Transformation Matrix Models of Machines

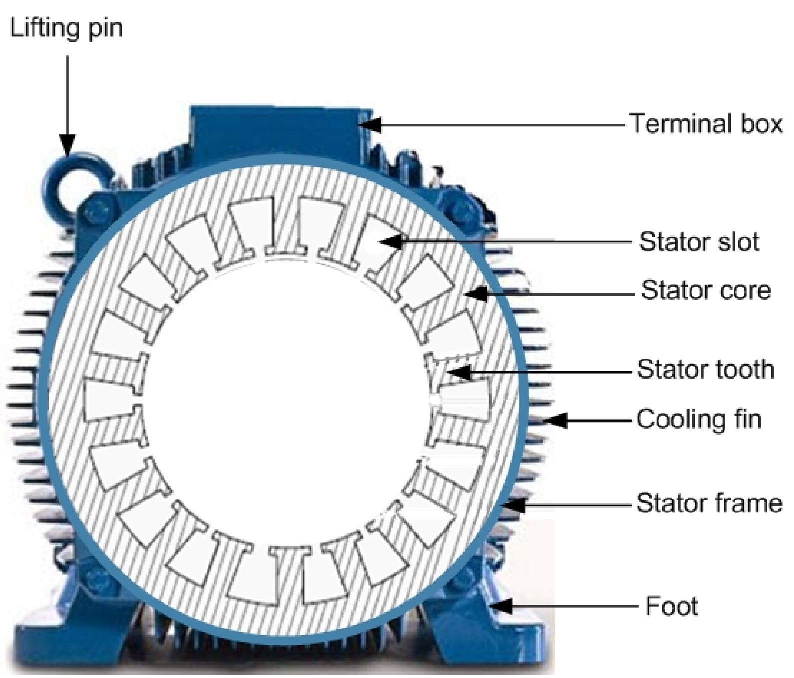
Day 13

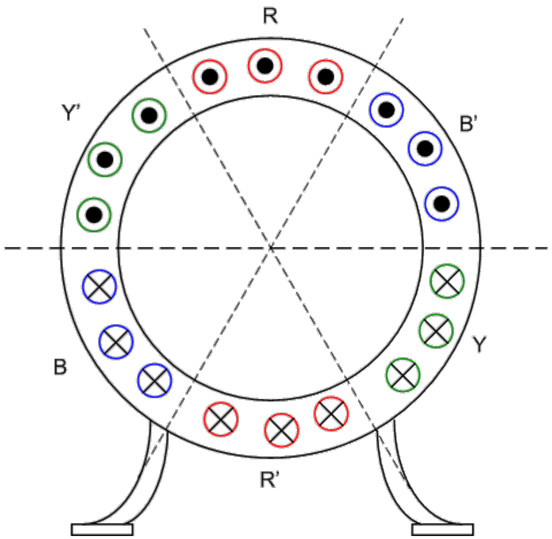
ILOs – Day13

- Draw and explain the linear transformation matrix model for
  - 3-phase induction machine
- Explain operating characteristics there from

Linear transformation matrix model for 3-ph IM

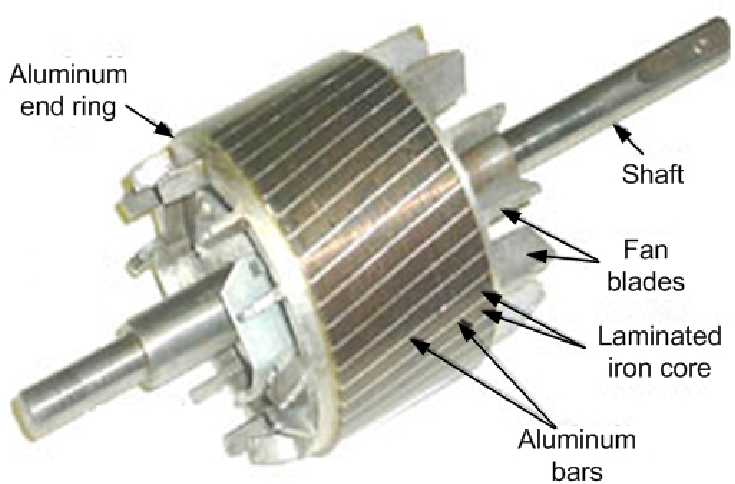
- **3-phase induction machine**
  - 3-phase winding in stator that produces rotating magnetic field (RMF) in the air gap

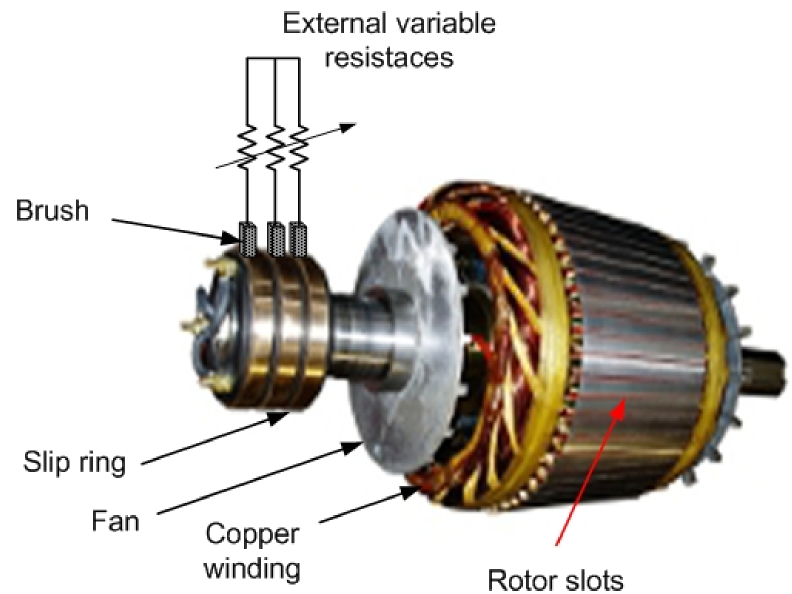




Linear transformation matrix model for 3-ph IM

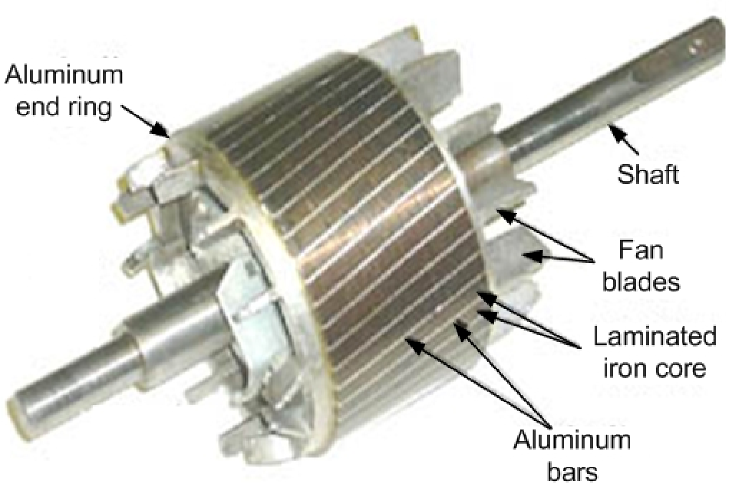
- **3-phase induction machine**
  - 3-phase winding in stator that produces rotating magnetic field (RMF) in the air gap
  - Whether it is squirrel cage type or wound rotor type, the rotor also has a 3-phase winding

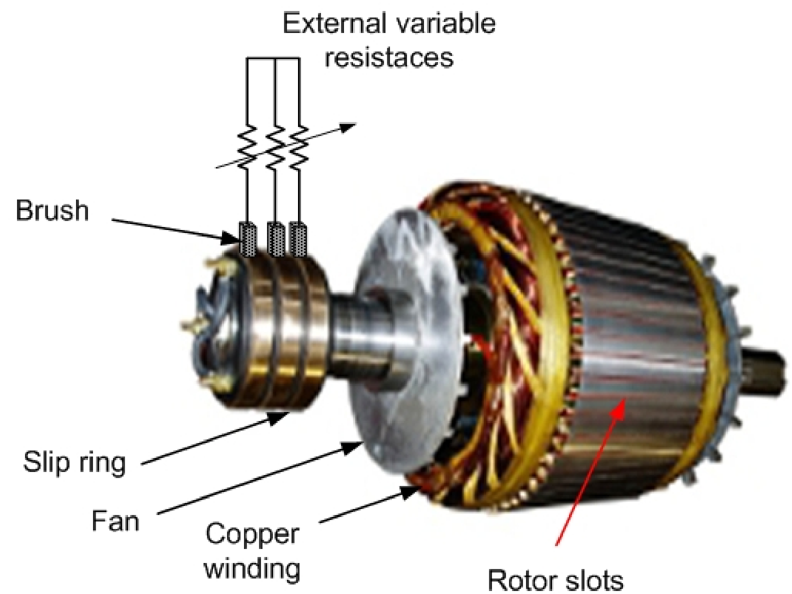




Linear transformation matrix model for 3-ph IM

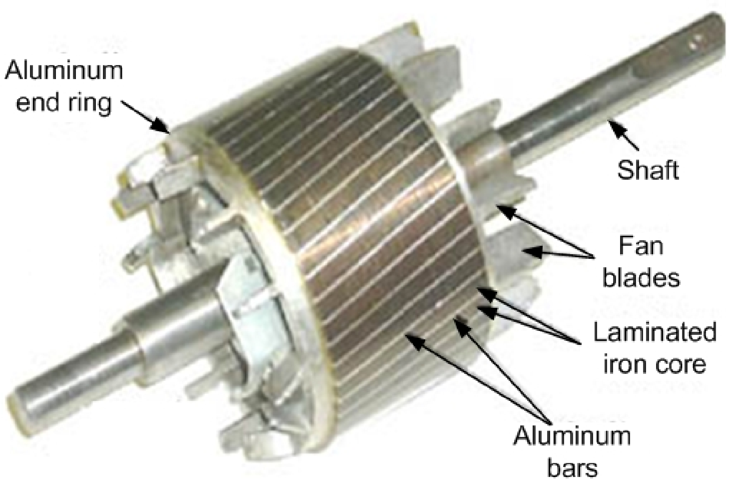
- **3-phase induction machine**
  - Both the stator and rotor windings carry alternating currents
  - Stator MMF rotates at synchronous speed w.r.t. stator
  - Currents induced in rotor also produces MMF that rotates at synchronous speed w.r.t. stator

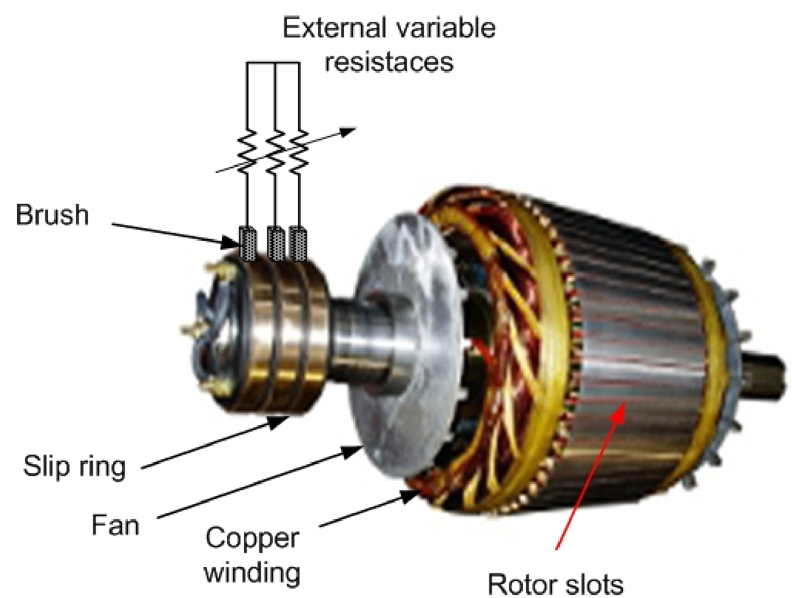




Linear transformation matrix model for 3-ph IM

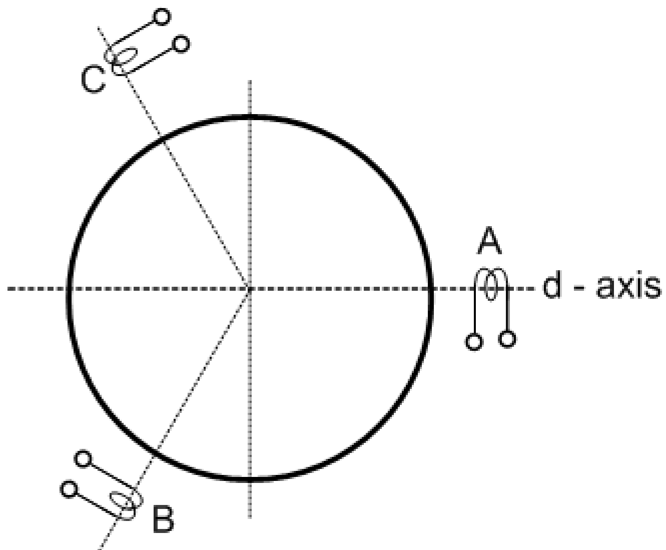
- **3-phase induction machine**
  - Stator and rotor MMF waves rotate at same speed and at the same direction
  - Under steady state condition, the relative speed between stator and rotor RMF is thus zero, i.e. they appear stationary to each other





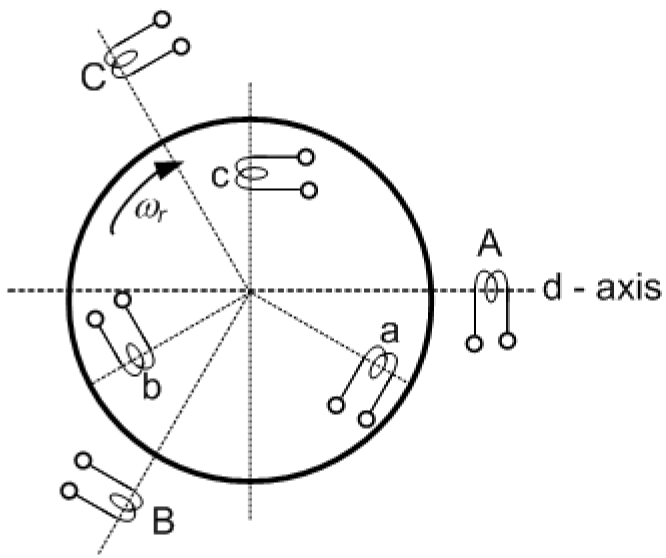
Linear transformation matrix model for 3-ph IM

- **3-phase induction machine**
  - 3-phase winding in stator that produces rotating magnetic field (RMF) in the air gap
    - Represented by three stator coils A, B, C placed 120^0 apart in space
    - Phase A coil can be taken along d-axis for convenience



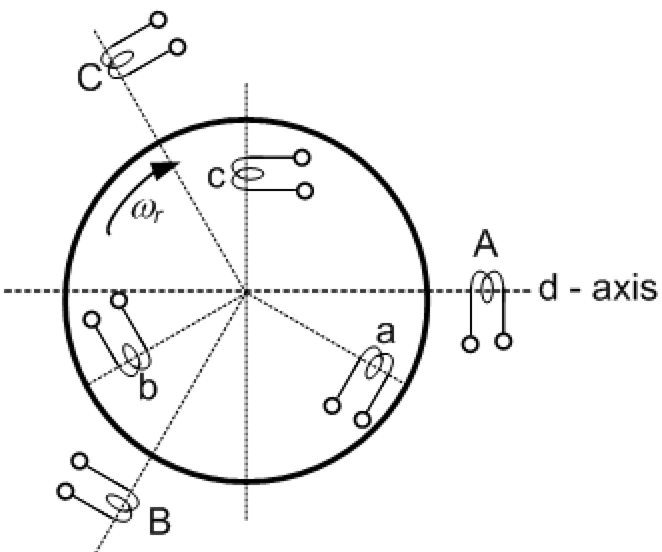
Linear transformation matrix model for 3-ph IM

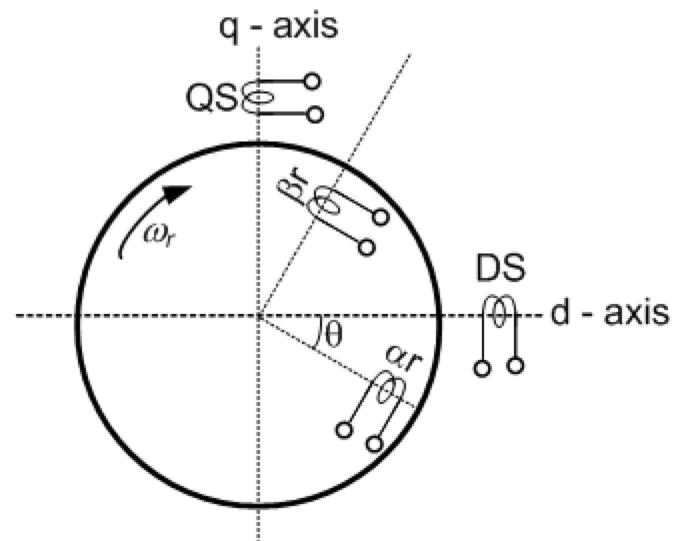
- **3-phase induction machine**
  - Whether it is squirrel cage type or wound rotor type, the rotor also has a 3-phase winding
    - Represented by three coils a, b, c placed 120^0 apart in space
    - Orientation of rotor coils (a, b, c) w.r.t the stator coils (A, B, C) is arbitrary



Linear transformation matrix model for 3-ph IM

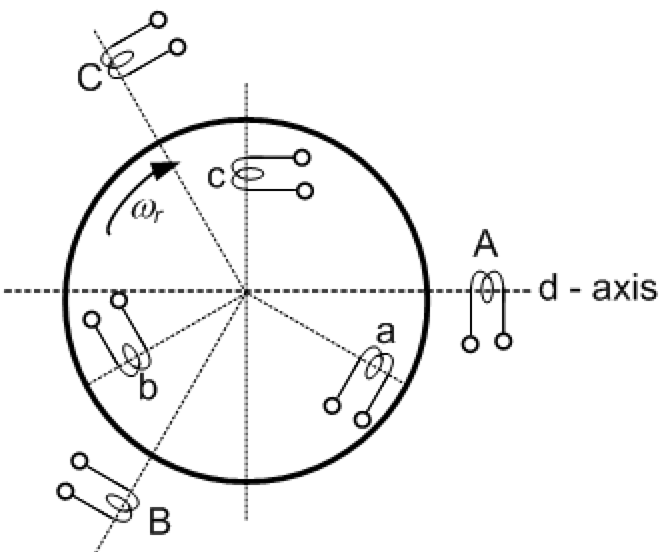
- **3-phase induction machine**
  - The 3-phase windings in stator and rotor both can be represented by their generalized equivalent 2-coil configuration
  - The *d-q* stationary coil axes are fixed along the stator structure
  - The rotor α-β rotating coils are assumed to make angle θ with *d-q* axes
  - The 2-phase coils in stator and rotor produce RMF that are identical with those produced by the original 3-phase windings *A, B, C* and *a, b, c* respectively

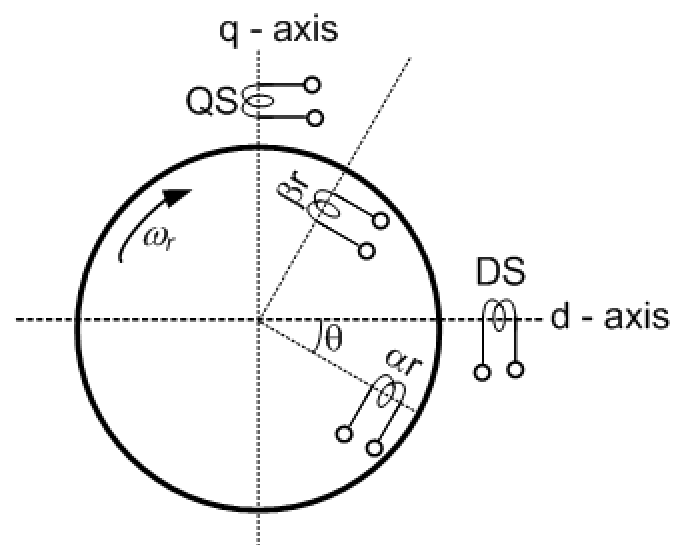




Linear transformation matrix model for 3-ph IM

- **3-phase induction machine**
  - Note that stator coils have been represented along the stationary *d*-*q* axes
  - Whereas the rotor coils have been represented along the rotating axes α-β
  - The 3-phase stator *A*-phase coincides with α -phase or *d*-phase in stator
  - Thus, for stator, 3-phase to 2-phase transformations from (*A*, *B*, *C*) to (α-β) and (*A*, *B*, *C*) to (*d*-*q*) will be same





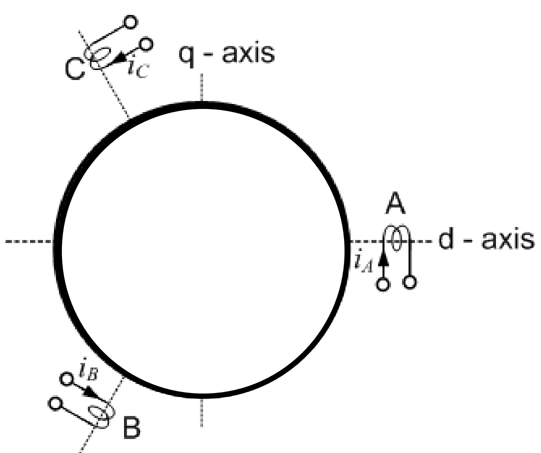
Transformation in stator

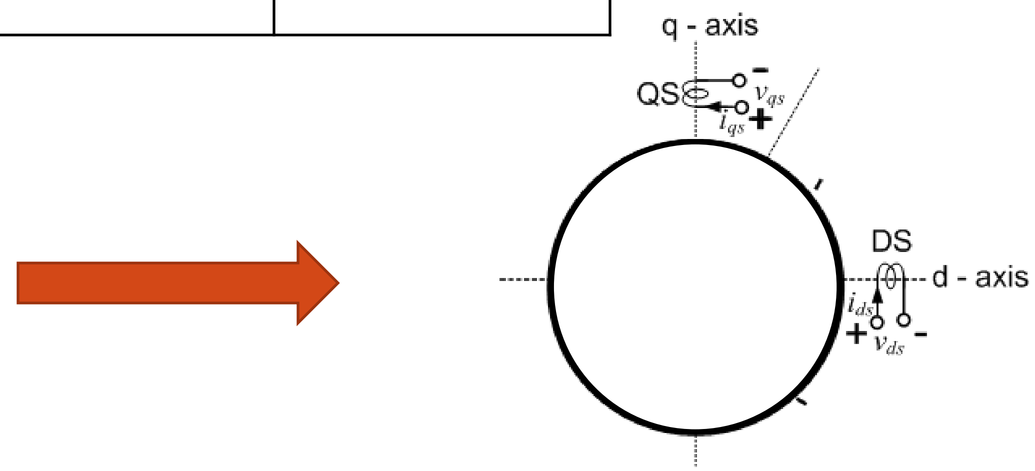
$(A, B, C) \rightarrow (\alpha s, \beta s, 0s) \rightarrow (ds, qs, 0s)$

Linear transformation matrix model for 3-ph IM

- **3-phase to 2-phase transformation in stator**
  - Transformations from (*A, B, C*) to (α-β-*0*) or (*d-q-0*) assuming angular displacement of θ between (*A, B, C*) and (α-β) or (d-q) axes
  - The generalized matrix form is:

$$\begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{0s} \end{bmatrix} = \begin{bmatrix} i_{\alpha s} \\ i_{\beta s} \\ i_{0s} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{array}{c} \\ \alpha \\ \beta \\ 0 \end{array} \begin{array}{ccc} A & B & C \\ \hline \cos\theta & \cos(\theta - 120^0) & \cos(\theta - 240^0) \\ -\sin\theta & -\sin(\theta - 120^0) & -\sin(\theta - 240^0) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{array} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix}$$

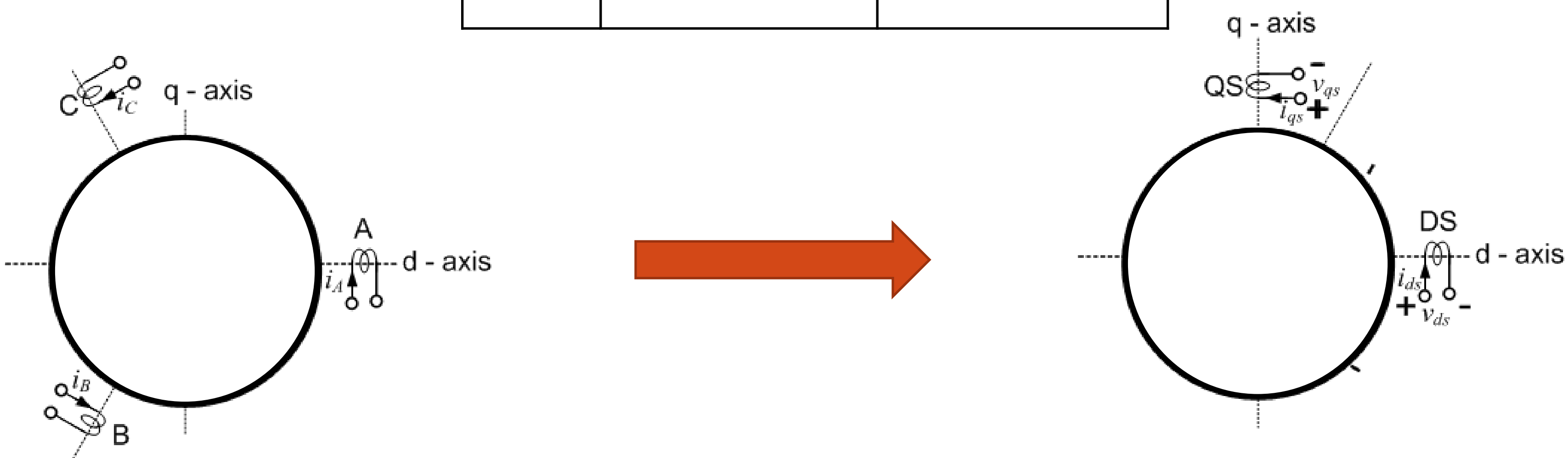




Linear transformation matrix model for 3-ph IM

- **3-phase to 2-phase transformation in stator**
  - However, since stator phase *A* is assumed to be along α–axis, or *d*-axis, the angle θ in this case is $\theta = 0^0$
  - The transformation matrix from (*A*, *B*, *C*) to (*d*-*q*-*0*) is derived as:

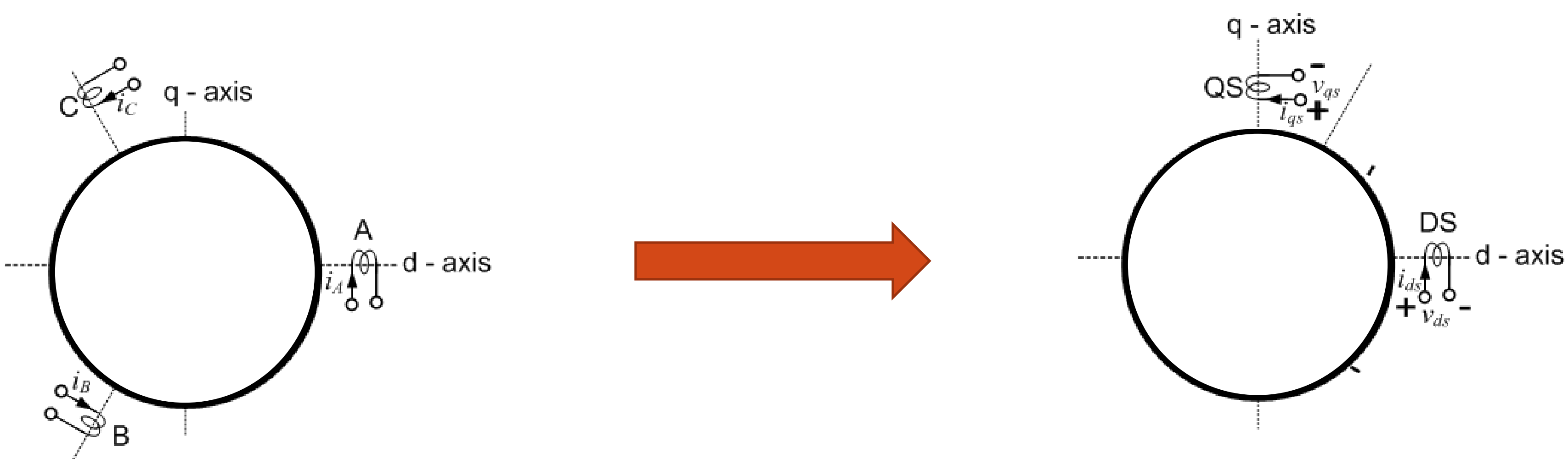
$$\begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{0s} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{array}{c} d \\ q \\ 0 \end{array} \overset{\begin{array}{ccc} A & B & C \end{array}}{\begin{bmatrix} \cos 0^0 & \cos\left(-120^0\right) & \cos\left(-240^0\right) \\ -\sin 0^0 & -\sin\left(-120^0\right) & -\sin\left(-240^0\right) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix}$$



Linear transformation matrix model for 3-ph IM

- **3-phase to 2-phase transformation in stator**
  - (A, B, C) to (d-q-0) transformation matrix:

$$\begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{0s} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{array}{c} \\ d \\ q \\ 0 \end{array} \begin{array}{ccc} A & B & C \\ \hline \cos 0^0 & \cos(-120^0) & \cos(-240^0) \\ -\sin 0^0 & -\sin(-120^0) & -\sin(-240^0) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{array} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{array}{c} \\ d \\ q \\ 0 \end{array} \begin{array}{ccc} A & B & C \\ \hline 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{array} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix}$$

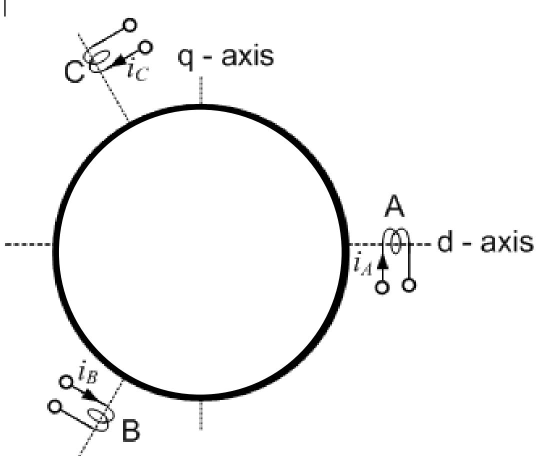


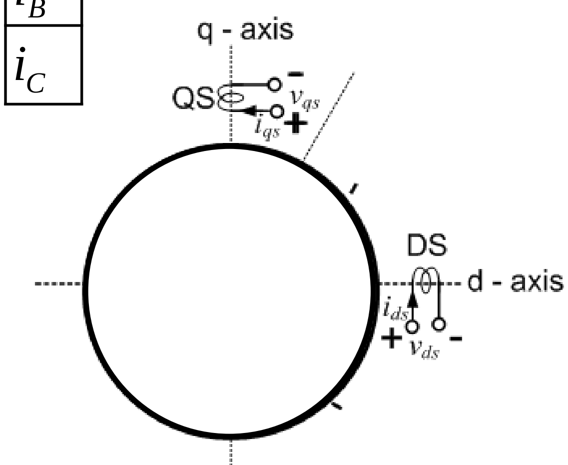
Linear transformation matrix model for 3-ph IM

- **3-phase to 2-phase transformation in stator**
  - Physical concepts of stator transformation:
    - Here I_m is the maximum value of stator current and α is the time phase angle of i_A w.r.t the time origin at $t=0$
    - Neglect zero sequence current for the time being
    - Park's transformation matrix:

$$i_A = I_m \cos(\omega t + \alpha)$$
$$i_B = I_m \cos(\omega t + \alpha - 240^0)$$
$$i_C = I_m \cos(\omega t + \alpha - 120^0)$$

$$\begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{matrix} d \\ q \end{matrix} \overset{\begin{matrix} A & \quad\quad B & \quad\quad\quad C \end{matrix}}{\begin{bmatrix} \cos 0^0 & \cos(-120^0) & \cos(-240^0) \\ -\sin 0^0 & -\sin(-120^0) & -\sin(-240^0) \end{bmatrix}} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix}$$





Linear transformation matrix model for 3-ph IM

- **3-phase to 2-phase transformation in stator**
  - Physical concepts of stator transformation:

$$i_{ds} = \sqrt{\frac{2}{3}}\left[i_A \cos 0^0 + i_B \cos\left(-120^0\right) + i_C \cos\left(-240^0\right)\right]$$

$$i_A = I_m \cos(\omega t + \alpha)$$
$$i_B = I_m \cos(\omega t + \alpha - 240^0)$$
$$i_C = I_m \cos(\omega t + \alpha - 120^0)$$

$$\cos A \cos B = \frac{1}{2}\cos(A - B) + \frac{1}{2}\cos(A + B)$$

Putting values of i_A, i_B, i_C:

$$i_{ds} = \sqrt{\frac{2}{3}} I_m \left[\cos 0^0 \cos(\omega t + \alpha) + \cos\left(-120^0\right)\cos\left(\omega t + \alpha - 240^0\right) + \cos\left(-240^0\right)\cos\left(\omega t + \alpha - 120^0\right)\right]$$

$$= \sqrt{\frac{2}{3}} I_m \frac{1}{2}\left[2\cos(\omega t + \alpha) + \cos\left(\omega t + \alpha - 360^0\right) + \cos\left(\omega t + \alpha - 120^0\right) + \cos\left(\omega t + \alpha - 360^0\right) + \cos\left(\omega t + \alpha + 120^0\right)\right]$$

$$= \sqrt{\frac{2}{3}} I_m \frac{1}{2}\left[2\cos(\omega t + \alpha) + \cos(\omega t + \alpha) + \cos\left(\omega t + \alpha - 120^0\right) + \cos(\omega t + \alpha) + \cos\left(\omega t + \alpha + 120^0\right)\right]$$

$$\begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{matrix} d \\ q \end{matrix} \overset{\begin{matrix} A & & B & & C \end{matrix}}{\begin{bmatrix} \cos 0^0 & \cos\left(-120^0\right) & \cos\left(-240^0\right) \\ -\sin 0^0 & -\sin\left(-120^0\right) & -\sin\left(-240^0\right) \end{bmatrix}} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix}$$

Linear transformation matrix model for 3-ph IM

- **3-phase to 2-phase transformation in stator**
  - Physical concepts of stator transformation:

$$i_{ds} = \sqrt{\frac{2}{3}} I_m \frac{1}{2}\left[2\cos(\omega t+\alpha)+\cos(\omega t+\alpha)+\cos\left(\omega t+\alpha-120^0\right)+\cos(\omega t+\alpha)+\cos\left(\omega t+\alpha+120^0\right)\right]$$

For a balanced 3-phase system:

$$\cos(\omega t+\alpha)+\cos\left(\omega t+\alpha-120^0\right)+\cos\left(\omega t+\alpha+120^0\right)=0$$

Thus: $i_{ds} = \sqrt{\frac{2}{3}} I_m \frac{1}{2}[3\cos(\omega t+\alpha)] = \sqrt{\frac{2}{3}} I_m \frac{3}{2}\cos(\omega t+\alpha) = \sqrt{\frac{3}{2}} I_m \cos(\omega t+\alpha)$

Similarly, it can be shown that: $i_{qs} = \sqrt{\frac{3}{2}} I_m \sin(\omega t+\alpha)$

  - The two *d*-*q* axes currents i_{ds} and i_{qs} are functions of time and are displaced from each other by 90^0 in time as well as space
  - The two phase system will thus produce exactly similar RMF in stator as the original 3-phase system of currents (i_A, i_B, i_C) that rotates in the same direction and with the same speed

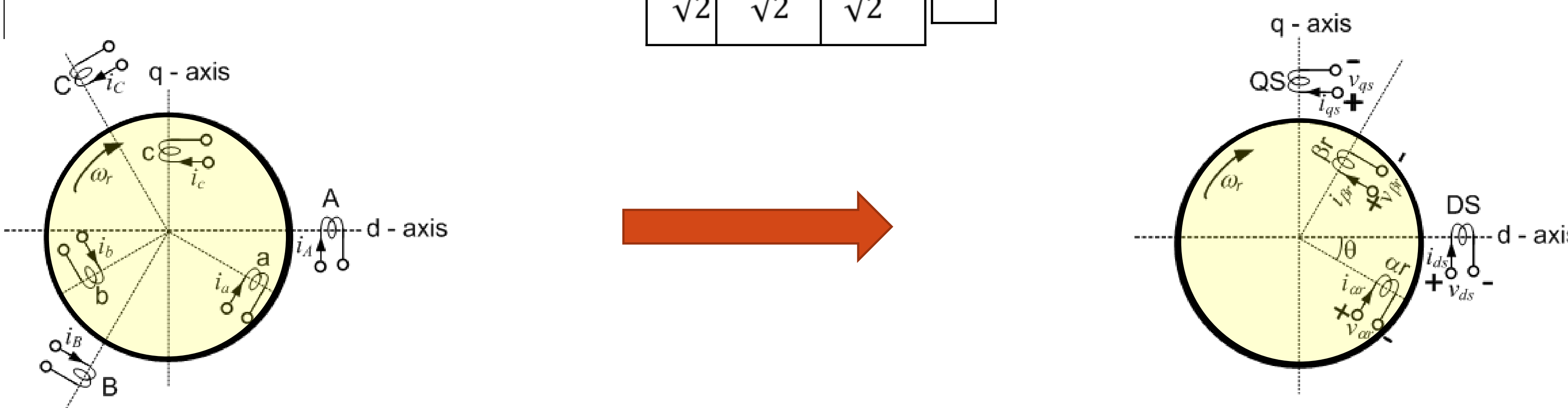
Transformation in rotor

$(a, b, c) \rightarrow (\alpha r, \beta r, 0r) \rightarrow (dr, qr, 0r)$

Linear transformation matrix model for 3-ph IM

- **3-phase to 2-phase transformation in rotor**
  - Transformations from 3-phase rotor coils (a, b, c) to rotating axes (α-β-0) requires no change of space frame, since both are on the rotating member
  - The generalized matrix form is:

$$\begin{bmatrix} i_{\alpha r} \\ i_{\beta r} \\ i_{0r} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{array}{c} \\ \alpha \\ \beta \\ 0 \end{array} \begin{array}{ccc} a & b & c \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{array} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

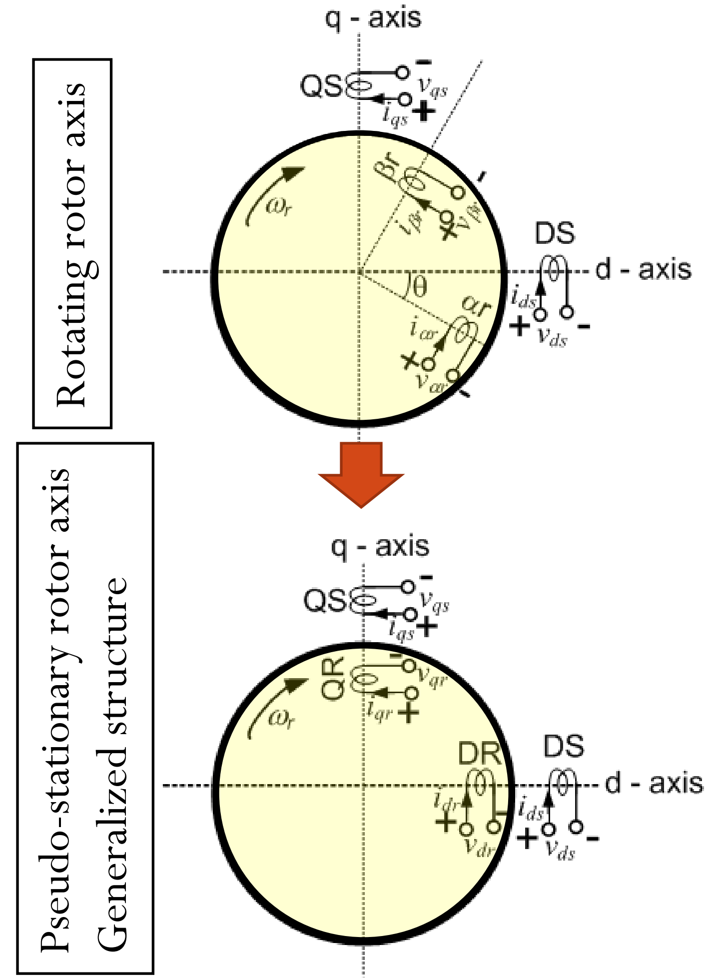


Linear transformation matrix model for 3-ph IM

- **3-phase to 2-phase transformation in rotor**
  - Transformations from rotating axes (α-β-0) to stationary axes (*d-q-0*) to represent in primitive generalized model:

$$\begin{bmatrix} i_{\alpha r} \\ i_{\beta r} \\ i_{0r} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{array}{c} \\ \alpha \\ \beta \\ 0 \end{array} \begin{array}{ccc} a & b & c \\ \hline 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{array} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$$\begin{bmatrix} i_{dr} \\ i_{qr} \\ i_{0r} \end{bmatrix} = \begin{array}{c} \\ d \\ q \\ 0 \end{array} \begin{array}{ccc} \alpha & \beta & 0 \\ \hline \cos\theta & \sin\theta & \\ -\sin\theta & \cos\theta & \\ & & 1 \end{array} \begin{bmatrix} i_{\alpha r} \\ i_{\beta r} \\ i_{0r} \end{bmatrix}$$



Linear transformation matrix model for 3-ph IM

- **(a, b, c) to (α, β,0) to (d, q,0) axes transformation**

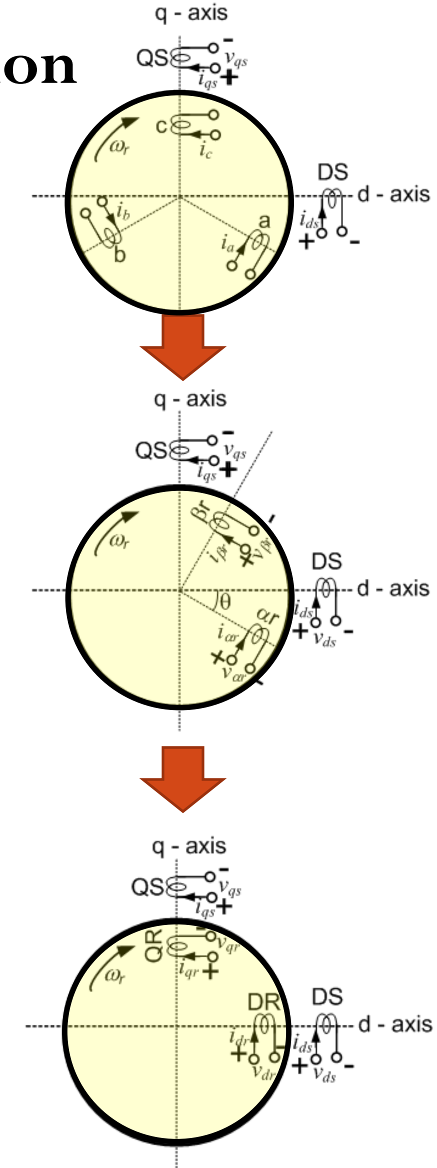
$(\alpha,\beta,0)$ to (d, q, 0) $\qquad\qquad$ (a,b,c) to $(\alpha,\beta,0)$

$$\begin{bmatrix} i_{dr} \\ i_{qr} \\ i_{0r} \end{bmatrix} = \begin{matrix} d \\ q \\ 0 \end{matrix} \overset{\begin{matrix}\alpha & \quad\beta & \quad 0\end{matrix}}{\begin{bmatrix} \cos\theta & \sin\theta & \\ -\sin\theta & \cos\theta & \\ & & 1 \end{bmatrix}} \sqrt{\frac{2}{3}} \overset{\begin{matrix} a & \qquad b & \qquad c \end{matrix}}{\begin{bmatrix} \cos 0^0 & \cos 120^0 & \cos 240^0 \\ \sin 0^0 & \sin 120^0 & \sin 240^0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

or,

$$\begin{bmatrix} i_{dr} \\ i_{qr} \\ i_{0r} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{matrix} d \\ q \\ 0 \end{matrix} \overset{\begin{matrix} a & \qquad\qquad b & \qquad\qquad c \end{matrix}}{\begin{bmatrix} \cos\theta & \cos\left(\theta-120^0\right) & \cos\left(\theta-240^0\right) \\ -\sin\theta & -\sin\left(\theta-120^0\right) & -\sin\left(\theta-240^0\right) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

(a,b,c) to (d, q, 0)



Analysis of IM: Performance equations

Using primitive generalized model

Linear transformation matrix model for 3-ph IM

- **Performance equations**
  - The voltage equations for generalized representation is:

$$\begin{bmatrix} v_{ds} \\ v_{qs} \\ v_{dr} \\ v_{qr} \end{bmatrix} = \begin{array}{c} ds \\ qs \\ dr \\ qr \end{array} \overset{\begin{array}{cccc} ds & qs & dr & qr \end{array}}{\begin{bmatrix} r_{ds}+L_{ds}p & & M_d p & \\ & r_{qs}+L_{qs}p & & M_q p \\ M_d p & -M_q\omega_r & r_{dr}+L_{dr}p & -\omega_r L_{qr} \\ M_d\omega_r & M_q p & \omega_r L_{dr} & r_{qr}+L_{qr}p \end{bmatrix}} \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}$$

Voltage matrix

Impedance matrix

Current matrix

  - Stator and rotor both have balanced winding:

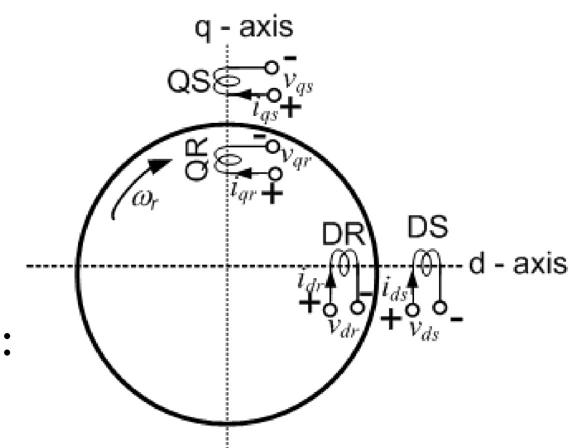
$$r_{ds} = r_{qs} = r_s \qquad r_{dr} = r_{qr} = r_r$$

  - Assuming uniform air gap:

$$L_{ds} = L_{qs} = L_s \qquad L_{dr} = L_{qr} = L_r$$

  - Coils DS-QS and DR-QR being assumed identical:

$$M_d = M_q = M$$



Linear transformation matrix model for 3-ph IM

- **Performance equations**
  - The voltage equations for generalized representation is:

$$
\begin{bmatrix} v_{ds} \\ v_{qs} \\ v_{dr} \\ v_{qr} \end{bmatrix}
=
\begin{array}{c|cccc}
 & ds & qs & dr & qr \\
\hline
ds & r_s + L_s p & & Mp & \\
qs & & r_s + L_s p & & Mp \\
dr & Mp & -M\omega_r & r_r + L_r p & -\omega_r L_r \\
qr & M\omega_r & Mp & \omega_r L_r & r_r + L_r p
\end{array}
\begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}
$$

Voltage matrix

Impedance matrix

Current matrix

  - Stator and rotor both have balanced winding:

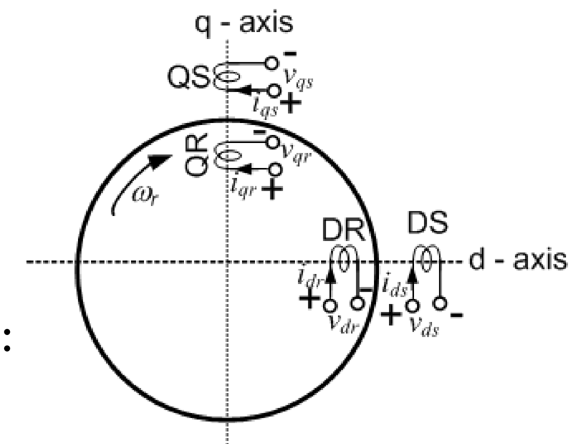
$$r_{ds} = r_{qs} = r_s \qquad r_{dr} = r_{qr} = r_r$$

  - Assuming uniform air gap:

$$L_{ds} = L_{qs} = L_s \qquad L_{dr} = L_{qr} = L_r$$

  - Coils DS-QS and DR-QR being assumed identical:

$$M_d = M_q = M$$



Linear transformation matrix model for 3-ph IM

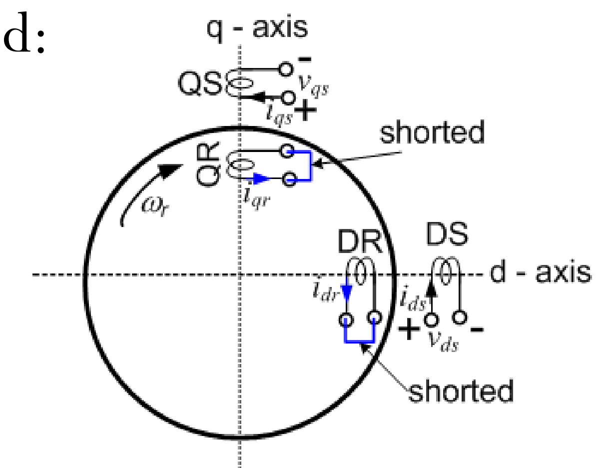
- **Performance equations**
  - The voltage equations for generalized representation is:

$$
\begin{bmatrix} v_{ds} \\ v_{qs} \\ v_{dr} \\ v_{qr} \end{bmatrix}
=
\begin{array}{c} ds \\ qs \\ dr \\ qr \end{array}
\begin{bmatrix}
r_s + L_s p & & Mp & \\
 & r_s + L_s p & & Mp \\
Mp & -M\omega_r & r_r + L_r p & -\omega_r L_r \\
M\omega_r & Mp & \omega_r L_r & r_r + L_r p
\end{bmatrix}
\begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}
$$

Voltage matrix　　Impedance matrix　　Current matrix

  - In an induction motor, the rotor bars are all shorted:

$\therefore v_{ar} = v_{br} = v_{cr} = 0 \qquad v_{\alpha r} = v_{\beta r} = 0 \qquad v_{dr} = v_{qr} = 0$



Linear transformation matrix model for 3-ph IM

- **Performance equations**
  - The voltage equations for generalized representation is:

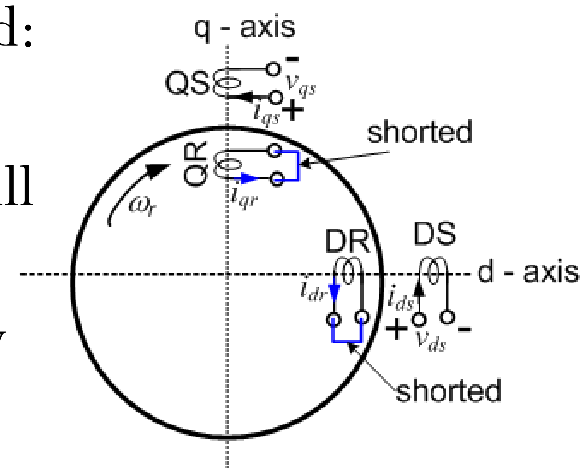
$$\begin{bmatrix} v_{ds}\\ v_{qs}\\ 0\\ 0\end{bmatrix} = \begin{matrix} ds\\ qs\\ dr\\ qr\end{matrix}\begin{bmatrix} r_s+L_s p & & Mp & \\ & r_s+L_s p & & Mp \\ Mp & -M\omega_r & r_r+L_r p & -\omega_r L_r \\ M\omega_r & Mp & \omega_r L_r & r_r+L_r p\end{bmatrix}\begin{bmatrix} i_{ds}\\ i_{qs}\\ i_{dr}\\ i_{qr}\end{bmatrix}$$

Voltage matrix — Impedance matrix — Current matrix

  - In an induction motor, the rotor bars are all shorted:

$\therefore v_{ar} = v_{br} = v_{cr} = 0 \qquad v_{\alpha r} = v_{\beta r} = 0 \qquad v_{dr} = v_{qr} = 0$

  - Since the rotor is shorted, the currents i_{dr} and i_{qr} will reverse their directions
  - The negative signs of i_{dr} and i_{qr} will be taken care by suitably changing the signs of corresponding impedance matrix elements



Linear transformation matrix model for 3-ph IM

- **Performance equations: Torque**
  - The developed torque expression is given by:

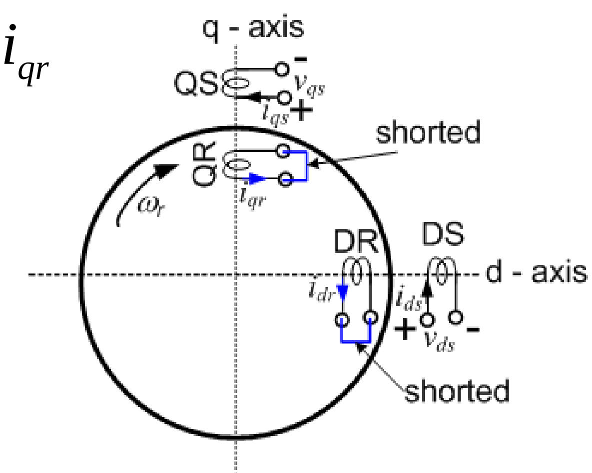
$$T_e = [i_t][G][i]$$

Terms containing ω

$$= \begin{bmatrix} i_{ds} & i_{qs} & i_{dr} & i_{qr} \end{bmatrix} \begin{bmatrix} & & & \\ & & & \\ & -M & & L_r \\ M & & -L_r & \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}$$

$$T_e = [i_t][G][i] = -i_{qs} M i_{dr} + i_{dr} L_r i_{qr} + i_{ds} M i_{qr} - i_{dr} L_{dr} i_{qr}$$

$$= M\left(i_{ds} i_{qr} - i_{dr} i_{qs}\right)$$



- Note that from the torque expression it is clear how stator and rotor currents that are in coils 90^0 apart in space are working together to produce torque
- (+) torque component indicates motoring action, and (-) torque component indicates generating action