DSE CLASS

CONDENSED MATTER PHYSICS

Lecture-4

27/09/2020

The London equations

in 1935, shortly after the discovery that magnetic fields are expelled from superconductors two brothers Fritz and Heinz London proposed equations which are consistent with the Meissner effect and can be used with Maxwell's equations to predict how the magnetic field and surface current vary with distance from the surface of a superconductor.

10. Fritz London and Heinz London

The equation is based on **two-fluid model**.

This model considers the free electrons within a superconductor as two *distinct, noninteracting* fluids.

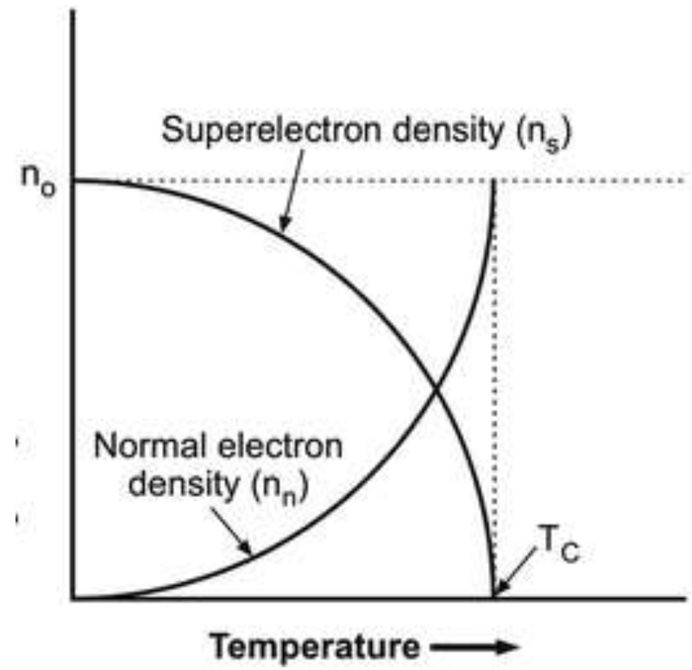
One fluid consists of '**normal**' electrons, number density n_n, and other is '**superconducting**' electrons, or **superelectrons**, which form a fluid with number density n_s.

the total carrier density, $n_o = n_s + n_n$

$n_o = n_s$ at T=0

$n_o = n_n$ at T>Tc

$J=Jn+Js,$



n_s density of the superfluid component of velocity $\mathbf{v}_s$
n_n density of the normal component of velocity $\mathbf{v}_n$

‘**normal**’ electrons behave in exactly the same way as the free electrons in a normal metal.

They are accelerated by an electric field **E**, but are frequently scattered by impurities and defects in the ion lattice and by thermal vibrations of the lattice.

The current density $\mathbf{J}_n$ due to flow of these electrons is

$$\mathbf{J}_n = -n_n e \langle \mathbf{v}_n \rangle = \frac{n_n e^2 \tau}{m} \mathbf{E}$$

The **superconducting electrons** are not scattered by impurities, defects or thermal vibrations, so they are freely accelerated by an electric field.

If the velocity of a superconducting electron is $\mathbf{v}_s$, then its equation of motion is

$$m\frac{d\mathbf{v}_s}{dt} = -e\mathbf{E}.$$

Combining this with the expression for the current density, $J_s = -n_s e\mathbf{v}_s$, we find that

$$\frac{\partial \mathbf{J}_s}{\partial t} = -n_s e\frac{\partial \mathbf{v}_s}{\partial t} = \frac{n_s e^2}{m}\mathbf{E}.$$

This is London's 1st equation

$$\frac{\partial \vec{J}_s}{\partial t} = \frac{n_s e^2}{m} \vec{E}$$ superelectrons

$$\vec{J}_n = \sigma_n \vec{E}$$ normal electrons

Scattering of the normal electrons leads to a constant current in a constant electric field,

whereas the absence of scattering of the electrons in a superconductor cause steady increase in current density at constant electric field.

However, if we consider a constant current flowing in the superconductor, then

$$\frac{\partial \vec{J}_s}{\partial t} = 0, \text{ so } \mathbf{E} = \mathbf{0}.$$

Therefore the normal current density J_n must be zero — all of the steady current in a superconductor is carried by the superconducting electrons.

Of course, with no electric field within the superconductor, there will be no potential difference across it, and so it has zero resistance.

Maxwell–Faraday equation

$$\bar{\nabla} \times \bar{E} = -\frac{\partial \vec{B}}{\partial t}$$

Replacing E form London's 1st equation $\frac{\partial \vec{J}_s}{\partial t} = \frac{n_s e^2}{m} \vec{E}$

$$\frac{m}{n_s e^2} \bar{\nabla} \times \frac{\partial \vec{J}_s}{\partial t} = -\frac{\partial \vec{B}}{\partial t}$$

$$\Rightarrow \frac{\partial}{\partial t}\left(\frac{m}{n_s e^2} \bar{\nabla} \times \bar{J}_s + \bar{B} \right) = 0$$

$$\frac{m}{n_s e^2} \bar{\nabla} \times \bar{J}_s + \bar{B} = \text{Constant} \quad = 0$$

This is known as **London's second equation.**

Ampère's law,

$$\bar{\nabla} \times \bar{B} = \mu_0 \vec{J}_s$$

$$\bar{\nabla} \times \bar{\nabla} \times \bar{B} = \mu_0 \ \bar{\nabla} \times \vec{J}_s$$

$$\bar{\nabla} \times \bar{\nabla} \times \bar{B} = \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \bar{B}) -: \nabla^2 \bar{B} = -: \nabla^2 \bar{B}$$

again

$$\frac{m}{n_s e^2} \bar{\nabla} \times \bar{J}_s + \bar{B} = 0$$

$$-: \nabla^2 \bar{B} = -\frac{\mu_0 n_s e^2}{m} \mathbf{B}$$

Thus $$\nabla^2 \bar{B} - \frac{\mu_0 n_s e^2}{m} \bar{B} = 0$$

$$\nabla^2 \bar{B} - \frac{1}{\lambda^2}\mathbf{B} = 0$$

where

$$\lambda = \left(\frac{m}{\mu_0 n_s e^2} \right)^{1/2}.$$

London's penetration depth

PENETRATION DEPTH

Magnetic fields are expelled from the interior of a type I superconductor by the formation of surface currents.

In reality, they penetrate the surface to a small extent.

Solving the equation $$\nabla^2 \bar{B} - \frac{1}{\lambda^2}\mathbf{B} = 0$$

we get

$$B(x) = B_0 e^{-x/\lambda}$$

where external magnetic field is parallel to the surface of the sample.

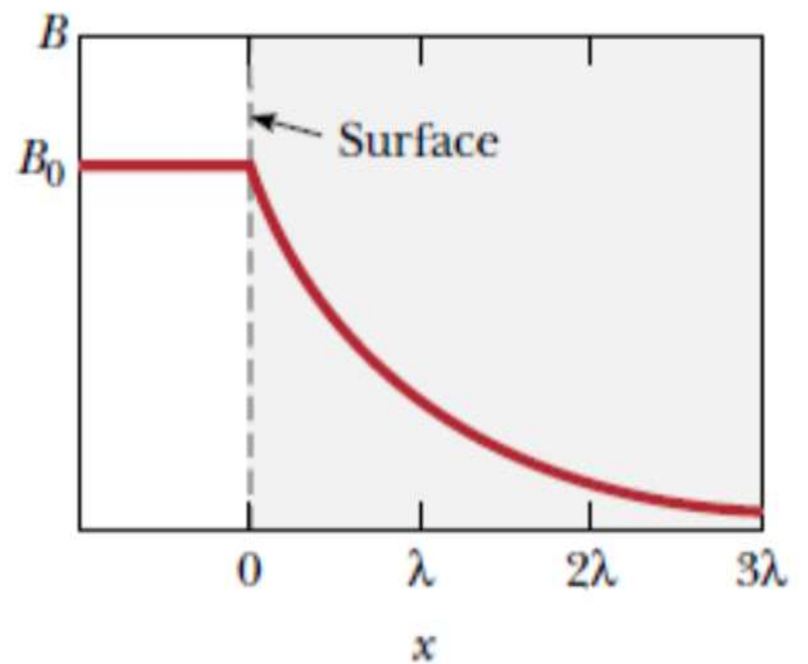
Thus the London equations lead to the prediction of an exponential decay of the magnetic field within the superconductor

The variation of magnetic field with distance inside a type I superconductor

Within this thin layer, which is about 100 nm thick, the magnetic field B decreases exponentially from its external value to zero, according to the expression

B0 is the value of the magnetic field at the surface.

x is the distance from the surface to some interior point, and is a parameter called the penetration depth λ.



Assuming all of the free electrons are superconducting if we consider a typical free electron density in a metal

$n_s = 10^{29} m^{-3}$,

then estimated value of λ will be 20 nm.

The number density of superconducting electrons depends on temperature, so the penetration depth is temperature dependent. For $T \ll T_c$

, all of the free electrons are superconducting, but the number density falls steadily with increasing temperature until it reaches zero at the critical temperature.

Since $\lambda \propto n_s^{-1/2}$ according to the London model, the penetration depth increases as the temperature approaches the critical temperature.

Temperature dependence of penetration depth

$$\lambda(T) = \frac{\lambda(0)}{\left[1 - (T/T_c)^4\right]^{1/2}},$$

where $\lambda(0)$ is the value of the penetration depth at $T = 0$ K.