DSE CLASS

CONDENSED MATTER PHYSICS

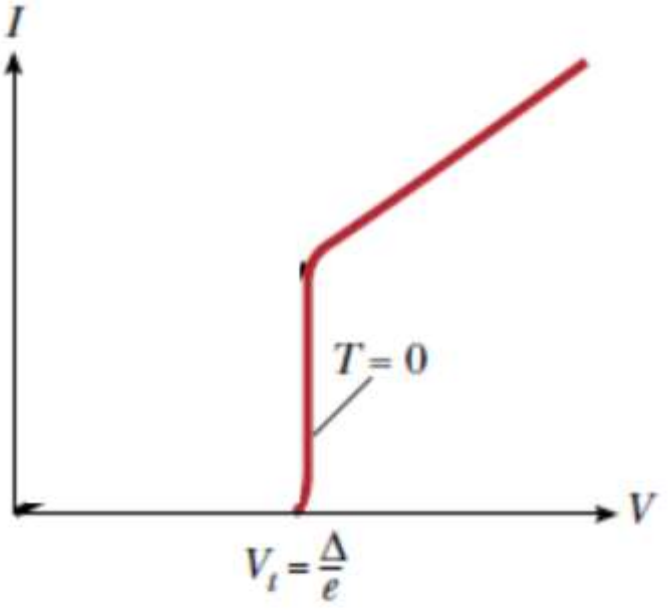
Lecture-9

10/12/2020

- The passage of electrons through a thin (<50 A) insulating barrier is a well-known example of quantum mechanical tunneling.
- The current-voltage (I-V) characteristic of such a barrier is ohmic (linear) at low bias.
- In accordance with the Pauli exclusion principle, the current is proportional to the number of electron states per unit energy in the conductors on either side of the barrier.

Giaever discovered that if the electrode is superconducting the curve becomes highly non-linear, with the current nearly zero for voltages up to V = 2Δ/e.

Above V = 2Δ/e, the current becomes linear.



Single electrons not available for the tunneling process, hence, I = 0.

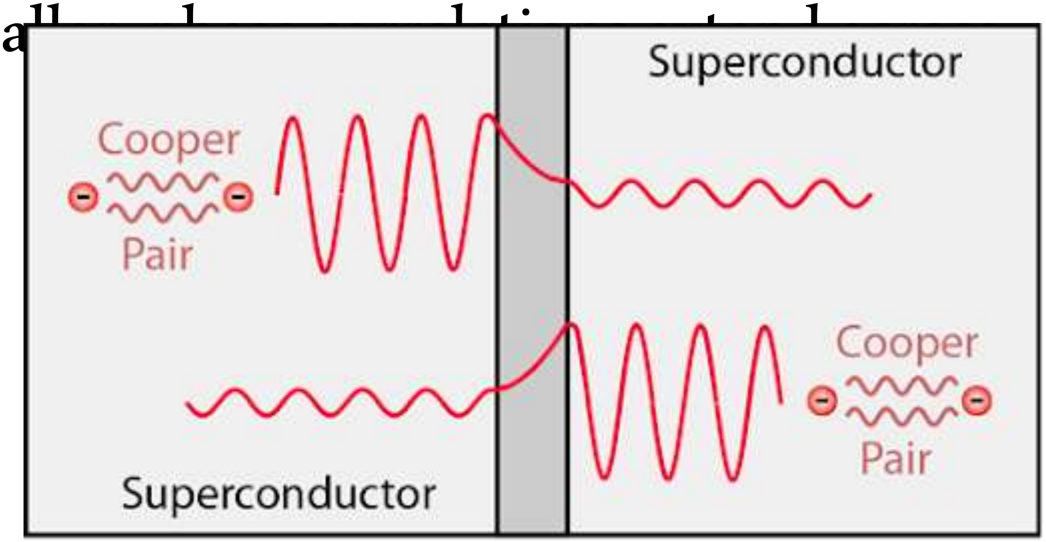
As voltage is raised to the energy gap, pairs are broken up into normal single electrons, which exhibit ohmic tunneling

The theoretical prediction of Josephson is that not only quasiparticles tunnel can through insulating barrier, but the Cooper pairs can do so.

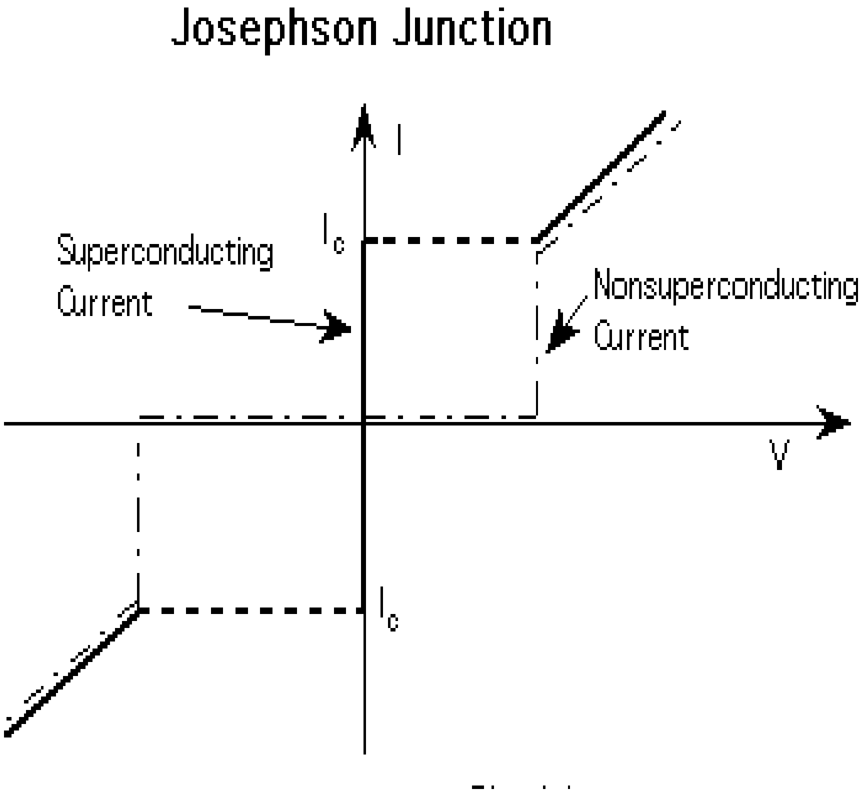
The condensed state of the Cooper pair bosons in each superconductor can be described by a wave function with a single phase.

As the barrier becomes sma

the insulating space.



- The flow of current between the superconductors in the absence of an applied voltage is called a *Josephson current*,
- the movement of cooper pairs across the barrier is known as *Josephson tunneling.*



dc Josephson current

$$J = J_o sin(\theta_1 - \theta_2)\ or\ J = J_o sin(\delta)$$

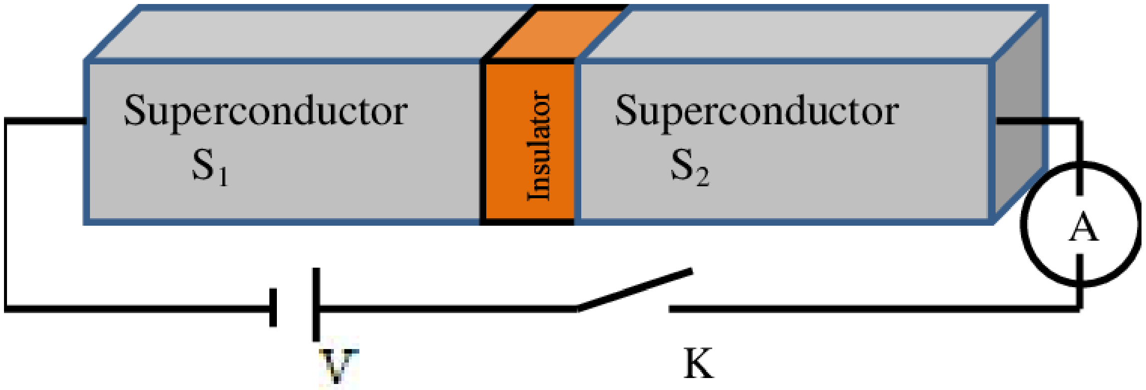
Josephson voltage

$$\frac{d\delta}{dt} = \frac{2eV}{\hbar}$$

AC Josephson Effect

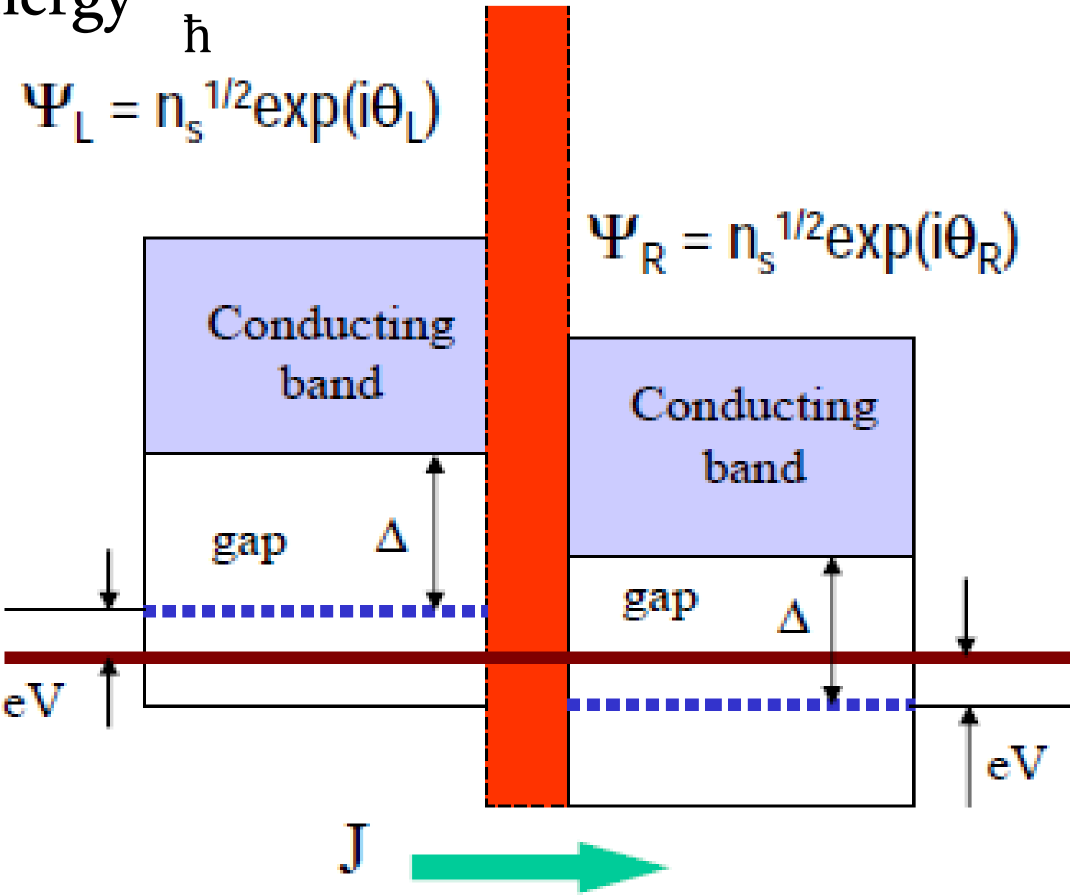
When a dc voltage V is applied across the Josephson junction, :

the dc voltage generates an alternating current I



Cooper pairs experience a potential energy difference of $qV=-2eV$

while passing through the barrier, which corresponds to a photon energy $\frac{2eV}{\hbar}$



Then wave function at left and right edge of the insulator

$$\psi_1 = \sqrt{n_1}e^{i\left(\theta_1+\frac{2eVt}{\hbar}\right)}\ and\ \psi_2 = \sqrt{n_2}e^{i(\theta_2)}$$

If we consider both superconductors to be of same material then

$$n_1 = n_2 = n_s$$

$$\psi_1 = \sqrt{n_s}e^{i\left(\theta_1+\frac{2eVt}{\hbar}\right)}\ and\ \psi_2 = \sqrt{n_s}e^{i(\theta_2)}$$

Now apply boundary condition as before and obtain expressions for A and B

$$\psi\left(-\frac{d}{2}\right) = \sqrt{n_1}e^{i\left(\theta_1+\frac{2eVt}{\hbar}\right)}$$

$$and\ \psi\left(+\frac{d}{2}\right) = \sqrt{n_2}e^{i(\theta_2)}$$

So the current density obtained

$$J = J_o sin\left(\theta_1 - \theta_2 - \frac{2eV}{\hbar}t\right) = J_o\ sin\left(\frac{2eV}{\hbar}t + \delta\right)$$

Hence supercurrent now varies sinusoidally with angular frequency $\frac{2eV}{\hbar}$.

This is a.c. Josephson effect.

It is used to determine $\frac{e}{\hbar}$ value from voltage and frequency of the

emitted radiation when cooper pair crosses the barrier.

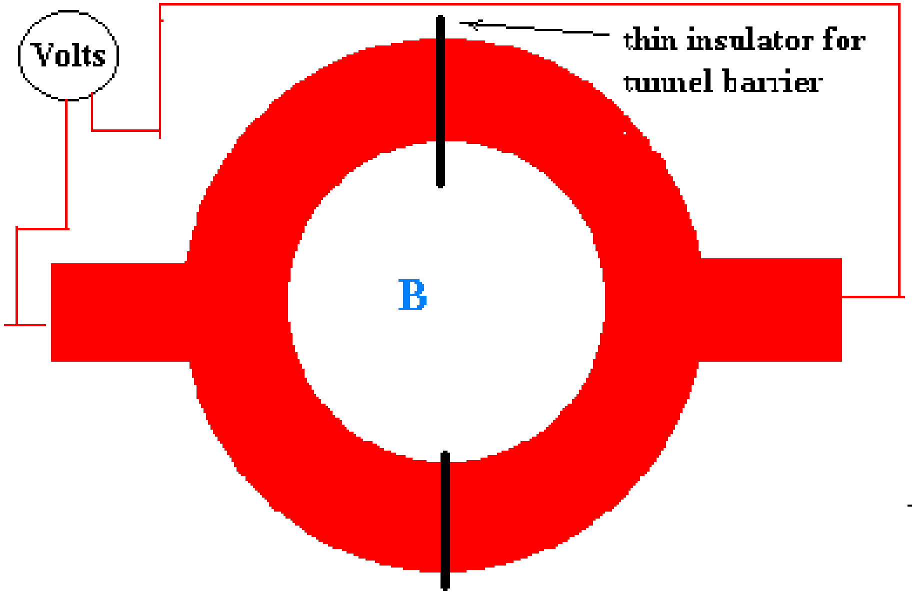
For V= 1 μV., $\omega \cong 3 \times \frac{10^9\,rad}{}$, $f \cong 500\ MHz$

sine term is immensely large and time-averages to zero.

Thus the tunneling supercurrent is zero if V≠0

Superconducting Quantum Interference Device

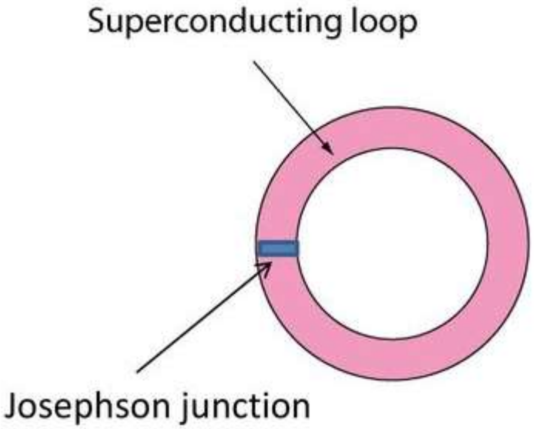
or *SQUID*



We have equations that describe just one Josephson junction, Here we need to apply these to a loop with two Josephson Junctions in a magnetic field.

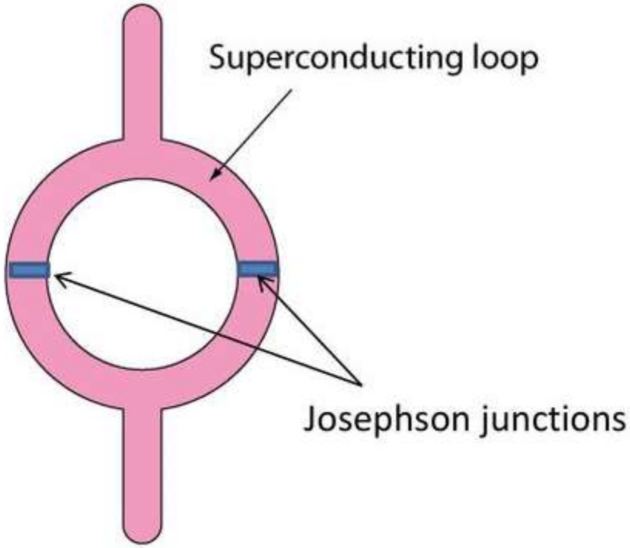
One can determine the flux present in the loop between the junctions with extreme precision

Superconducting loop + Josephson junctions



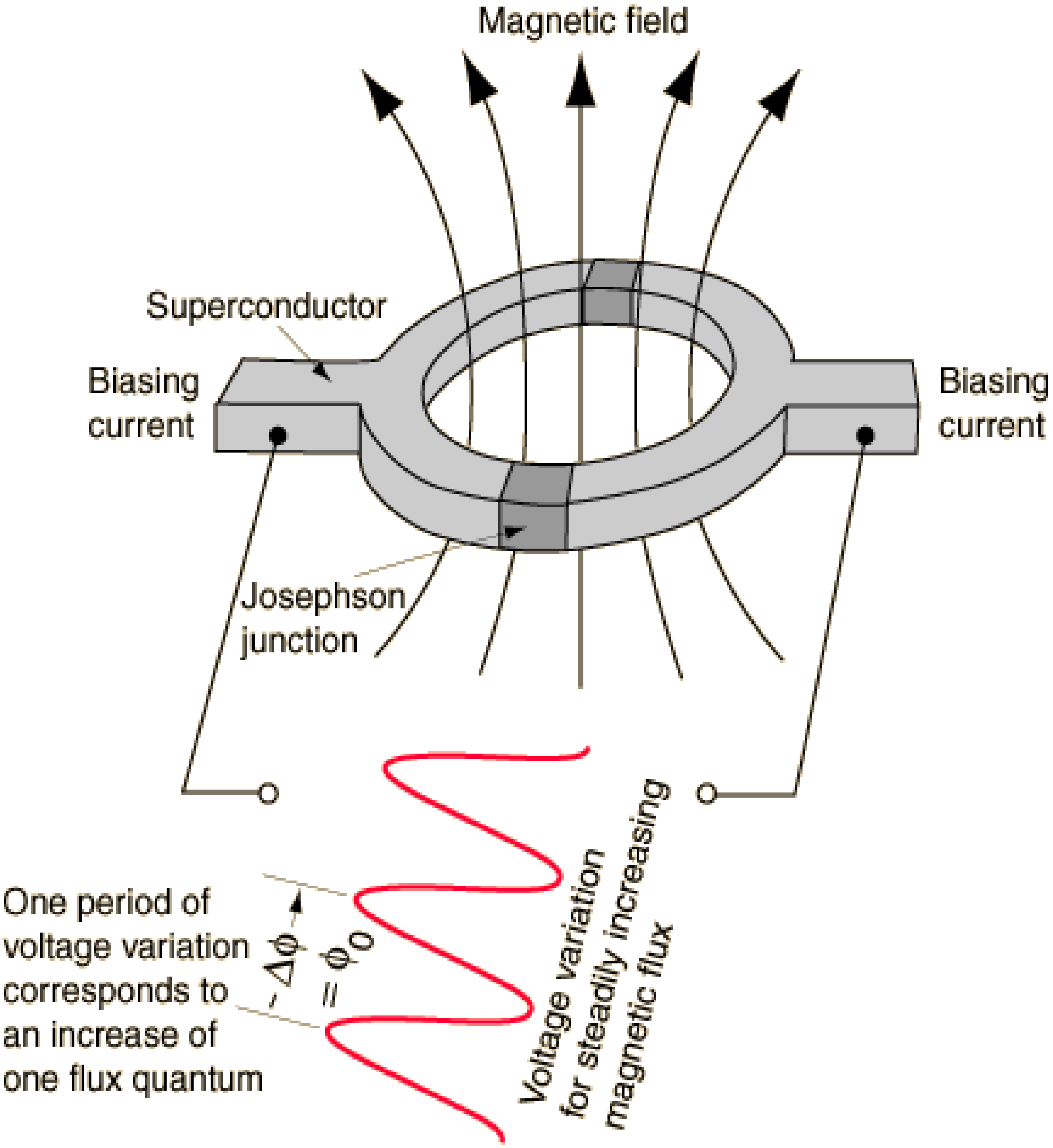
Superconducting loop

Josephson junction



Superconducting loop

Josephson junctions


Magnetic field
Superconductor
Biasing current
Biasing current
Josephson junction
One period of voltage variation corresponds to an increase of one flux quantum
$\Delta\phi = \phi_0$
Voltage variation for steadily increasing magnetic flux

When a Josephson junction is subjected to a magnetic field, the maximum critical current in the junction depends on the magnetic flux through the junction.

The tunneling current under these conditions is predicted to be periodic in the number of flux quanta through the junction.

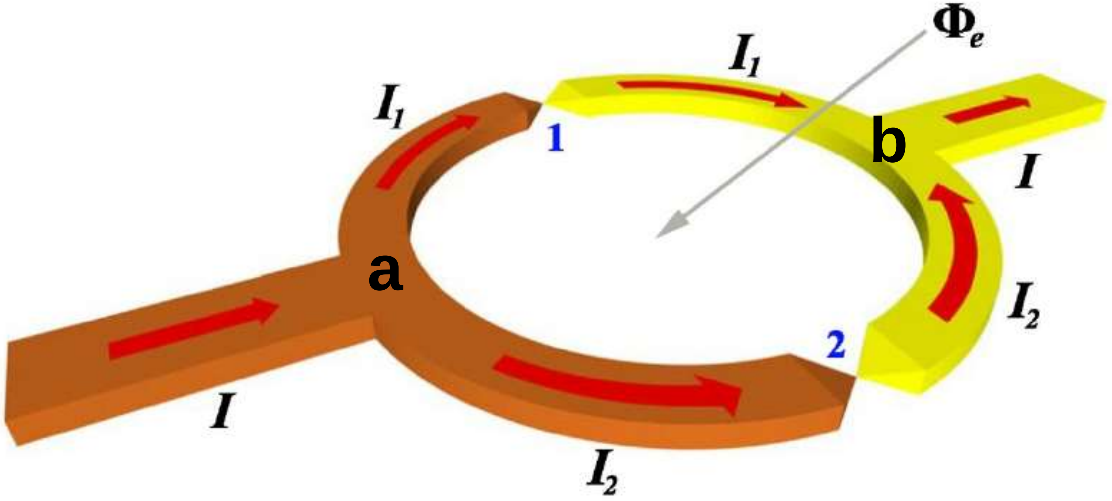
The probability current in the electromagnetic field is given by

$$J = -\frac{i\hbar q}{2M}(\psi^{*}\nabla\psi - \psi\nabla\psi^{*}) - \frac{q^{2}A}{M}\psi^{*}\psi$$

For Cooper pair q=-2e, M=2m and $\psi = \sqrt{n_s}e^{i(\theta)}$

probability current now can be written as

$$J = -\left[\frac{\hbar e}{m}\nabla\theta + \frac{2e^{2}A}{m}\right]n_p$$



Perform line intergral along path 1

$$\int_a^b J.dl = -n_p\left[\frac{\hbar e}{m}\int_a^b \nabla\theta.dl + \frac{2e^2}{m}\int_a^b A.dl\right]$$

$$\int_a^b J.dl = -n_p\left[\frac{\hbar e}{m}(\nabla\theta)_1 + \frac{2e^2}{m}\int_a^b A.dl\right]$$

$(\nabla\theta)_1$ is change in phase across junction 1

In a superconductor the current induced by an external magnetic field flows mostly on the surface, so the current density J will be zero at some point in the volume.

So

$$(\nabla\theta)_1 = -\frac{2e}{\hbar}\int_a^b A.dl$$

Similarly

$$(\nabla\theta)_2 = -\frac{2e}{\hbar}\int_a^b A.dl$$

$$(\nabla\theta)_2 - (\nabla\theta)_1 = \frac{2e}{\hbar}\oint A.dl = \frac{2e}{\hbar}\varphi$$

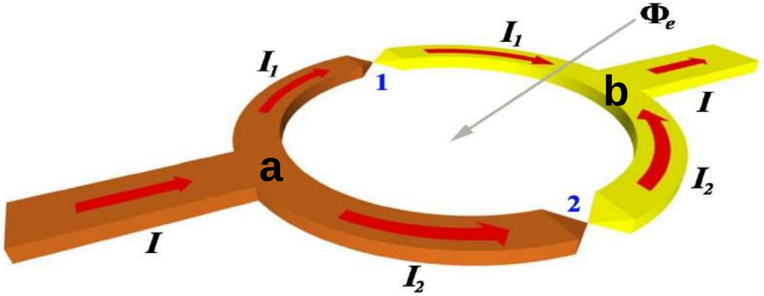
where φ is the magnetic flux

$$(\nabla\theta)_1 = \theta_0 - \frac{e}{\hbar}\varphi \qquad (\nabla\theta)_2 = \theta_0 + \frac{e}{\hbar}\varphi$$

The current density in each of the two branches.

$$J_1 = J_0 sin\left[\theta_0 - \frac{e}{\hbar}\varphi\right] \qquad J_2 = J_0 sin\left[\theta_0 + \frac{e}{\hbar}\varphi\right]$$

The current is now the sum of the two branches.



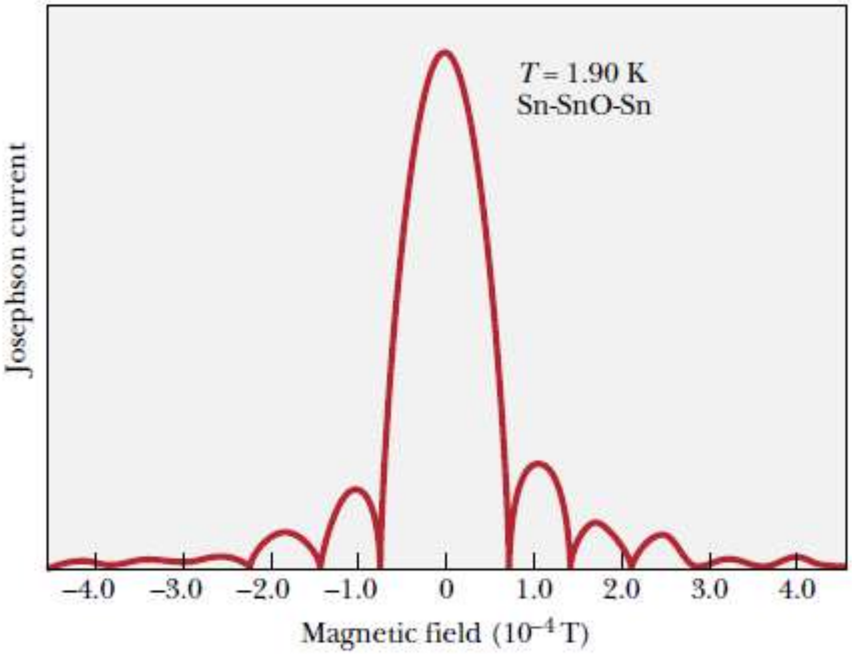
$$I = I_0\left[\sin\left(\theta_0 - \frac{e}{\hbar}\varphi\right) + \sin\left(\theta_0 + \frac{e}{\hbar}\varphi\right)\right] = 2I_0 \sin\theta_0 \cos\frac{e}{\hbar}\varphi$$

I_0 is max supercurrent supported by either junctions.

Current is thus oscillating function of magnetic flux through the loop.

Its magnitude is max when $\frac{e}{\hbar}\varphi$ is multiple of π

ie. φ is multiple of $\frac{\pi\hbar}{e}$.



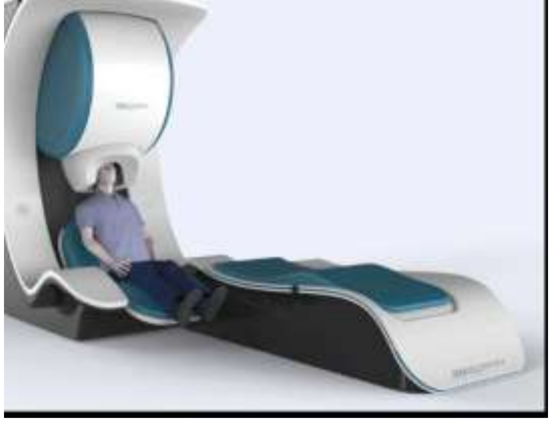
So if a superconducting circuit is constructed with two Josephson junctions in parallel, one can observe interference effects similar to the interference of light waves in Young's double-slit experiment.

This result means that we can determine an unknown magnetic flux by measuring the current.

A typical value for the area of a SQUID used as a magnetometer is A= 1 sq. cm.

In this case a change in the magnetic flux density 10^{-11} T is half period in the current which is easily measurable.

Biomagnetism:



Magnetocardiography (MCG).

Magnetoencephalography (MEG).

Ultra-Low-Field MRI, combined ULF-MRI/MEG.

Super-Paramagnetic Relaxometry for Cancer Diagnostics.

Immunoassay for Diagnosis of Alzheimer’s Disease .

Geophysics

Transient Electromagnetics (TEM) for

Mineral Exploration

Ionospheric Detection

Non-Destructive Testing

Materials Characterization

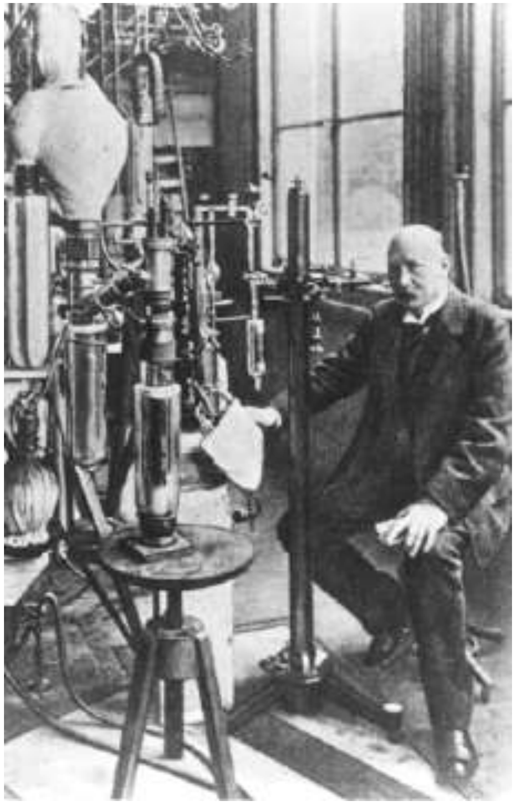
Integrated Circuit Inspection

Defect and Failure Analysis

Astrophysics —Cosmic Microwave Background (CMB) and X-Ray Observatories

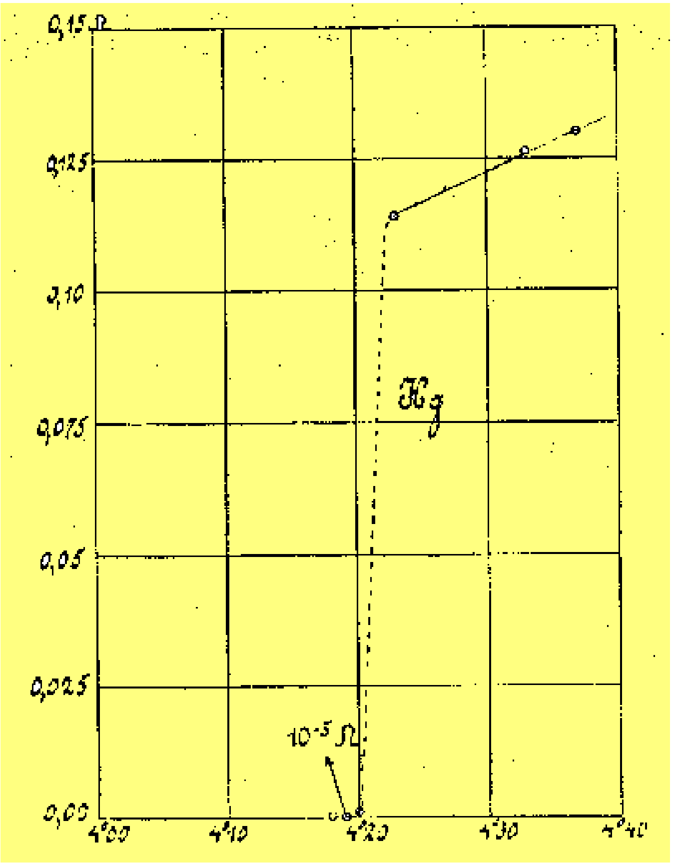
SUMMARY

Superconductivity- discovery



- Liquid Helium (4K) (1908). ***Boiling point*** 4.22K.
- Superconductivity in Hg T_C=4.2K (1911)

H. Kamerlingh Onnes 1913 (Nobel preis 1913)



Resistivity R=0 below T_C; ($R<10^{-23}$ Ω·cm, 10^{18} times smaller than for Cu)

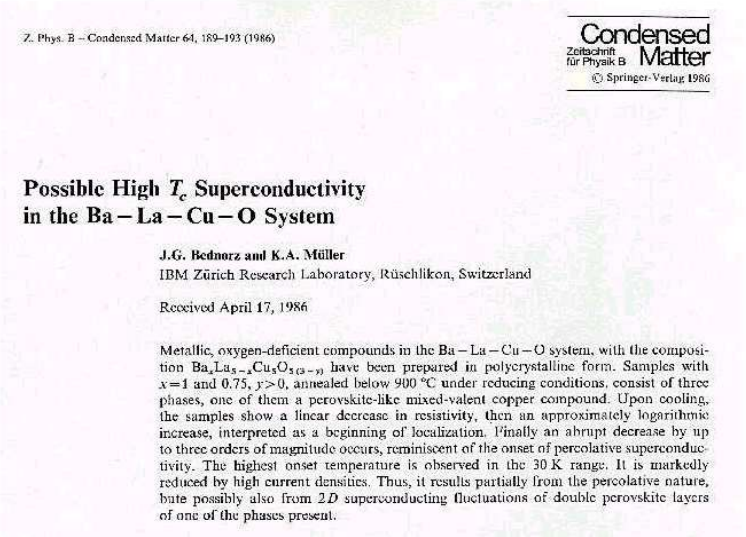
Further discoveries

1911-1986: “Low temperature superconductors” Highest T_C=23K for Nb_3Ge

1986 (January): High Temperature Superconductivity $(LaBa)_2 CuO_4$ T_C=35K

K.A. Müller und G. Bednorz (IBM Rüschlikon) (Nobel preis 1987)

Professor Dr. Dr. h. c. mult. Karl Alex Müller (links) und
Dr. Johannes Georg Bednorz



Z. Phys. B – Condensed Matter 64, 189–193 (1986)

Condensed Matter
Zeitschrift für Physik B
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Possible High T_c Superconductivity in the Ba – La – Cu – O System

J.G. Bednorz and K.A. Müller
IBM Zürich Research Laboratory, Rüschlikon, Switzerland

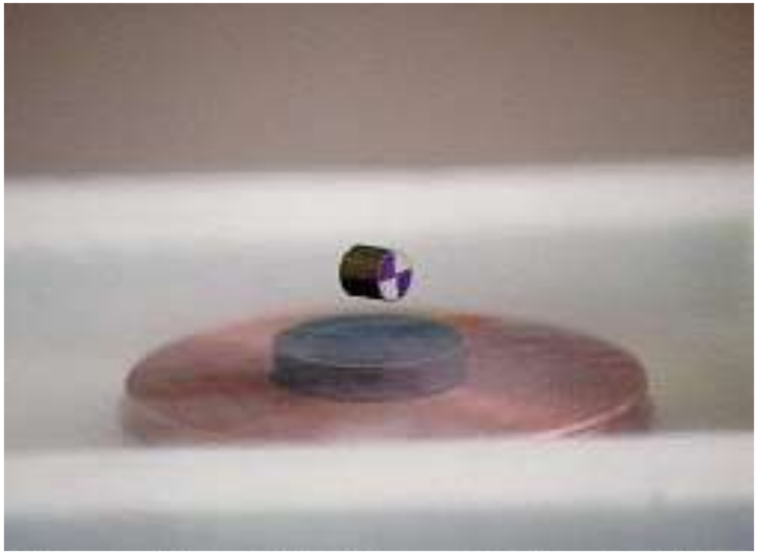
Received April 17, 1986

Metallic, oxygen-deficient compounds in the Ba – La – Cu – O system, with the composition $Ba_xLa_{5-x}Cu_5O_{5(3-y)}$ have been prepared in polycrystalline form. Samples with $x=1$ and 0.75, $y>0$, annealed below 900 °C under reducing conditions, consist of three phases, one of them a perovskite-like mixed-valent copper compound. Upon cooling, the samples show a linear decrease in resistivity, then an approximately logarithmic increase, interpreted as a beginning of localization. Finally an abrupt decrease by up to three orders of magnitude occurs, reminiscent of the onset of percolative superconductivity. The highest onset temperature is observed in the 30 K range. It is markedly reduced by high current densities. Thus, it results partially from the percolative nature, bute possibly also from 2D superconducting fluctuations of double perovskite layers of one of the phases present.

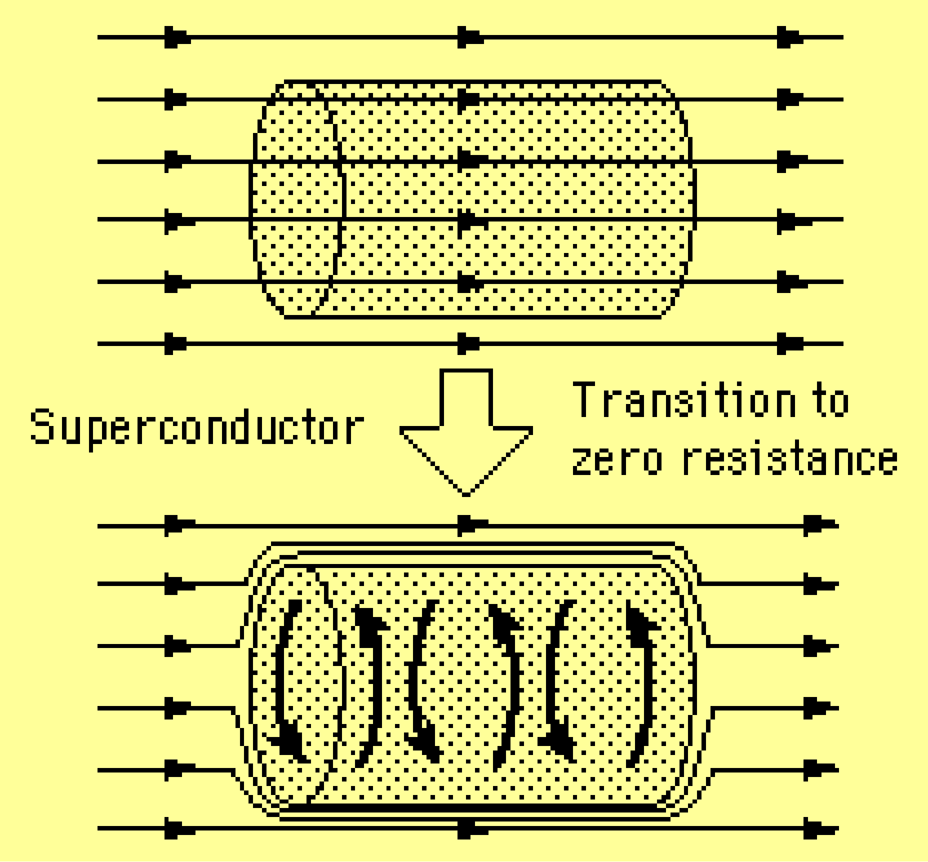
Meissner-Ochsenfeld-effect

A superconductor is a perfect diamagnet. Superconducting material expels magnetic flux from the interior.

W. Meissner, R. Ochsenfeld (1933)

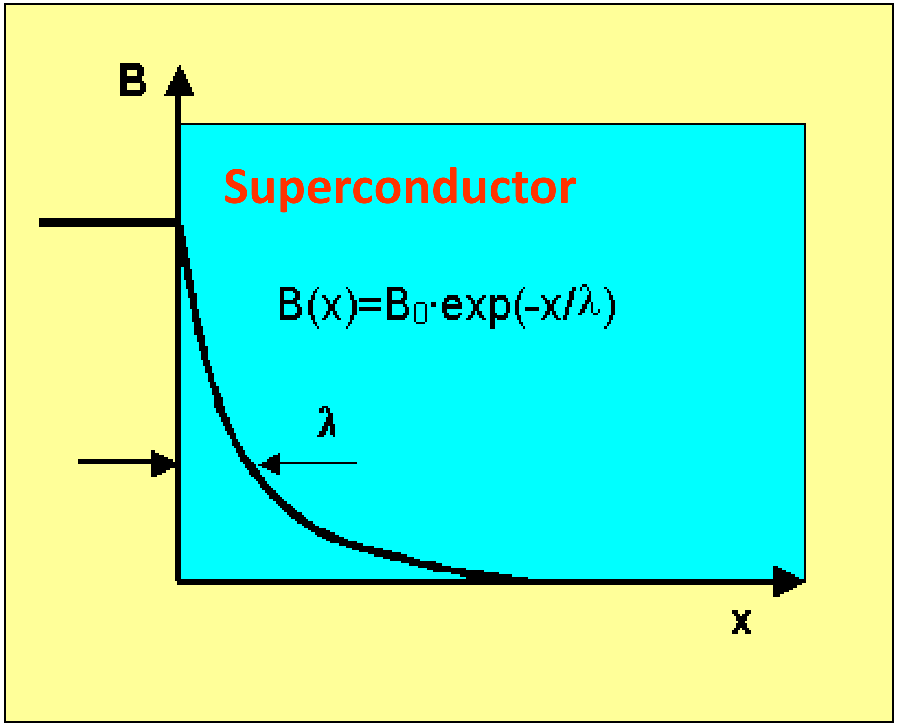


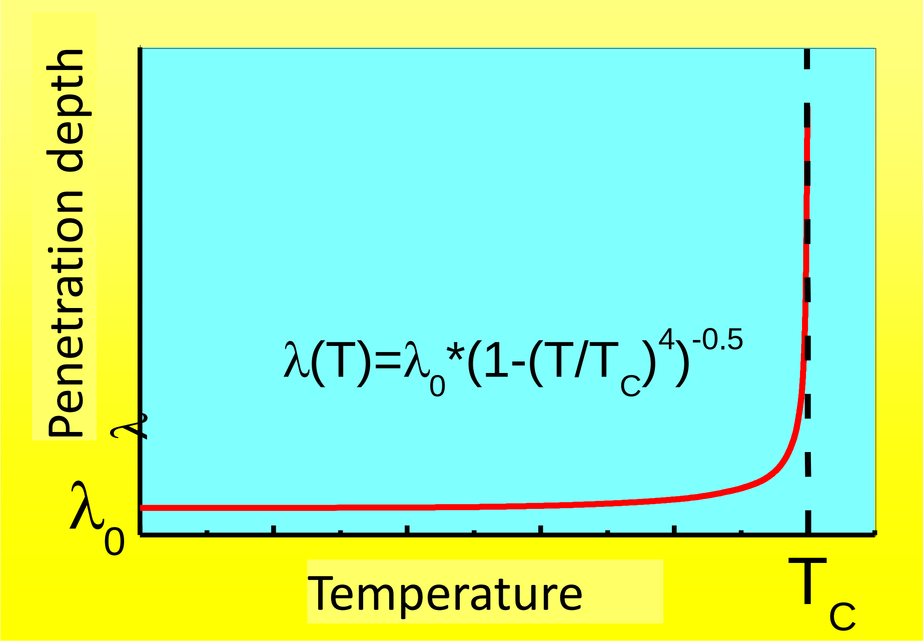
Magnetic levitation



On the surface of a superconductor ($T<T_C$) screening current will be induced. This creates a magnetic field compensating the outside one.

Penetration depth

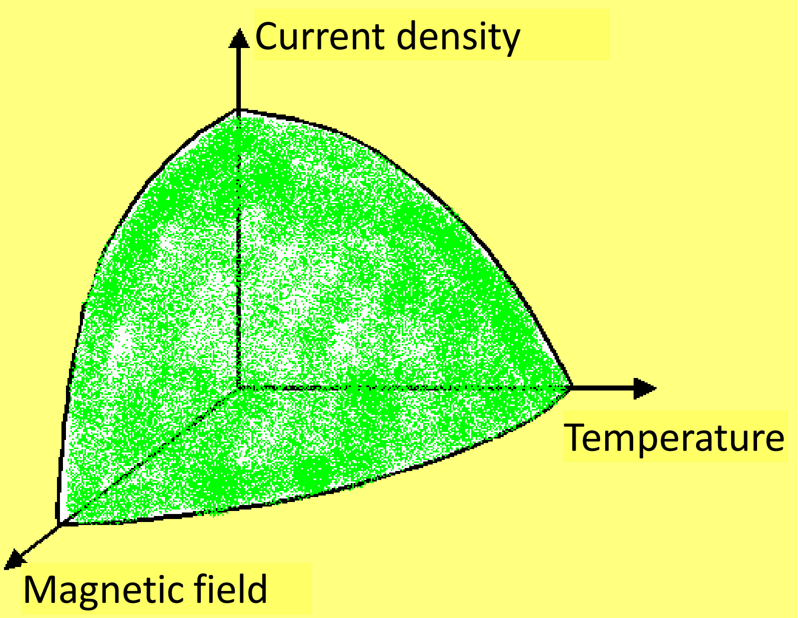




λ depicts the distance where B(x) is e-time smaller than on the surface

What destroys superconductivity?

A current: produces magnetic field which in turn destroys superconductivity.



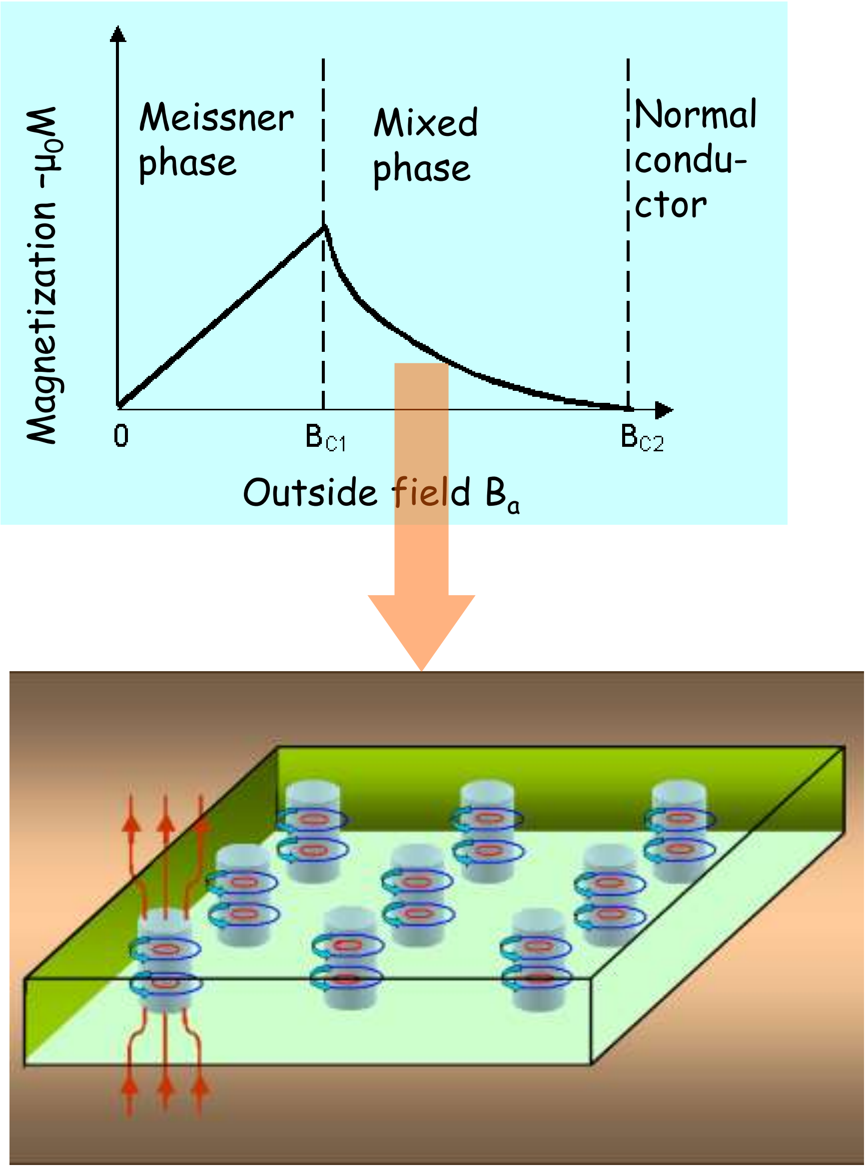
High temperatures: strong thermal vibration of the lattice predominate over the electron-phonon coupling.

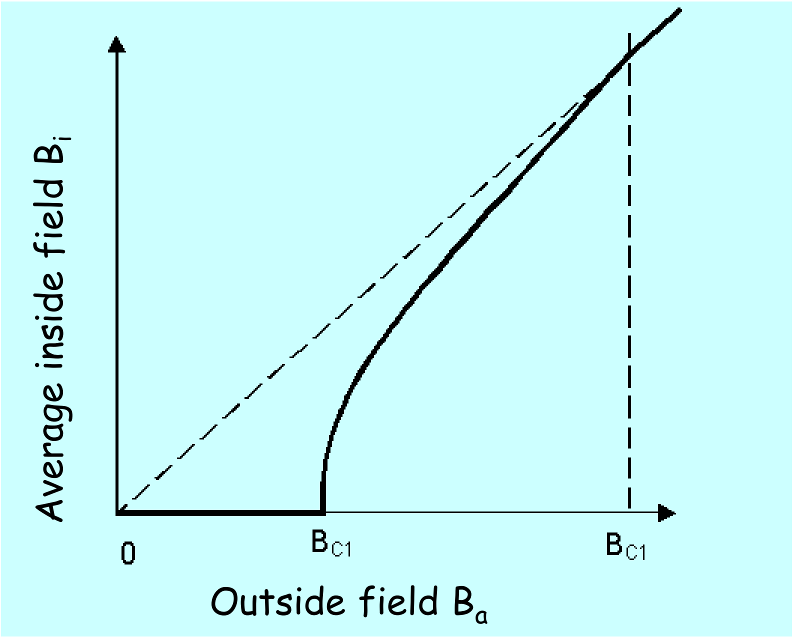
Magnetic field: the spins of the C-P will be directed parallel.

(should be antiparallel in C-P)

Superconductor type II in a magnetic field

$B_i=B_a+\mu_0 M$



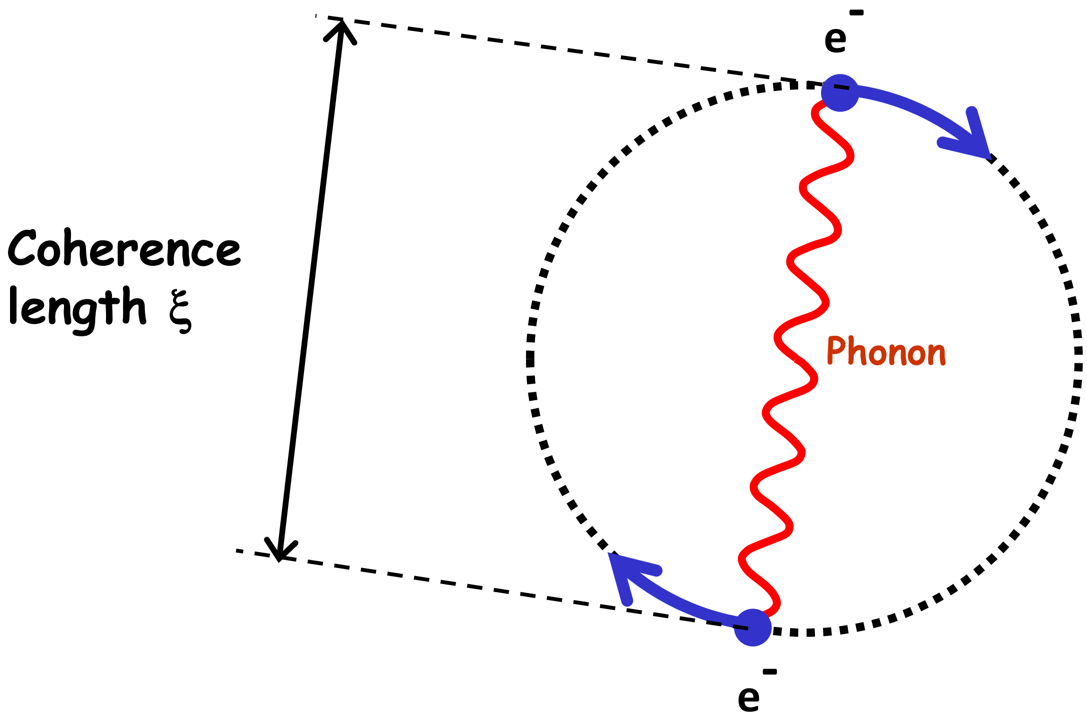


Vortex-lattice in superconductor type II. Magnetic flux of a vortex is quantized:
$\Phi_0=h/2e\cong 2.07\cdot 10^{-15}\,\mathrm{Tm}^2$

John Bardeen, Leon Neil Cooper, John Robert Schrieffer

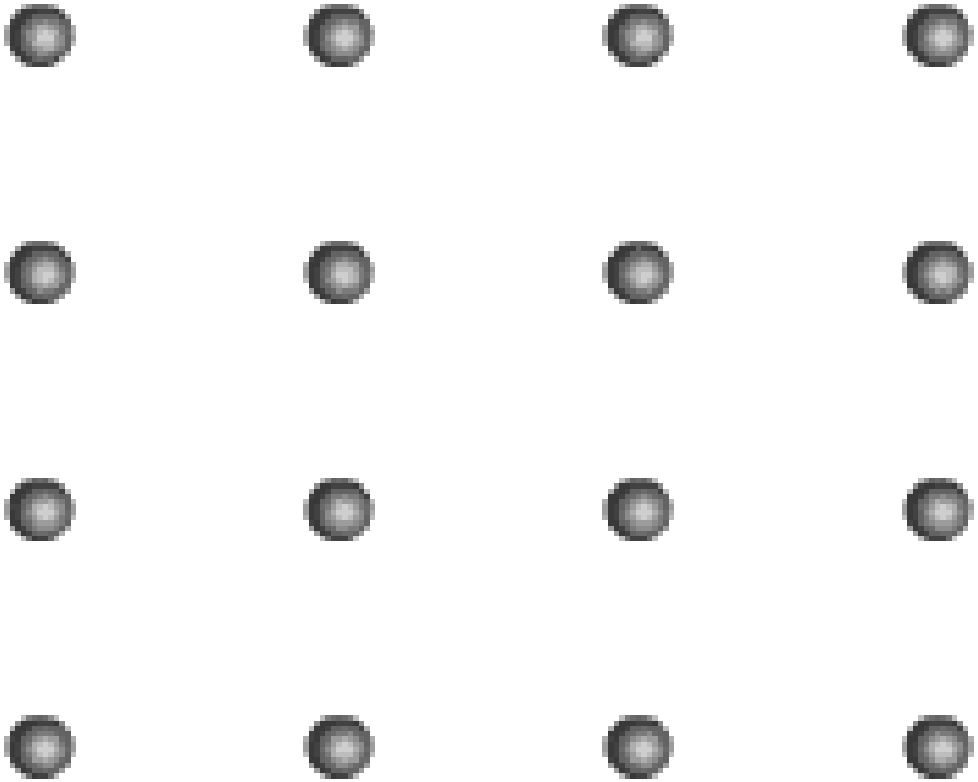
Nobel Prize in Physics 1972

"for their jointly developed theory of superconductivity, called the BCS-theory"



Cooper pair model

Motion of Cooper pairs:



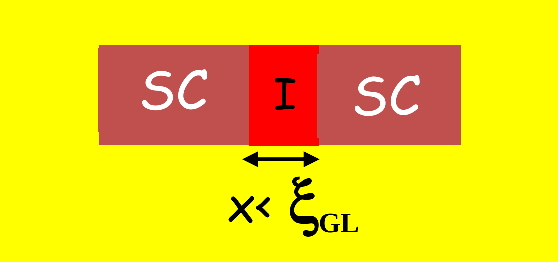
Nobel Prize in Physics 1973

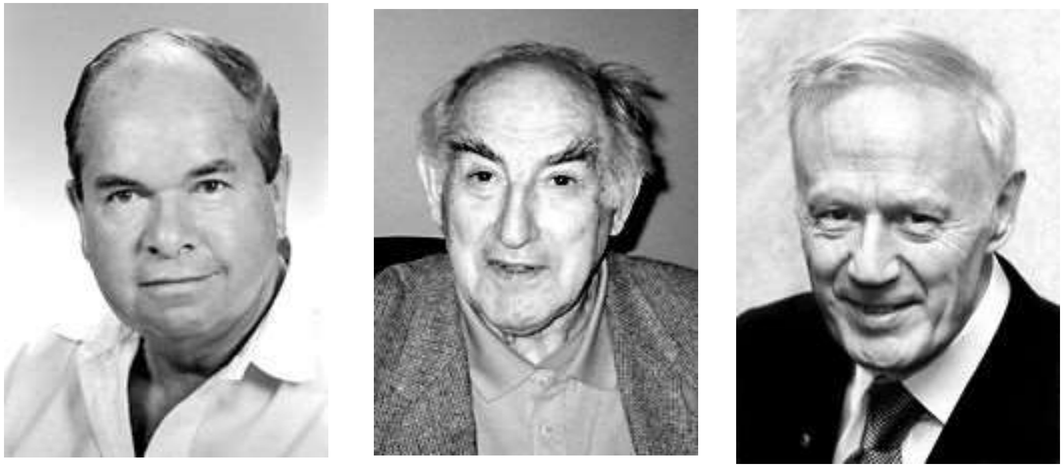
"for his theoretical predictions of the properties of a supercurrent through a tunnel barrier, in particular those phenomena which are generally known as the Josephson effects".

Brian David Josephson

Josephson discovered in 1963 tunnelling effect being 23-years old PhD student

The **superconducting tunnel Josephson) junction** (**superconductor–insulator–superconductor tunnel junction** (SIS) — is an electronic device consisting of two superconductors separated by a very thin layer of insulating material





Alexei A. Abrikosov, Vitaly L. Ginzburg, Anthony J. Leggett

*Nobel Prize in Physics **2003***

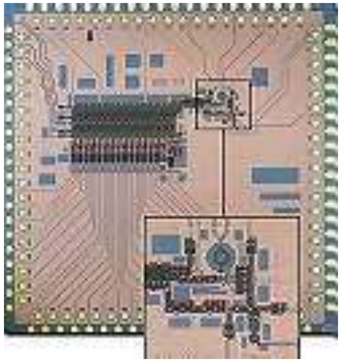
"for pioneering contributions to the theory of superconductors and superfluids".

Applications

- Large distance power transmission ($\rho = 0$)
- Switching device (easy destruction of superconductivity)
- Sensitive electrical equipment (small V variation ➔ large constant current)
- Memory / Storage element (persistent current)
- Highly efficient small sized electrical generator and transformer

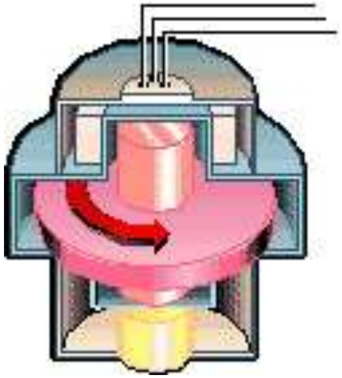
Medical Applications

- NMR – Nuclear Magnetic Resonance – Scanning
- Brain wave activity – brain tumour, defective cells
- Separate damaged cells and healthy cells
- Superconducting solenoids – magneto hydrodynamic power generation – plasma maintenance



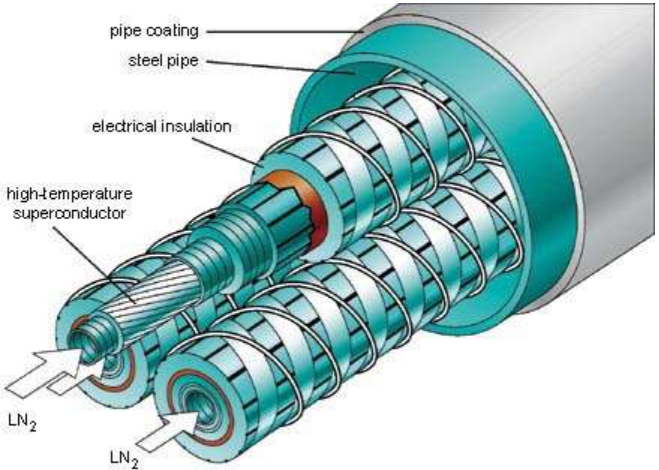
Application. Industry.

Magnetic bearing

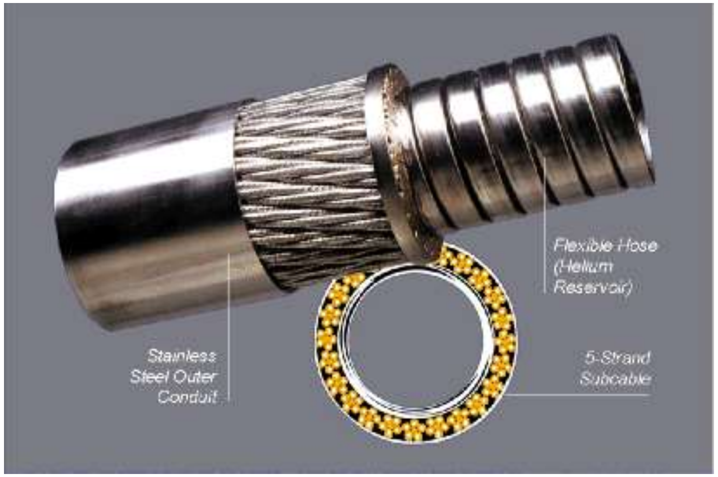


A flywheel in a vacuum chamber – energy accumulator.

MagLev – train (magnetic levitation)

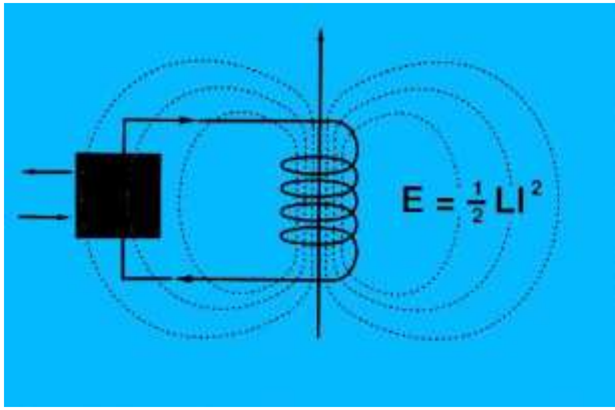


HT$_C$ Cable



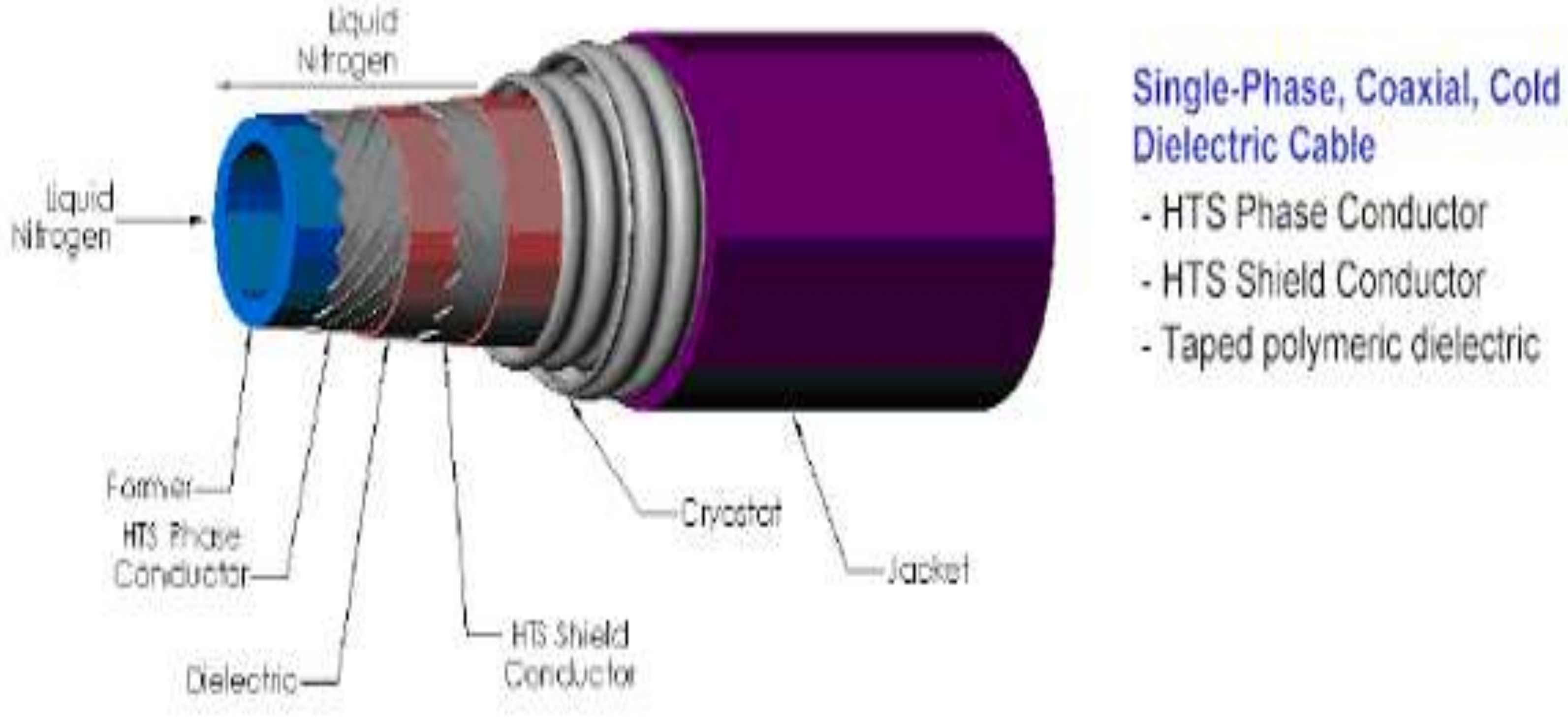
SMES: Superconducting Magnetic Energy Storage

Saves energy in form of magnetic field produced by a superconducting coil.



$$E = \frac{1}{2} L I^2$$

Cable – transmits 3 to 5 times more energy than copper wire

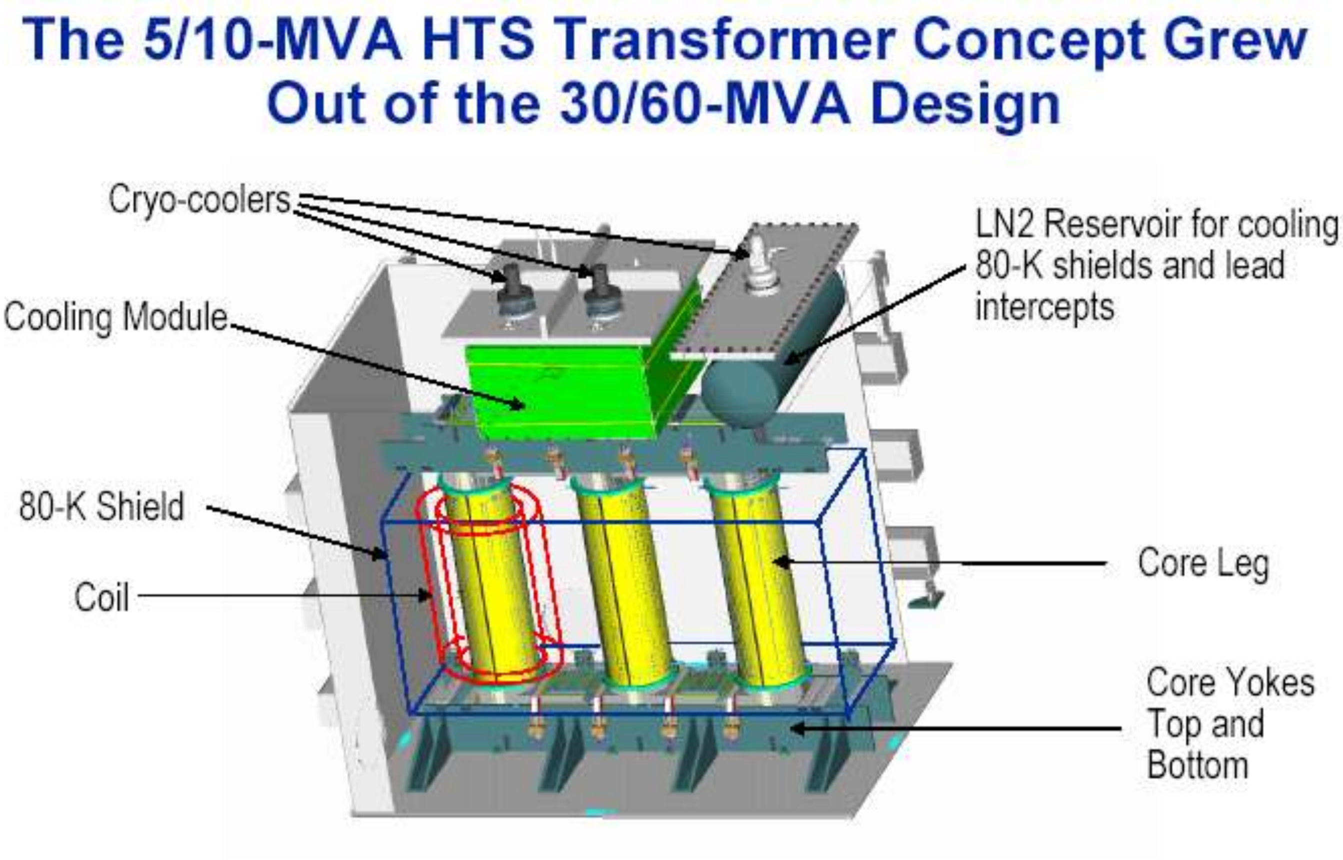


Single-Phase, Coaxial, Cold Dielectric Cable

- HTS Phase Conductor
- HTS Shield Conductor
- Taped polymeric dielectric

Source: Southwire

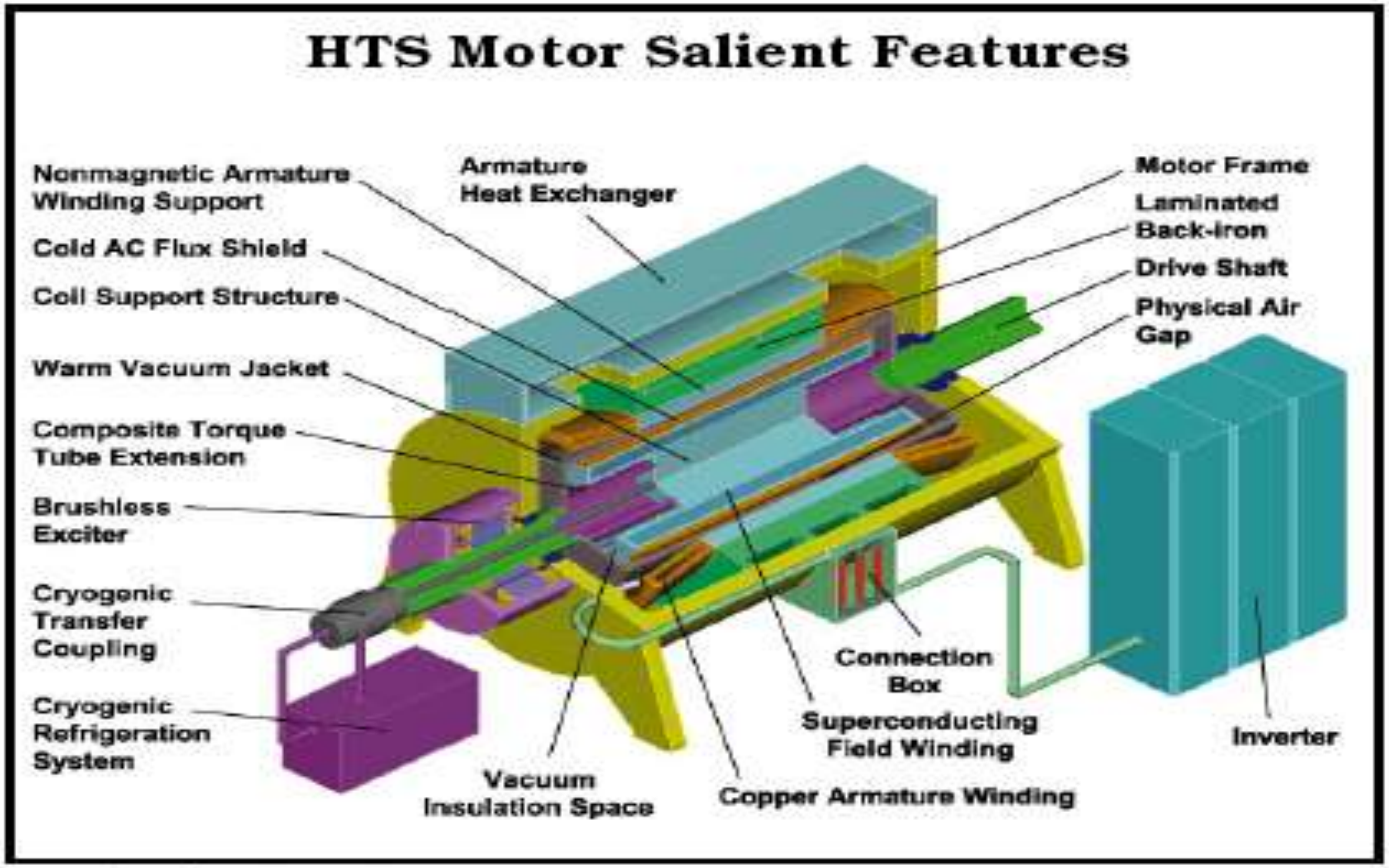
Transformer- 2 times overload capacity without insulation damage and environmentally friendly due to lack of oil used in operation.



The 5/10-MVA HTS Transformer Concept Grew Out of the 30/60-MVA Design

Source: Waukesha Electric Systems

HTS Motor – requires half the space of copper based motors

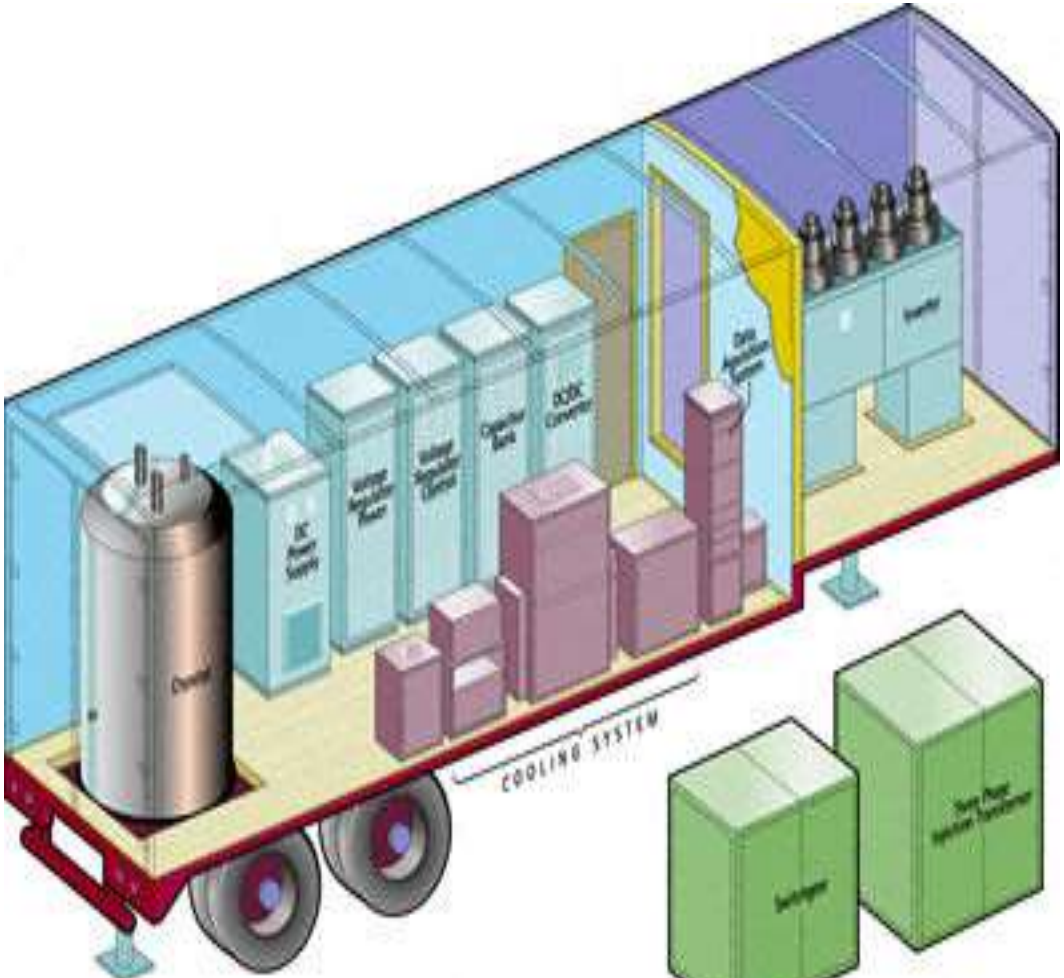


(Rockwell Automation)

Source: Rockwell

SMES

(Superconducting Magnetic Energy Storage)



Source: American Superconductor