DSE CLASS

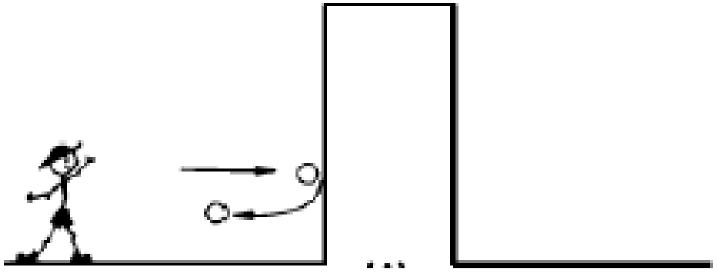
CONDENSED MATTER PHYSICS

Lecture-8

3/12/2020

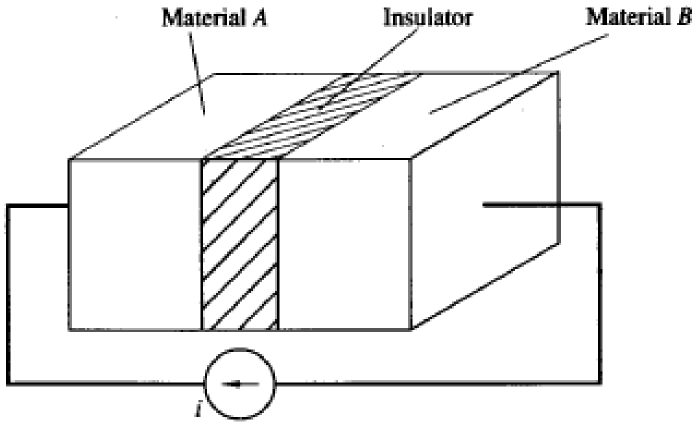
Single-Particle Tunneling

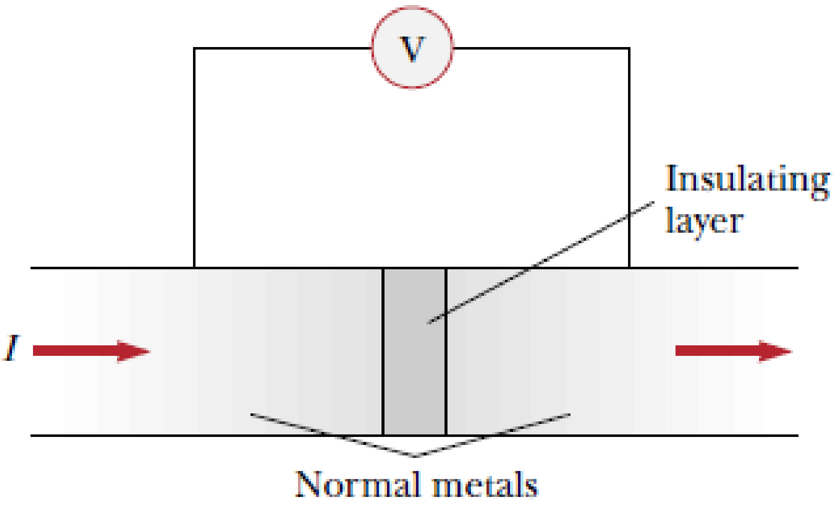
Tunneling is a phenomenon in quantum mechanics that enables a particle to penetrate and go through a barrier even though classically it has insufficient energy to go over the barrier.

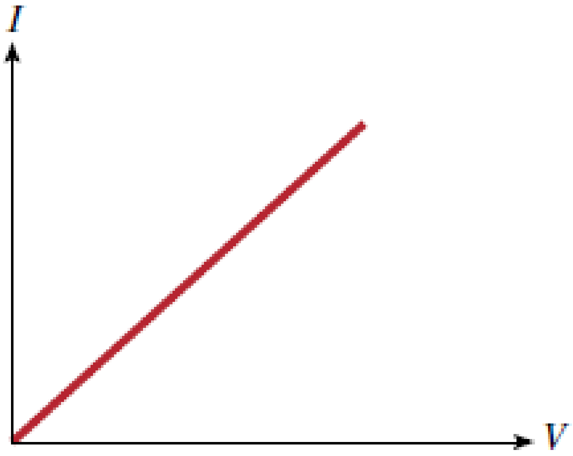


If two metals are separated by an insulator, the insulator normally acts as a barrier to the motion of electrons between the two metals.

However, if the insulator is made sufficiently thin (less than about 2 nm), there is a small probability that electrons will tunnel from one metal to the other.



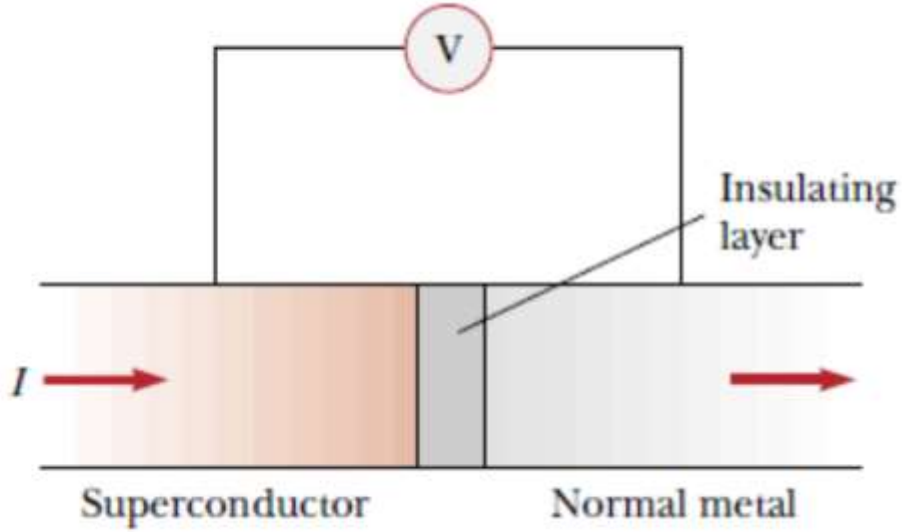




If a potential difference *V* is applied between the two metals, electrons can pass from one metal to the other, and a current is set up.

For small applied voltages, the current-voltage relationship is linear

If one of the metals is replaced by a superconductor and maintained at a temperature below Tc, something quite unusual occurs.

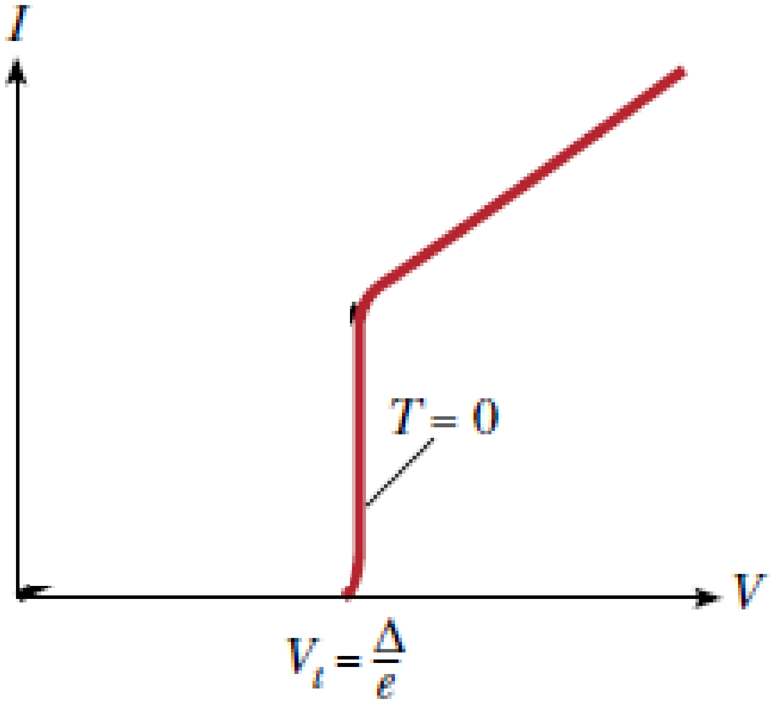


As V increases, no current is observed until V reaches a

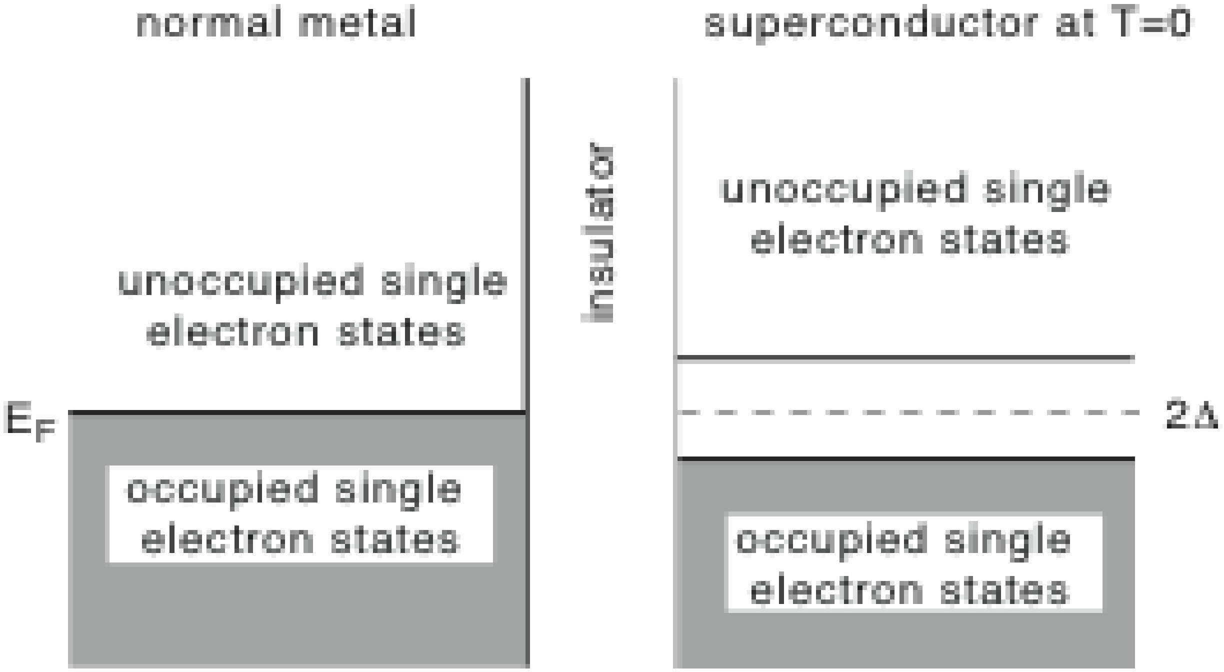
threshold $V_t = E_g/2e = \Delta/e$,

value

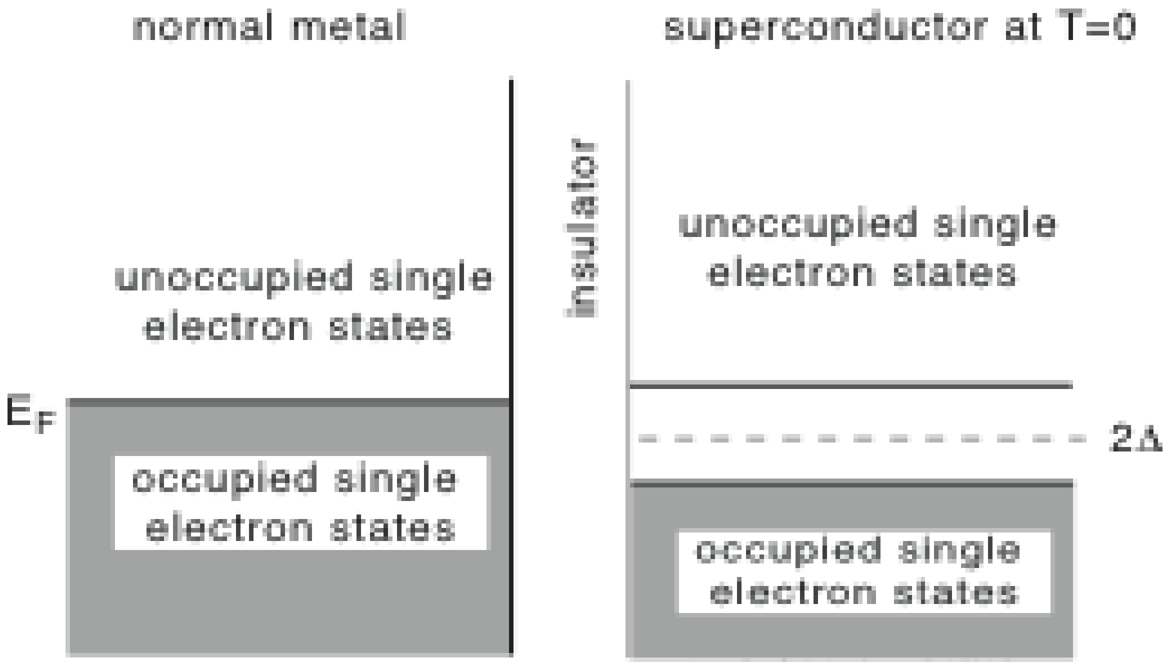
where Δ is half the energy gap.



U=0


normal metal
superconductor at T=0
insulator
unoccupied single electron states
unoccupied single electron states
E_F
2Δ
occupied single electron states
occupied single electron states

$0<U<\Delta/e$



normal metal

superconductor at $T=0$

insulator

unoccupied single electron states

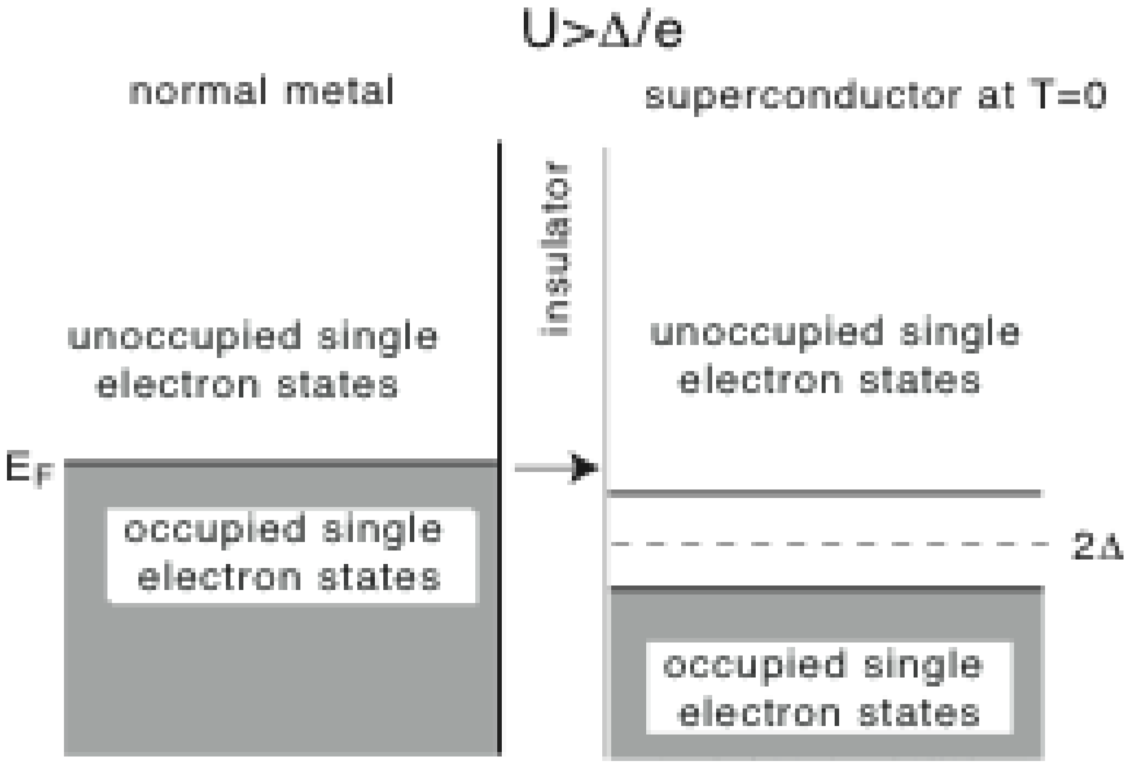
E_F

occupied single electron states

unoccupied single electron states

2Δ

occupied single electron states



$U > \Delta/e$

normal metal

superconductor at $T=0$

insulator

unoccupied single electron states

unoccupied single electron states

E_F

occupied single electron states

2Δ

occupied single electron states

There is another possible process which involves the destruction of a Cooper pair.

The voltage source provides the energy required to break a Cooper pair and free an electron to tunnel.

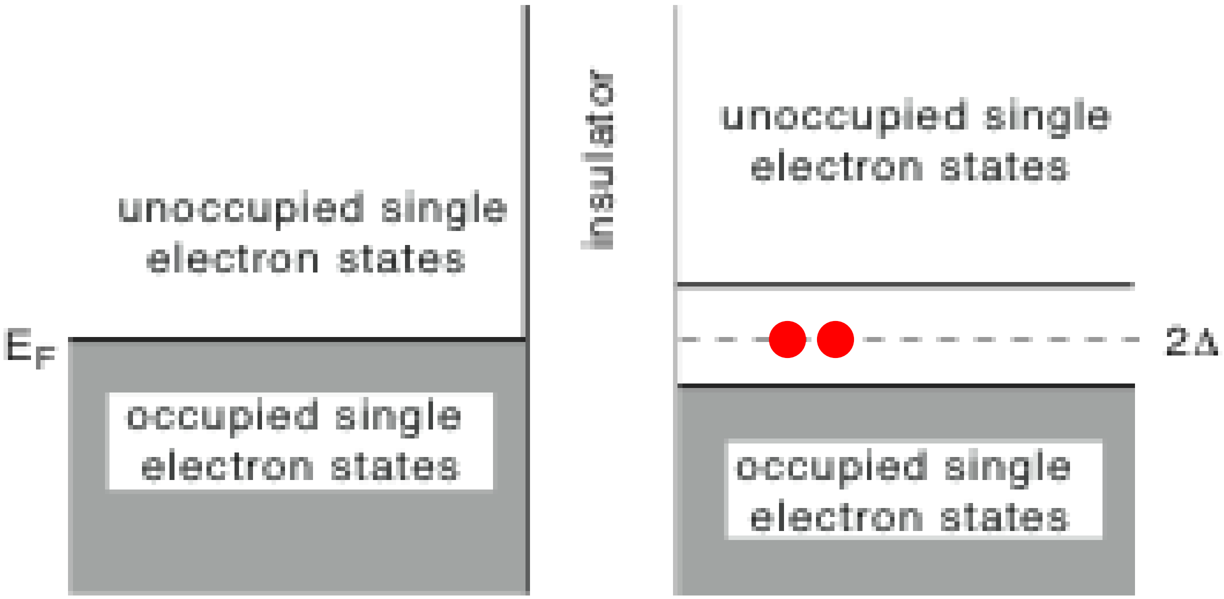
The factor of one half comes from the fact that we are dealing with single-particle tunneling, and the energy required is one-half the binding energy of a pair, 2Δ.

If $eV \geq 0.5Eg$, then tunneling can occur between the normal metal and the superconductor.

U=0

normal metal

superconductor at T=0



insulator

unoccupied single electron states

E_F

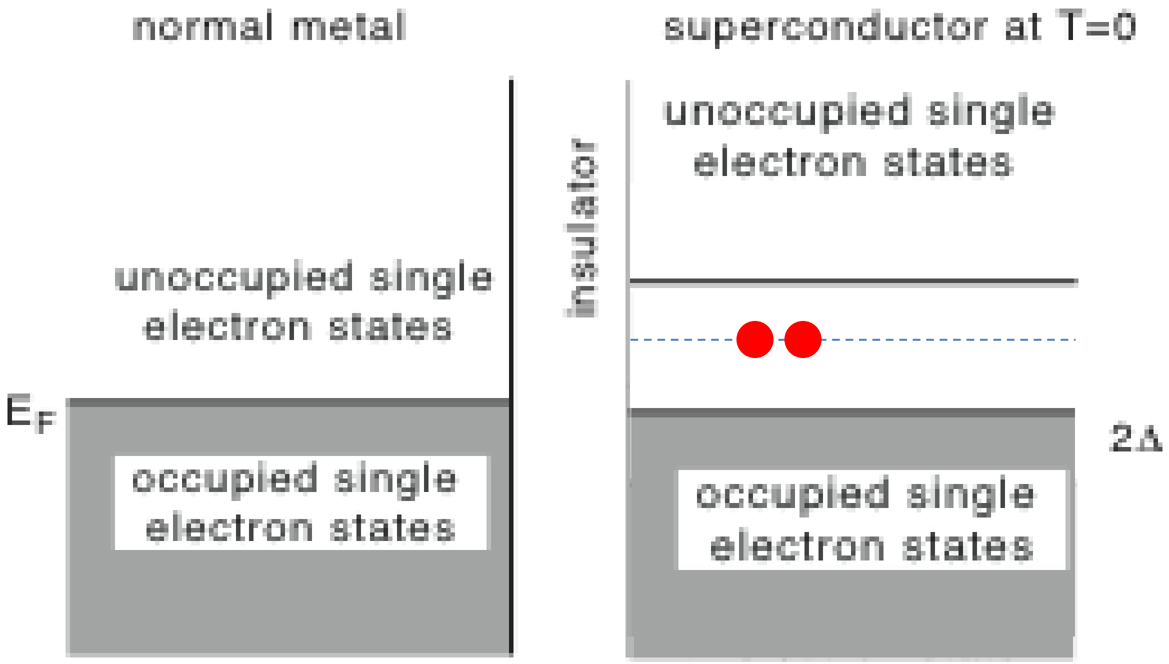
occupied single electron states

unoccupied single electron states

2Δ

occupied single electron states

$0<U<\Delta/e$



normal metal

superconductor at $T=0$

insulator

unoccupied single electron states

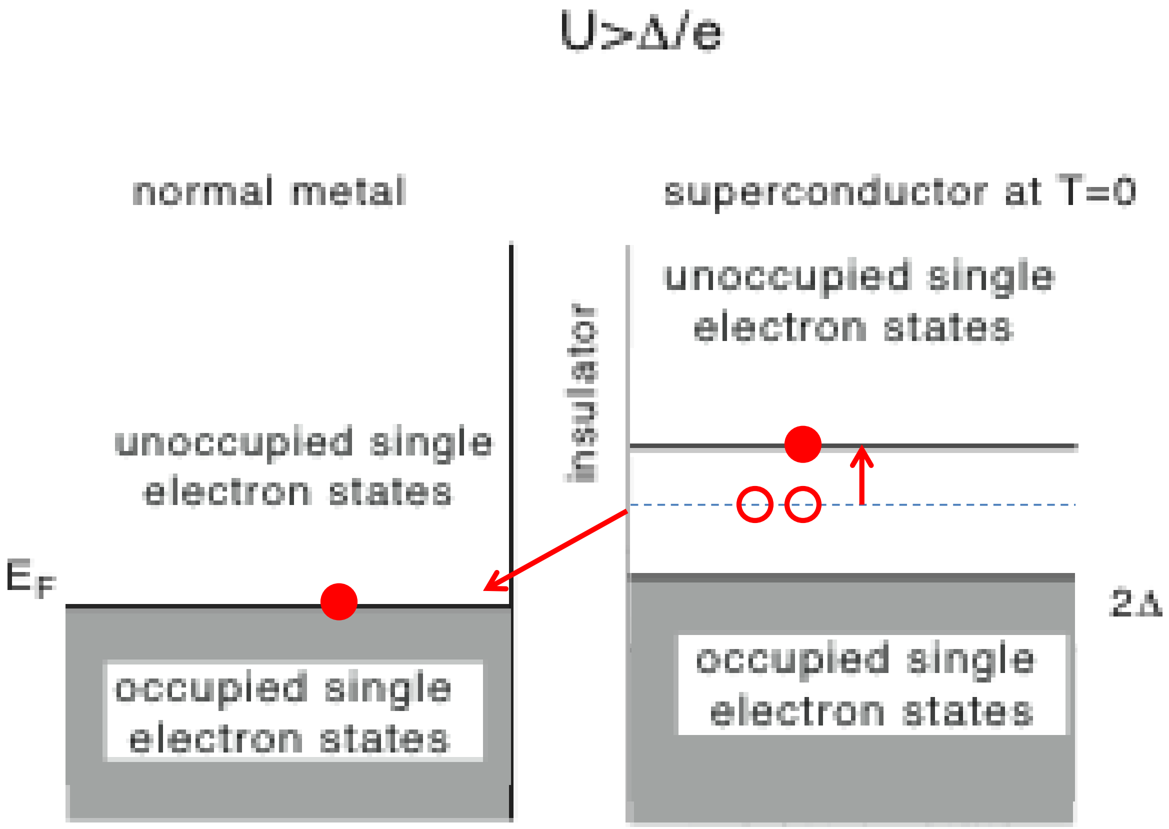
unoccupied single electron states

E_F

occupied single electron states

occupied single electron states

2Δ



one electron tunnels into the metal and gives the resulting energy to the other electron.

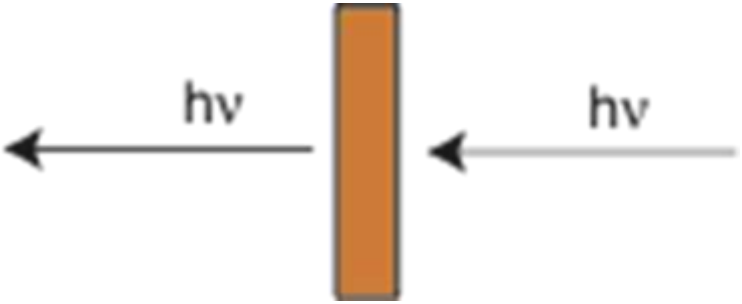
single-particle tunneling provides a direct experimental measurement of the energy gap.

The value of obtained from such experiments is in good agreement with the results of electronic heat capacity measurements.

Absorption of Electromagnetic Radiation

Another experiment used to measure the energy gaps of superconductors is the absorption of electromagnetic radiation.

In superconductors, photons can be absorbed by the material when their energy is greater than the gap energy. $h\nu > 2\Delta$



Relevant light wavelength in the infrared.

JOSEPHSON TUNNELING

Now we consider tunneling between two superconductors separated by a thin insulator.

In 1962 Brian Josephson proposed that, in addition to single particles, Cooper pairs can tunnel through such a junction. Josephson predicted that pair tunneling can occur without any resistance, producing a direct current when the applied voltage is zero and an alternating current when a dc voltage is applied across the junction.

The Nobel Prize in Physics

1973

"for their experimental discoveries regarding tunneling phenomena in semiconductors and superconductors, respectively"

"for his theoretical predictions of the properties of a supercurrent through a tunnel barrier, in particular those phenomena which are generally known as the Josephson effects-

Leo Esaki

◔ 1/4 of the prize

Japan

Ivar Giaever

◔ 1/4 of the prize

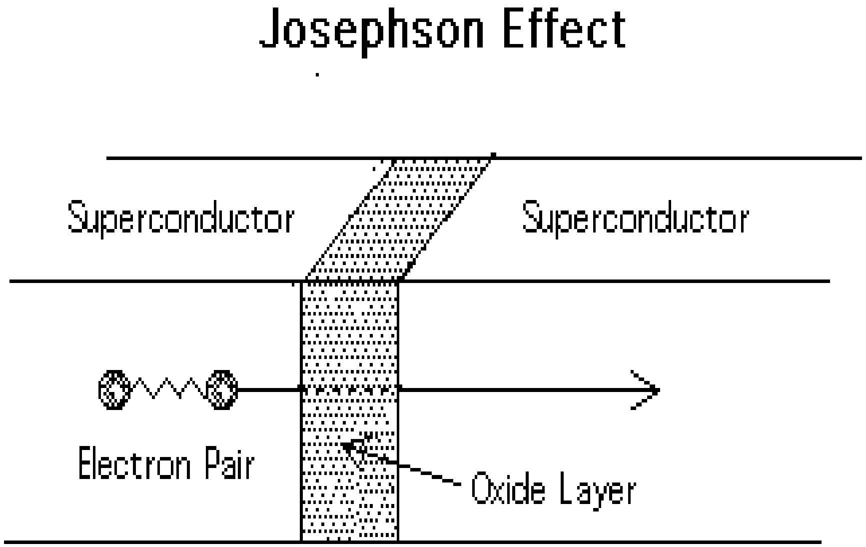
USA

Brian David Josephson

◑ 1/2 of the prize

United Kingdom

A **Josephson junction** is an superconductor-insulator-superconductor (SIS) layer structure placed between two electrodes



→ finite supercurrent at zero applied voltage

→ oscillation of supercurrent at constant applied voltage

Josephson effects

Josephson Tunneling

Tunneling arises because the electron waves in a metal do not cut off sharply at the surface but leaks into the “forbidden” barrier region.

Within this distance there is a small but finite probability that an electron will be found outside the metal.

Therefore, when a piece of metal is placed very close to another, electrons have a finite probability of penetrating the potential barrier formed by the insulating layer between the two metals.

Josephson Tunneling

The coherent quantum mechanical wave associated with the Cooper pairs leaks from the superconductor on each side into the insulating region.

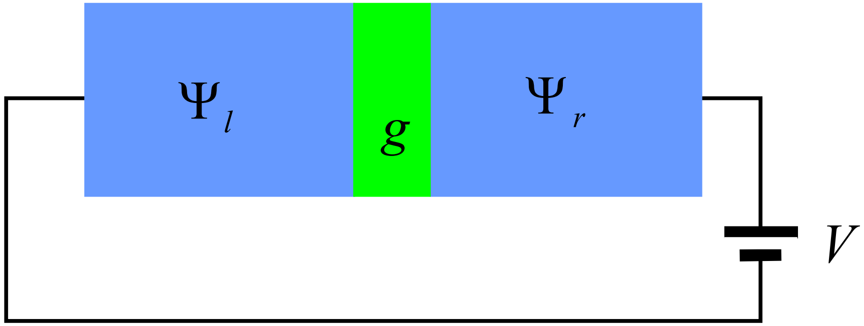
Josephson suggested that if the barrier is sufficiently thin, the waves on each side must overlap and their phases should lock together.

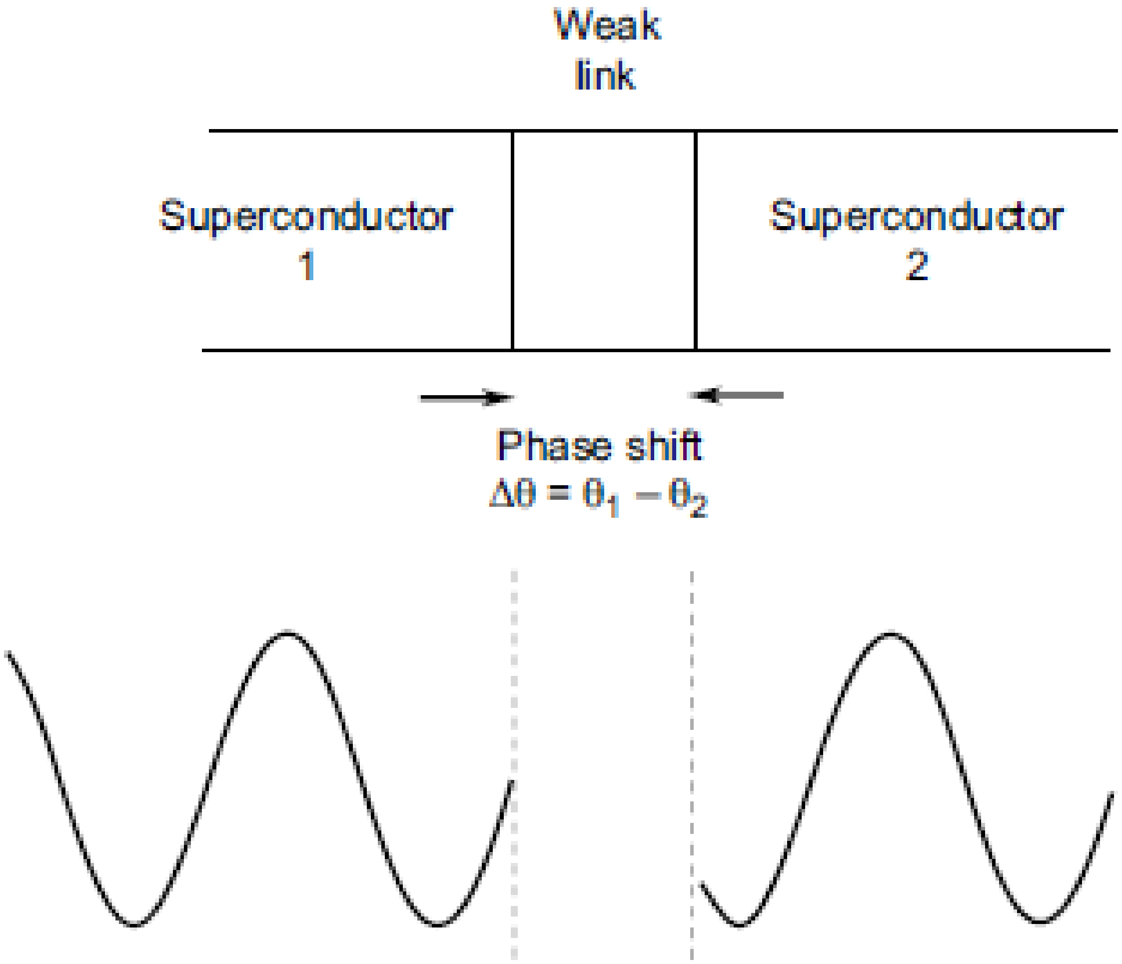
Under these circumstances the Cooper pairs can tunnel through the barrier without breaking up.

In a given superconductor, the pairs could be represented by a wavefunction $\psi_j = \sqrt{n_j}\, e^{i\theta_j}$

where θ is the phase and is the same for every pair.

If the superconductor to the left of the insulating layer has a phase θ1 and that to the right has a phase θ2,





Weak link

Superconductor 1

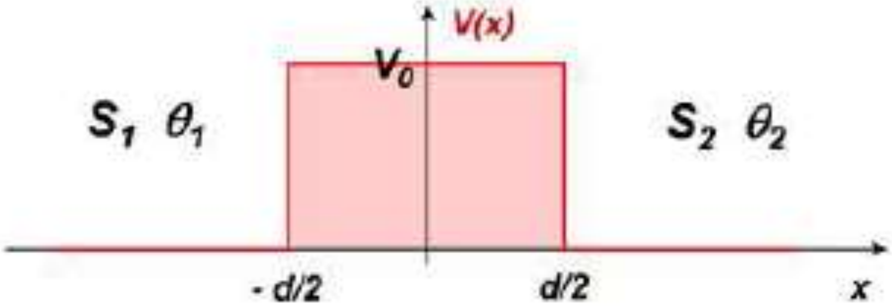
Superconductor 2

Phase shift

$\Delta\theta = \theta_1 - \theta_2$

$\Psi = |\Psi_1| e^{i\theta_1}$

$\Psi = |\Psi_2| e^{i\theta_2}$



$$-\frac{\hbar^2}{2m^*}\nabla^2\psi(\mathbf{r}) = (E_0 - V_0)\psi(\mathbf{r})$$

- in superconductors $\psi_{1,2} = \sqrt{n_{1,2}}\, e^{i\theta_{1,2}}$

- in insulator: $\psi(x) = A\cosh(\kappa x) + B\sinh(\kappa x)$

$$\kappa = \sqrt{\frac{2m_s(V_0 - E_0)}{\hbar^2}}$$

coefficients A and B are determined by the boundary conditions at $x = \pm d/2$:

$$\psi(-d/2) = \sqrt{n_1}\, e^{i\theta_1}$$

$$\psi(+d/2) = \sqrt{n_2}\, e^{i\theta_2}$$

Here you consider both the superconductors of same material and have same pair concentration. So $n_1 = n_2$

Obtain the expressions for A and B

Obtain the current density in the insulator by replacing ψ of the insulator in the expression for current density

$$\mathbf{j}_p(\mathbf{r}) = \frac{-ie\hbar}{2m}\left[\psi^*(\vec{\nabla}\psi) - (\vec{\nabla}\psi^*)\psi\right]$$

Josephson showed that at zero voltage there appears across the junction a supercurrent satisfying the relationship

$$J = J_0 \sin(\theta_1 - \theta_2)$$

J_0 is the maximum supercurrent density across the junction that can be induced to flow under zero-voltage, that is the critical current I_c.

J_0 value is determined mainly by properties of insulating layer. The value decreases with increase in layer thickness.

$$J = J_0 \sin(\theta_1 - \theta_2)$$

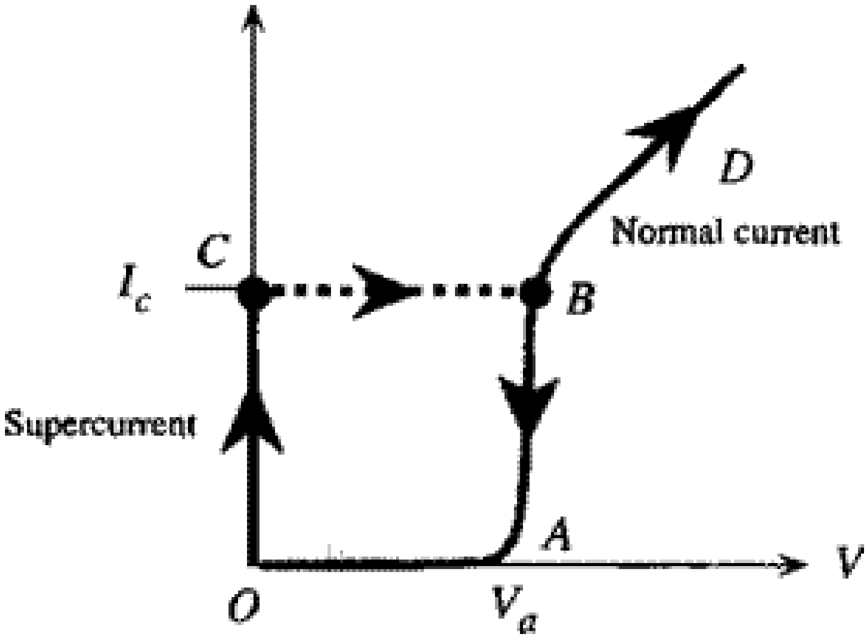
If the coupled superconductors are linked to a current source by an external circuit, the tunneling current flows without an applied voltage.

The maximum critical current, *J0*, corresponds to a phase difference of π/2,

It is proportional to the strength of the coupling across the barrier.

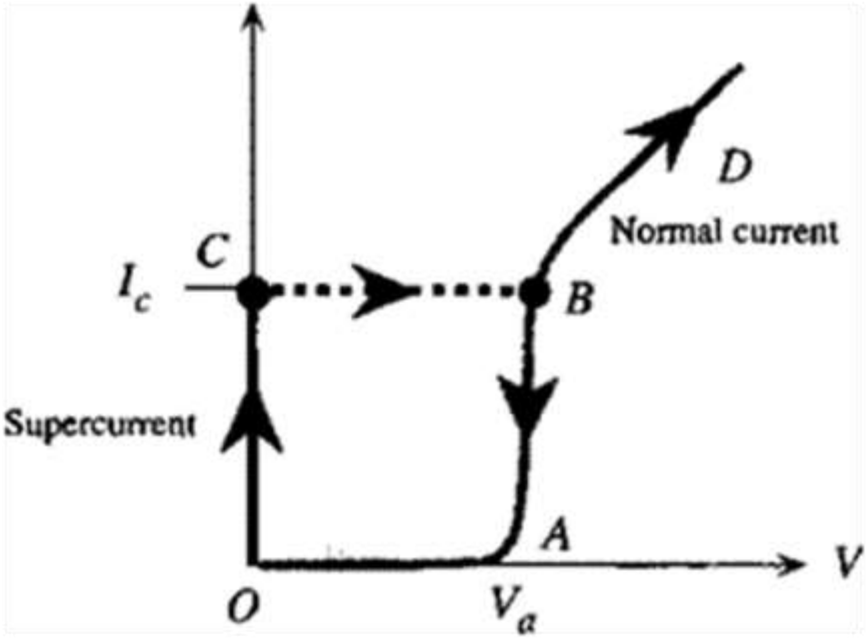
It is determined by the dimensions of the barrier region, the materials and the

temperature.

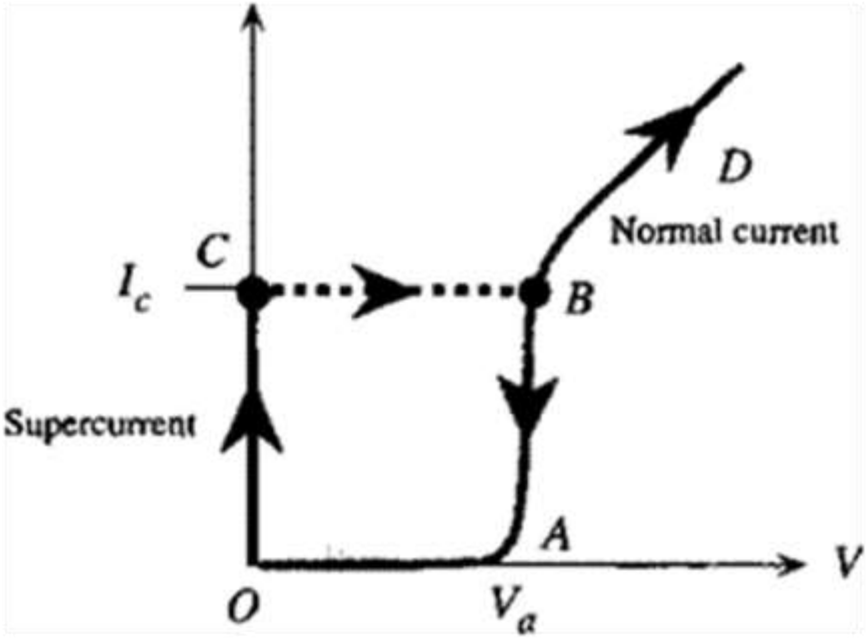


I-V characteristics of a Josephson junction for positive currents when the current is controlled by an external circuit.

Ic is the max. supercurrent that can be supported by junction with zero potential difference



With no voltage across the junction, one can have a large current, but if any voltage is applied, the current oscillates and its average goes to zero. The current will remain zero as the DC voltage is raised above the gap voltage, V = 2Δ/e.



The normal tunneling current in the range OA is negligible and rises suddenly when the voltage exceeds V_a.

The reason is that a certain amount of voltage (corresponding to a potential energy eV_a) is needed to provide the necessary energy to disassociate the tunneling single electron from its Cooper pair.