DSE CLASS

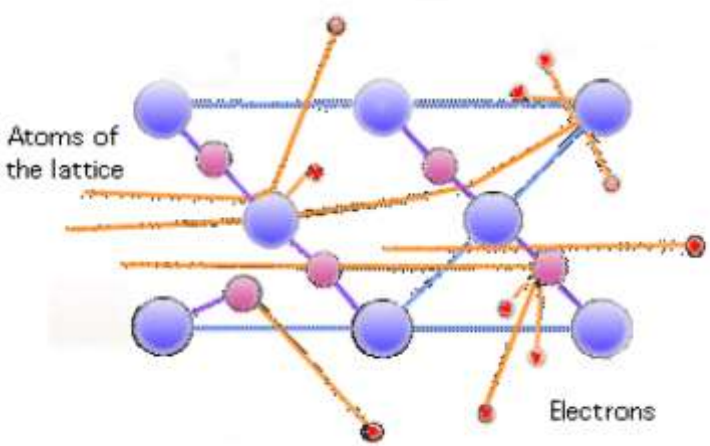
CONDENSED MATTER PHYSICS

Lecture-7

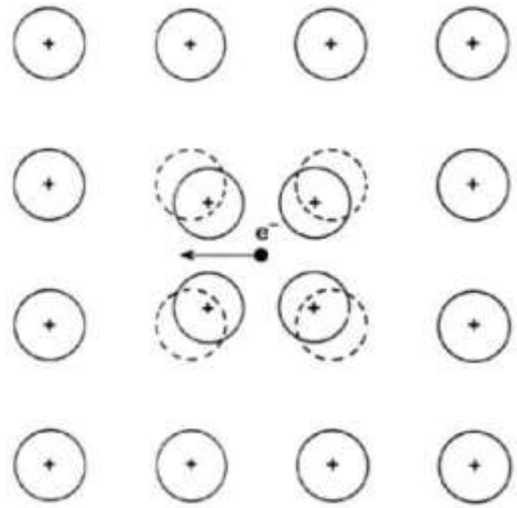
15/10/2020

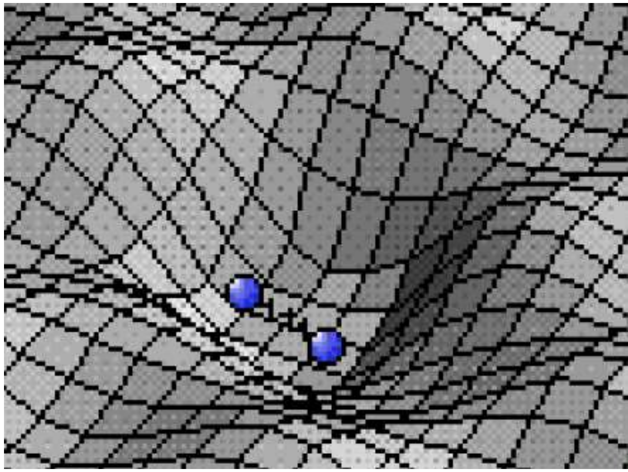
BCS theory

Normal conducting state



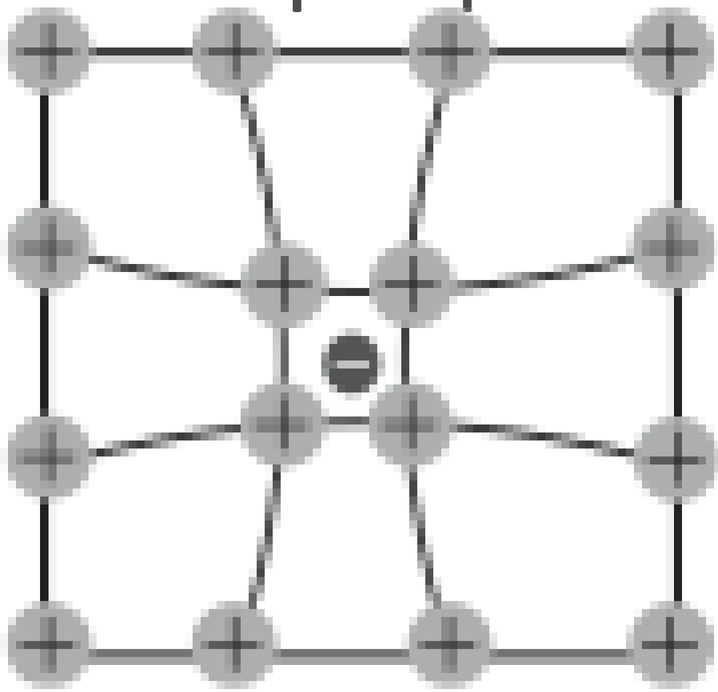
Superconducting state





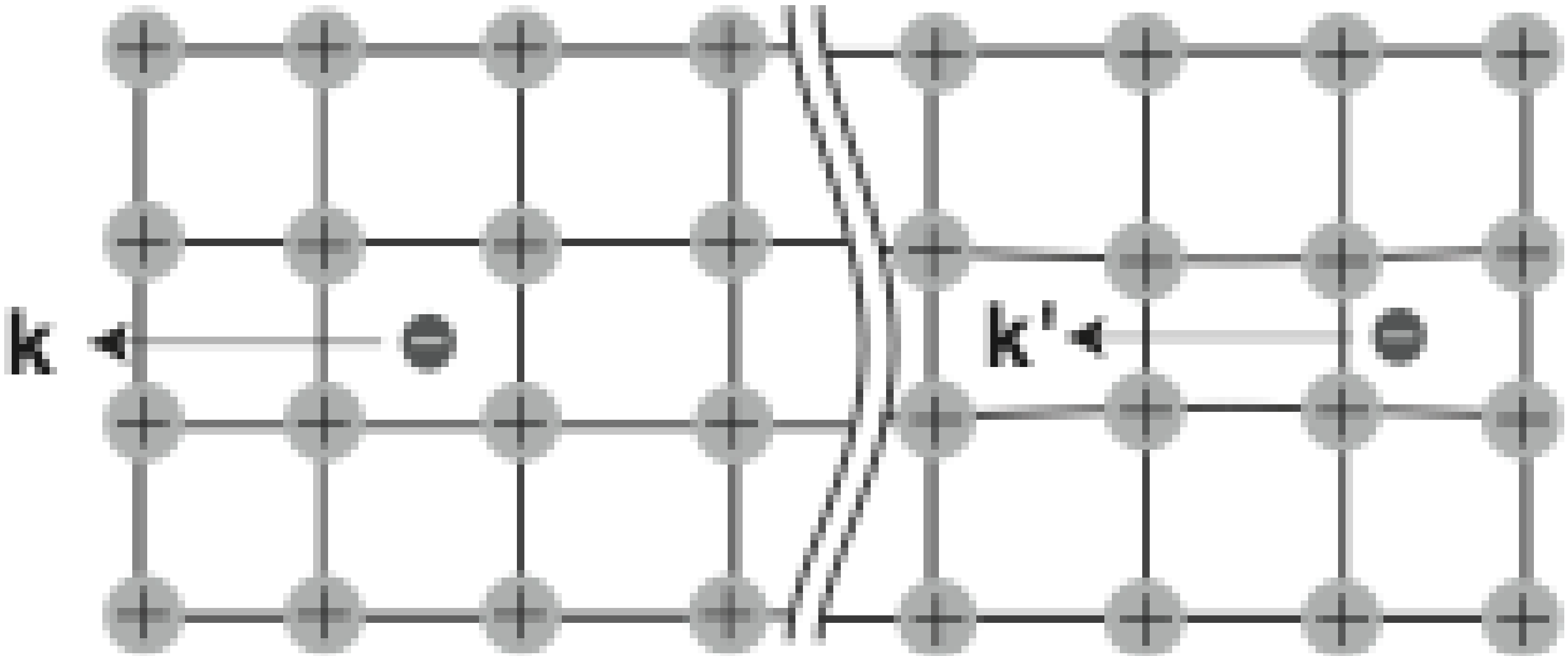
- $T_c \sim 1/\sqrt{M_{isotopic}}$ -> phonons should play a role in superconductivity
- Creation of Cooper pairs (over-screening effect)
  - An e^- attracts the surrounding ion creating a region of increased positive charge
  - The lattice oscillations enhance the attraction of another passing by e^- (Cooper pair)
  - The interaction is strengthened by the surrounding sphere of conduction e^-
  - In a superconductor the net effect of e^-e^- attraction through phonon interaction and the e^-e^- coulombian repulsion is attractive and the Cooper pair becomes a singlet state with zero momentum and zero spin

The electron-phonon interaction/ Cooper pairs

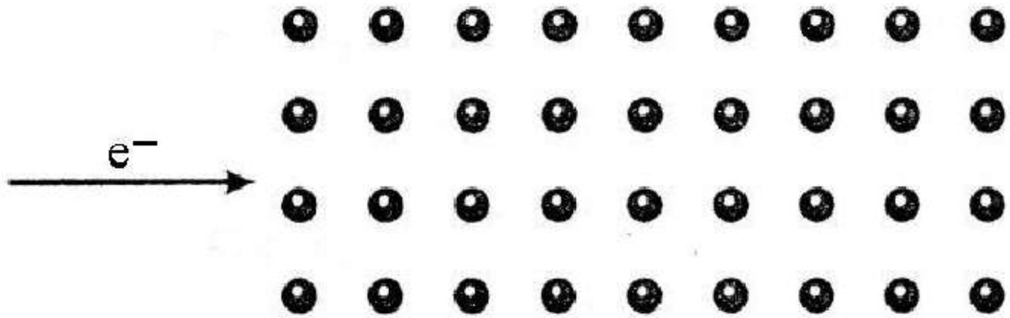


- Polarization of the lattice by one electron leads to an attractive potential for another electron.

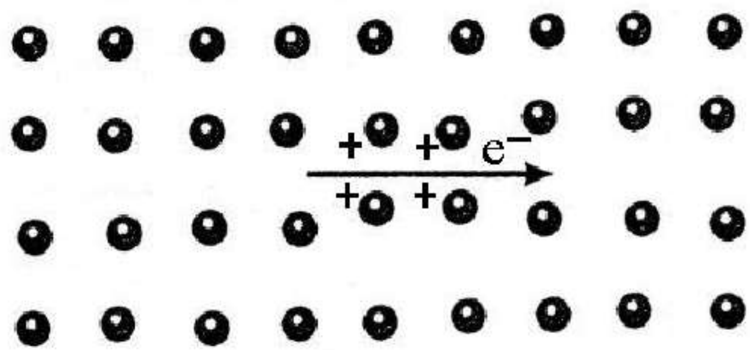
The electron-phonon interaction / Cooper pairs



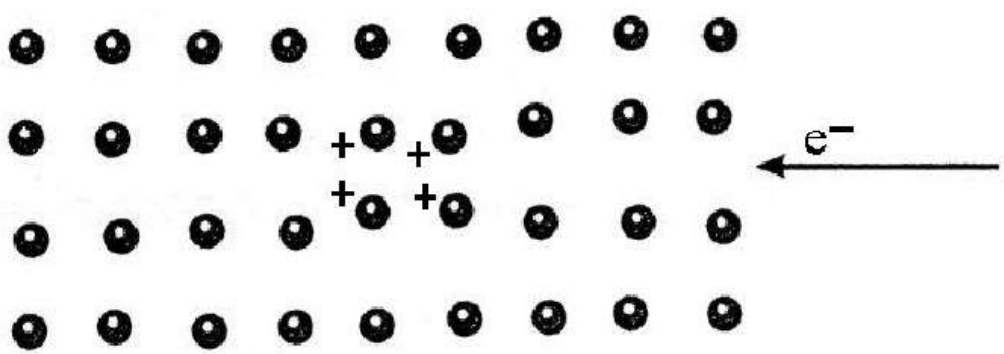
- Polarization of the lattice by one electron leads to an attractive potential for another electron.



First electron approach a section of the crystal lattice



First electrons defoms part of the lattice electrostatically.



Second electron is attracted to the net positive charge of the deformation

The attractive potential between electrons is much smaller than the kinetic energy of the two electrons.

So it should not normally be able to bind the electrons together.

However, in this case, the two electrons are not in free space.

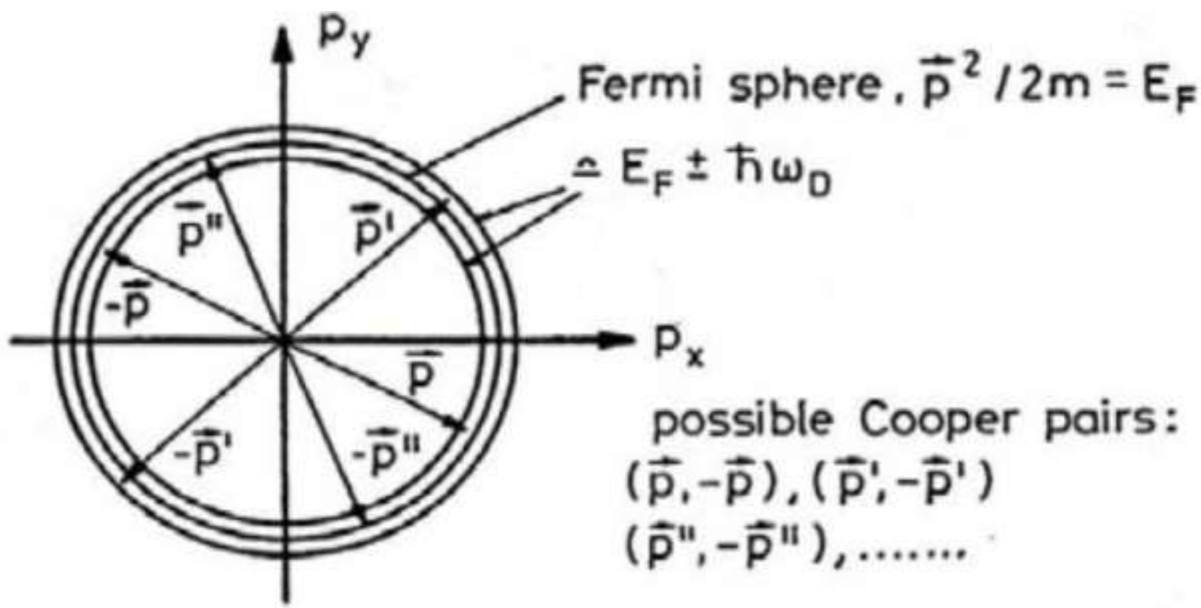
They ... d

up to

In the ... vo

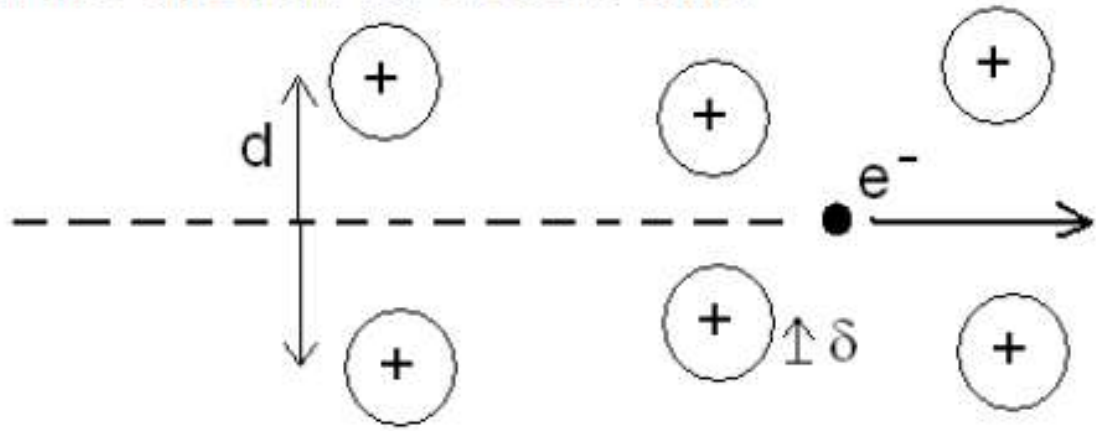
electr

form



What is the "distance" between the electrons forming Cooper pairs?

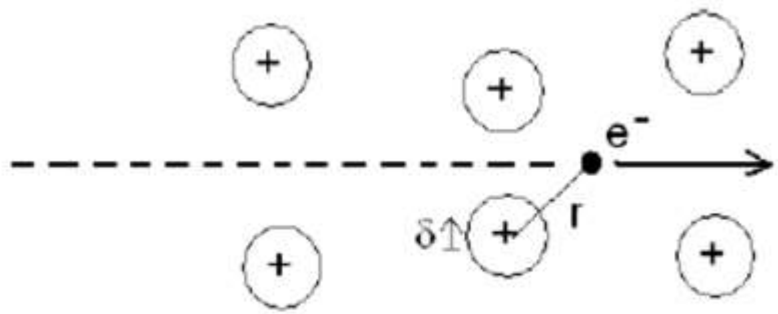
When an electron passes a positive ion, the ion experiences a force from the electron for a short time.



This impulse cause the ion to move and oscillate at the Debye frequency.

The average charge in a lattice is zero. The potential energy of an electron is due to an ion is

$$U = -\frac{e^2}{4\pi\epsilon_0 r}.$$



If the ion is displaced by an electron's attraction, the net potential is approximately the change in the ion's potential:

$$\delta U \approx \frac{dU}{dr}\delta r \approx -\frac{e^2}{4\pi\epsilon_0 r^2}\delta.$$

This net potential is only present near the ion. Further away, it will not be felt because other electrons would move around and cancel it (screening).

So another electron would feel this potential only when r is about d, the distance between ions.

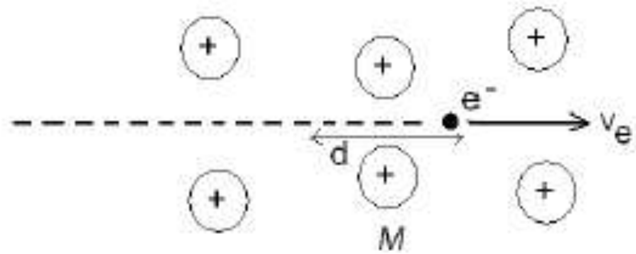
For small displacement of the ion, the ion's motion is simple harmonic.

We know then that: maximum velocity = amplitude x frequency.

$$v_0 \approx \delta \times \omega_D.$$

So when the ion receives a sudden attraction (impulse) from a passing

To find v_0, we need to know the impulse (force x time) from the electron. The force is active only when the distance is within a distance of about d from the ion.



So the force and time, are respectively,

$$F \approx \frac{e^2}{4\pi\epsilon_0 d^2} \text{ and } \tau \approx \frac{d}{v_e},$$

where v_e is the velocity of the electron.

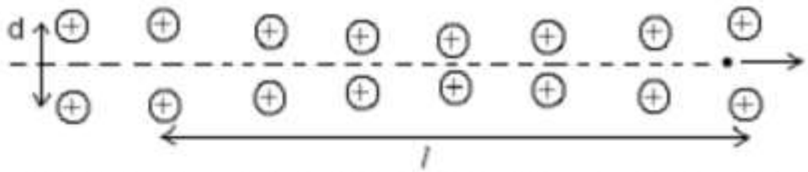
If the electron is at the Fermi level E_F, the velocity can be obtained from:

$$E_F = \frac{1}{2} m v_e^2.$$

Then the ion's velocity is

$$v_0 = \frac{F\tau}{M}$$

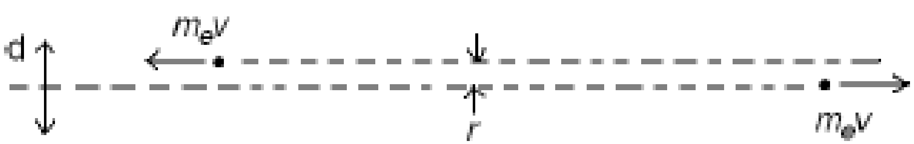
where M is the mass of the ion.



The attraction only exists in the narrow region between adjacent ions, and behind the passing electron. Also, it would only last until the displaced ion returns to its rest position. The time for this is the half of the period $2\pi/\omega_D$. So a passing electron with velocity v_e leaves behind a trail of displaced ions of length

$$l = v_e \times \frac{\pi}{\omega_D}.$$

This is also the length, or extent, of the attractive region. This attraction can only be felt by another electron travelling along nearly the same pa



The lattice atoms move much slower than the electrons.

The polarization is retarded

$$\frac{2\pi}{\omega_D} \approx 10^{-13}\mathrm{s}$$

Establishing the polarization takes time τ

$v_F \approx 10^6\,\mathrm{ms}^{-1}$

a typical electron close to the Fermi surface moves with velocity

which is much larger than the velocity of the ions

$10^{-7}\mathrm{m}$

So by the time the ions have polarized themselves, 1st electron has gone

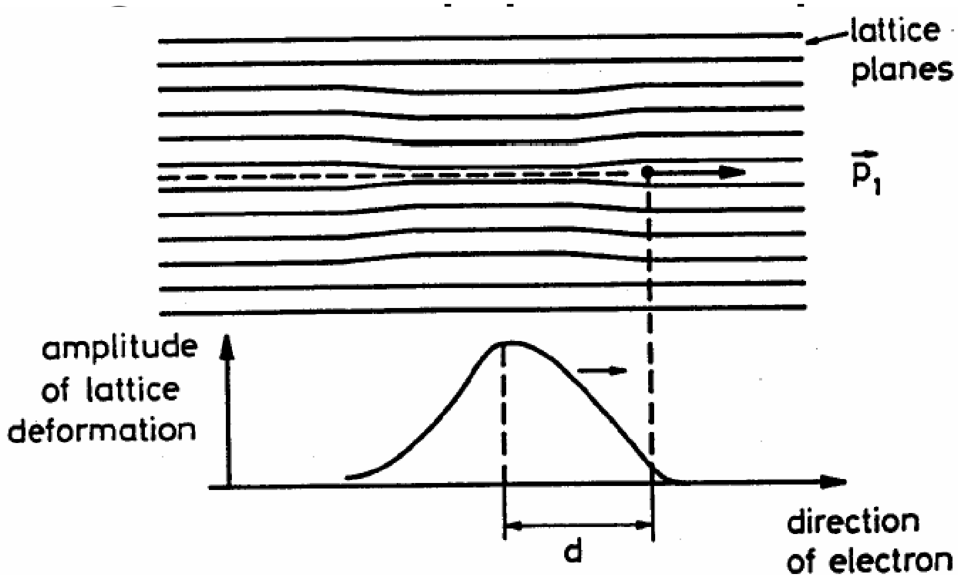
$v_F\,\tau =$ and 2nd electron have moved

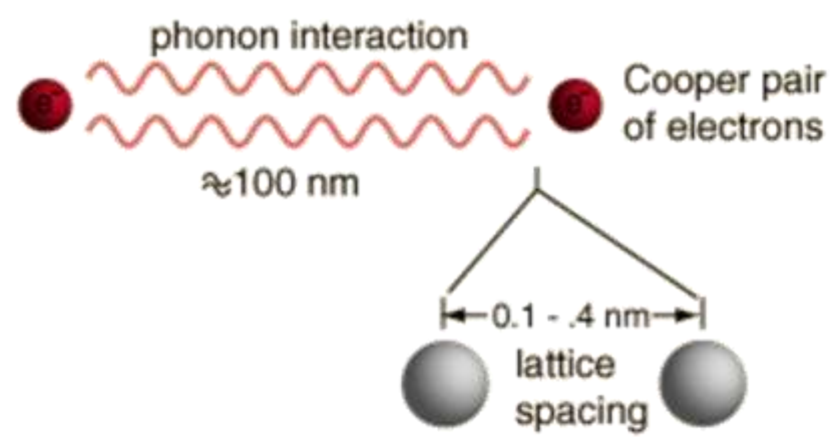
towards concentration of positive charge before the

This gives rise to an effective attraction between the two electrons as shown, which may be large enough to overcome the repulsive Coulomb interaction.

So distance between the electrons forming the

10^{-7}m



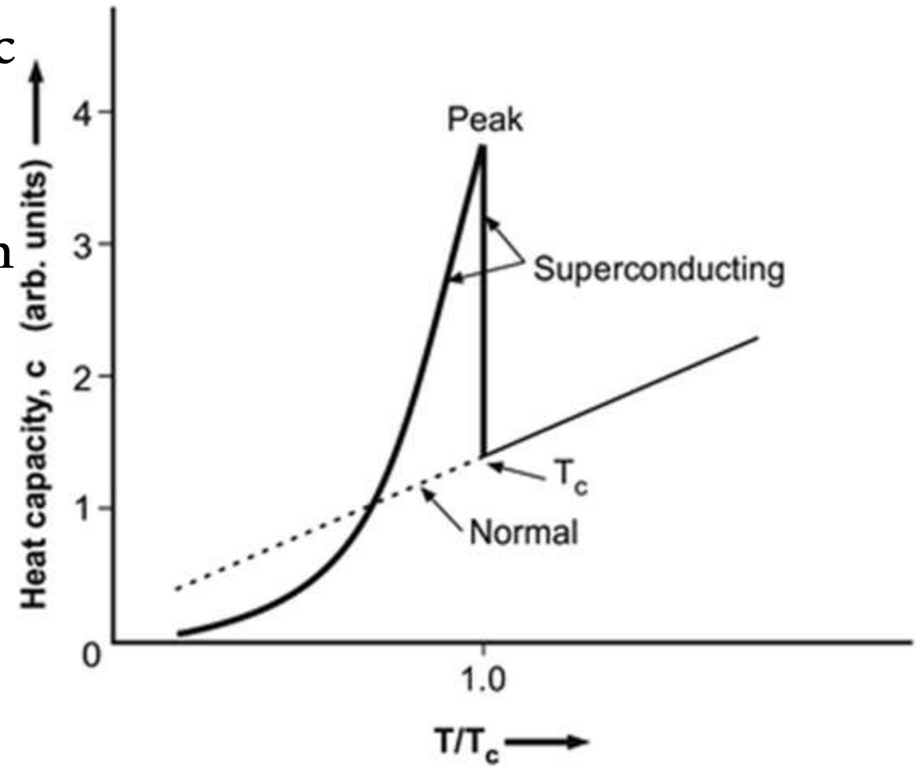


ELECTRONIC SPECIFIC HEAT

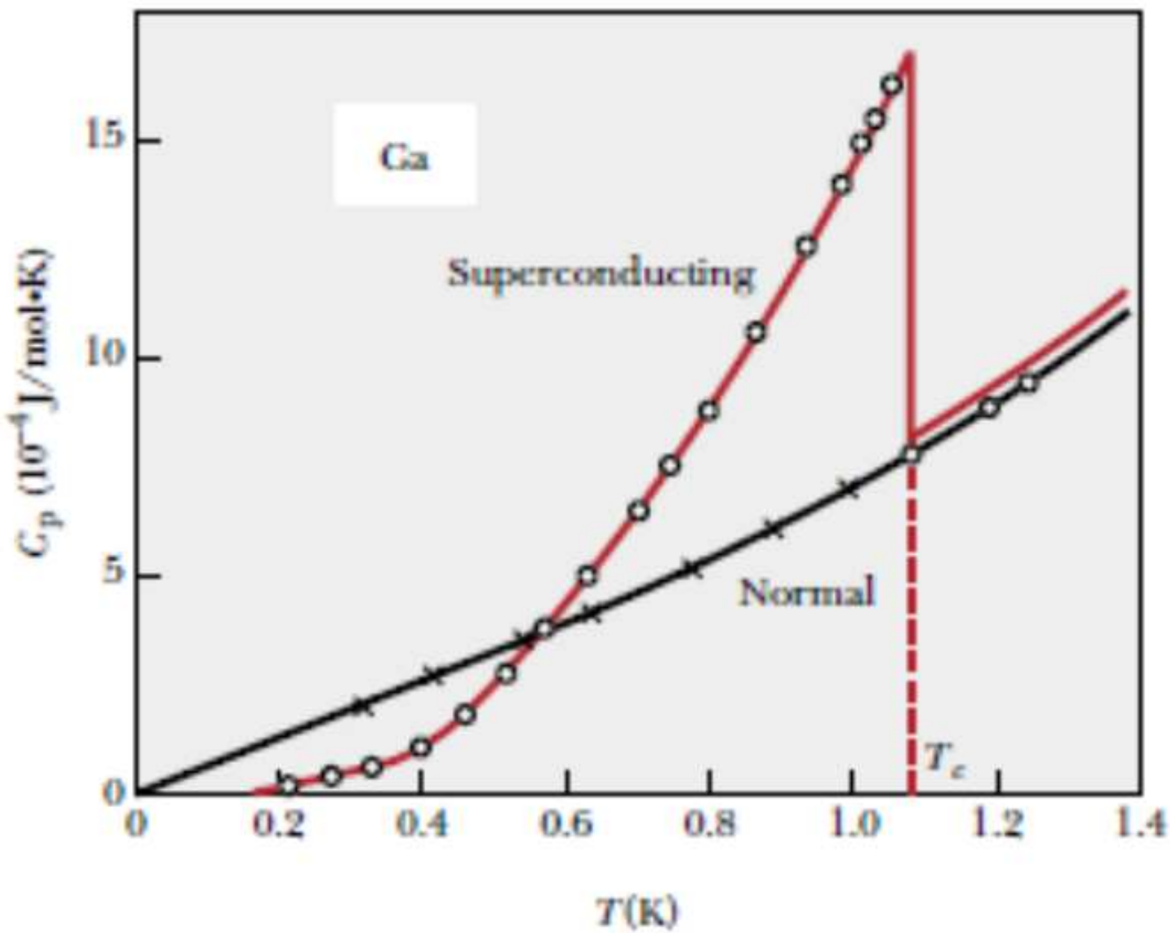
The thermal properties of superconductors have been extensively studied and compared with those of the same materials in the normal state. When a small amount of thermal energy is added to a normal metal, some of the energy is used to excite lattice vibrations, and the remainder is used to increase the kinetic energy of the conduction electrons.

The electronic specific heat C is defined as the ratio of the thermal energy absorbed by the electrons to the increase in temperature of the system.

At low temperatures, the electronic specific heat of the material in the normal state, C_n, varies with temperature as AT



As the temperature is lowered starting from $T > T_c$, the specific heat first jumps to a very high value at T_c and then falls below the value for the normal state at very low temperatures.



Electronic specific heat versus temperature for superconducting gallium (in zero applied magnetic field) and normal gallium (in a 0.020-T magnetic field).

Specific Heat of a Superconductor

If we plot only the electronic contribution to the heat capacity of gallium in the superconducting state we find ...

$$C_S^{el} \propto \gamma T_c \exp^{(-\Delta/k_B T)}$$

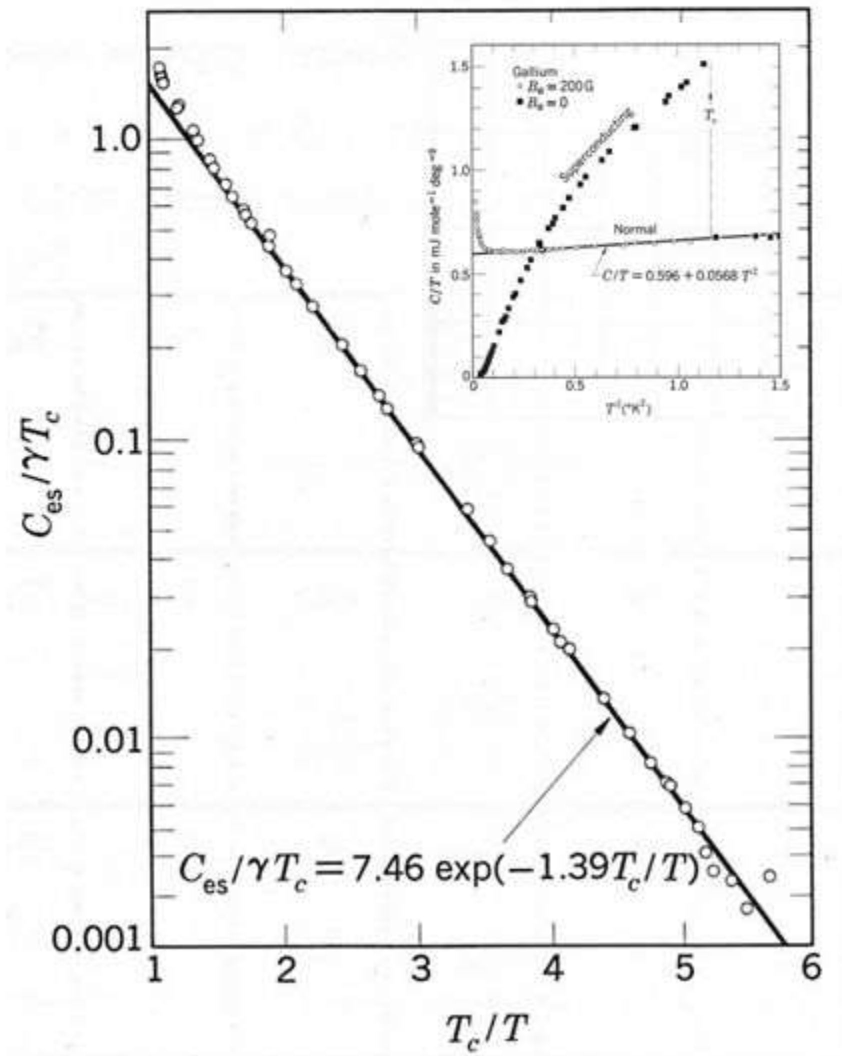
This form is quite general for almost all superconductors

It implies that there is an *energy gap* in the electronic states at the Fermi energy

This gap is 2Δ wide

Δ generally scales with T_c

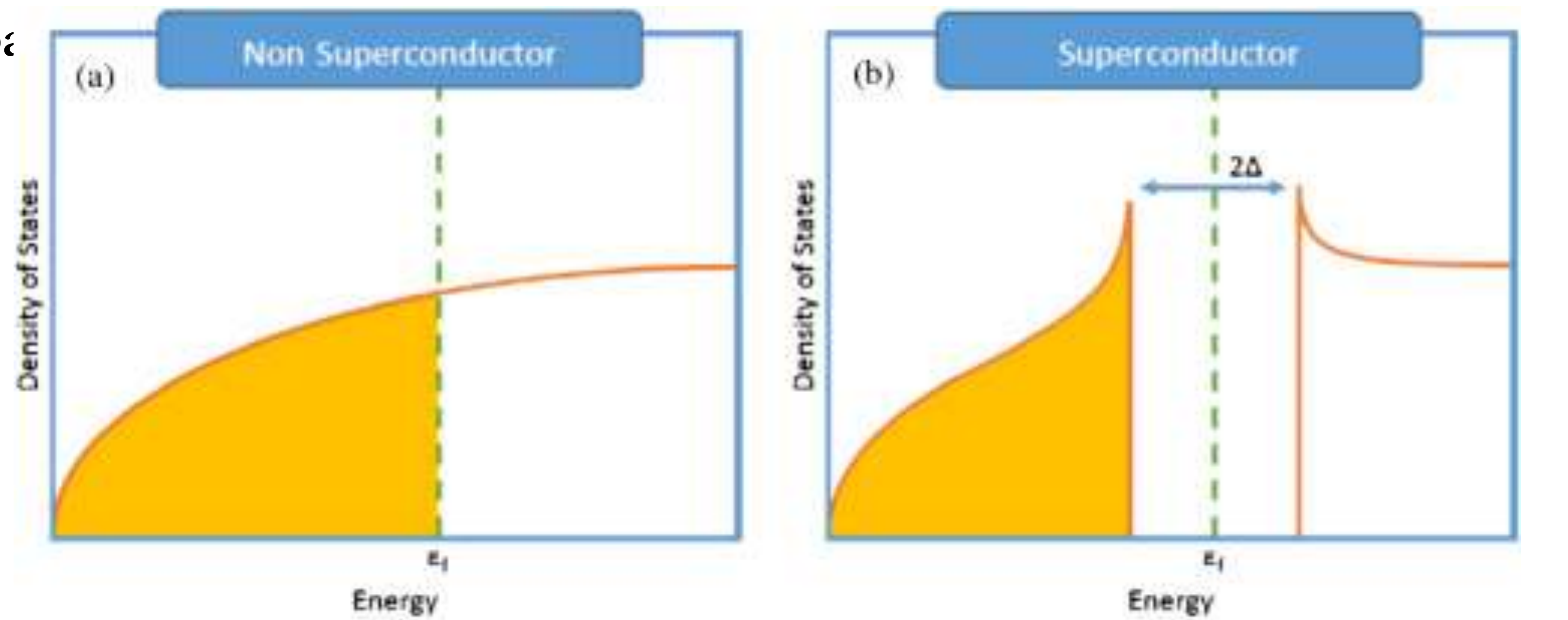
$2\Delta \sim 3.5 k_B T_c$ ~1-10meV



In a normal conductor, the Fermi energy E_F represents the largest kinetic energy the free electrons can have at 0 K.

The energy gap in a superconductor is very small, of the order of $k_B T_c$ (10^{-3} eV) at 0 K, as compared with the energy gap in semiconductors (1 eV) or the Fermi energy of a metal (5 eV).

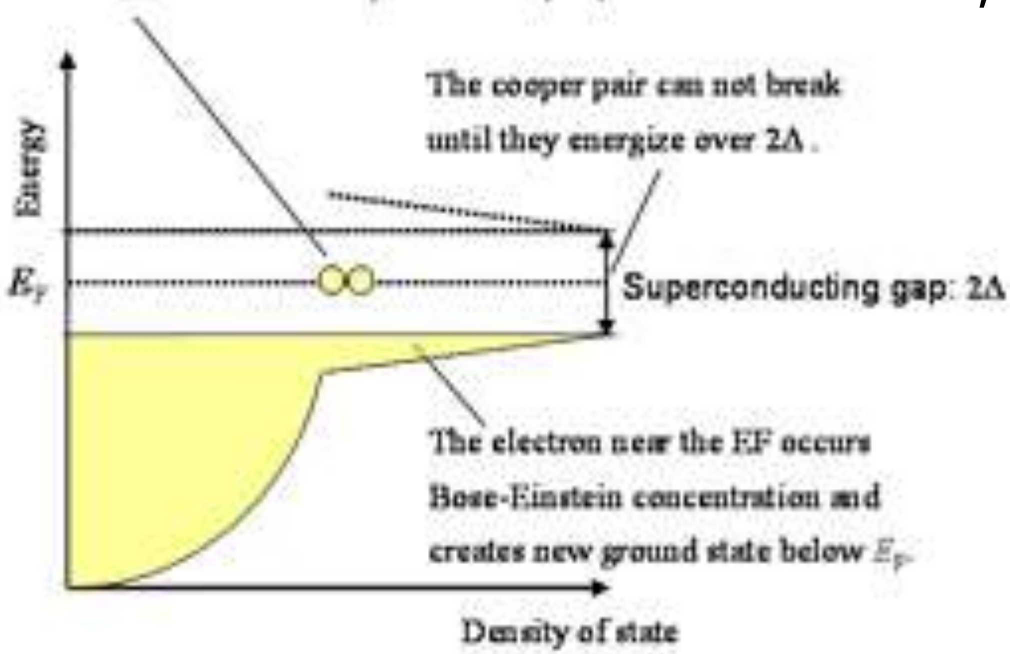
The energy gap represents the energy needed to break apart a Cooper pa



only electrons in the region $\pm\Delta$ about E_F contribute to BCS ground state

If we take 2Δ as the energy needed to break the Cooper pair, then we would expect that it is close to $k_B T_c$, because the Cooper pairs should start disappearing around Tc.
This is consistent with measurements, which give 2Δ ~$k_B T_c$

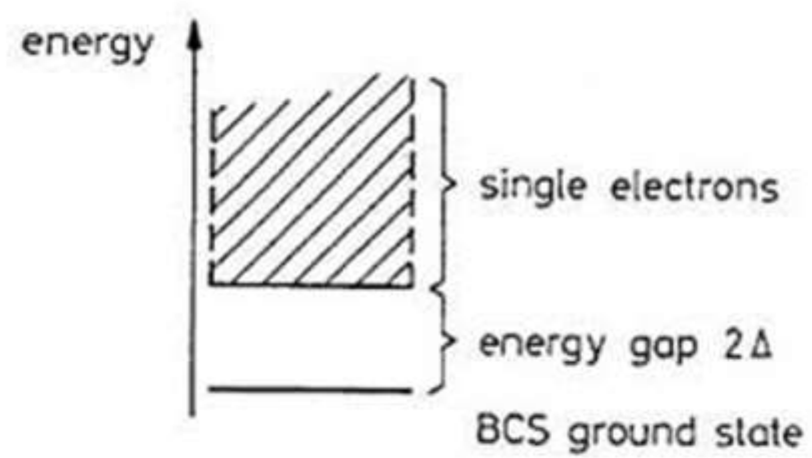
.



The electrons in the pair have opposite spin, so that resultant spin of the Cooper pair is zero - it is a boson.

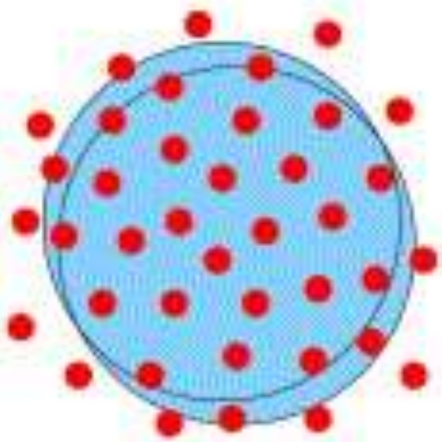
So, like the Bose-Einstein condensate, the Cooper pairs can condense into the ground state and form a condensate.

However, because of the considerable overlap, it is normally called a BCS condensate instead.



Δ is the energy needed to excite one electron from the BCS condensate.

BCS	BEC
weak coupling	strong coupling
large pair size **k**-space pairing	small pair size **r**-space pairing
strongly overlapping Cooper pairs	ideal gas of preformed pairs



The BCS theory predicts that at $T = 0$ K,

$2\Delta = 3.52\, k_B T_c$

Thus superconductors that have large energy gaps have relatively high critical temperatures.

The exponential dependence of the electronic heat capacity by $\exp(-\Delta / k_B T)$, contains an experimental factor, $\Delta = E_g/2$, used to determine the value of E_g

at $T = 0$ K

Superconductor	E_g (meV)
Al	0.34
Ga	0.33
Hg	1.65
In	1.05
Pb	2.73
Sn	1.15
Ta	1.4
Zn	0.24
La	1.9
Nb	3.05

These pairs don’t cooperate only with each other, but also with other Cooper pairs.

Simply speaking, the pairs can overlap, or even include other pairs within a longer pairing.

This creates one cohesive group of paired electrons flowing with exquisite efficiency through the superconductor – as a single wave.

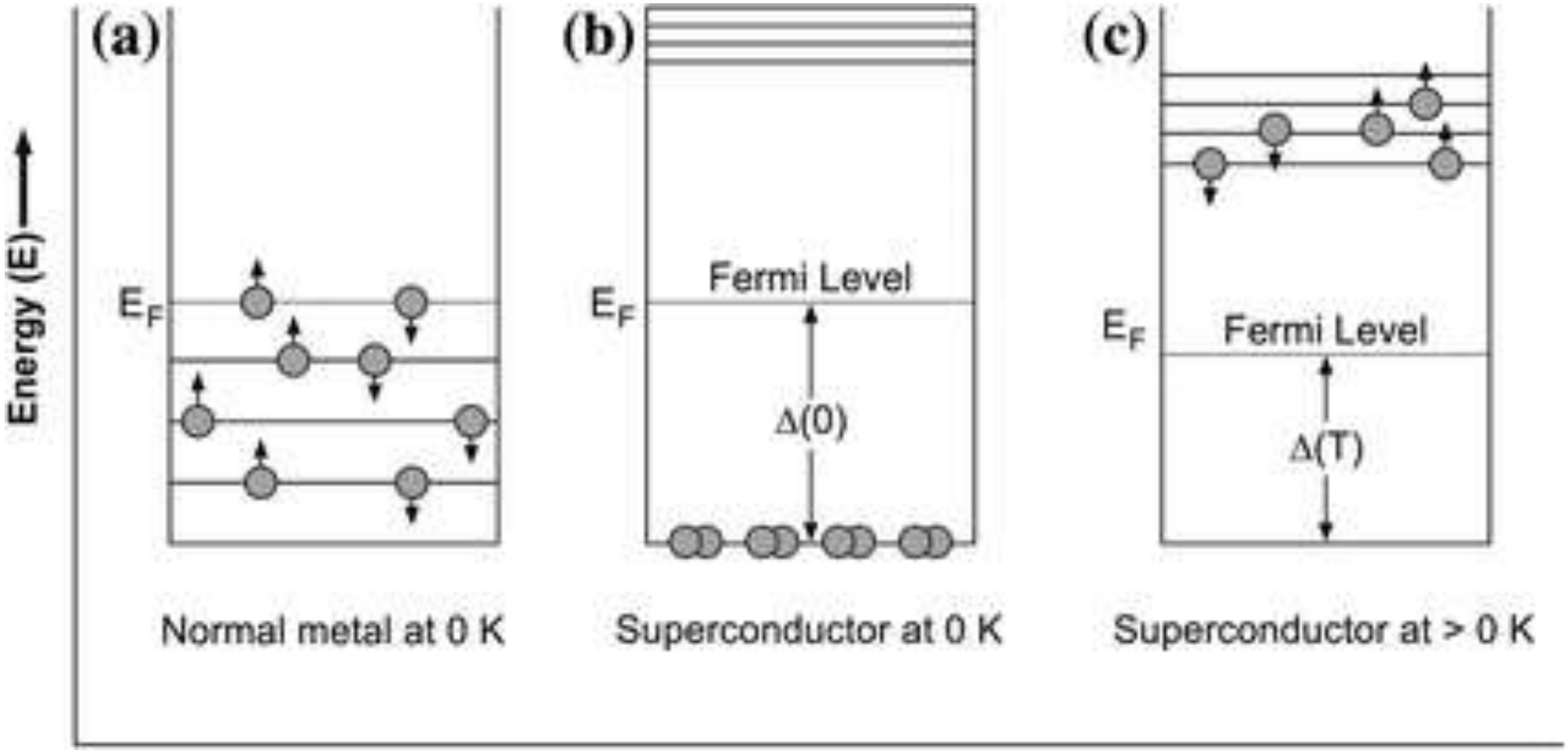
The transformation is such that materials in superconducting mode are considered to be

The motions of all of the Cooper pairs within a single superconductor are correlated; they constitute a system that functions as a single entity.

Application of an electrical voltage to the superconductor causes all Cooper pairs to move, constituting a current.

When the voltage is removed, current continues to flow indefinitely because the pairs encounter no opposition.

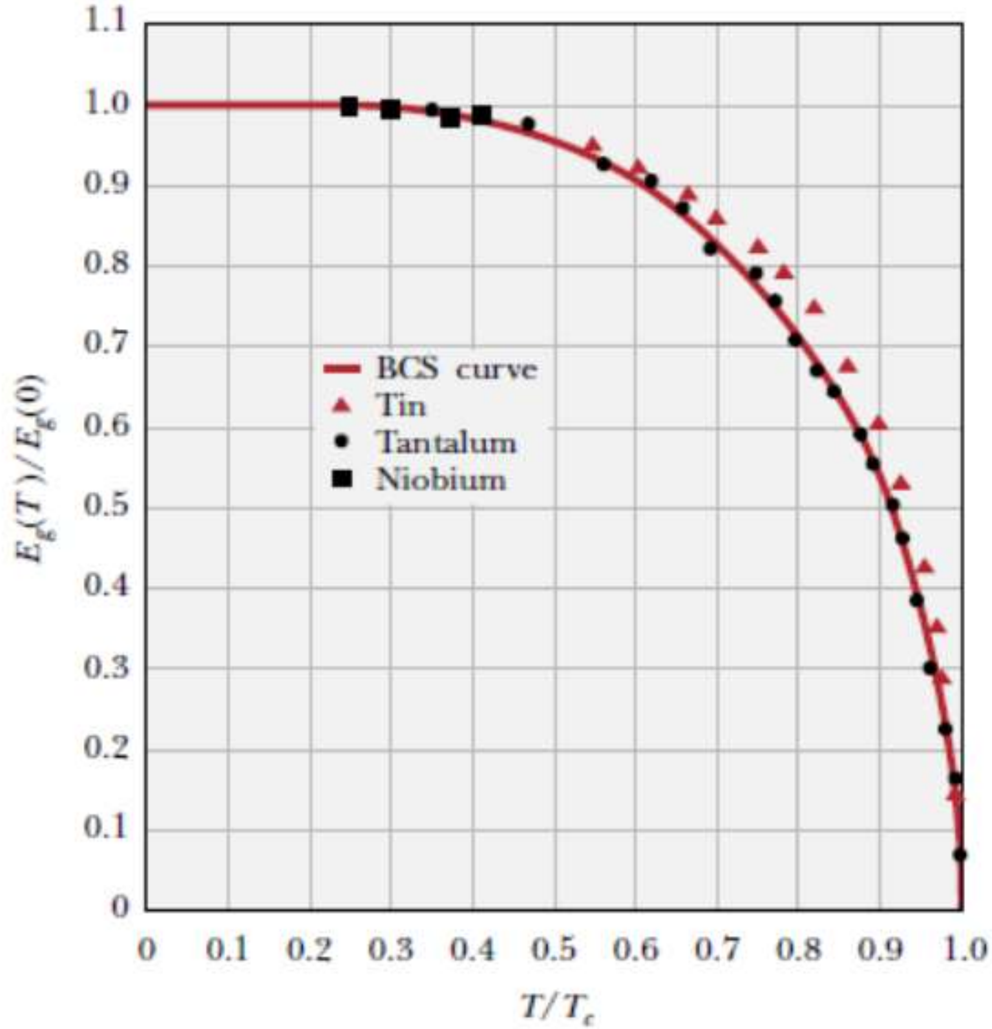
For the current to stop, all of the Cooper pairs would have to be halted at the same time, a very



This energy gap arises as a result of the interaction of the Cooper pairs to form a coherent state in which the superconducting electrons have a lower energy than they would have in the normal state.

A central prediction of the BCS theory is that the Cooper pairs form a condensed state whose lowest quantum state is stable below an energy gap of value 2 Δ, which separates the superconducting states from the normal ones

$$\frac{\Delta (T)}{\Delta (O)} = 1.76\ (1 - \frac{T}{T_c})^{2}$$



Energy gaps in superconductors, at $T = 0$

$E_g(0)$ in 10^{-4} eV.
$E_g(0)/k_B T_c$.

										Al	**Si**
										3.4	
										3.3	
Sc	**Ti**	**V**	**Cr**	**Mn**	**Fe**	**Co**	**Ni**	**Cu**	**Zn**	**Ga**	**Ge**
		16.							2.4	3.3	
		3.4							3.2	3.5	
Y	**Zr**	**Nb**	**Mo**	**Tc**	**Ru**	**Rh**	**Pd**	**Ag**	**Cd**	**In**	**Sn** (w)
		30.5	2.7						1.5	10.5	11.5
		3.80	3.4						3.2	3.6	3.5
La fcc	**Hf**	**Ta**	**W**	**Re**	**Os**	**Ir**	**Pt**	**Au**	**Hg** (α)	**Tl**	**Pb**
19.		14.							16.5	7.35	27.3
3.7		3.60							4.6	3.57	4.38

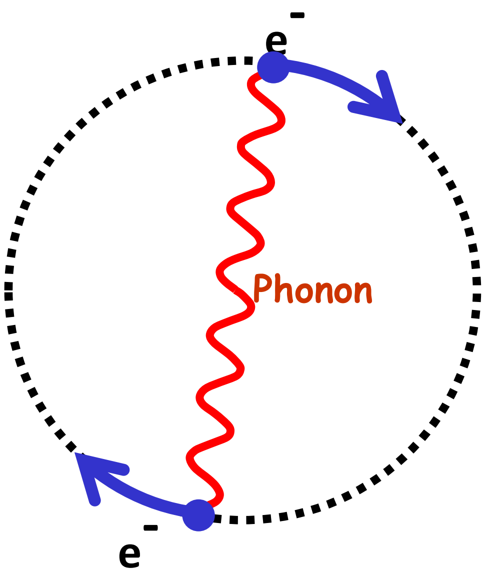
$E_g(0)/k_B T_c = 3.52$ **Weak electron-phonon coupling**

$E_g(0)/k_B T_c > 3.52$ **Strong electron-phonon coupling**

Creation of a C-Pairs diminishes energy of electrons. Breaking a pair (e.g. through interaction with impurity site) means increase of the energy.

A movement of the C-P when a supercurrent is flowing, is considered as a movement of a centre of the mass of two electrons creating C-P.

All the C-P are in the same quantum state with the same energy. A scattering by a lattice imperfection (impurity) can not change quantum state of all C-P at the same time (**collektive behaviour**).



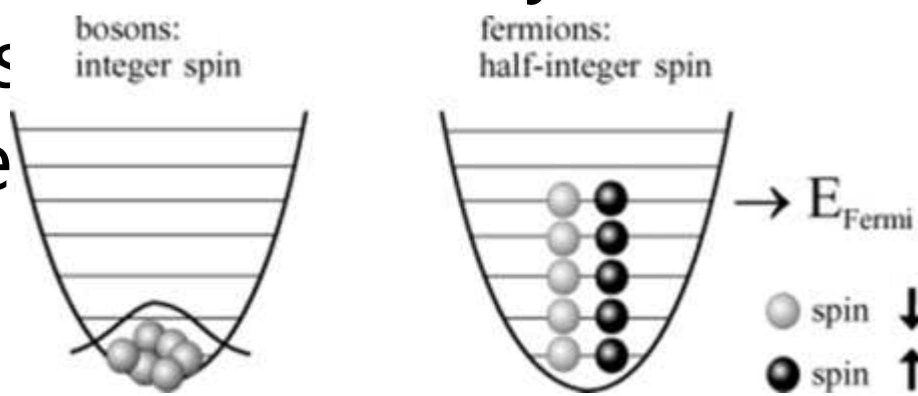
- As one electron moves through the lattice, the surrounding nuclei are attracted to it. The motion of the nuclei pulls the second electron in the same direction as the first.
- The nuclei then return to their original positions to avoid colliding with the second electron as it approaches.
- The pairs of electrons, called Cooper pairs, migrate through the crystal as a unit.
- A superconductor can be visualized as a complex dance of Cooper pairs which are all moving in time with each other and exchanging partners continuously..
- Electrons are able to travel through a solid with zero resistance

According to the BCS theory, as the temperature of the solid increases, the vibrations of the atoms in the lattice increase continuously, until eventually the electrons cannot avoid colliding with them.
The collisions result in the loss of superconductivity at higher temperatures.

-When a metal is cooled to the critical temperature, electrons in the metal form Cooper Pairs.

-Cooper Pairs are electrons which exchange phonons and become bound together.

-Bound electrons behave like bosons. Their wavefunctions don't obey Pauli exclus an all occupy the

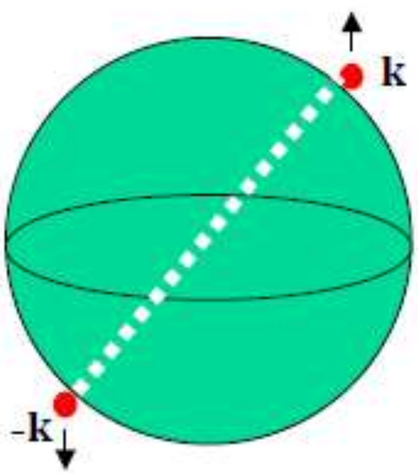


-The BCS theory of Superconductivity states that bound photons have slightly lower energy, which prevents lattice collisions and thus eliminates resistance.

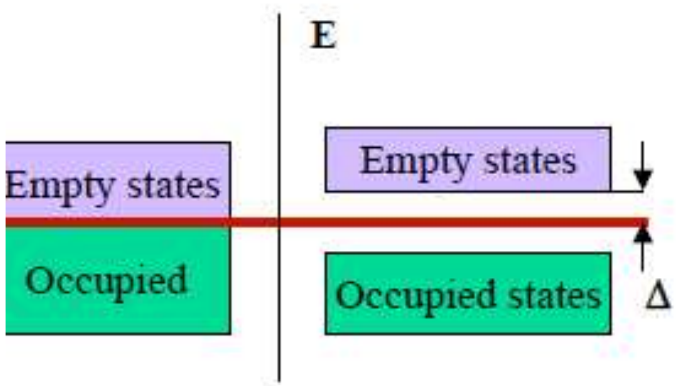
-As long as $kT <$ binding energy, then a current can flow without dissipation.

Cooper pairs and BCS theory of superconductivity

Bardeen-Cooper-Schrieffer (BCS) theory (1957). Nobel prize in 1972



Cooper pair on the Fermi surface



Normal metal

superconductor

- Attraction between electrons with antiparallel momenta **k** and spins due to exchange of lattice vibration quanta (phonons)
- Instability of the normal Fermi surface due to bound states of electron (Cooper) pairs
- Bose condensation of overlapping Cooper pairs into a coherent superconducting state.
- Superconducting gap Δ on the Fermi surface
- Critical temperature: $k_B T_c \approx 1.13\hbar\omega_D \exp(-1/\gamma)$, $\gamma \approx$ 0.1-1 is a dimensionless coupling constant

$$2\hbar\Delta = 3.52 k_B T_c, \quad T_c << T_D \sim 300K$$