Transformation Matrix Models of Machines

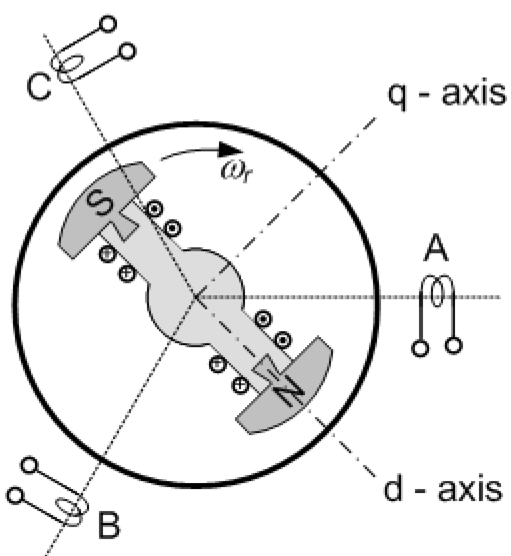
Day 12

ILOs – Day12

- Draw and explain the linear transformation matrix model for
  - 3-phase synchronous machine
- Explain operating characteristics there from

Linear transformation matrix model for 3-ph SM

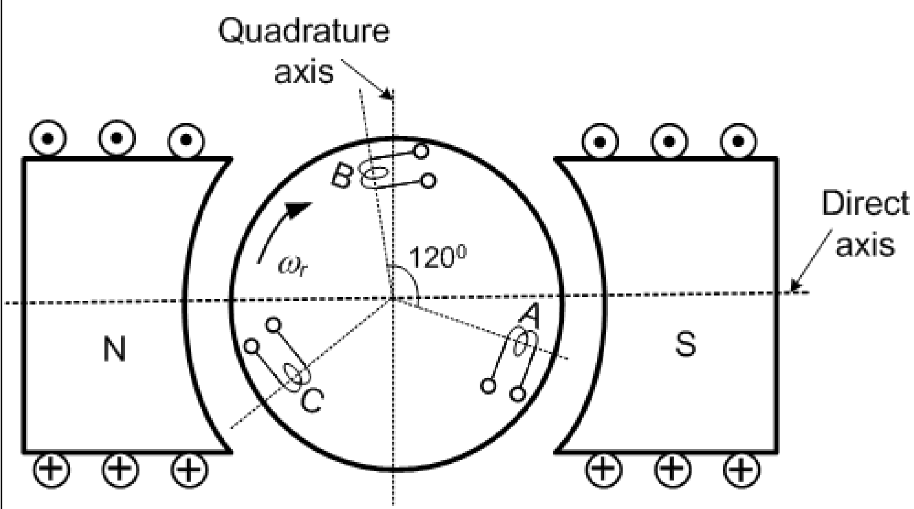
- **3-phase synchronous machine having field winding (DC) in rotor and 3-phase armature winding in stator**
- *d*-axis is always positioned along the pole axis
- *q*-axis is at 90^0 to the pole axis (i.e. along inter-polar axis)
- As the rotor rotates, the pole axis, and hence the *d*-*q* axes also rotate at same speed and in the same direction



- The rotor and the armature MMF both rotate at synchronous speed in CW direction w.r.t. the stationary stator
- However, the rotor and the armature MMF both are stationary w.r.t the field structure because they all rotate at the same speed and in the same direction

Linear transformation matrix model for 3-ph SM

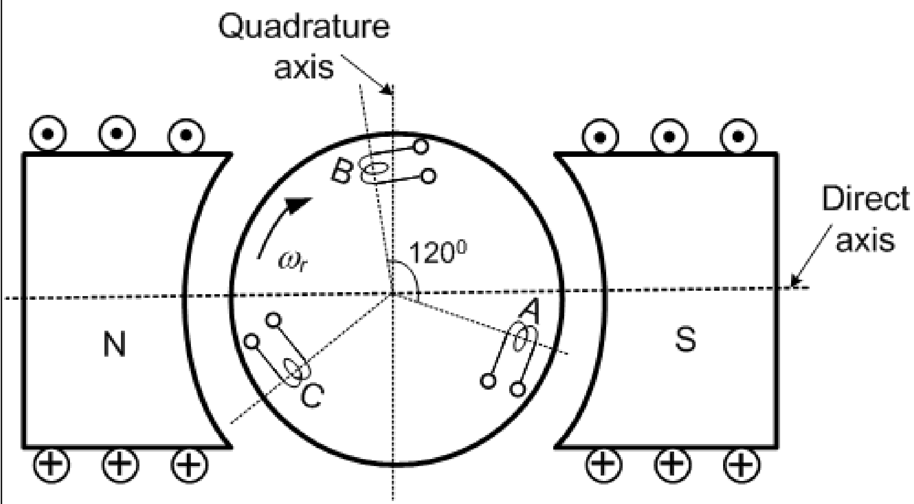
- **3-phase synchronous machine having field winding (DC) in stator and 3-phase armature winding in rotor**
- *d*-axis is always positioned along the pole axis
- *q*-axis is at 90^0 to the pole axis (i.e. along inter-polar axis)
- This configuration is more suitable for representation in generalized form



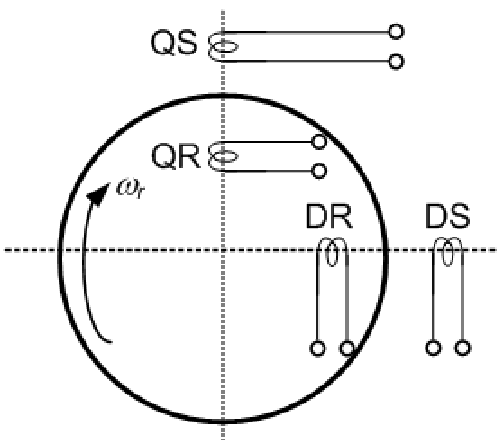
- Since the stator field pole axis is fixed, the rotor MMF axis must also remain fixed w.r.t. the stator
- Thus, with the rotor rotating in CW direction, the rotor MMF must rotate in ACW direction w.r.t. rotor body so that the MMF in air gap appear stationary to stator poles
- i.e. the rotor coils can be assumed to be pseudo-sationary

Linear transformation matrix model for 3-ph SM

- **3-phase synchronous machine having field winding (DC) in stator and 3-phase armature winding in rotor**
- We need to represent the 3-phase rotor, and DC type stator pole windings in the synchronous machine structure to fit to the generalized machine model
- The 3-phase rotor winding is to be resolved into 2-phase pseudo-stationary coils *DR* and *QR* along *d*- and *q*-axes respectively



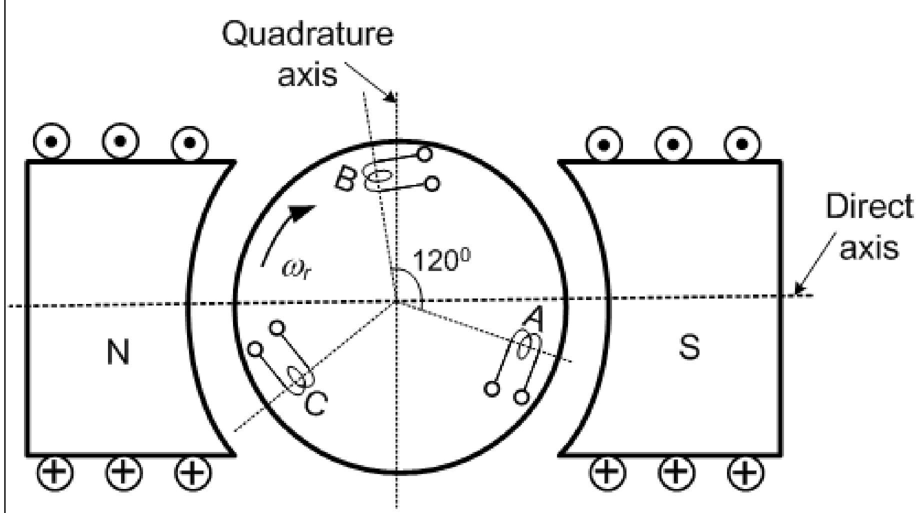
3-phase synchronous machine



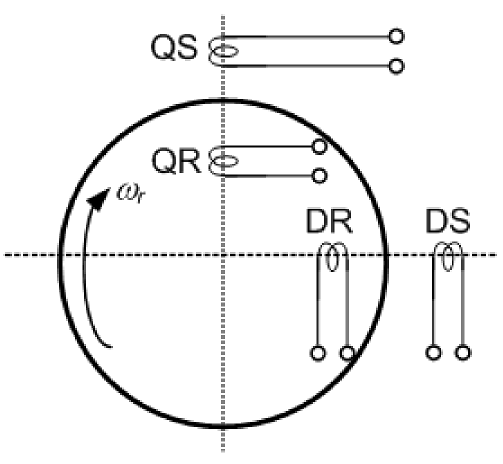
Generalized machine model

Linear transformation matrix model for 3-ph SM

- **3-phase synchronous machine having field winding (DC) in stator and 3-phase armature winding in rotor**
- The resultant field produced by combined action of the two equivalent rotor coils *DR* and *QR* is of the same nature (rotating at a constant speed ω_r with constant magnitude) as that produced by the armature 3-phase currents flowing in the 3-phase winding



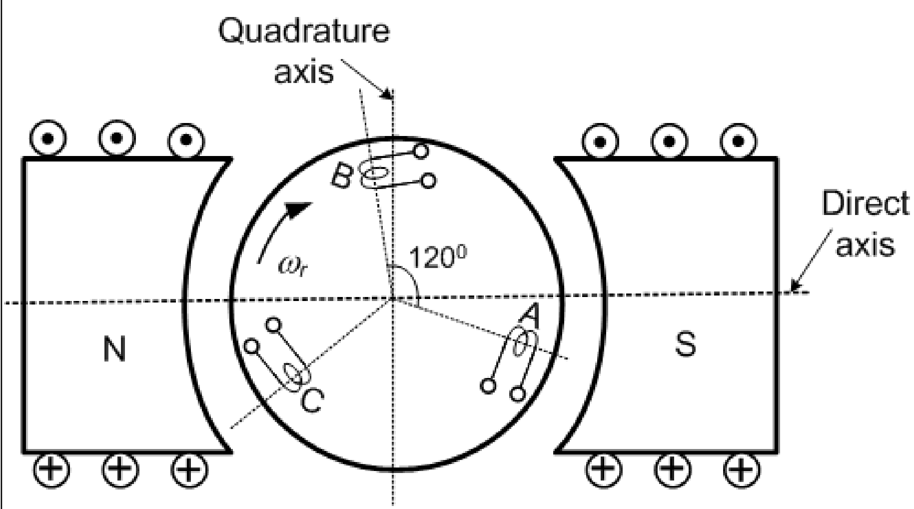
3-phase synchronous machine



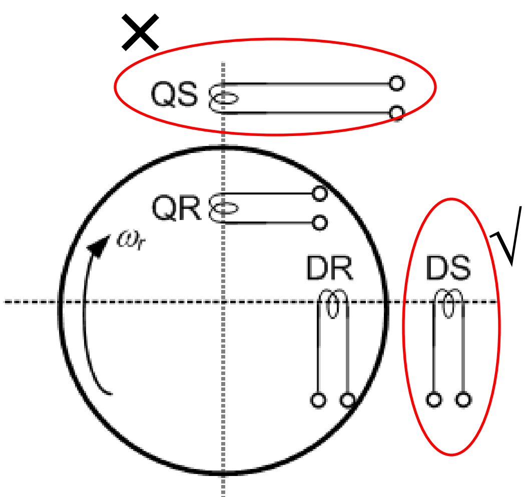
Generalized machine model

Linear transformation matrix model for 3-ph SM

- **3-phase synchronous machine having field winding (DC) in stator and 3-phase armature winding in rotor**
- The DC type stator pole winding is to be represented by the single coil *DS* along the *d*-axis
- In the generalized model of such a synchronous machine, the stator coil *QS* along *q*-axis will not be present



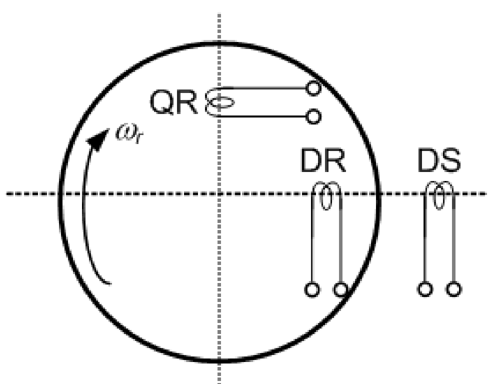
3-phase synchronous machine



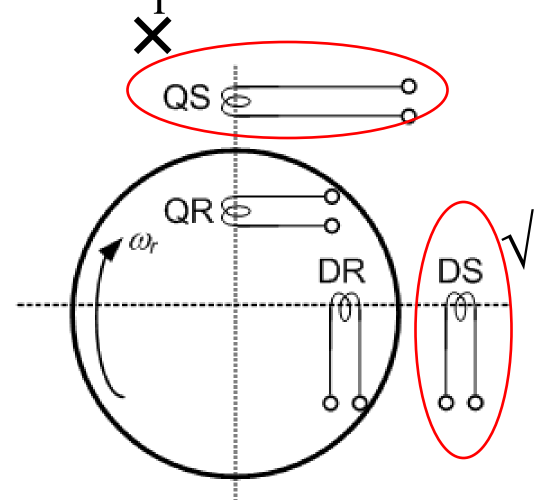
Generalized machine model

Linear transformation matrix model for 3-ph SM

- **3-phase synchronous machine having field winding (DC) in stator and 3-phase armature winding in rotor**
- The DC type stator pole winding is to be represented by the single coil *DS* along the *d*-axis
- In the generalized model of such a synchronous machine, the stator coil *QS* along *q*-axis will not be present
  - (since stator has fixed DC type field poles, and one equivalent coil is sufficient to represent it)



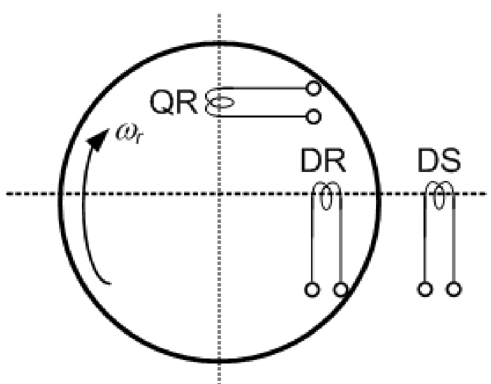
Generalized 2-qaxis model of 3-phase synchronous machine



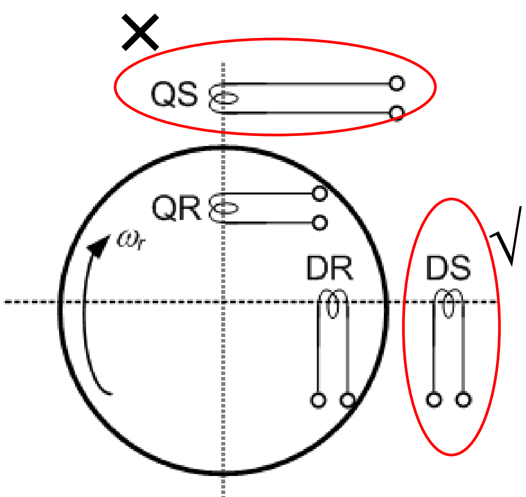
Generalized machine model

Linear transformation matrix model for 3-ph SM

- **Generalized 2-qaxis model of 3-phase synchronous machine**
- We will be deriving the necessary transformation matrix for the generalized 2-axis model of synchronous machine from transformation matrix of the generalized model
- Note that all parameters relating to *QS* will be absent in transformation matrix of the generalized 2-axis model of synchronous machine since the *QS* coil is not present



Generalized 2-qaxis model of 3-phase synchronous machine



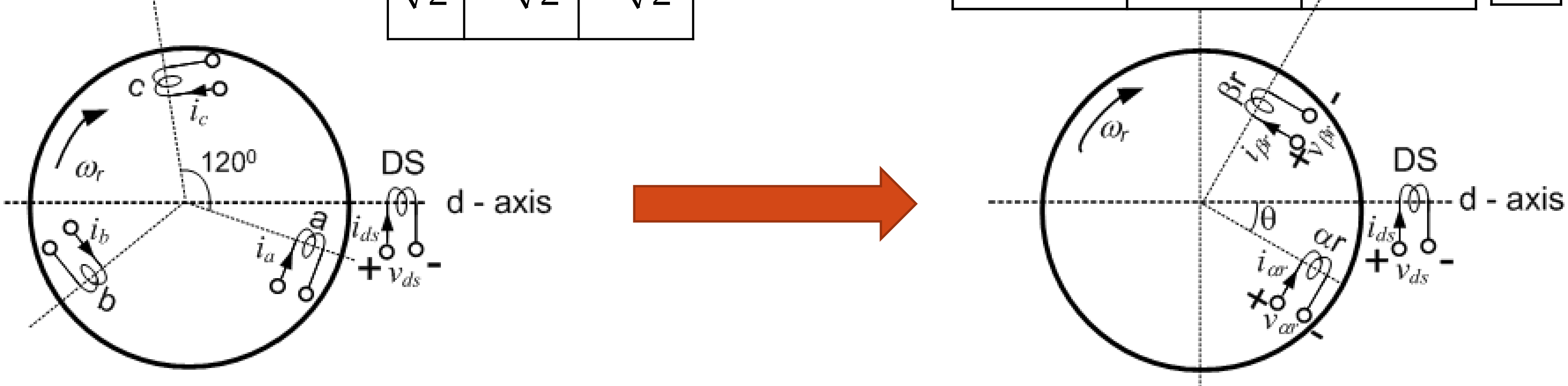
Generalized machine model

Transformation equations

Linear transformation matrix model for 3-ph SM

- **3-phase to 2-phase transformation in rotor**
  - Transformations from 3-phase rotor coils (a, b, c) to rotating axes (α-β-0) requires no change of space frame, since both are on the rotating member
  - The generalized matrix form is:

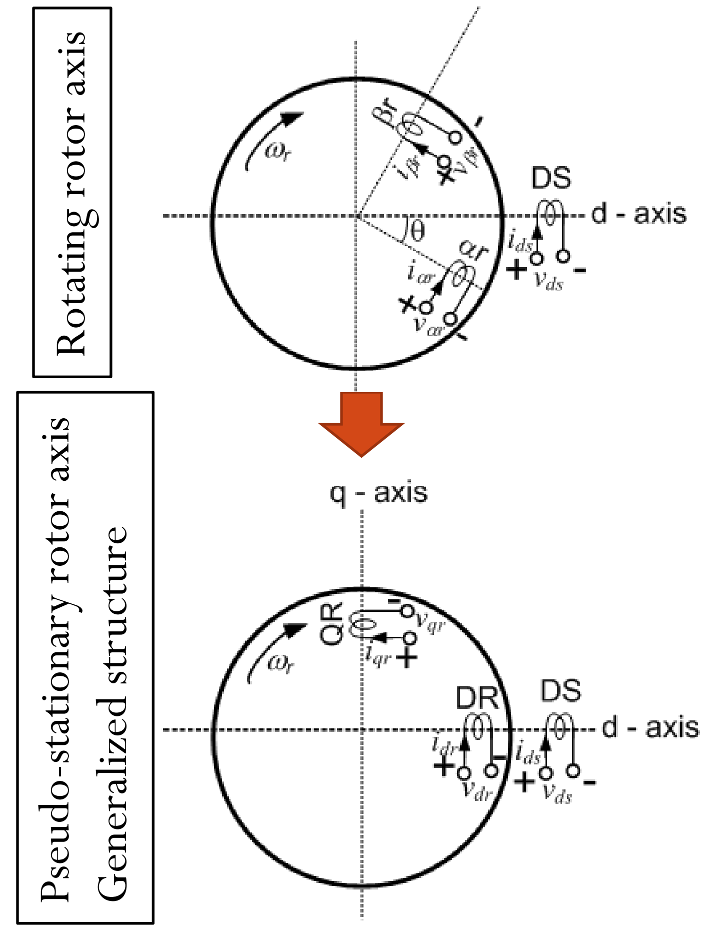
$$\begin{bmatrix} i_{\alpha r} \\ i_{\beta r} \\ i_{0r} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{array}{c} \\ \alpha \\ \beta \\ 0 \end{array} \begin{array}{ccc} a & b & c \\ \hline 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{array} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{array}{c} \\ \alpha \\ \beta \\ 0 \end{array} \begin{array}{ccc} a & b & c \\ \hline \cos 0^0 & \cos 120^0 & \cos 240^0 \\ \sin 0^0 & \sin 120^0 & \sin 240^0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{array} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$



Linear transformation matrix model for 3-ph SM

- **3-phase to 2-phase transformation in rotor**
  - Transformations from rotating axes (α-β-0) to stationary axes (*d*-*q*-*0*) to represent in primitive generalized model:

$$
\begin{array}{c}
\\
\begin{bmatrix} i_{dr} \\ i_{qr} \\ i_{0r} \end{bmatrix}
\end{array}
=
\begin{array}{c}
\\
\begin{matrix} d \\ q \\ 0 \end{matrix}
\end{array}
\begin{array}{c}
\begin{matrix} \alpha \quad & \quad \beta \quad & \quad 0 \end{matrix} \\
\begin{bmatrix} \cos\theta & \sin\theta & \\ -\sin\theta & \cos\theta & \\ & & 1 \end{bmatrix}
\end{array}
\begin{array}{c}
\\
\begin{bmatrix} i_{\alpha r} \\ i_{\beta r} \\ i_{0r} \end{bmatrix}
\end{array}
$$



Linear transformation matrix model for 3-ph SM

- **(a, b, c) to (α, β,0) to (d, q,0) axes transformation**

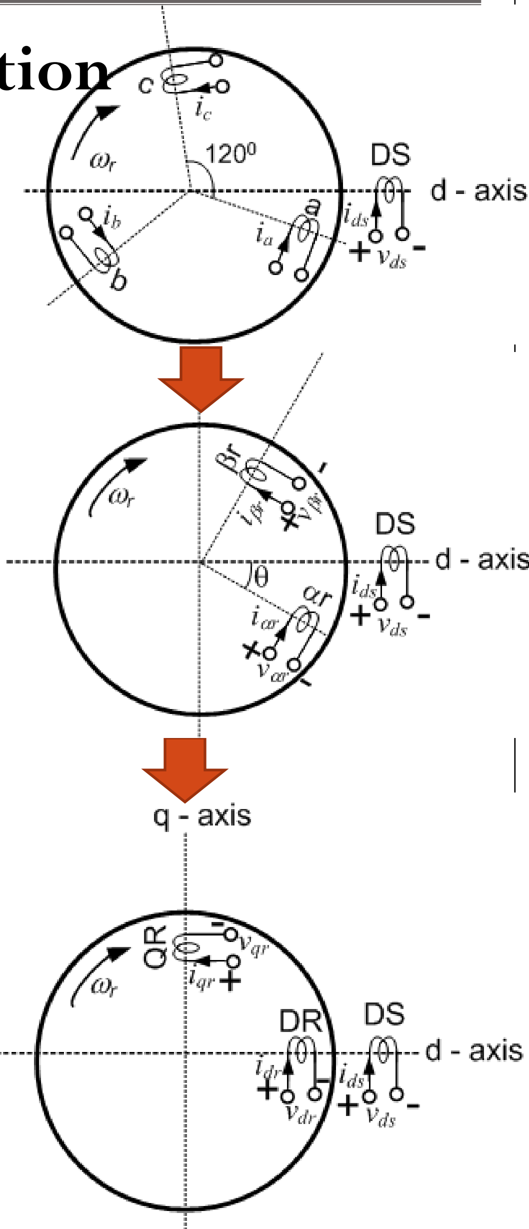
(α,β,0) to (d, q, 0) — (a,b,c) to (α,β,0)

$$
\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} =
\begin{array}{c} d \\ q \\ 0 \end{array}
\overset{\begin{array}{ccc} \alpha & \beta & 0 \end{array}}{\begin{bmatrix} \cos\theta & \sin\theta & \\ -\sin\theta & \cos\theta & \\ & & 1 \end{bmatrix}}
\sqrt{\frac{2}{3}}
\overset{\begin{array}{ccc} a & b & c \end{array}}{\begin{bmatrix} \cos 0^0 & \cos 120^0 & \cos 240^0 \\ \sin 0^0 & \sin 120^0 & \sin 240^0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}}
\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}
$$

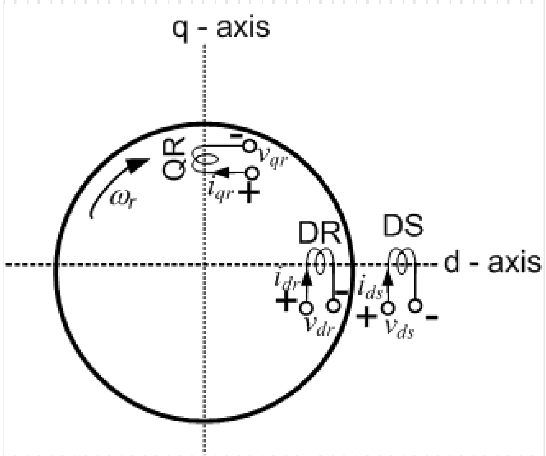
or,

$$
\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} = \sqrt{\frac{2}{3}}
\begin{array}{c} d \\ q \\ 0 \end{array}
\overset{\begin{array}{ccc} a & b & c \end{array}}{\begin{bmatrix} \cos\theta & \cos\left(\theta-120^0\right) & \cos\left(\theta-240^0\right) \\ -\sin\theta & -\sin\left(\theta-120^0\right) & -\sin\left(\theta-240^0\right) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}}
\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}
$$

(a,b,c) to (d, q, 0)



Voltage equations from generalized model



Linear transformation matrix model for 3-ph SM

- **Impedance matrix for generalized 2-axis model of 3-phase synchronous machine**

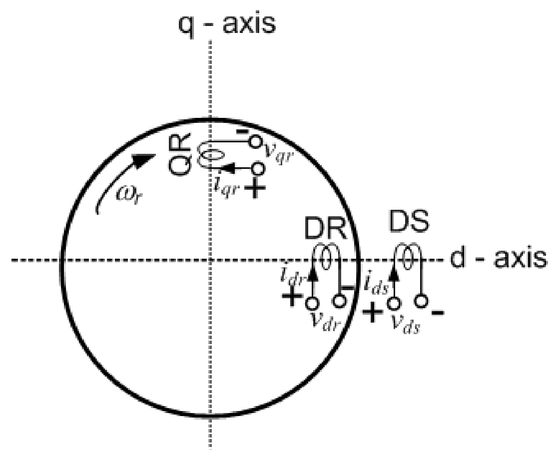
Generalized machine model

Omit

$$
\begin{matrix} \\ ds \\ qs \\ dr \\ qr \end{matrix}
\begin{bmatrix} v_{ds} \\ v_{qs} \\ v_{dr} \\ v_{qr} \end{bmatrix} =
\begin{array}{cccc}
ds & qs & dr & qr \\
\hline
r_{ds} + L_{ds} p & & M_d p & \\
 & r_{qs} + L_{qs} p & & M_q p \\
M_d p & -M_q \omega_r & r_{dr} + L_{dr} p & -\omega_r L_{qr} \\
M_d \omega_r & M_q p & \omega_r L_{dr} & r_{qr} + L_{qr} p
\end{array}
\begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}
$$

Generalized 2-qaxis model of 3-phase synchronous machine

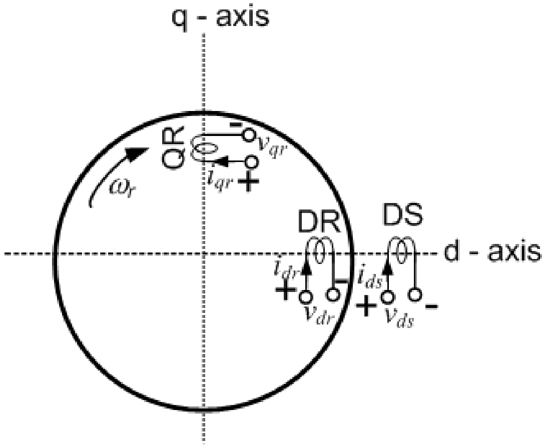
$$
\begin{bmatrix} v_{ds} \\ v_{dr} \\ v_{qr} \end{bmatrix} =
\begin{matrix} ds \\ dr \\ qr \end{matrix}
\begin{array}{ccc}
ds & dr & qr \\
\hline
r_{ds} + L_{ds} p & M_d p & 0 \\
M_d p & r_{dr} + L_{dr} p & -\omega_r L_{qr} \\
M_d \omega_r & \omega_r L_{dr} & r_{qr} + L_{qr} p
\end{array}
\begin{bmatrix} i_{ds} \\ i_{dr} \\ i_{qr} \end{bmatrix}
$$



Linear transformation matrix model for 3-ph SM

- **Imoedance matrix for generalized 2-axis model of 3-phase synchronous machine**

$$
\begin{array}{c} \\ \begin{bmatrix} v_{ds} \\ v_{dr} \\ v_{qr} \end{bmatrix} \end{array} = \begin{array}{c} \\ ds \\ dr \\ qr \end{array} \begin{array}{ccc} ds & dr & qr \\ \hline r_{ds}+L_{ds}p & M_d p & 0 \\ M_d p & r_{dr}+L_{dr}p & -\omega_r L_{qr} \\ M_d\omega_r & \omega_r L_{dr} & r_{qr}+L_{qr}p \end{array} \begin{array}{c} \\ \begin{bmatrix} i_{ds} \\ i_{dr} \\ i_{qr} \end{bmatrix} \end{array}
$$



- Represent the field winging by *F* in place of *DS*
- Represent the armature windings by *D* and *Q* in place of *DR* and *QR*
- For balanced and uniformly distributed armature winding: $r_{dr} = r_{qr} = r_a$

$$
\begin{array}{c} \\ \begin{bmatrix} v_f \\ v_d \\ v_q \end{bmatrix} \end{array} = \begin{array}{c} \\ f \\ d \\ q \end{array} \begin{array}{ccc} f & d & q \\ \hline r_f+L_f p & M_d p & 0 \\ M_d p & r_a+L_d p & -\omega_r L_q \\ M_d\omega_r & \omega_r L_d & r_a+L_q p \end{array} \begin{array}{c} \\ \begin{bmatrix} i_f \\ i_d \\ i_q \end{bmatrix} \end{array}
$$

Linear transformation matrix model for 3-ph SM

- **Impedance matrix for generalized 2-axis model of 3-phase synchronous machine**

$$\begin{bmatrix} v_f \\ v_d \\ v_q \end{bmatrix} = \begin{array}{c} f \\ d \\ q \end{array} \overset{\begin{array}{ccc} f & d & q \end{array}}{\begin{bmatrix} r_f + L_f p & M_d p & 0 \\ M_d p & r_a + L_d p & -\omega_r L_q \\ M_d \omega_r & \omega_r L_d & r_a + L_q p \end{bmatrix}} \begin{bmatrix} i_f \\ i_d \\ i_q \end{bmatrix}$$

- Hence, the voltage equations are:

$$v_f = (r_f + L_f p) i_f + M_d p i_d$$
$$v_d = M_d p i_f + (r_a + L_d p) i_d - \omega_r L_q i_q$$
$$v_q = M_d \omega_r i_f + \omega_r L_d i_d + (r_a + L_q p) i_q$$

- These relationships are valid for both steady state as well as transient analysis of synchronous machines

Steady state analysis of synchronous machine

Linear transformation matrix model for 3-ph SM

- **Steady state analysis of 3-phase synchronous machine**
  - Voltage equations for motor operation

$$v_f = (r_f + L_f p)i_f + M_d p i_d$$
$$v_d = M_d p i_f + (r_a + L_d p)i_d - \omega_r L_q i_q$$
$$v_q = M_d \omega_r i_f + \omega_r L_d i_d + (r_a + L_q p)i_q$$

  - At steady state operation, the transient term (derivative, p) is omitted
  - Thus, steady state voltage equations are reduced to:

$$V_f = r_f I_f$$
$$V_d = r_a I_d - \omega L_q I_q$$
$$V_q = M_d \omega I_f + \omega L_d I_d + r_a I_q$$

Linear transformation matrix model for 3-ph SM

- **Steady state analysis of 3-phase synchronous machine**
  - Voltage equations at steady state

$$V_f = r_f I_f$$
$$V_d = r_a I_d - \omega L_q I_q$$
$$V_q = M_d \omega I_f + \omega L_d I_d + r_a I_q$$

$\omega L_d = X_d =$ Direct axis synchronous reactance

$\omega L_q = X_q =$ Quadrature axis synchronous reactance

$$V_f = r_f I_f$$
$$V_d = r_a I_d - X_q I_q$$
$$V_q = M_d \omega I_f + X_d I_d + r_a I_q$$

Linear transformation matrix model for 3-ph SM

- **Steady state analysis of 3-phase synchronous machine**
  - Voltage equations at steady state

$$V_f = r_f I_f$$
$$V_d = r_a I_d - X_q I_q$$
$$V_q = M_d \omega I_f + X_d I_d + r_a I_q$$

$M_d \omega I_f = E_f =$ Induced EMF

$$V_f = r_f I_f$$
$$V_d = r_a I_d - X_q I_q$$
$$V_q = E_f + X_d I_d + r_a I_q$$

Transient analysis of synchronous machine

Linear transformation matrix model for 3-ph SM

- **Transient analysis of 3-phase synchronous machine**

We will do in Chapter 3