DSE 4A CLASS

Lecture-3

06/05/2021

To provide a wide-ranging calibration service at NIST for 17 decades of resistance from 10^{-4} Ω to 10^{12} Ω used seven stand-alone measurement systems for comparing standard resistors.

1) Automated current comparator for 1 Ω measurements.

2) Automated current comparator for 10 Ω and 100 Ω measurements.

3) Current comparator for measurements < 1 Ω.

4) Automated unbalanced bridge for 1 kΩ to 1 MΩ measurements.

5) Guarded double-ratio bridge for special 10 kΩ measurements.

6) Guarded Wheatstone bridge for 10^7 Ω to 10^{10} Ω measurements.

7) Capacitance-discharge (CD) system for 10^7 Ω to 10^{12} Ω measurements.

The current comparator systems (1 to 3) are based on a current-ratio method.

The **bridge systems** (4 to 6) are based on a **resistance-ratio** method.

The CD system(7)are based on a loss-of-charge method.

Bridge Circuit

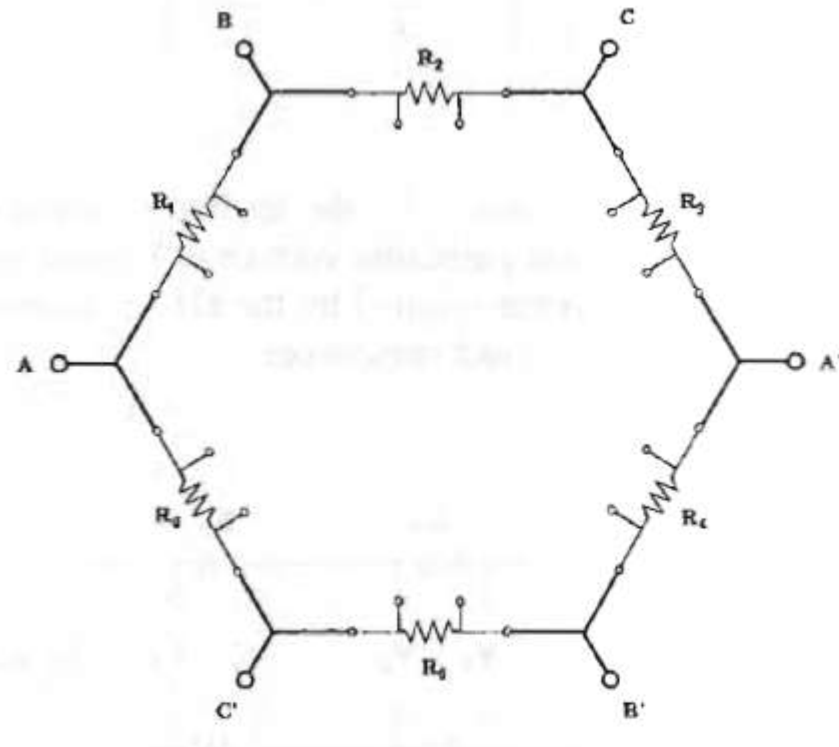
Bridge Circuit is a null method, operates on the principle of comparison. That is a known (standard) value is adjusted until it is equal to the unknown value.

Bridge Circuit

- **DC Bridge (Resistance)**
  - Wheatstone Bridge
  - Kelvin Bridge
  - Megaohm Bridge
- **AC Bridge**
  - **Inductance**
    - Maxwell Bridge
    - Hay Bridge
    - Owen Bridge
    - Etc.
  - **Capacitance**
    - Schering Bridge
  - **Frequency**
    - Wien Bridge

An automated measurement system, based on the **unbalanced-bridge technique**, was developed in 1989 to replace the manual system used to measure resistors in the range 1 kΩ to 1 MΩ

This technique is referred to as the "**ring method**" since the resistors are connected in a ring configuration



A string of resistors 2(n+2) [n>1] connected in a closed electrical circuit where the first and last resistors are interconnected.

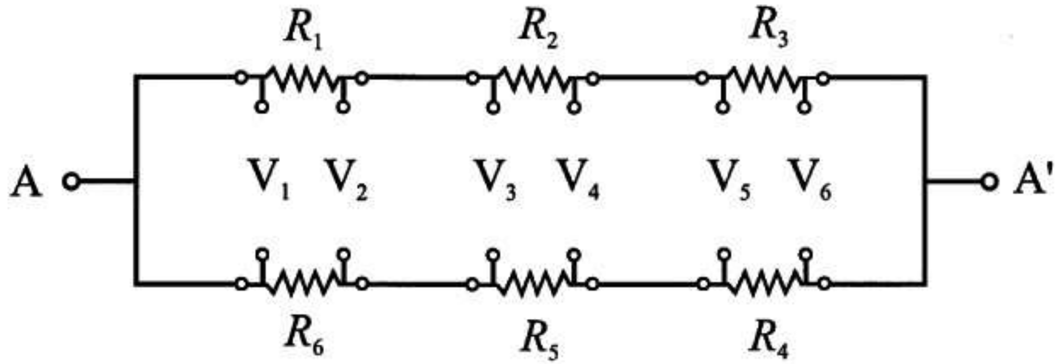
A voltage is applied across opposite corners of the ring (A and A ').

Then, DVM is used to measure voltages between opposite potential terminals of the resistors that are at nearly equal potentials..

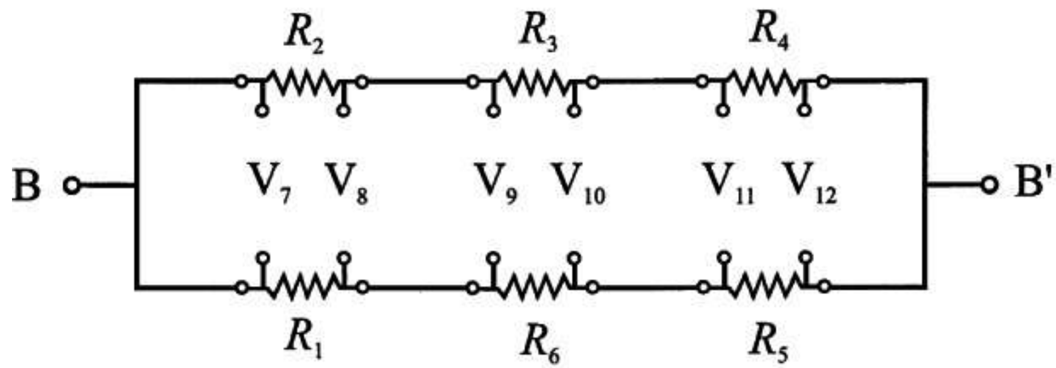
Next, the applied voltage points across the ring are rotated in a clockwise (or counterclockwise) direction to the next pair of resistor connection points.

Again voltage measurements are taken between corresponding terminals of the resistors that are at nearly equal potentials.

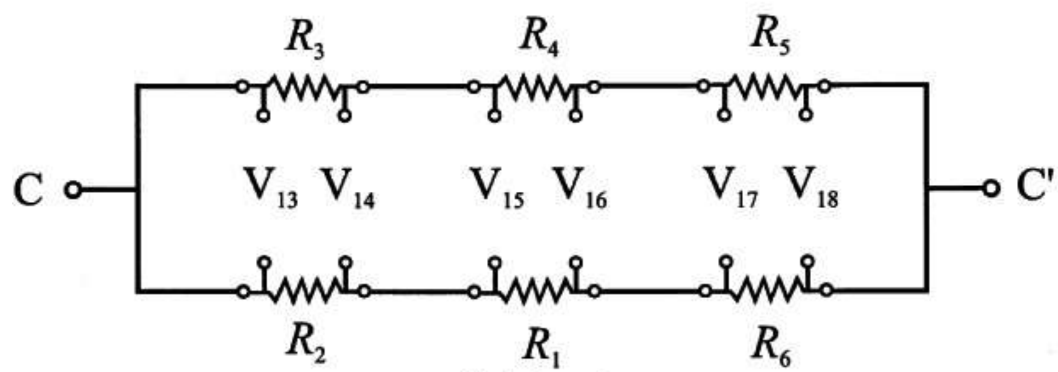
This measurement process is repeated "n + 2" times.



(a) Subset 1



(b) Subset 2



(c) Subset 3

From the "n + 2" subsets of voltage measurements, one obtains a set of "3(n + 2)" linear equations that can be solved.

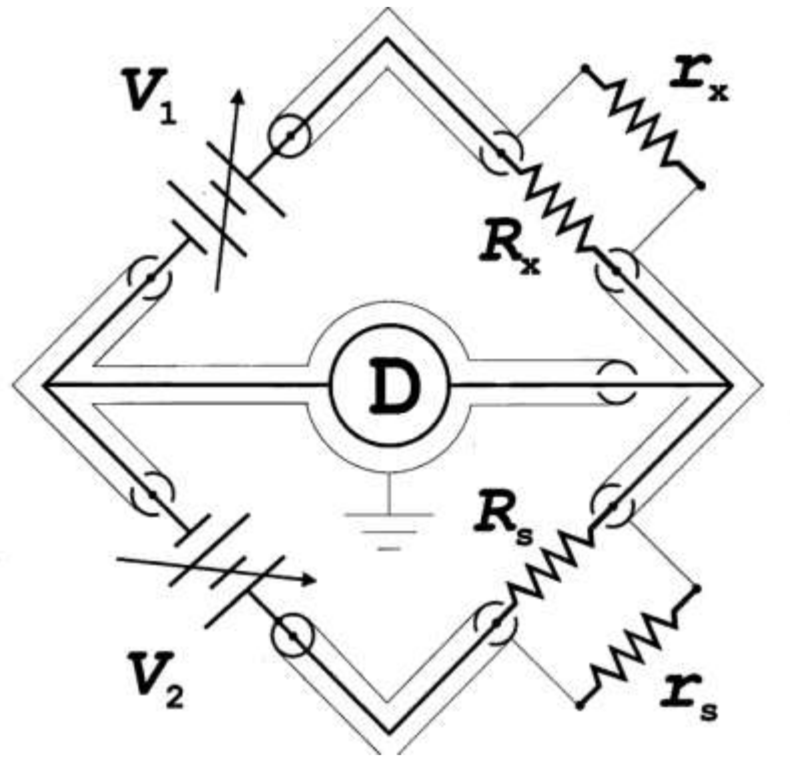
Two of the resistors are working standards (R3 and R6), one is a

check standard (R4), and the remaining three are unknowns or resistors under test (R), R2, and R5).

The three pairs of applied voltage points are designated as AA', BB', and CC'.

Three subsets of voltage measurements are taken to determine the values of the unknown resistors

In **1996**, an **automated guarded bridge** was developed for calibrating multi megohm standard resistors from **10** MΩ to **100** TΩ. Two of the ratio arms are replaced by programmable voltage



The outputs of sources V_1 and V_2 drive bridge resistors R_x and R_s and guard resistors r_x and r_s

Multiple ratios up to 1000/1 can be selected by adjusting the outputs of the voltage sources.

An electrometer with a resolution of ±3 fA in the current mode is used as the detector to measure the difference in currents ΔI flowing through R_x and R_s.

Initial voltage sources are set tc $E_{1\mathrm{Est}}$ an $\dot{E}_2$

Estimated output of source V_1 required to drive the bridge to null can be calculated using $E'_{1\mathrm{Est}} = [\Delta I + E_2/R_{\mathrm{S}}]R_{\mathrm{X}}$

Source V_1 then set t $E'_{1\mathrm{Est}}$

This reduces ΔI to lower value $\Delta I'$

A linear fit is then applied to determine exact setting of V_1 to reach a null based on two iterations of $E_{1\mathrm{Est}}$ and ΔI

$$E_1 = [\Delta I \cdot E'_{1\mathrm{Est}} - \Delta I' \cdot E_{1\mathrm{Est}}]/[\Delta I - \Delta I'].$$

Unknown value of R_x can then be solved by equation

$$E_1/E_2 = R_\mathrm{X}/R_\mathrm{S}$$

The circuit we now know as the Wheatstone Bridge was actually first described by Samuel Hunter Christie in 1833.

British scientist, physicist and mathematician.

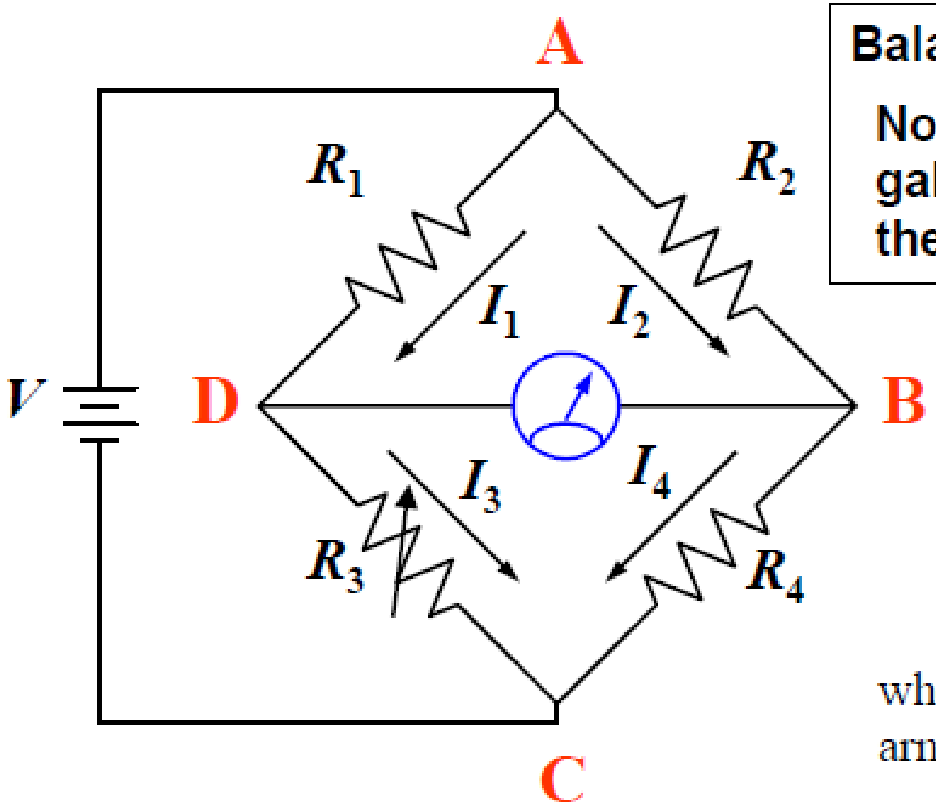
In 1833 Samuel Christie proposed a method for measuring unknown resistance in electric circuits.
He called it the **diamond method** because of the shape of electrical circuit in his experiment.

However, Sir Charles Wheatstone invented many uses for this circuit once he found the description in 1843. As a result, this circuit is known generally as the **Wheatstone Bridge**.

Sir **Charles Wheatstone** was an English physicist

Inventor of first commercially successful telegraph, English concertina, stereoscope, and the Playfair cipher.

Wheatstone Bridge



Balance condition:

No potential difference across the galvanometer (there is no current through the galvanometer)

Under this condition: $V_{AD} = V_{AB}$

$$I_1 R_1 = I_2 R_2$$

And also $V_{DC} = V_{BC}$

$$I_3 R_3 = I_4 R_4$$

where I_1, I_2, I_3, and I_4 are current in resistance arms respectively, since $I_1 = I_3$ and $I_2 = I_4$

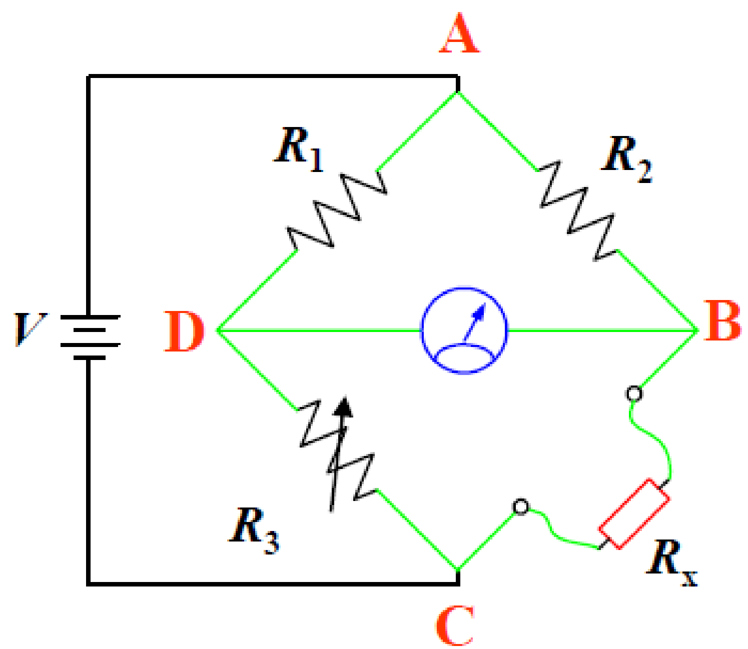
$$\frac{R_1}{R_3} = \frac{R_2}{R_4} \quad \text{or} \quad \boxed{R_x = R_4 = R_3 \frac{R_2}{R_1}}$$

Measurement Errors

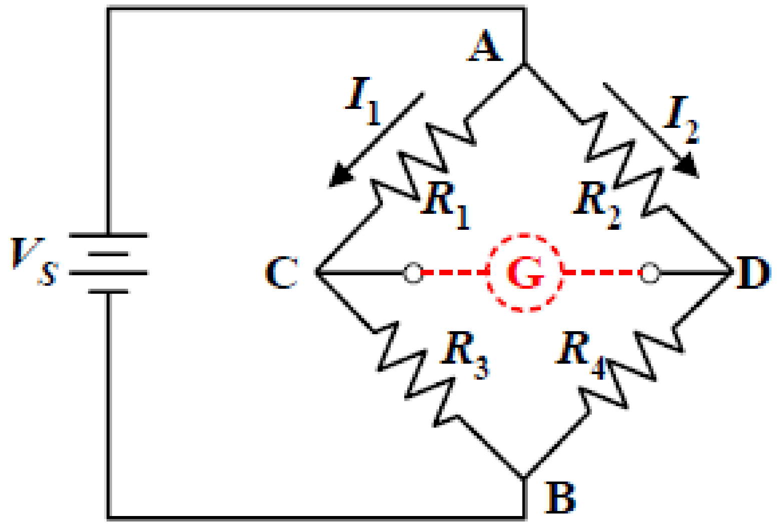
1. Limiting error of the known resistors

$$R_x = (R_3 \pm \Delta R_3)\left(\frac{R_2 \pm \Delta R_2}{R_1 \pm \Delta R_1}\right) \qquad R_x = R_3 \frac{R_2}{R_1}\left(1 \pm \frac{\Delta R_1}{R_1} \pm \frac{\Delta R_2}{R_2} \pm \frac{\Delta R_3}{R_3}\right)$$

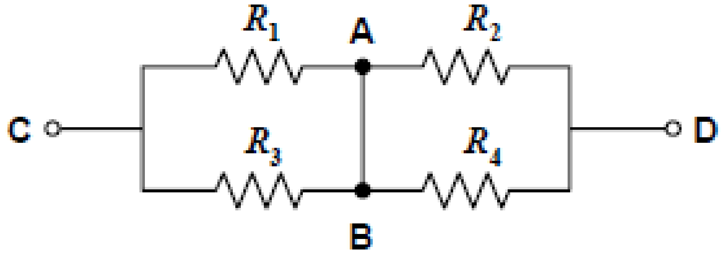
2. Insufficient sensitivity of Detector
3. Changes in resistance of the bridge arms due to the heating effect.
4. Thermal emf or contact potential in the bridge circuit.
5. Error due to the lead connection

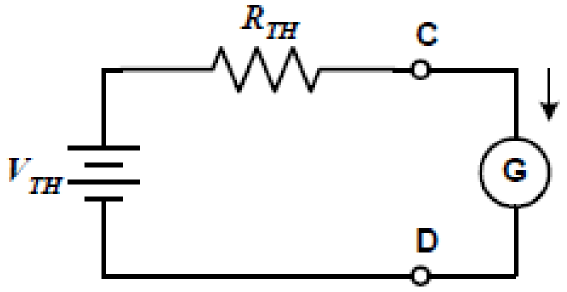


Consider a bridge circuit under a small unbalance condition



Thévenin Equivalent Circuit



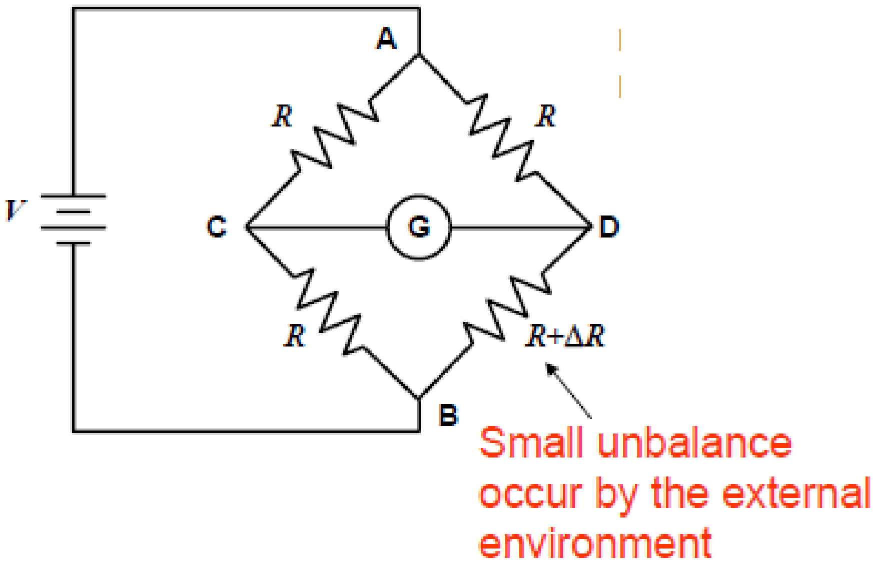


$$R_{TH} = R_1 // R_3 + R_2 // R_4$$

$$I_g = \frac{V_{TH}}{R_{TH} + R_g}$$

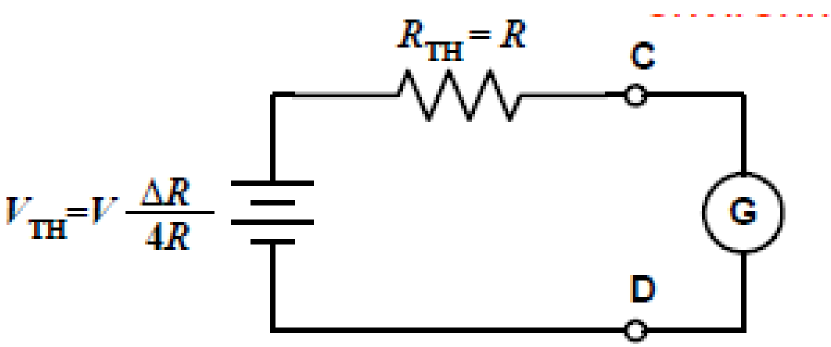
$$V_{TH} = V_{CD} = V\left(\frac{R_1}{R_1 + R_3} - \frac{R_2}{R_2 + R_4}\right)$$

Unbalance bridge



$$V_{TH} = V_{CD} \approx V\frac{\Delta R}{4R}$$

$$R_{TH} \approx R$$



$$I_g = V\frac{\Delta R}{4R(R + R_g)}$$

Sensitivity of Galvanometer

The amount of deflection depends on the sensitivity of the galvanometer.

This sensitivity can be expressed as amount of deflection per unit current.

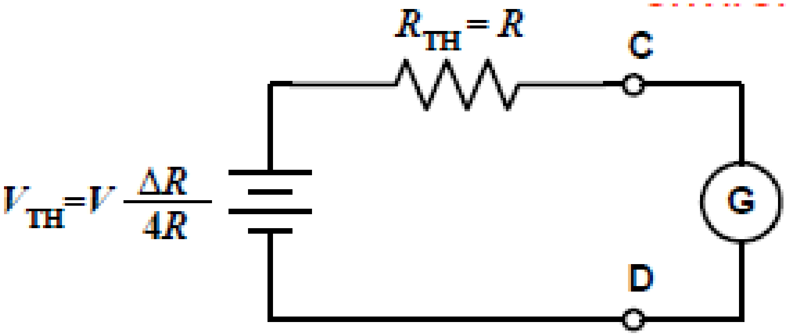
$$Sensitivity = \frac{deflection}{current}$$

Another way of representing the galvanometer sensitivity is the amount of deflection per unit voltage across the galvanometer. This is called voltage sensitivity of the galvanometer

$$Sensitivity = \frac{deflection}{voltage\ across\ galvanometer}$$

If bridge is unbalanced an emf appears across galvanometer which is basically equal to the Thevenin equivalent voltage between the galvanometer nodes.

$$\theta = S_V V_{Th} = S_V V \frac{\Delta R}{4R}$$



θ is the galvanometer deflection and S_V is voltage sensitivity of the galvanometer.

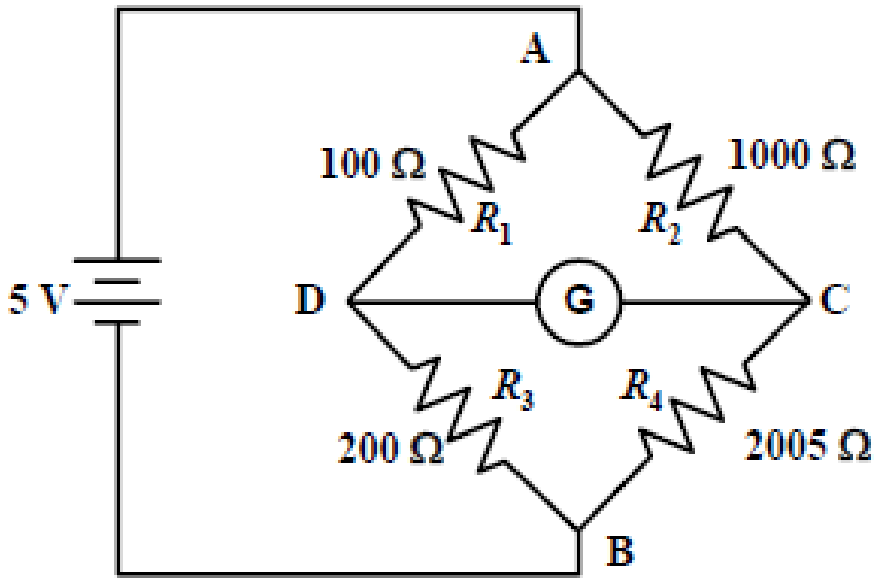
The current sensitivity of galvanometer is expressed as

$$S_V = \frac{S_i}{R_{Th} + R_g}$$

The **bridge sensitivity** is defined as the amount of deflection of the galvanometer per unit fractional change in the unknown resistance.

$$\boldsymbol{S_B = \frac{\theta}{\Delta R/R}}$$

$$\boldsymbol{S_B = S_V \frac{V_{Th}}{\Delta R/R} = S_V V \frac{\Delta R/4R}{\Delta R/R} = \frac{S_V V}{4}}$$



Thévenin Voltage

The galvanometer current

Thévenin Resistance