Chapter 3
Transient Analysis of Synchronous Machines

Day 17

ILOs – Day17

- Use transformation matrix of generalized theory of synchronous machine to develop the expressions for transient 3-phase currents in alternators due to short circuit faults
- Also plot these transient currents

Transient currents in synchronous generator

- **Sudden short circuit at alternator 3-phase output terminals**
  - Expressions for armature d-axis, armature q-axis, and field currents

$$i_d(t) = \frac{v_{q0}}{X_d'}[1 - \cos \omega t]$$

$$i_q(t) = \frac{v_{q0}}{X_q} \sin \omega t$$

$$i_f(t) = I_{f0}\left[\frac{X_d}{X_d'} - \left(\frac{X_d}{X_d'} - 1\right)\cos \omega t\right]$$

Transient currents in synchronous generator

- **Sudden short circuit at alternator 3-phase output terminals**
  - Take help of the transformation matrix

$$i_d(t)=\frac{v_{q0}}{X_d'}\left[1-\cos\omega t\right]$$

$$i_q(t)=\frac{v_{q0}}{X_q}\sin\omega t$$

$$i_f(t)=I_{f0}\left[\frac{X_d}{X_d'}-\left(\frac{X_d}{X_d'}-1\right)\cos\omega t\right]$$

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}=\sqrt{\frac{2}{3}}\begin{array}{c} \\ a \\ b \\ c \end{array}\begin{array}{ccc} d & q & 0 \\ \cos\theta & -\sin\theta & \frac{1}{\sqrt{2}} \\ \cos\left(\theta-120^0\right) & -\sin\left(\theta-120^0\right) & \frac{1}{\sqrt{2}} \\ \cos\left(\theta-240^0\right) & -\sin\left(\theta-240^0\right) & \frac{1}{\sqrt{2}} \end{array}\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix}$$

Refer to Chapter 1, Day 9: Rotating to Stationary axis transformation

  - For phase *a* (neglecting zero sequence current)

$$i_a=\sqrt{\frac{2}{3}}\left[i_d\cos\theta-i_q\sin\theta\right]$$

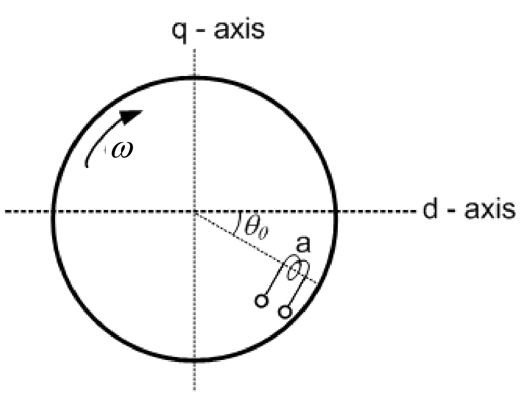
Transient currents in synchronous generator

- **Sudden short circuit at alternator 3-phase output terminals**
  - Take help of the transformation matrix

$$i_d(t) = \frac{v_{q0}}{X_d'}\left[1-\cos\omega t\right] \qquad i_q(t) = \frac{v_{q0}}{X_q}\sin\omega t \qquad i_f(t) = I_{f0}\left[\frac{X_d}{X_d'} - \left(\frac{X_d}{X_d'}-1\right)\cos\omega t\right]$$

$$i_a = \sqrt{\frac{2}{3}}\left[i_d\cos\theta - i_q\sin\theta\right]$$

- At the instant of short circuit, let the axis of phase *a* makes an angle θ_0 with the d-axis



Transient currents in synchronous generator

- **Sudden short circuit at alternator 3-phase output terminals**
  - Take help of the transformation matrix

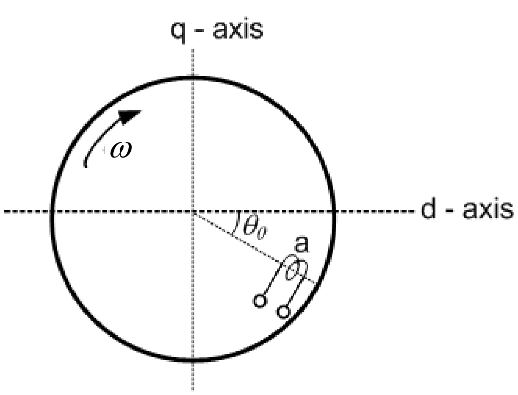
$$i_d(t)=\frac{v_{q0}}{X_d'}\left[1-\cos\omega t\right]$$

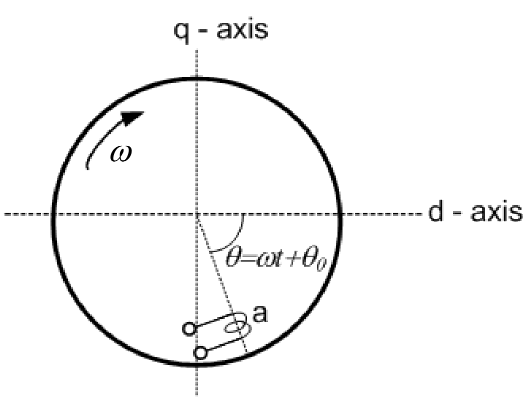
$$i_q(t)=\frac{v_{q0}}{X_q}\sin\omega t$$

$$i_f(t)=I_{f0}\left[\frac{X_d}{X_d'}-\left(\frac{X_d}{X_d'}-1\right)\cos\omega t\right]$$

$$i_a=\sqrt{\frac{2}{3}}\left[i_d\cos\theta-i_q\sin\theta\right]$$

- At the instant of short circuit, let the axis of phase *a* makes an angle θ_0 with the d-axis
- After time *t* sec, the angle θ between *a* phase and *d-axis* is given by: $\theta=\omega t+\theta_0$





Transient currents in synchronous generator

- **Sudden short circuit at alternator 3-phase output terminals**
  - Take help of the transformation matrix

$$i_d(t)=\frac{v_{q0}}{X_d'}\left[1-\cos\omega t\right]$$

$$i_q(t)=\frac{v_{q0}}{X_q}\sin\omega t$$

$$i_f(t)=I_{f0}\left[\frac{X_d}{X_d'}-\left(\frac{X_d}{X_d'}-1\right)\cos\omega t\right]$$

$$i_a=\sqrt{\frac{2}{3}}\left[i_d\cos\theta-i_q\sin\theta\right] \qquad \theta=\omega t+\theta_0$$

Putting the values of i_d, i_q and θ in the expression for i_a:

$$\Rightarrow i_a=\sqrt{\frac{2}{3}}\left[\frac{v_{q0}}{X_d'}(1-\cos\omega t)\cos(\omega t+\theta_0)-\frac{v_{q0}}{X_q}\sin\omega t\sin(\omega t+\theta_0)\right]$$

$$\Rightarrow i_a=\sqrt{\frac{2}{3}}\left[\frac{v_{q0}}{X_d'}\cos(\omega t+\theta_0)-\frac{v_{q0}}{X_d'}\cos\omega t\cos(\omega t+\theta_0)-\frac{v_{q0}}{X_q}\sin\omega t\sin(\omega t+\theta_0)\right]$$

Transient currents in synchronous generator

- **Sudden short circuit at alternator 3-phase output terminals**

$$i_a = \sqrt{\frac{2}{3}}\left[\frac{v_{q0}}{X_d'}\cos(\omega t+\theta_0) - \frac{v_{q0}}{X_d'}\cos\omega t\cos(\omega t+\theta_0) - \frac{v_{q0}}{X_q}\sin\omega t\sin(\omega t+\theta_0)\right]$$

- Take help of the trigonometric identities

$$\cos A\cos B = \frac{1}{2}\left[\cos(A-B)+\cos(A+B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A-B)-\cos(A+B)\right]$$

$$i_a = \sqrt{\frac{2}{3}}\left[\begin{array}{l}\frac{v_{q0}}{X_d'}\cos(\omega t+\theta_0) - \frac{v_{q0}}{X_d'}\times\frac{1}{2}\left[\cos(\omega t-\omega t-\theta_0)+\cos(\omega t+\omega t+\theta_0)\right]\\ -\frac{v_{q0}}{X_q}\times\frac{1}{2}\left[\cos(\omega t-\omega t-\theta_0)-\cos(\omega t+\omega t+\theta_0)\right]\end{array}\right]$$

$$\Rightarrow i_a = \sqrt{\frac{2}{3}}\left[\begin{array}{l}\frac{v_{q0}}{X_d'}\cos(\omega t+\theta_0) - \frac{v_{q0}}{X_d'}\times\frac{1}{2}\left[\cos\theta_0+\cos(2\omega t+\theta_0)\right]\\ -\frac{v_{q0}}{X_q}\times\frac{1}{2}\left[\cos\theta_0-\cos(2\omega t+\theta_0)\right]\end{array}\right]$$

Transient currents in synchronous generator

- **Sudden short circuit at alternator 3-phase output terminals**

$$i_a = \sqrt{\frac{2}{3}}\begin{bmatrix} \frac{v_{q0}}{X_d'}\cos(\omega t+\theta_0) - \frac{v_{q0}}{X_d'}\times\frac{1}{2}\left[\cos\theta_0 + \cos(2\omega t+\theta_0)\right] \\ -\frac{v_{q0}}{X_q}\times\frac{1}{2}\left[\cos\theta_0 - \cos(2\omega t+\theta_0)\right] \end{bmatrix}$$

$$\Rightarrow i_a = \sqrt{\frac{2}{3}}\left[\frac{v_{q0}}{X_d'}\cos(\omega t+\theta_0) - \frac{v_{q0}}{2}\left(\frac{1}{X_d'}+\frac{1}{X_q}\right)\cos\theta_0 - \frac{v_{q0}}{2}\left(\frac{1}{X_d'}-\frac{1}{X_q}\right)\cos(2\omega t+\theta_0)\right]$$

  - From Day 16 lesson, we know that before the short circuit i.e. at $t = 0-$ $v_{d0} = 0$ and $v_{q0} = M_d \omega I_{f0}$
  - Now, d-axis voltage being zero, thus, at no-load, the entire 3-phase induced EMF ($\sqrt{3}E_{f0}$) must be fully represented by the q-axis voltage in armature

$\therefore v_{q0} = M_d \omega I_{f0} = \sqrt{3}E_{f0}$ Where E_{f0} = per phase no-load induced EMF

Transient currents in synchronous generator

- **Sudden short circuit at alternator 3-phase output terminals**

$$i_a = \sqrt{\frac{2}{3}}\left[\frac{v_{q0}}{X_d'}\cos(\omega t+\theta_0)-\frac{v_{q0}}{2}\left(\frac{1}{X_d'}+\frac{1}{X_q}\right)\cos\theta_0-\frac{v_{q0}}{2}\left(\frac{1}{X_d'}-\frac{1}{X_q}\right)\cos(2\omega t+\theta_0)\right]$$

$$v_{q0} = \sqrt{3}E_{f0}$$

$$\Rightarrow i_a = \frac{\sqrt{2}E_{f0}}{X_d'}\cos(\omega t+\theta_0)-\frac{\sqrt{2}E_{f0}}{2}\left(\frac{1}{X_d'}+\frac{1}{X_q}\right)\cos\theta_0-\frac{\sqrt{2}E_{f0}}{2}\left(\frac{1}{X_d'}-\frac{1}{X_q}\right)\cos(2\omega t+\theta_0)$$

- The other two line currents i_b and i_c can be simply obtained by replacing θ_0 in the above equation by (θ_0-120^0) and (θ_0-240^0) respectively

Transient currents in synchronous generator

- **Sudden short circuit at alternator 3-phase output terminals**

$$i_a = \frac{\sqrt{2}E_{f0}}{X_d'}\cos(\omega t+\theta_0) - \frac{\sqrt{2}E_{f0}}{2}\left(\frac{1}{X_d'}+\frac{1}{X_q}\right)\cos\theta_0 - \frac{\sqrt{2}E_{f0}}{2}\left(\frac{1}{X_d'}-\frac{1}{X_q}\right)\cos(2\omega t+\theta_0)$$

Fundamental frequency component

DC component

2nd harmonic component

The transient line currents thus consist of:

i) A fundamental frequency AC component (1st term)
ii) A DC component (2nd term) – independent of time t
iii) A 2nd harmonic (double the fundamental frequency) AC component (3rd term)

Magnitude of all three components depend on the pre-fault excitation voltage E_{f0}, i.e. the pre-fault no-load induced EMF, i.e. the pre-fault field current I_{f0}

Transient currents in synchronous generator

- **Sudden short circuit at alternator 3-phase output terminals**

$$i_a = \frac{\sqrt{2}E_{f0}}{X_d'}\cos(\omega t + \theta_0) - \frac{\sqrt{2}E_{f0}}{2}\left(\frac{1}{X_d'} + \frac{1}{X_q}\right)\cos\theta_0 - \frac{\sqrt{2}E_{f0}}{2}\left(\frac{1}{X_d'} - \frac{1}{X_q}\right)\cos(2\omega t + \theta_0)$$

Additional observations

a) Magnitude of the fundamental component (1st term) depends on the *d-axis* transient reactance X_d'

b) Magnitude of the DC component (2nd term), in addition to the *d*- and *q*-axis reactances, depends on θ_0 i.e. the instant at which the short circuit occurs

c) Magnitude of the 2nd harmonic component (3rd term) depends on the term $\left(\frac{1}{X_d'} - \frac{1}{X_q}\right)$ which is called *transient saliency* (due to difference in *d*- and *q-axis* reactances)

Transient currents in synchronous generator

- **Sudden short circuit at alternator 3-phase output terminals**

$$i_a = \frac{\sqrt{2}E_{f0}}{X_d'}\cos(\omega t+\theta_0) - \frac{\sqrt{2}E_{f0}}{2}\left(\frac{1}{X_d'}+\frac{1}{X_q}\right)\cos\theta_0 - \frac{\sqrt{2}E_{f0}}{2}\left(\frac{1}{X_d'}-\frac{1}{X_q}\right)\cos(2\omega t+\theta_0)$$

Some more observations

If the short circuit occurs at $\theta_0 = 90^0$, i.e. at the instant when the coil axis is perpendicular to *d-axis*, then DC component of current (2^{nd} term) is zero

However, DC components will still be in the other two line currents i_b and i_c since they are with 120^0 and 240^0 phase displacement from i_a

Transient currents in synchronous generator

- **Sudden short circuit at alternator 3-phase output terminals**

$$i_d(t) = \frac{v_{q0}}{X_d'}\left[1 - \cos\omega t\right]$$

DC component

Fundamental frequency component

$$i_a = \frac{\sqrt{2}E_{f0}}{X_d'}\cos(\omega t + \theta_0) - \frac{\sqrt{2}E_{f0}}{2}\left(\frac{1}{X_d'} + \frac{1}{X_q}\right)\cos\theta_0 - \frac{\sqrt{2}E_{f0}}{2}\left(\frac{1}{X_d'} - \frac{1}{X_q}\right)\cos(2\omega t + \theta_0)$$

Fundamental frequency component

DC component

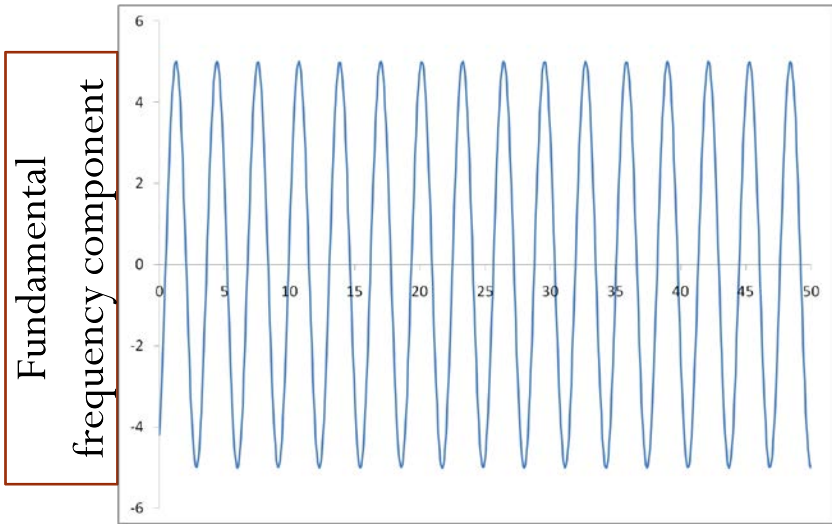
2^{nd} harmonic component

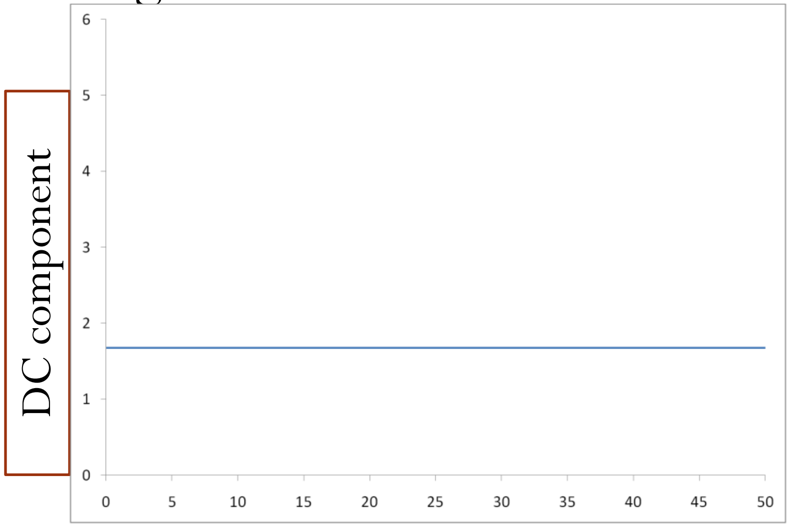
Some interesting observations of *d-q* to *a-b-c* transformation

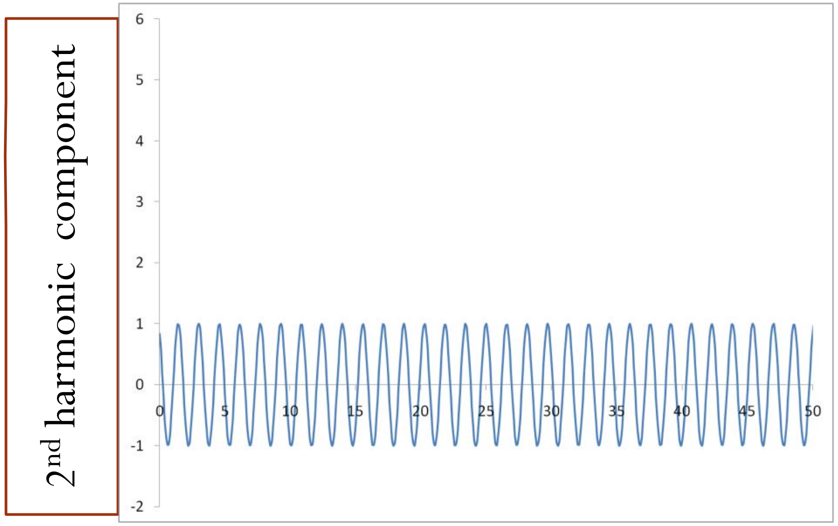
- DC component of $i_d(t)$ is transformed to fundamental frequency component of $i_a(t)$
- Fundamental frequency component of $i_d(t)$ is transformed to DC and 2^{nd} harmonic components of $i_a(t)$

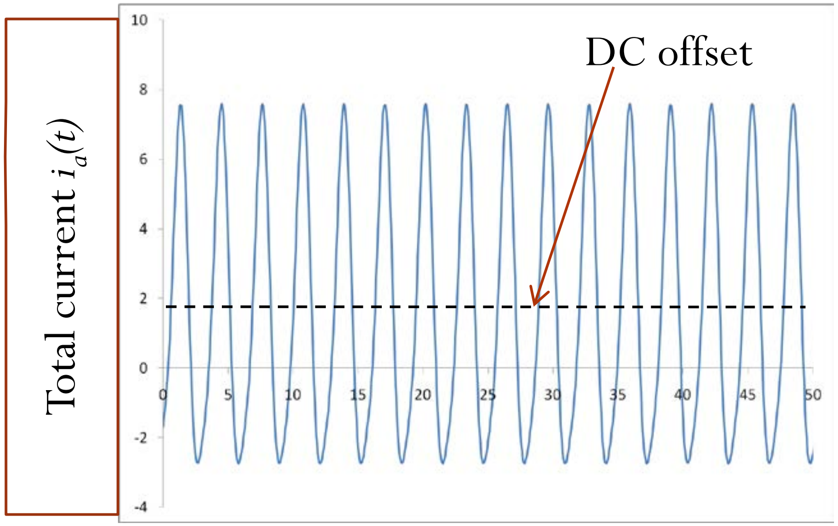
Transient currents in synchronous generator

- Plot of transient line current: Resistances neglected



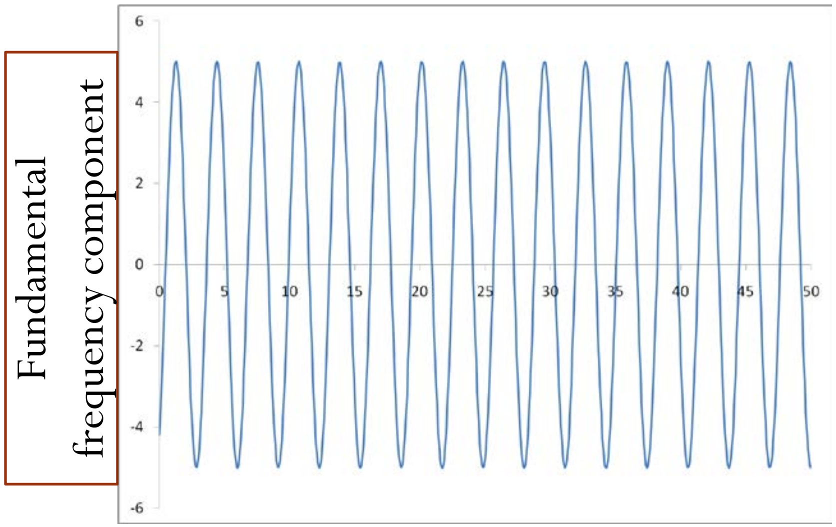


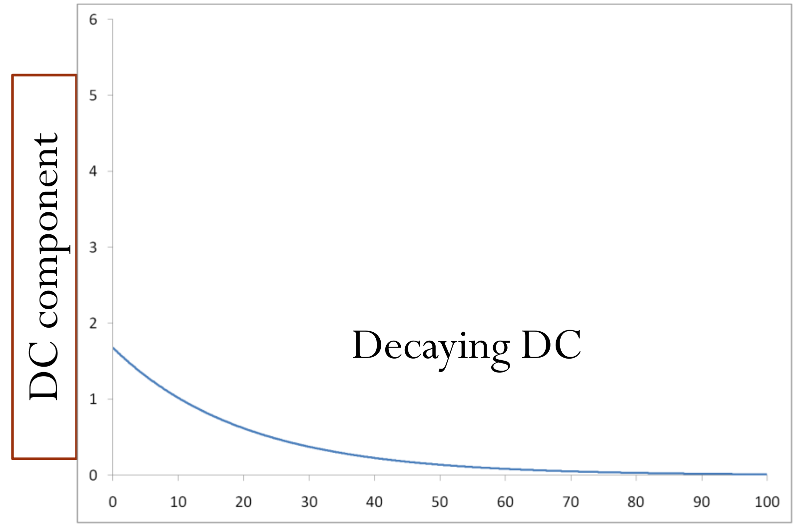


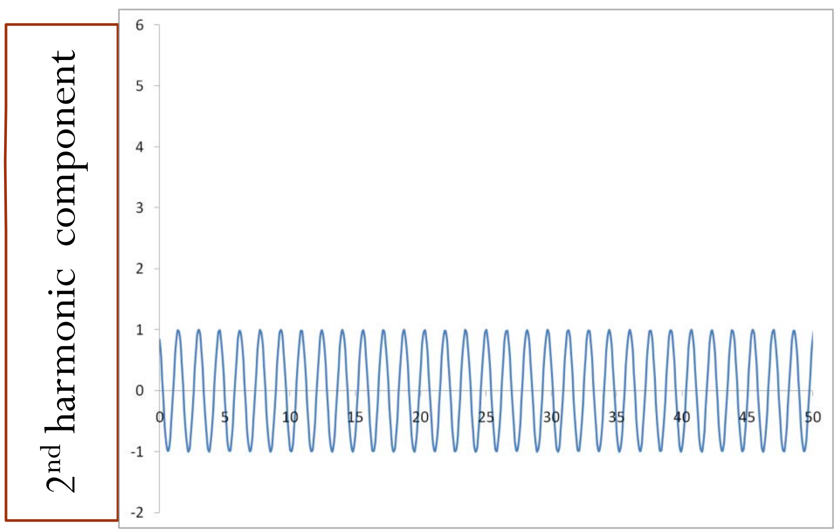


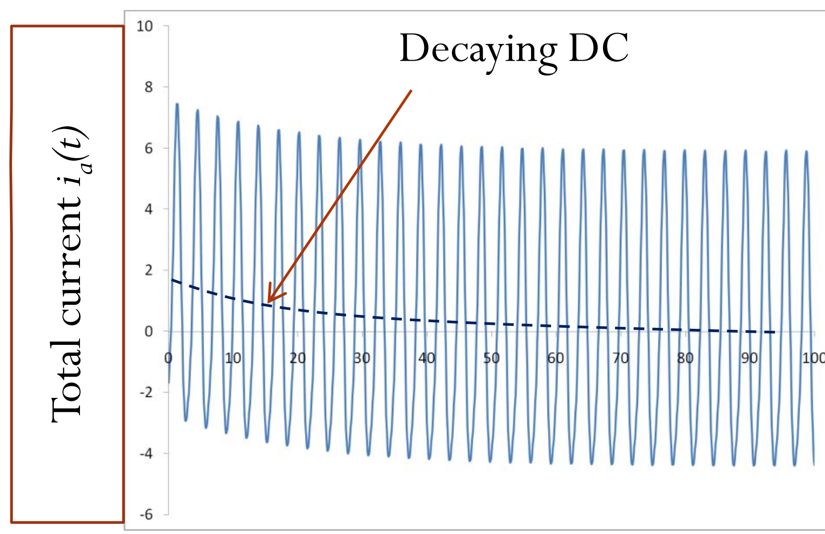
Transient currents in synchronous generator

- Plot of transient line current: Resistances **not** neglected (DC will decay)



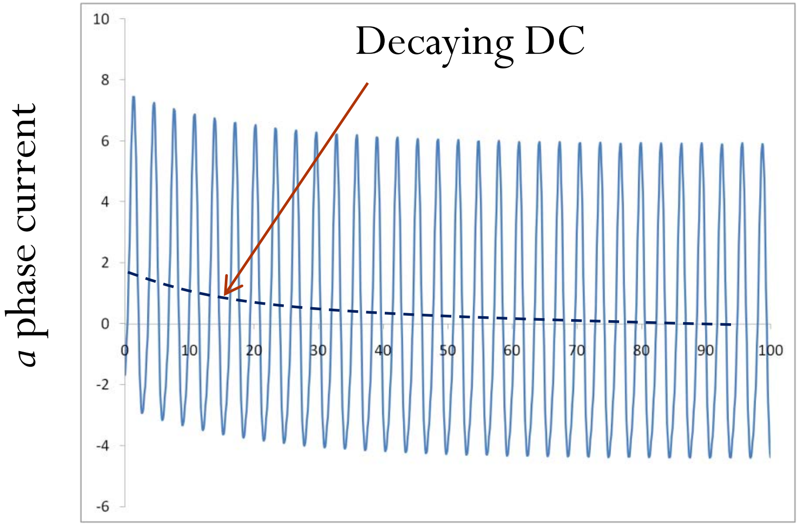






Transient currents in synchronous generator

- Plot of transient line current: Resistances **not** neglected (DC will decay)



b phase current and *c* phase currents will be displaced from *a* phase current by 120^0 and 240^0 respectively (replace θ_0 in the equation by $\left(\theta_0 - 120^0\right)$ and $\left(\theta_0 - 240^0\right)$ respectively)