Transformation Matrix Models of Machines

Day 10

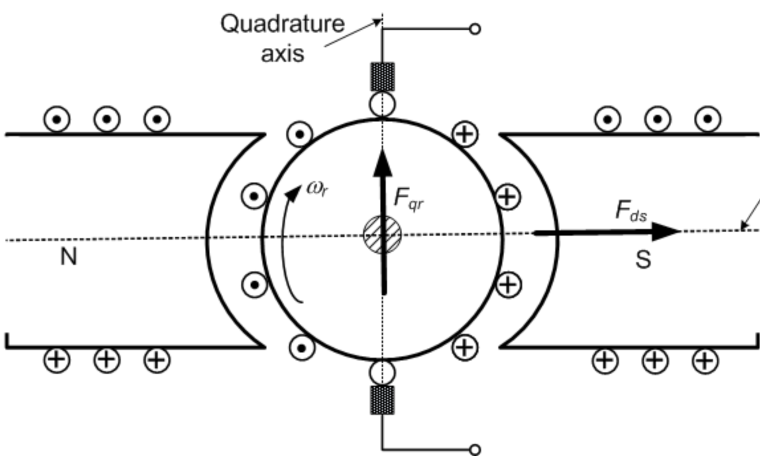
ILOs – Day10

- Draw and explain the linear transformation matrix model for
  - DC machine
- Explain operating characteristics there from

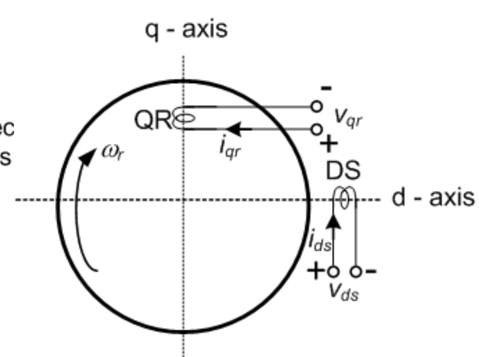
Linear transformation matrix model for DC machine

Linear transformation matrix model for DC machine

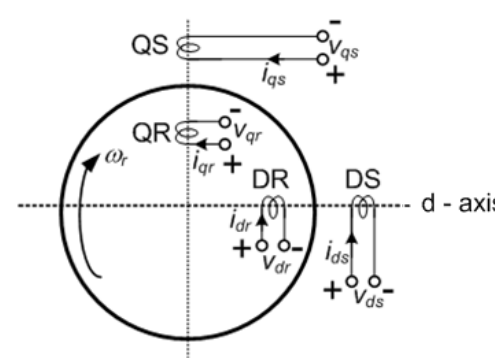
- DC machines have salient poles in stator
- Armature winding in rotor is distributed in slots and connected to the commutator segments making it pseudo-stationary
- *d*-axis of the primitive machine is along field poles
- *q*-axis is along brush axis (inter-polar axis)
- Thus, for DC machines, the 2-pole equivalent structure, and Kron's primitive machine structure are nearly identical
- **Hence, no transformation is necessary**



2-pole DC machine



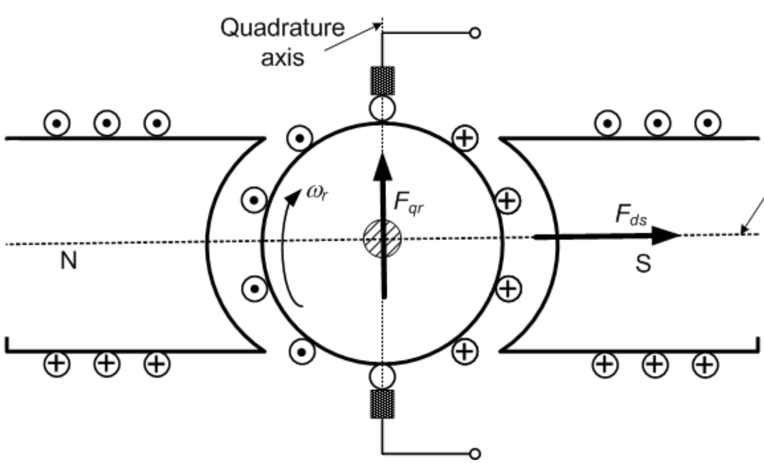
Basic 2-pole equivalent



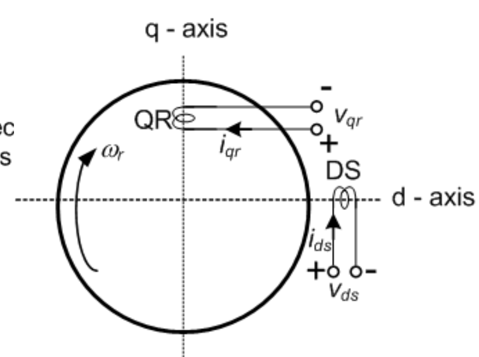
Kron's primitive model

Linear transformation matrix model for DC machine

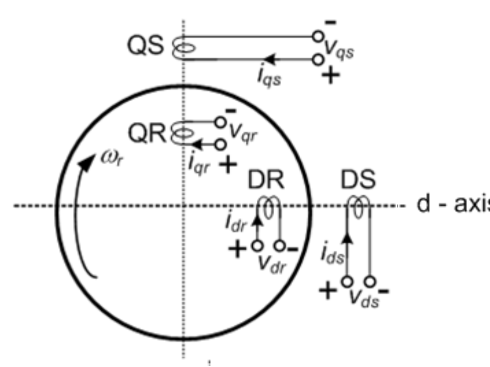
- However, some more assumptions are necessary for using the generalized mathematical model to a real DC machine
  - The effect of armature MMF along q-axis on total *d*-axis flux is neglected, i.e. the cross-magnetizing and de-magnetizing effects of armature reaction are neglected
    - The commutation process is such that the stationary armature MMF wave is always along *q*-axis
  - Effects of saturation is neglected, i.e. magnetic circuit is assumed to stay linear



2-pole DC machine



Basic 2-pole equivalent

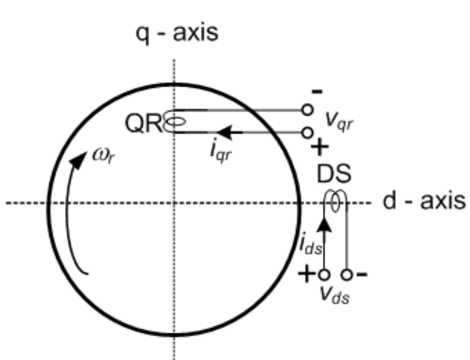


Kron's primitive model

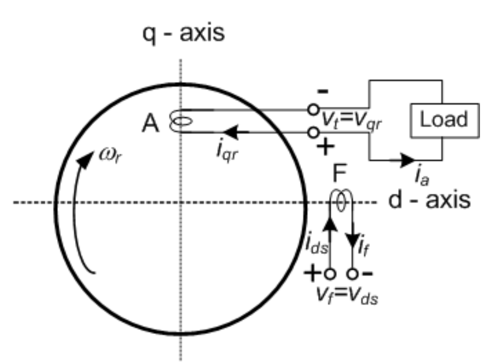
Linear transformation matrix model for DC machine

- Steady state and transient analysis of DC generators
  - With electrical load connected to the generator armature terminals, the output quantities are voltage, current, and power
  - Voltage equations of Kron's generalized model:

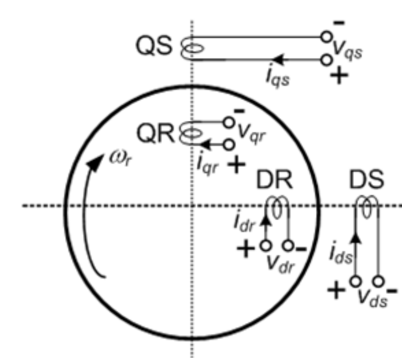
$$
\begin{array}{c} \\ \end{array}
\begin{bmatrix} v_{ds} \\ v_{qs} \\ v_{dr} \\ v_{qr} \end{bmatrix}
=
\begin{array}{c|cccc}
 & ds & qs & dr & qr \\ \hline
ds & r_{ds} + L_{ds} p & & M_d p & \\
qs & & r_{qs} + L_{qs} p & & M_q p \\
dr & M_d p & -M_q \omega_r & r_{dr} + L_{dr} p & -\omega_r L_{qr} \\
qr & M_d \omega_r & M_q p & \omega_r L_{dr} & r_{qr} + L_{qr} p
\end{array}
\begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}
$$



Basic 2-pole DC machine equivalent



Load connected to armature

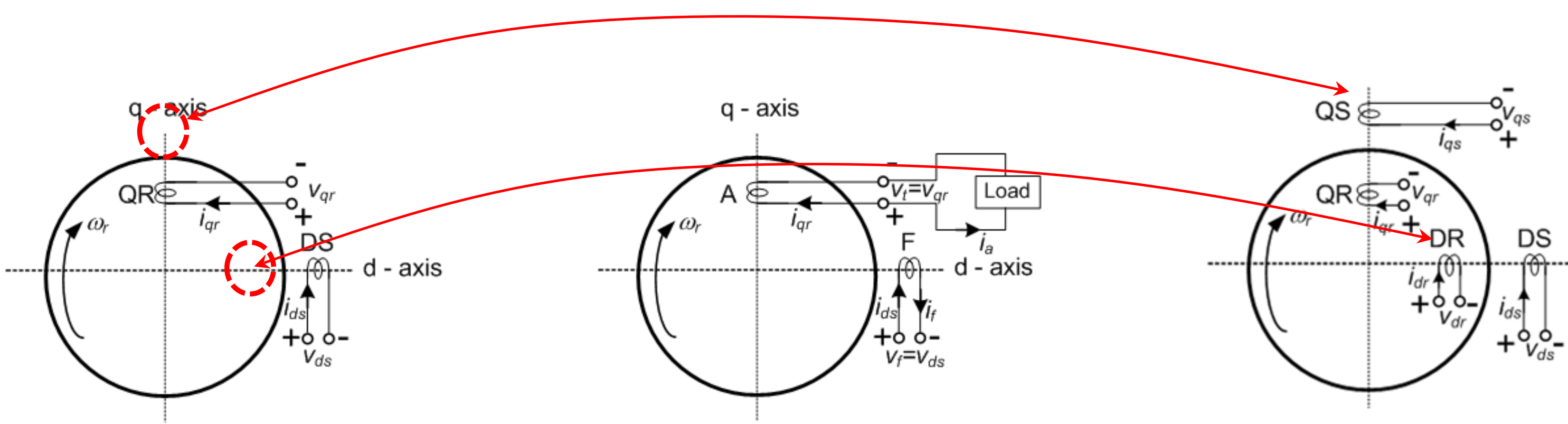


Generalized Kron's machine

Linear transformation matrix model for DC machine

- Steady state and transient analysis of DC generators

Basic 2-pole DC machine equivalent	Generalized Kron's machine
Stator has only DS coil, but QS coil is not present	Stator has both DS and QS coils
Rotor has only QR coil, but DR coil is not present	Rotor has both DR and QR coils

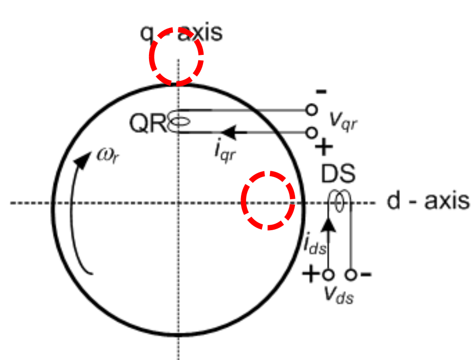


Basic 2-pole DC machine equivalent | Load connected to armature | Generalized Kron's machine

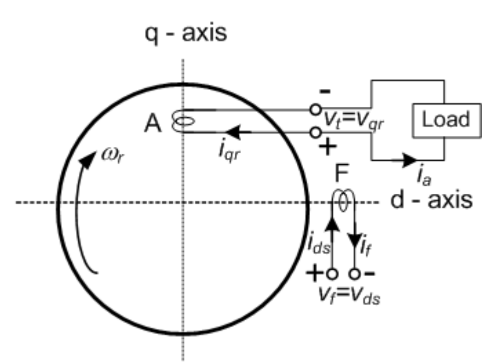
Linear transformation matrix model for DC machine

- Steady state and transient analysis of DC generators
  - Comparing with the DC machine model, the matrices can be suitably modified so that rows and columns relating to *qs* and *dr* can be omitted

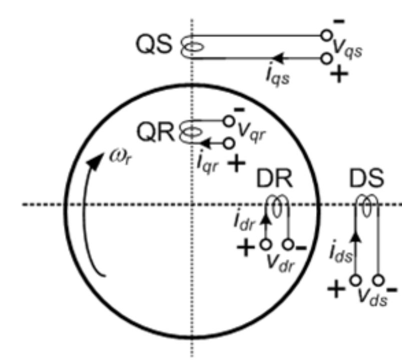
		ds	*qs*	*dr*	*qr*	
v_{ds}	*ds*	$r_{ds} + L_{ds} p$		$M_d p$		i_{ds}
v_{qs}	*qs*		$r_{qs} + L_{qs} p$		$M_q p$	i_{qs}
v_{dr}	*dr*	$M_d p$	$-M_q \omega_r$	$r_{dr} + L_{dr} p$	$-\omega_r L_{qr}$	i_{dr}
v_{qr}	*qr*	$M_d \omega_r$	$M_q p$	$\omega_r L_{dr}$	$r_{qr} + L_{qr} p$	i_{qr}



Basic 2-pole DC machine equivalent



Load connected to armature



Generalized Kron's machine

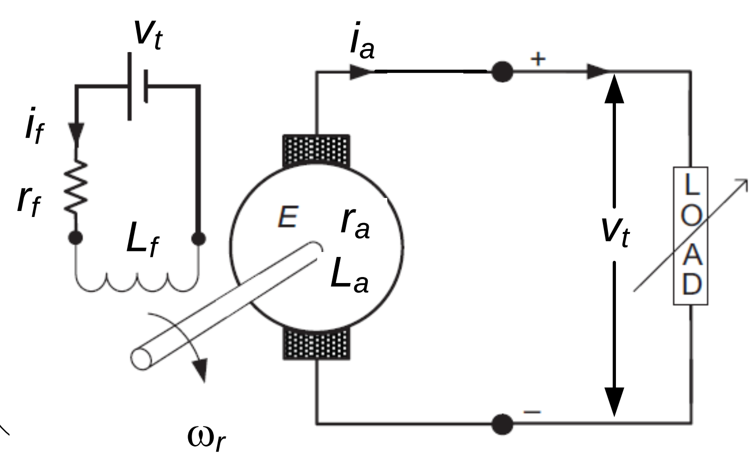
Linear transformation matrix model for DC machine

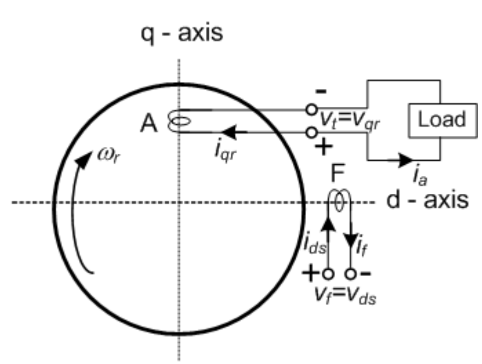
- Steady state and transient analysis of DC generators
  - Comparing with the DC machine model, the matrices can be suitably modified so that rows and columns relating to *qs* and *dr* can be omitted:

$$\begin{array}{c} \\ \begin{bmatrix} v_{ds} \\ v_{qr} \end{bmatrix} = \end{array} \begin{array}{c} \\ ds \\ qr \end{array} \begin{array}{cc} ds & qr \\ \begin{bmatrix} r_{ds} + L_{ds} p & 0 \\ M_d \omega_r & r_{qr} + L_{qr} p \end{bmatrix} \end{array} \begin{array}{c} \\ \begin{bmatrix} i_{ds} \\ i_{qr} \end{bmatrix} \end{array}$$

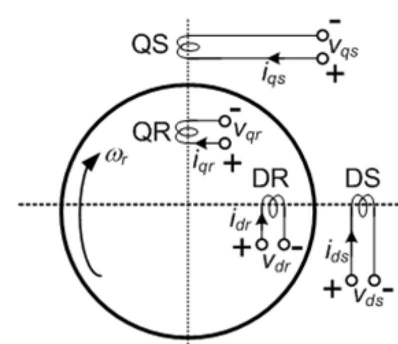
  - Circuit equations:

$$\begin{array}{l|l|l|l|} i_{qr} = -i_a & v_{qr} = v_t & r_{qr} = r_a & L_{qr} = L_a \\ i_{ds} = i_f & v_{ds} = v_f & r_{ds} = r_f & L_{ds} = L_f \end{array} \quad \Rightarrow \quad \begin{bmatrix} v_f \\ v_t \end{bmatrix} = \begin{array}{c} ds \\ qr \end{array} \begin{array}{cc} ds & qr \\ \begin{bmatrix} r_f + L_f p & 0 \\ M_d \omega_r & r_a + L_a p \end{bmatrix} \end{array} \begin{bmatrix} i_f \\ -i_a \end{bmatrix}$$





Load connected to armature



Generalized Kron's machine

Linear transformation matrix model for DC machine

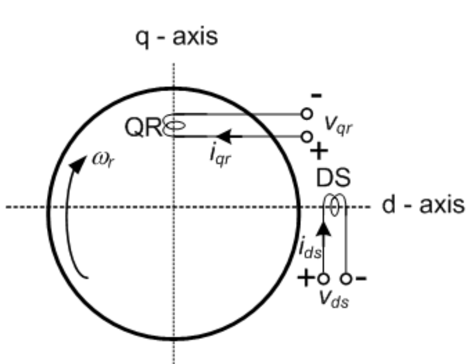
- Steady state and transient analysis of DC generators
  - Voltage equations

$$
\begin{array}{c} \\ \begin{array}{|c|} \hline v_f \\ \hline v_t \\ \hline \end{array} \end{array}
=
\begin{array}{c} \\ \begin{array}{c} ds \\ qr \end{array} \end{array}
\begin{array}{c} \begin{array}{cc} \quad ds \quad & \quad qr \quad \end{array} \\ \begin{array}{|c|c|} \hline r_f + L_f p & 0 \\ \hline M_d \omega_r & r_a + L_a p \\ \hline \end{array} \end{array}
\begin{array}{c} \\ \begin{array}{|c|} \hline i_f \\ \hline -i_a \\ \hline \end{array} \end{array}
$$

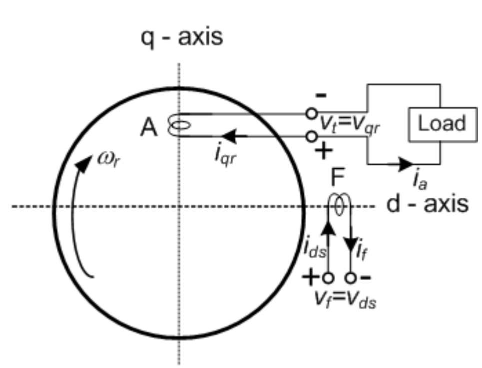
$$v_f = \left(r_f + L_f p\right) i_f$$

$$v_t = M_d \omega_r i_f - \left(r_a + L_a p\right) i_a$$

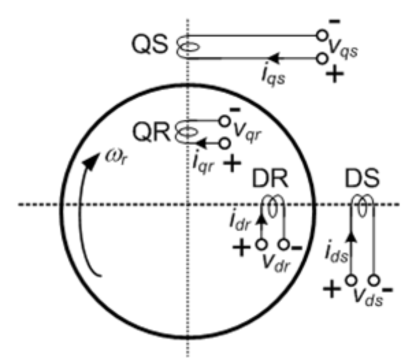
  - These equations are valid for both steady state as well as transient conditions



Basic 2-pole DC machine equivalent



Load connected to armature



Generalized Kron's machine

Steady state analysis of DC machine

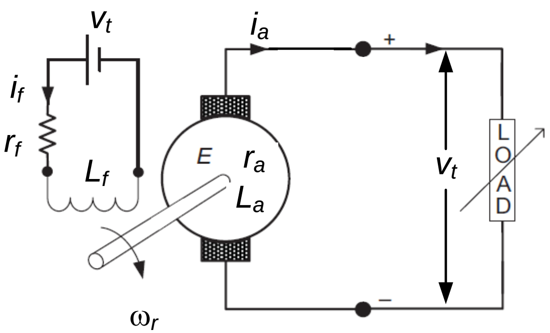
Linear transformation matrix model for DC machine

- **Steady state analysis of DC generators**
  - Voltage equations

$$v_f = (r_f + L_f p)\, i_f$$
$$v_t = M_d \omega_r i_f - (r_a + L_a p)\, i_a$$

  - At steady state operation, the transient term (derivative, p) is omitted
  - Thus, steady state voltage equations are reduced to:

$$V_f = r_f I_f$$
$$V_t = M_d \omega_r I_f - r_a\, I_a$$



DC generator equations we are familiar with

$$V_f = r_f I_f$$
$$V_t = E - r_a\, I_a$$

  - At no-load, the armature current is zero, i.e. $I_a = 0$
  - Thus, no-load terminal voltage (induced EMF) is: $V_t = M_d \omega_{r0} I_f = E_{a0}$
  - E_{ao} is the armature terminal voltage at no-load (induced EMF) and at a constant speed ω_{ro}

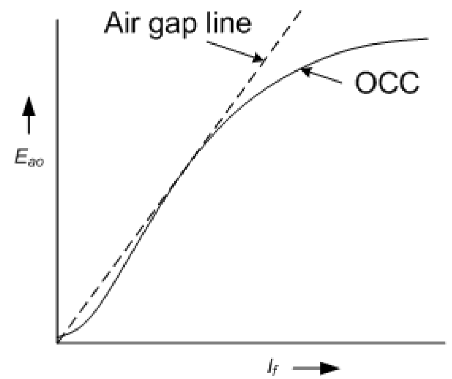
Linear transformation matrix model for DC machine

- **Steady state analysis of DC generators: No-load characteristics**

$$E_{a0} = M_d \omega_{r0} I_f$$

$$y = mx$$

  - The plot of E_{ao} vs. I_f gives the OCC, or saturation curve, or magnetization curve



  - Initially, the open circuit armature voltage E_{ao} increases linearly with field current
  - But, at higher value of I_f, when the magnetic path saturates, the *rotational mutual inductance* or *motional inductance* $\boldsymbol{M_d}$ begins to decrease
  - The OCC plot starts to bend horizontal indicating saturation

Linear transformation matrix model for DC machine

- **Steady state analysis of DC generators: Load characteristics**
  - Armature EMF at no-load: $E_{a0} = M_d \omega_{r0} I_f$
  - Hence, armature generated EMF at any other speed:

$$V_f = r_f I_f$$
$$V_t = M_d \omega_r I_f - r_a I_a$$

$$E_a = \frac{E_{a0}}{\omega_{r0}} \times \omega_r$$
$$= M_d \omega_r I_f$$
$$= K_g I_f$$

Generator constant
$K_g = M_d \omega_r$

  - Thus, armature terminal voltage at any load current I_a is:

$$V_t = E_a - r_a I_a$$
$$= M_d \omega_r I_f - r_a I_a$$

$$V_t = K_g I_f - r_a I_a$$

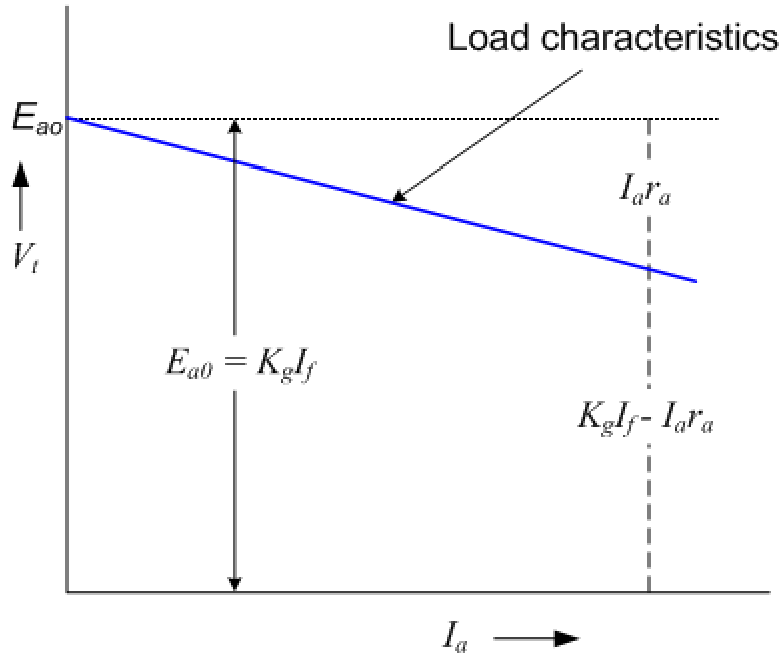
Linear transformation matrix model for DC machine

- **Steady state analysis of DC generators: Load characteristics**

$$V_t = K_g I_f - r_a I_a$$

$$y = C - mx$$

- The plot of V_t vs. I_a gives the load, or external characteristics of a separately excited DC generator



Transient analysis of DC machine

Linear transformation matrix model for DC machine

- **Transient analysis of DC generators**
  - Voltage equations $v_f = \left(r_f + L_f p\right) i_f$

    $$v_t = M_d \omega_r I_f - \left(r_a + L_a p\right) i_a$$

  - The generator is running at no-load ($i_a = 0$)with a constant no-load speed ω_{r0}
  - There is **sudden application of step field excitation $V_f U(t)$**
  - The field circuit transient equation in Laplace domain is:

$$\frac{V_f}{s} = \left(r_f + L_f s\right) I_f(s) \Rightarrow I_f(s) = \frac{V_f}{s\left(r_f + L_f s\right)}$$

  - The armature circuit equation under no-load condition in Laplace domain is:

$$V_t(s) = M_d \omega_{r0} I_f(s) = E_{a0}(s)$$

$$\Rightarrow E_{a0}(s) = M_d \omega_{r0} I_f(s) = M_d \omega_{r0} \frac{V_f}{s\left(r_f + L_f s\right)}$$

Linear transformation matrix model for DC machine

- **Transient analysis of DC generators**

$$E_{a0}(s) = M_d \omega_{r0} I_f(s) = M_d \omega_{r0} \frac{V_f}{s(r_f + L_f s)}$$

$$\Rightarrow E_{a0}(s) = M_d \omega_{r0} \frac{V_f}{s r_f \left(1 + \frac{L_f}{r_f} s\right)} = M_d \omega_{r0} \frac{V_f}{s r_f (1 + \tau_f s)} = \frac{M_d \omega_{r0} I_f}{s(1 + \tau_f s)}$$

$$\tau_f = \frac{L_f}{r_f} = \text{field time constant}$$

$$M_d \omega_{r0} I_f = E_{a0} = \text{no-load armature terminal voltage (induced EMF) at } \omega_{r0}$$

$$\Rightarrow E_{a0}(s) = \frac{E_{a0}}{s(1 + \tau_f s)}$$

- Expression for generator terminal voltage at no-load under transient condition can be obtained by taking Laplace inverse of the above eqn.

Linear transformation matrix model for DC machine

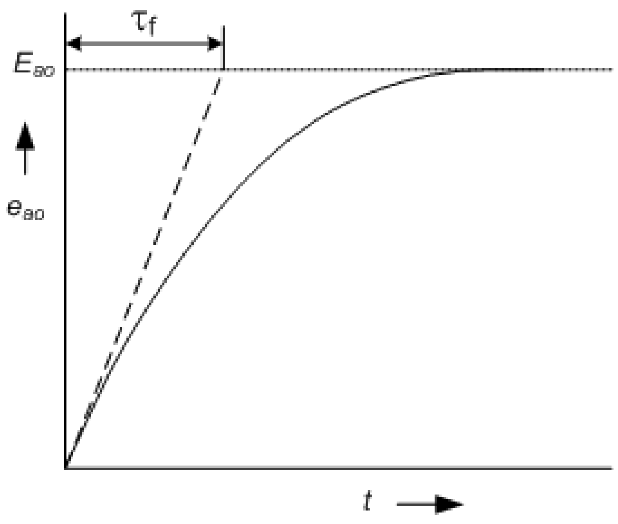
- **Transient analysis of DC generators**

$$E_{a0}(s) = \frac{E_{a0}}{s(1+\tau_f s)}$$

- Expression for generator terminal voltage at no-load under transient condition can be obtained by taking Laplace inverse of the above eqn.

$$e_{a0}(i) = E_{a0}\left(1 - e^{-\frac{t}{\tau_f}}\right)$$

- The transient characteristics looks like:



Linear transformation matrix model for DC machine

- **Transient analysis of DC generators with load**
  - Voltage equations $v_f = \left(r_f + L_f p\right) i_f$

    $$v_t = M_d \omega_r I_f - \left(r_a + L_a p\right) i_a$$
  - The generator is running at load $(i_a \neq 0)$with a speed ω_r, load is $R_L + jX_L$
  - There is **sudden application of step field excitation** $V_f U(t)$
  - The field circuit transient equation in Laplace domain is:

$$\frac{V_f}{s} = \left(r_f + L_f s\right) I_f(s) \Rightarrow I_f(s) = \frac{V_f}{s\left(r_f + L_f s\right)}$$

  - The armature circuit equation in Laplace domain is:

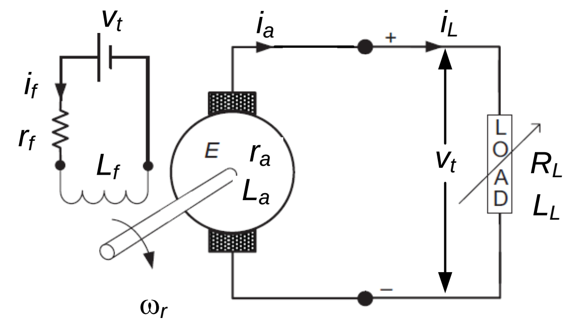
$$V_t(s) = M_d \omega_r \frac{V_f}{s\left(r_f + L_f s\right)} - \left(r_a + L_a s\right) I_a(s)$$

$$\Rightarrow V_t(s) = M_d \omega_r \frac{V_f}{s r_f \left(1 + \tau_f s\right)} - \left(r_a + L_a s\right) I_a(s)$$

Linear transformation matrix model for DC machine

- **Transient analysis of DC generators with load**

$$V_t(s) = M_d\omega_r \frac{V_f}{sr_f(1+\tau_f s)} - (r_a + L_a s) I_a(s)$$



  - The load current expression in Laplace domain:

$$I_L(s) = I_a(s) = \frac{V_t(s)}{(R_L + L_L s)}$$

$$\Rightarrow \quad I_a(s)(R_L + L_L s) = V_t(s) = M_d\omega_r \frac{V_f}{sr_f(1+\tau_f s)} - (r_a + L_a s) I_a(s)$$

$$\Rightarrow \quad I_a(s)(R_L + L_L s) = \frac{M_d\omega_r I_f}{s(1+\tau_f s)} - (r_a + L_a s) I_a(s)$$

$M_d\omega_r I_f = E_a =$ Induced EMF at ω_r

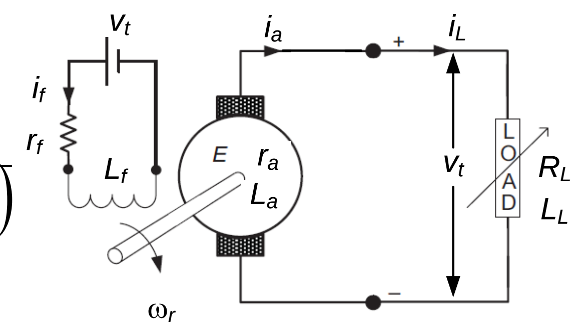
$$\Rightarrow \quad I_a(s)(R_L + L_L s) = \frac{E_a}{s(1+\tau_f s)} - (r_a + L_a s) I_a(s)$$

$$\Rightarrow \quad I_a(s)(R_L + L_L s + r_a + L_a s) = \frac{E_a}{s(1+\tau_f s)}$$

Linear transformation matrix model for DC machine

- **Transient analysis of DC generators with load**

$$I_a(s)(R_L + L_L s + r_a + L_a s) = \frac{E_a}{s(1+\tau_f s)}$$



$$\Rightarrow \quad I_a(s)[(R_L + r_a) + s(L_L + L_a)] = \frac{E_a}{s(1+\tau_f s)}$$

$$\Rightarrow \quad I_a(s)(R_L + r_a)\left[1 + s\frac{(L_L + L_a)}{(R_L + r_a)}\right] = \frac{E_a}{s(1+\tau_f s)}$$

$$\Rightarrow \quad I_a(s)(R_L + r_a)[1 + s\tau] = \frac{E_a}{s(1+\tau_f s)}$$

$$\tau = \frac{(L_L + L_a)}{(R_L + r_a)} = \text{Effective armature circuit time constant}$$

$$\Rightarrow \quad I_a(s) = \frac{E_a}{s(R_L + r_a)(1+s\tau_f)(1+s\tau)}$$

Terminal voltage: $V_t(s) = (R_L + L_L s) I_a(s) = \dfrac{E_a(R_L + L_L s)}{s(R_L + r_a)(1+s\tau_f)(1+s\tau)}$

Linear transformation matrix model for DC machine

- **Transient analysis of DC generators with load**

Terminal voltage: $V_t(s) = (R_L + L_L s) I_a(s) = \dfrac{E_a (R_L + L_L s)}{s(R_L + r_a)(1 + s\tau_f)(1 + s\tau)}$

To get V_t in time domain, we need to take Laplace inverse of the above equation (partial fractions)

Armature current: $I_a(s) = \dfrac{E_a}{s(R_L + r_a)(1 + s\tau_f)(1 + s\tau)}$

To get I_a in time domain, we need to take Laplace inverse of the above equation (partial fractions)