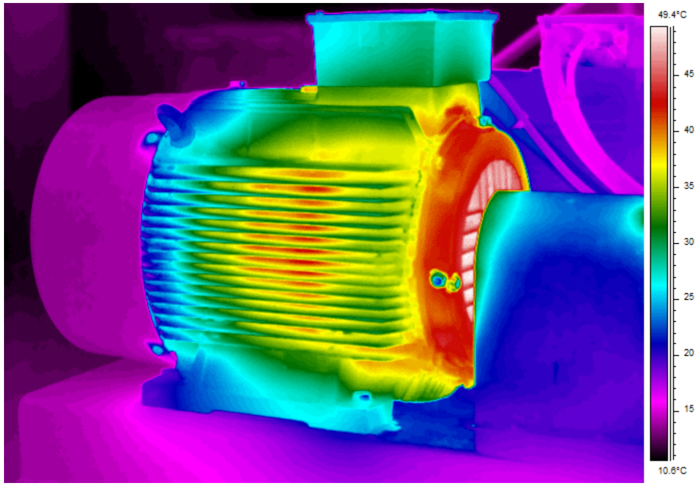
- Derive the expressions for heating and cooling in electrical machines
- Plot the heating and cooling curves in electrical machines
- Analyze the heating and cooling conditions in electrical machines

Heating & Cooling curves

- In an ideal **homogeneous body**, internally heated and surface cooled, has a maximum surface temperature-rise proportional directly to the power loss and inversely to that of the cooling
- Though an electric machine is far from being homogeneous, we consider it to be for the sake of simplicity
- The temperature of a machine initially rises from cold condition (=ambient temperature) as it runs with steady load
- As the temperature rises, active parts of the machine starts dissipating the heat by processes of conduction, convection, and radiation
- More the temperature rise, better will be the cooling
- The rate of temperature rise is exponential till a steady state temperature is reached

Heating & Cooling curves

- Thermal images of heated machines



Heating curve

Heating curve

- Heat developed in body of the machine in small time dt:

$$Q_{dev} = Q \times dt \quad W\text{-}s \text{ or } J$$

- Heat energy stored in the body during this time as the body temperature rises by $d\theta$:

$$Q_{st} = G \times h \times d\theta \quad J$$

- Heat dissipated during this time as the body temperature reaches $\theta\,^0$C above ambient:

$$Q_{diss} = \lambda \times S \times \theta \times dt \quad W\text{-}s \text{ or } J$$

- Q= Power loss or heat developed, W
- G = Mass of the machine, kg
- h = specific heat, J/kg-^{0}C
- S = cooling surface area, m^2
- λ = specific heat dissipation, W/m^2-^{0}C
- $c=1/\lambda$ = cooling coefficient
- θ = temperature rise at any time t, ^{0}C
- θ_m = final steady temperature rise while heating, ^{0}C
- θ_i = initial temperature rise over ambient medium, ^{0}C
- τ_h = heating time constant, s
- τ_c = cooling time constant, s
- t = time, s

- Since heat developed = heat stored + heat dissipated:

$$\Rightarrow Q \times dt = G \times h \times d\theta + \lambda \times S \times \theta \times dt$$

Heating curve

$$Q \times dt = G \times h \times d\theta + \lambda \times S \times \theta \times dt$$

- The machine reaches steady state temperature $\theta = \theta_m$ as $t \rightarrow \infty$
- There is no further temperature rise

$$d\theta = 0$$

- Hence, no further energy is stored in the body of the material

$$G \times h \times d\theta = 0$$

- Amount of heat production = amount of heat dissipation

$$Q \times dt = \lambda \times S \times \theta_m \times dt$$

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$$\Rightarrow \theta_m = \frac{Q}{\lambda S} = \frac{Qc}{S}$$ = The steady state temperature rise

Heating curve

$$Q \times dt = G \times h \times d\theta + \lambda \times S \times \theta \times dt$$

$$\Rightarrow dt = \frac{d\theta}{\frac{Q}{Gh} - \frac{\lambda S}{Gh}\theta}$$

- Solving for the above differential equation by putting $\theta_m = \frac{Q}{\lambda S}$ and applying suitable boundary conditions:

$$t = \frac{Gh}{\lambda S}\log_e \frac{\theta_m - \theta}{\theta_m - \theta_i}$$

$$\Rightarrow t = \tau_h \log_e \frac{\theta_m - \theta}{\theta_m - \theta_i}$$

$\tau_h = \frac{Gh}{\lambda S}$ = The ***heating time constant***

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* For detailed solution please refer to appendix

Heating curve

$$t = \tau_h \log_e \frac{\theta_m - \theta}{\theta_m - \theta_i}$$

$$\Rightarrow \frac{\theta_m - \theta}{\theta_m - \theta_i} = e^{-\frac{t}{\tau_h}}$$

$$\Rightarrow \theta = \theta_m \left(1 - e^{-\frac{t}{\tau_h}}\right) + \theta_i e^{-\frac{t}{\tau_h}}$$

- If the machine starts from cold condition, i.e. no temperature ***rise*** above ambient: $\theta_i = 0$

$$\theta = \theta_m \left(1 - e^{-\frac{t}{\tau_h}}\right)$$

This is the ***temperature rise equation with time***

- Q= Power loss or heat developed, W
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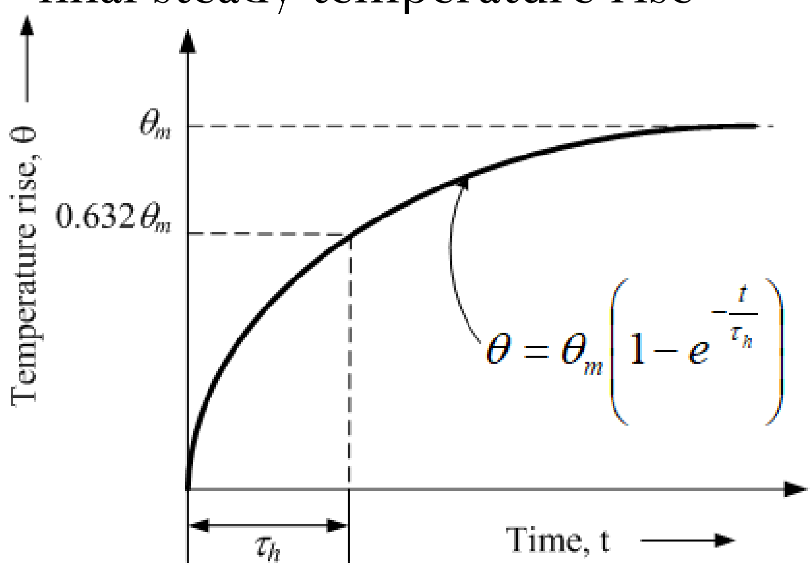
Heating curve

$$\theta = \theta_m\left(1 - e^{-\frac{t}{\tau_h}}\right)$$

- Putting $t = \tau_h$ in the above equation:

$$\theta = \theta_m\left(1 - e^{-1}\right) = 0.632\theta_m$$

- Thus, the heating time constant τ_h can be defined as the time taken by the machine to attain 63.2% of its final steady temperature rise



This is the ***heating curve of a machine***

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Heating curve

Some interesting observations:

- Heating time constant: $\tau_h = \dfrac{Gh}{S\lambda}$
- The time constant is inversely proportional to the specific heat dissipation λ
- For a well ventilated machine, the value of λ is high and hence the time constant τ_h is small
- Thus, a well ventilated machine will reach its steady state temperature quickly
- Conversely, the value of heating time constant is large for a poorly ventilated machine and hence it takes lot of time to reach its steady state temperature rise

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Heating curve

Some interesting observations:

- Heating time constant: $\tau_h = \frac{Gh}{S\lambda}$
- The time constant is directly proportional to volume and inversely proportional to the surface area of the machine
- Thus, overall, the heating time constant is proportional to 1st power of linear dimension
- Hence, a large sized machine will have higher value of heating time constant
- Thus, large sized machines will take more time to reach its steady state temperature

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Conventional machines have heating time constants anywhere between half an hour to 3-4 hours

Cooling curve

Cooling curve

- The cooling curve of a machine can be derived from the same starting heat balance equation:

$$Q \times dt = G \times h \times d\theta + \lambda \times S \times \theta \times dt$$

- After the machine has been switched off after running for some time, it starts to cool down
- The machine reaches steady state temperature $\theta = \theta_n$ after cooling as $t \rightarrow \infty$
- There is no further temperature drop

$$d\theta = 0$$

- Hence, no further energy is stored in the body of the material

$$G \times h \times d\theta = 0$$

- Amount of heat production = Amount of heat dissipation

$$Q \times dt = \lambda \times S \times \theta_n \times dt$$

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$$\theta_n = \frac{Q}{\lambda S}$$

Cooling curve

- Proceeding in the same way as with heating, the expression for temperature rise with time during cooling can be derived to be:

$$\theta = \theta_n \left(1 - e^{-\frac{t}{\tau_c}}\right) + \theta_i e^{-\frac{t}{\tau_c}}$$

Where θ_n is the final steady state temperature rise after cooling

$$\theta_n = \frac{Q}{\lambda S}$$

τ_c is the ***cooling time constant***

$$\tau_c = \frac{Gh}{\lambda S}$$

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Note that all three equations are same for heating as well as cooling. However, the value of λ is usually different under cooling condition from that under heating condition

Cooling curve

$$\theta = \theta_n\left(1 - e^{-\frac{t}{\tau_c}}\right) + \theta_i e^{-\frac{t}{\tau_c}}$$

- If the machine is shut down, no heat is produced further, and it continues to cool down till its final steady state temperature rise reaches 0^0; i.e.

$$\theta_n = 0$$

- Thus, temperature rise expression during cooling is:

$$\theta = \theta_i e^{-\frac{t}{\tau_c}}$$

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This is the ***Cooling curve equation of a machine***

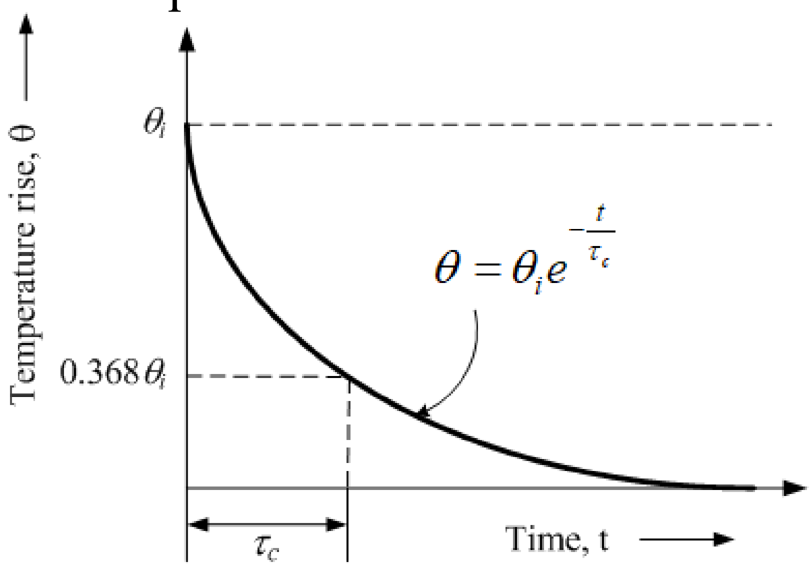
Cooling curve

$$\theta = \theta_i e^{-\frac{t}{\tau_c}}$$

- Putting $t = \tau_c$ in the above equation:

$$\theta = \theta_i e^{-1} = 0.368\theta_i$$

- Thus, the cooling time constant τ_c can be defined as the time taken by the machine for its temperature rise to drop to 36.8% of its initial value



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This is the ***cooling curve of a machine***

Cooling curve

Some interesting observations:

- As the machine is switched off, it cools down
- Thus during cooling the machine no longer rotates
- The cooling process is thus not as efficient as when it was running, and hence the value of λ is lower during cooling than during heating
- Thus, cooling time constant is usually larger owing to poorer ventilation condition when the machine cools
- In self-cooled motors, the cooling time constant is about 2-3 times greater than the heating time constant

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Cooling curve

Some interesting observations:

- Final steady state temperature rise after cooling:
$$\theta_n = \frac{Q}{\lambda S}$$
- The final steady temperature after cooling thus depends on the power loss Q
- But when the machine is shut down, there is no power loss, $Q = 0$
- Thus, $\theta_n = 0$, meaning that the machine temperature finally comes down to ambient temperature after cooling

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