Transient Analysis & Stability of Synchronous Machines

Day 18

ILOs – Day18

- Derive dynamic electromechanical transient equations for synchronous generator
- Assess transient stability of synchronous generator
  - Derive swing equation
  - Apply equal area criterion
- Explain the process of re-synchronization by synchronizing power

Transient stability of synchronous generator

- **Transient stability**
  - A synchronous generator connected to an infinite bus runs stably at synchronous speed under steady state stable running condition:
    - The mechanical input power from the prime mover exactly balances the generator losses and electrical output power
    - Relative velocity between stator and rotor fields is zero, i.e. they are at synchronism
    - The torque angle (load angle δ) is constant
  - Any disturbance to this balance results in a change of speed and rotor angle
  - The generator is transiently stable only if after some temporary fluctuations the speed returns to synchronism and the rotor angle again to a constant value
  - In the unstable condition the speed does not return to synchronism and the rotor angle consequently varies continuously
  - The calculation of the swing curve of rotor angle (how the rotor angle δ varies during the transient period) as a function of time, is therefore the prime objective of any transient stability study

Transient stability of synchronous generator

- **Transient stability**
  - Consider a synchronous generator connected to infinite bus
  - A sudden increase in the output load reduces the prime mover speed momentarily
  - The load angle δ increases to supply the additional load
  - Before the rotor settles to a new steady value of load angle, the rotor oscillates around its final equilibrium position
  - As a result, the rotor speed fluctuates around synchronous speed before finally attaining synchronous speed again.
  - This phenomena of rotor oscillation around its final synchronized position is called *hunting*
  - Study of nature and extent of hunting is the prime objective of transient stability investigation

Transient stability of synchronous generator

- **Disturbances in a synchronous machine**
  - A sudden change in load
  - A fault in the mechanical prime mover (for generator)
  - A fault in the main supply system (for motor)
  - A fault in the field excitation system
  - Presence of harmonic torques in addition to main torque
- **Effect of hunting**
  - Mechanical stress on shaft
  - Fluctuations in output voltage
  - Increase in losses and rise in machine temperature
  - Too much oscillation may cause machine to fall out of step (lose synchronism)

Transient stability of synchronous generator

- **Dynamic electromechanical equations:**
  - For a generator, neglecting friction & damping, the torque balance equation is: $T_{sh} = T_e + T_i$
    - T_{sh} is the mechanical input torque (shaft torque)
    - T_e is the electromagnetic torque developed (back torque) in rotor
    - T_i is the inertia torque that opposes the rotation
  - Similarly, the power balance equation is obtained by multiplying both sides of the torque balance equation by synchronous speed:

$$P_{sh} = P_e + P_i$$

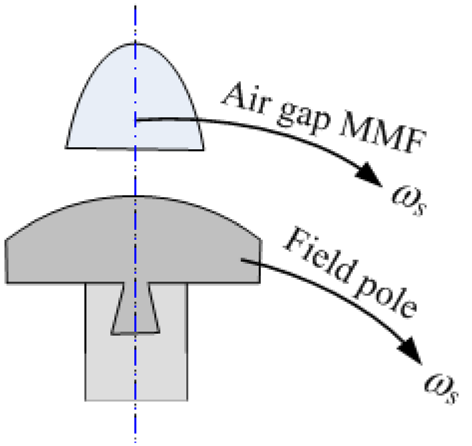
    - P_{sh} is the mechanical input power (shaft power)
    - P_e is the electromagnetic power developed (back power) in rotor
    - P_i is the inertia power that opposes the rotation

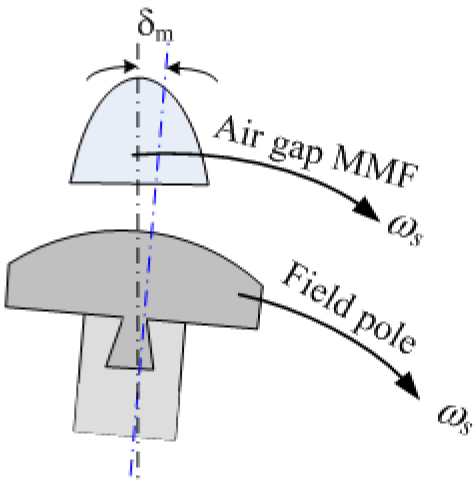
Transient stability of synchronous generator

- **Dynamic electromechanical equations:**
  - Rotor positions w.r.t. air gap flux during hunting:
    - At time $t=0$, and at no-load, let the field pole (salient pole) axis coincides with the axis of resultant air-gap flux wave (sinusoidal)
    - A sudden increase in the shaft power input P_{sh} will tend to take the field pole axis ahead of the air-gap flux axis by an angle δ_m
    - Thus we have the relation:

$$\theta_m = \omega_s t + \delta_m$$

Where, ω_s = synchronous speed in rad/s
$\omega_s t$ = space travelled by the rotating air gap flux wave in t sec after the sudden disturbance
θ_m = space angle travelled by field poles in t sec after the sudden disturbance





Transient stability of synchronous generator

- **Dynamic electromechanical equations:**

$$\theta_m = \omega_s t + \delta_m$$

Converting the mechanical angle δ_m to equivalent electrical angle δ for a machine with P number of poles, we have:

$$\delta_m = \frac{2}{P}\delta$$

Thus,

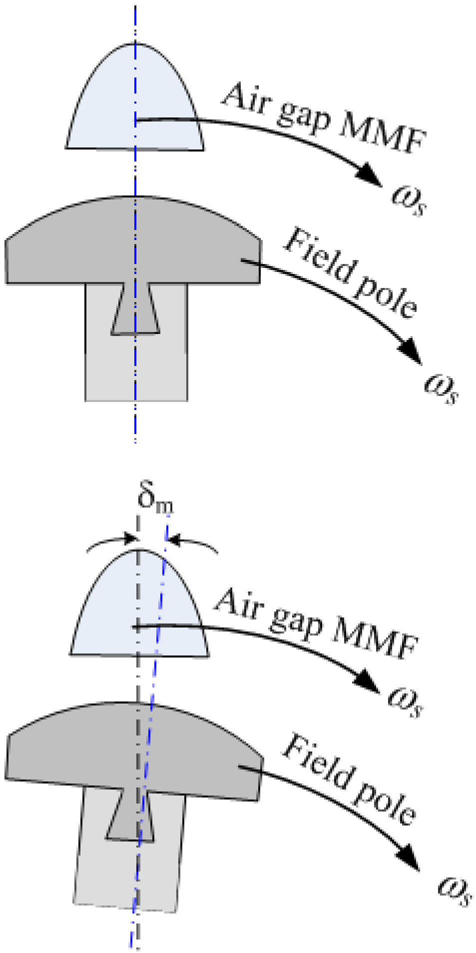
$$\theta_m = \omega_s t + \frac{2}{P}\delta$$

Taking 1st derivative on both sides,

$$\frac{d\theta_m}{dt} = \omega_s + \frac{2}{P}\frac{d\delta}{dt}$$

Taking 2nd derivative on both sides for a constant synchronous speed ω_s:

$$\frac{d^2\theta_m}{dt^2} = \frac{2}{P}\frac{d^2\delta}{dt^2}$$



Transient stability of synchronous generator

- **Dynamic electromechanical equations:**

$$P_{sh} = P_e + P_i \qquad \frac{d^2\theta_m}{dt^2} = \frac{2}{P}\frac{d^2\delta}{dt^2}$$

Inertia power (or angular momentum) P_i as per definition is given by:

$$P_i = \omega_s J \frac{d^2\theta_m}{dt^2}$$

Where, $J =$ Polar moment of inertia of the rotating system in kgm^2

$$\Rightarrow P_i = \frac{2\pi N_s}{60} J \frac{2}{P}\frac{d^2\delta}{dt^2}$$

$$\Rightarrow P_i = \left(2\pi n_s \frac{2}{P} J\right)\frac{d^2\delta}{dt^2}$$

Where, $n_s = N_s/60 =$ speed in rps

$$\Rightarrow P_i = P_j \frac{d^2\delta}{dt^2}$$

Where, $P_j = \left(2\pi n_s \frac{2}{P} J\right)$ angular momentum in watts /electrical radian per sec^2 or in Joules/electrical radian per sec

Thus, $$\boxed{P_{sh} = P_j \frac{d^2\delta}{dt^2} + P_e}$$

Transient stability of synchronous generator

- **Dynamic electromechanical equations:**

$$P_{sh} = P_j \frac{d^2\delta}{dt^2} + P_e$$

The electrical power P_e in a synchronous machine is given by: $P_e = \frac{E_f V_t}{X_s} \sin\delta = P_m \sin\delta$

Where, E_f is the excitation voltage (induced EMF), V_t is the terminal voltage and X_s is the synchronous reactance

The dynamic power balance equation can thus be written as:

$$P_j \frac{d^2\delta}{dt^2} + P_m \sin\delta = P_{sh}$$

If damping (friction) is also to be considered, then the dynamic power balance equation can be updated as:

$$P_j \frac{d^2\delta}{dt^2} + K_d \frac{d\delta}{dt} + P_m \sin\delta = P_{sh}$$

Where, K_d is the viscous damping coefficient

Transient stability of synchronous generator

- **Dynamic electromechanical equations:**

$$P_j \frac{d^2\delta}{dt^2} + K_d \frac{d\delta}{dt} + P_m \sin\delta = P_{sh}$$

- Solution of the above differential equation is tedious because of its non-linear nature.
- However, for small angular oscillations it is possible to linearize the above equation and arrive at a solution.
- If damping is neglected and the machine is assumed to connected to an infinite bus, the electro-mechanical dynamic differential equation is reduced to:

$$P_j \frac{d^2\delta}{dt^2} + P_m \sin\delta = P_{sh}$$

- This is called the ***swing equation*** of a synchronous machine

Transient stability of synchronous generator

- **Solution of the swing equation:**

$$P_j \frac{d^2\delta}{dt^2} + P_m \sin\delta = P_{sh}$$

From the above equation we get:

$$\frac{d^2\delta}{dt^2} = \frac{P_{sh} - P_m \sin\delta}{P_j}$$

Multiplying both sides of the above equation by $\frac{d\delta}{dt}$

$$\left(\frac{d\delta}{dt}\right)\frac{d^2\delta}{dt^2} = \left(\frac{P_{sh} - P_m \sin\delta}{P_j}\right).\frac{d\delta}{dt}$$

$$\Rightarrow \frac{1}{2}\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 = \left(\frac{P_{sh} - P_m \sin\delta}{P_j}\right).\frac{d\delta}{dt}$$

$$\Rightarrow d\left(\frac{d\delta}{dt}\right)^2 = \frac{2(P_{sh} - P_m \sin\delta)}{P_j}d\delta$$

Transient stability of synchronous generator

- **Solution of the swing equation:**

$$d\left(\frac{d\delta}{dt}\right)^2 = \frac{2\left(P_{sh} - P_m \sin\delta\right)}{P_j} d\delta$$

On integrating the above equation we get:

$$\int d\left(\frac{d\delta}{dt}\right)^2 = \int \frac{2\left(P_{sh} - P_m \sin\delta\right)}{P_j} d\delta$$

$$\Rightarrow \left(\frac{d\delta}{dt}\right)^2 = \int \frac{2\left(P_{sh} - P_m \sin\delta\right)}{P_j} d\delta$$

$$\Rightarrow \frac{d\delta}{dt} = \sqrt{\int \frac{2\left(P_{sh} - P_m \sin\delta\right)}{P_j} d\delta}$$

Transient stability of synchronous generator

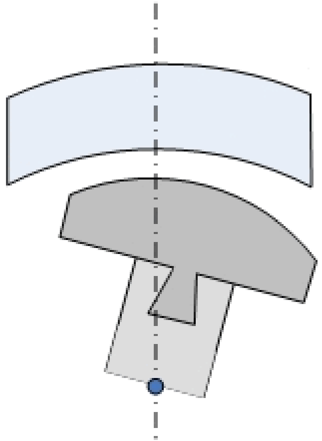
- **Solution of the swing equation:**

$$\frac{d\delta}{dt} = \sqrt{\int \frac{2(P_{sh} - P_m \sin\delta)}{P_j} d\delta}$$

  - Here, the term $\frac{d\delta}{dt}$ is the relative **velocity** between stator and rotor fields
  - Before any disturbance since the machine was running at steady synchronous speed, this relative velocity must have been zero
  - After disturbance, the machine will experience some oscillations of δ between stator and rotor fields
  - Finally, once such oscillation dies out and the rotor attains synchronous speed again, the relative velocity drops to zero: $\frac{d\delta}{dt} = 0$

$$\Rightarrow \sqrt{\int \frac{2(P_{sh} - P_m \sin\delta)}{P_j} d\delta} = 0$$

$$\Rightarrow \int (P_{sh} - P_m \sin\delta) d\delta = 0$$

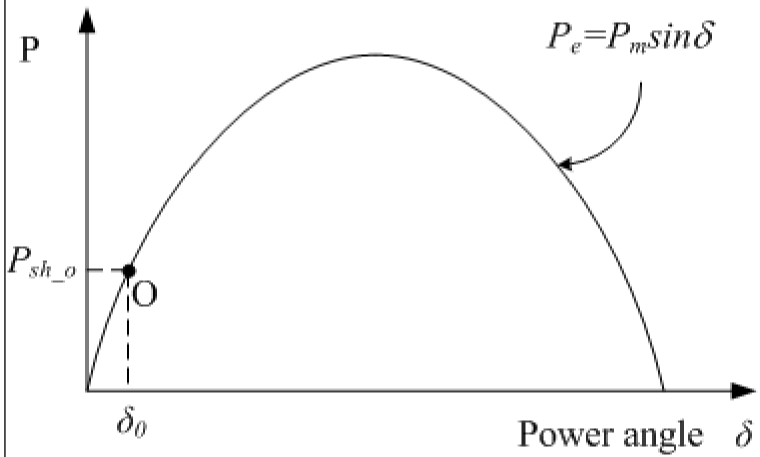


Transient stability of synchronous generator

- **Swing curve and condition for stability:**

$$\int (P_{sh} - P_m \sin\delta) d\delta = 0$$

- The electrical power varies sinusoidally with power angle δ as: $P_e = P_m \sin\delta$
- The shaft power input is independent of power angle δ
- The combined plot will thus look like:



- P_{sh_o} is the shaft power input before disturbance
- Intersection of P_e curve and P_{sh_o} line is the initial operating point "O" at power angle δ_0

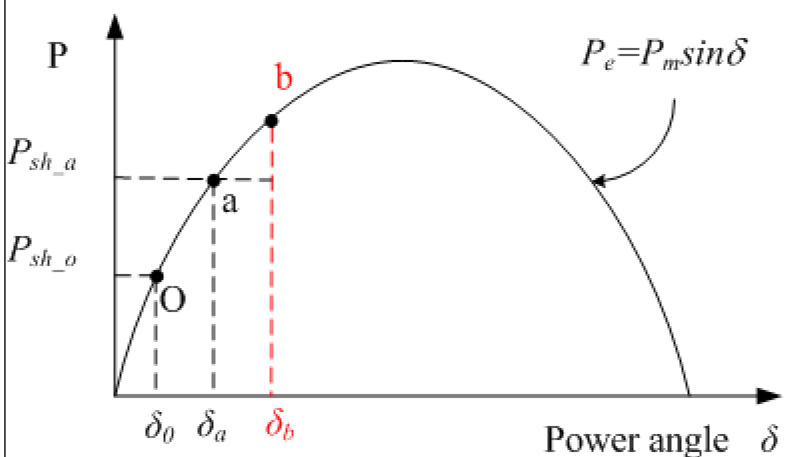
$$P_{e0} = P_m \sin\delta_0 = P_{sh\_0}$$

Transient stability of synchronous generator

- **Swing curve and condition for stability:**

$$\int \left(P_{sh} - P_m \sin\delta\right) d\delta = 0$$

- There is a sudden increase in the input shaft power to P_{sh_a}
- This causes the power angle to rise to δ_a and $\frac{d\delta}{dt}$ is increasing (acceleration)



- The new intersection point is "a"
- Acceleration will tend to bring up the machine to the new operating point "a" from "O"
- But due to inertia, the rotor may overshoot
- After crossing "a", the machine decelerates and reaches "b"
- At "b", the electrical power output being more than shaft input, the machine slows down

- The machine will now again try to come back to "a"
- After few such oscillations, machine will finally settle at the new steady point "a"

Transient stability of synchronous generator

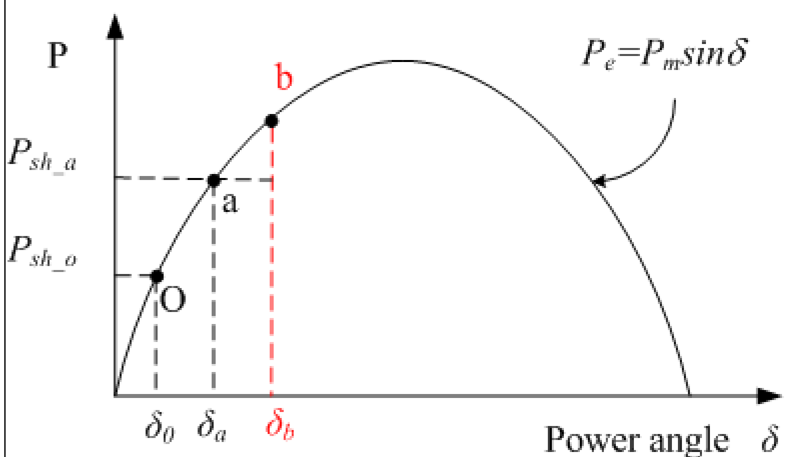
- **Swing curve and condition for stability:**

$$\int \left(P_{sh} - P_m \sin\delta\right) d\delta = 0$$

- For this system to be stable, we must have $\int_{\delta_0}^{\delta_b} \left(P_{sh} - P_m \sin\delta\right) d\delta = 0$

$$\text{or,} \quad \int_{\delta_0}^{\delta_a} \left(P_{sh\_a} - P_m \sin\delta\right) d\delta + \int_{\delta_a}^{\delta_b} \left(P_{sh\_a} - P_m \sin\delta\right) d\delta = 0$$

$$\text{or,} \quad \int_{\delta_0}^{\delta_a} \left(P_{sh\_a} - P_m \sin\delta\right) d\delta = \int_{\delta_a}^{\delta_b} \left(P_m \sin\delta - P_{sh\_a}\right) d\delta$$



Transient stability of synchronous generator

- **Swing curve and condition for stability:**

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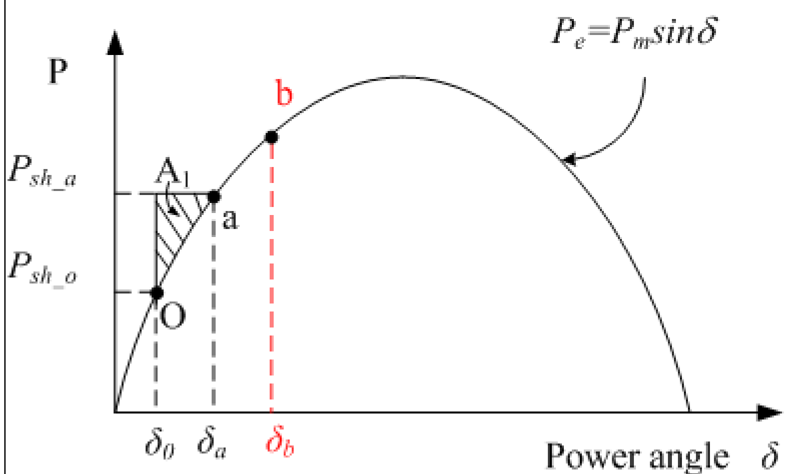


or, $$\int_{\delta_0}^{\delta_a} \left(P_{sh\_a} - P_m \sin\delta\right) d\delta + \int_{\delta_a}^{\delta_b} \left(P_{sh\_a} - P_m \sin\delta\right) d\delta = 0$$

or, $$\int_{\delta_0}^{\delta_a} \left(P_{sh\_a} - P_m \sin\delta\right) d\delta = \int_{\delta_a}^{\delta_b} \left(P_m \sin\delta - P_{sh\_a}\right) d\delta$$

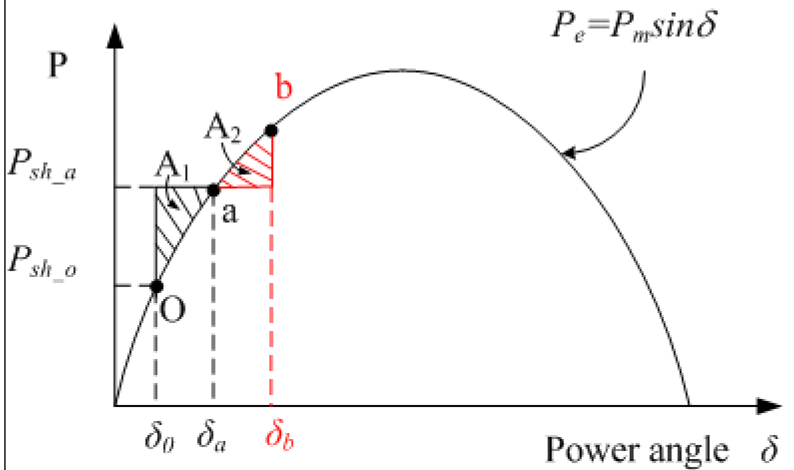
Note: $$\int_{\delta_0}^{\delta_a} \left(P_{sh\_a} - P_m \sin\delta\right) d\delta = \text{Area } A_1$$

Transient stability of synchronous generator

- **Swing curve and condition for stability:**

$$\int (P_{sh} - P_m \sin\delta)\, d\delta = 0$$

- For this system to be stable, we must have $\int_{\delta_0}^{\delta_b} (P_{sh} - P_m \sin\delta)\, d\delta = 0$



$$\text{or,}\quad \int_{\delta_0}^{\delta_a} (P_{sh\_a} - P_m \sin\delta)\, d\delta + \int_{\delta_a}^{\delta_b} (P_{sh\_a} - P_m \sin\delta)\, d\delta = 0$$

$$\text{or,}\quad \int_{\delta_0}^{\delta_a} (P_{sh\_a} - P_m \sin\delta)\, d\delta = \int_{\delta_a}^{\delta_b} (P_m \sin\delta - P_{sh\_a})\, d\delta$$

$$\text{Note:}\quad \int_{\delta_0}^{\delta_a} (P_{sh\_a} - P_m \sin\delta)\, d\delta = \text{Area } A_1$$

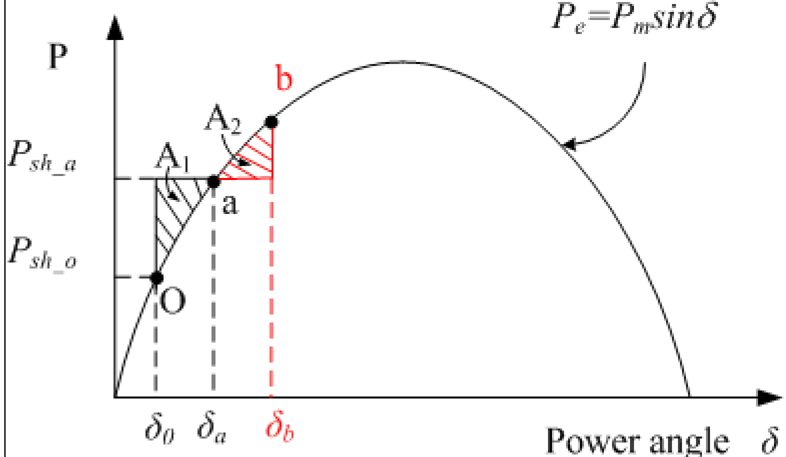
$$\text{and}\quad \int_{\delta_a}^{\delta_b} (P_m \sin\delta - P_{sh\_a})\, d\delta = \text{Area } A_2$$

Thus we have the relation: **Area A_1 = Area A_2**

Transient stability of synchronous generator

- **Swing curve and condition for stability:**

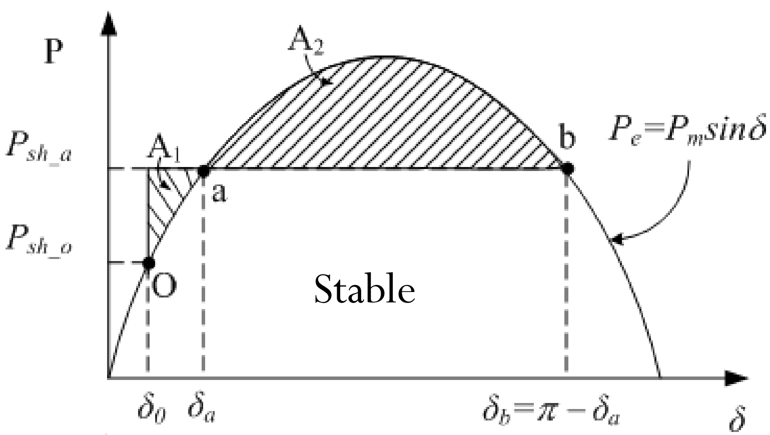
$$\textbf{Area } A_1 = \textbf{Area } A_2$$

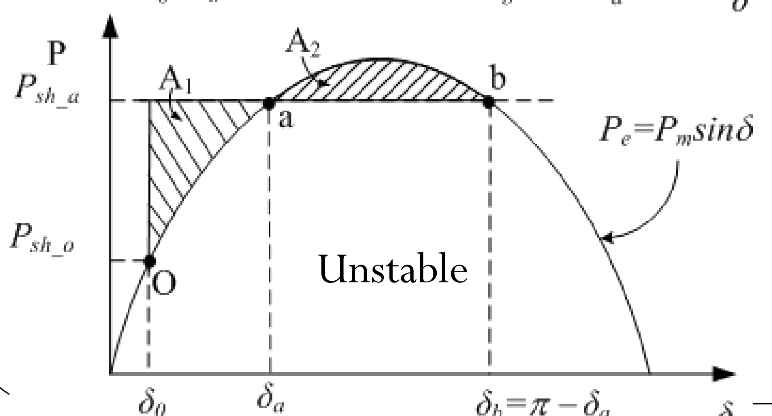


- Thus for maintaining stability, area A_1 should be equal to area A_2
- Once this criteria is satisfied, a machine will oscillate around its new steady state operating point for a while (hunting); but finally will settle down to its new operating point and continue to run stably
- This method for examining transient stability of a synchronous machine is called ***equal area criterion of stability***

Transient stability of synchronous generator

- **Conditions for stability:**
  - Suppose that the alternator is operating under steady state at a load angle (power angle) of δ_0
  - The shaft power input at this condition is P_{sh_0} and operating point is "O"
  - The shaft input power is now suddenly increased to P_{sh_a}
  - The overshoot point may reach point "b", which is **beyond peak of the curve**





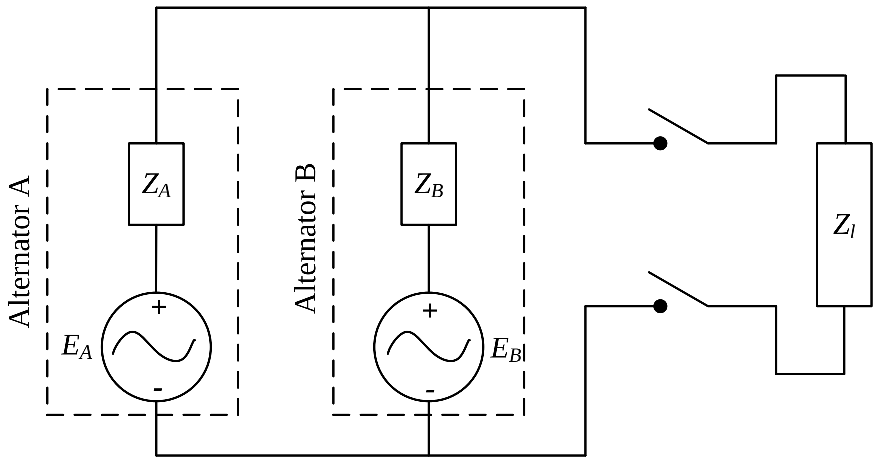
- Following are the conditions that will indicate whether the machine will be able to come back to stability after the transient oscillation period:
  - If area A_2 > area A_1, the system will reach steady state
  - If area A_2 < area A_1, the system will fail to come back to steady state and the machine will fall out of step (lose synchronism)
  - If area A_2 = area A_1, it is the case of marginal stability, i.e. synchronism will just be maintained (equal area criterion)

Transient stability of synchronous generator

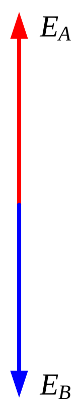
- **Re-synchronization & self-synchronization:**
  - When an alternator is connected to an infinite bus or to a 2^{nd} alternator is parallel, they are said to be in synchronism when their terminal voltages, phase and frequency are same
  - Due to sudden disturbance in any one of the two alternators in parallel, they may momentarily lose synchronism (e.g. their frequencies may no longer remain same)
  - However, if such deviation is not too large the machines may re-synchronize themselves (as we have seen previously when $A_2 > A_1$) after a brief period of ***hunting***
  - During this re-synchronization period, a circulating current flows between the two alternators connected in parallel and there is an exchange of so-called ***synchronizing power***
  - This synchronizing power helps bring the frequency of both machines to the same value, i.e. bring them back to synchronism

Transient stability of synchronous generator

- **Re-synchronization & self-synchronization:**
  - Let us consider two such two alternators which are connected in parallel but their connection to the load (Z_l) is kept open.



  - Let the induced EMF of both the generators are same in magnitude such that $E_A=E_B=E$
  - If the frequencies of the two machines are also same, then their EMFs E_A and E_B should be exactly 180° opposite in phase such that they oppose each other and there is no circulating current among them.



Transient stability of synchronous generator

- **Re-synchronization & self-synchronization:**
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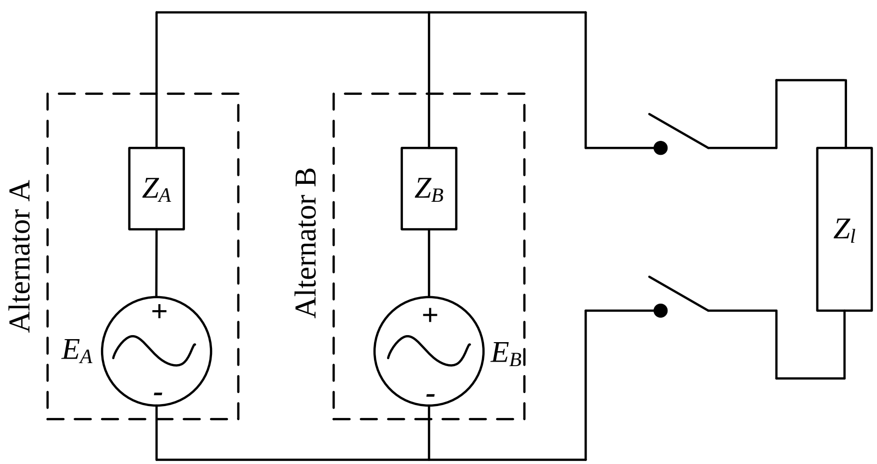

  - Let the induced EMF of both the generators are same in magnitude such that $E_A = E_B = E$
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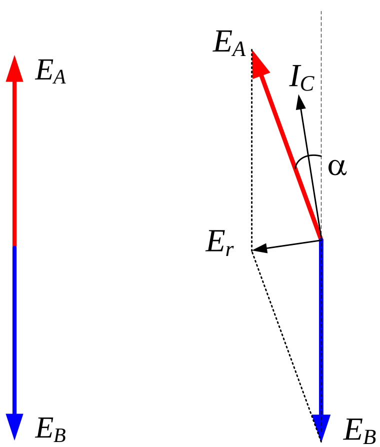
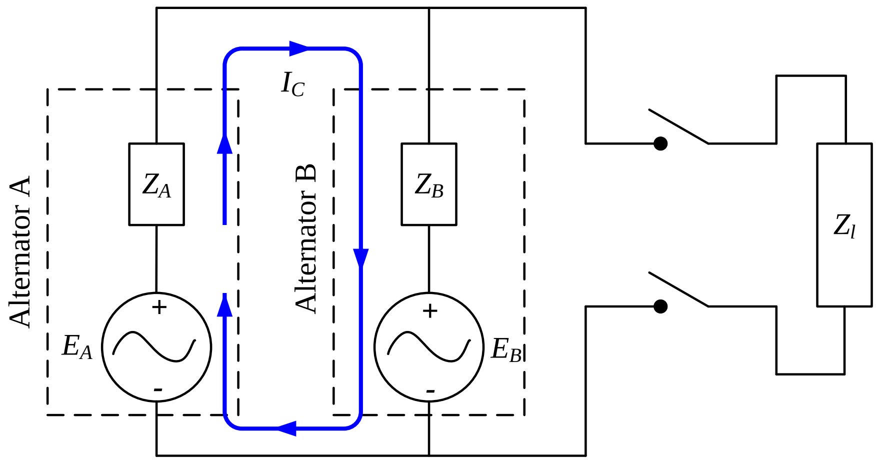


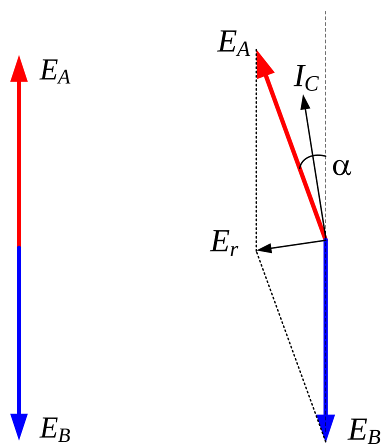
  - However, during any disturbance, these two frequencies may not be exactly the same
  - Let the speed of alternator *A* is slightly more than that of *B* such that frequency of alternator *A* is slightly more than *B*
  - Then, phasor $\overline{E_A}$, which was supposed to be in exact phase opposition to $\overline{E_B}$, is now ahead of its position of exact phase opposition by the small angle α as marked

Transient stability of synchronous generator

- **Re-synchronization & self-synchronization:**
  - Then, a resultant EMF $\boldsymbol{E_r}$ is created, which circulates the current I_C through the closed circuit formed by alternators *A* and *B*



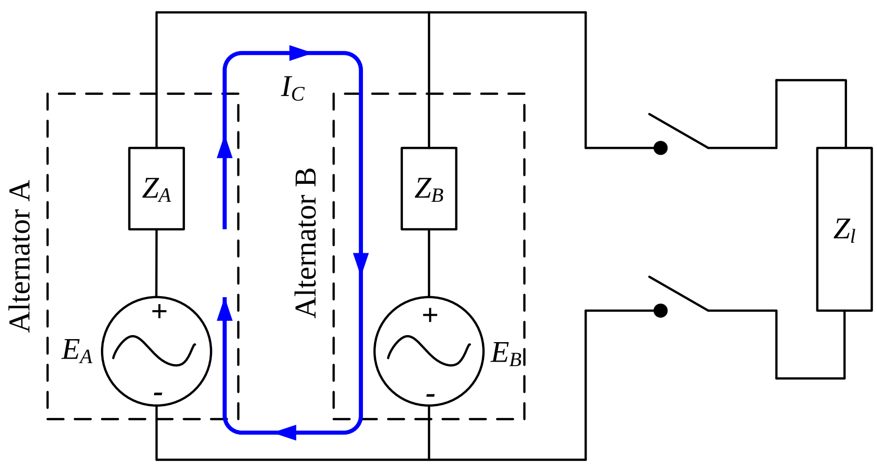
  - Thus, at the instant shown in the schematic diagram, the alternator *A* is delivering the circulating current I_C
  - The alternator *B* is accepting the circulating current I_C



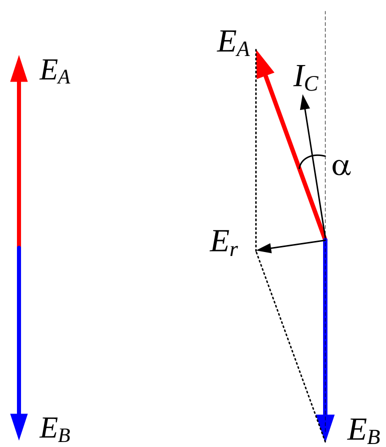
  - Therefore, it can be said that alternator *A* ***delivers*** electrical power (through positive I_C) and alternator *B* ***consumes*** electrical power (through negative I_C)

Transient stability of synchronous generator

- **Re-synchronization & self-synchronization:**
  - The machine *A* ***delivers*** electrical power and behaves as an alternator, while machine *B* starts to act as motor and it ***consumes*** electrical power



  - With machine *A* acting as a generator, the electromagnetic torque developed in it opposes the driving prime mover torque and decelerates it down
  - On the other hand, the electromagnetic torque developed in machine *B* accelerates the rotor of the machine due to its motoring action



  - Remember that *due to the sudden disturbance,* alternator *A* had started to run at a higher speed than *B*
  - But now, action of the circulating current is to bring down the speed of machine *A* and raise the speed of machine *B*
  - This will happen until the two machines attain the same speed again such that their frequencies become equal, i.e. till they are **re-synchronized**