

DSE CLASS

CONDENSED MATTER PHYSICS

Lecture-6

08/10/2020

According to classical physics, part of the resistivity of a material is due to collisions between free electrons and thermally displaced ions of the lattice, and part is due to scattering of electrons from impurities or defects.

Classical model could never explain the superconducting state, because the electrons in a material always suffer some collisions, and therefore resistivity can never be zero.

None could explain why electrons enter the superconducting state and why electrons in this state are not scattered by impurities and lattice vibrations.

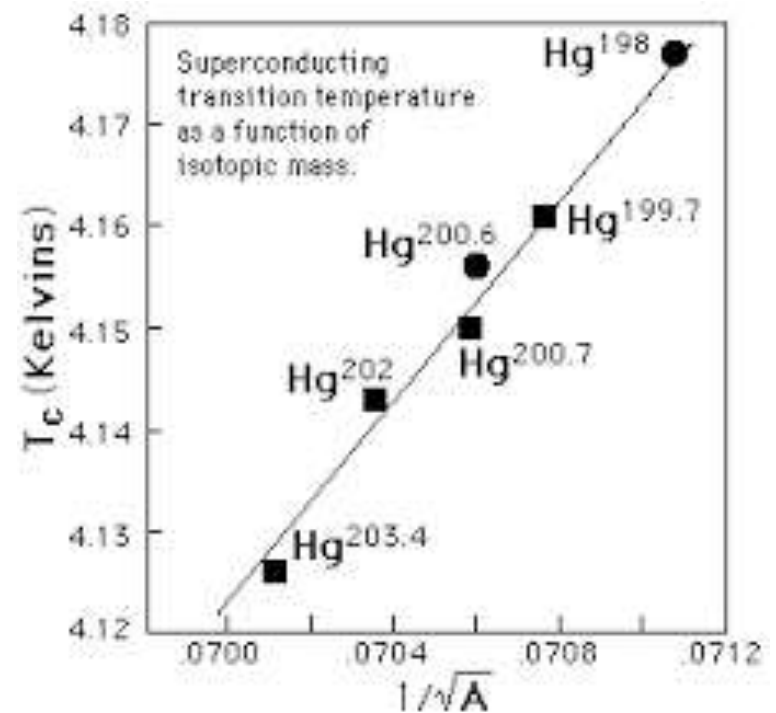
Isotope effect:

At T_c , transition from normal state to superconducting state occurs. This temperature of a material varies with the isotopic mass

$$T_c \propto 1 / M^{1/2}$$

$$T_c \cdot \sqrt{M} = \text{Constant}$$

$$\text{i.e., } M_1^{1/2} T_{c1} = M_2^{1/2} T_{c2} = M_3^{1/2} T_{c3}$$



The isotope effect

One of the final clues to the origin of the superconducting energy gap, and indeed to the origin of superconductivity itself was the discovery of the isotope effect

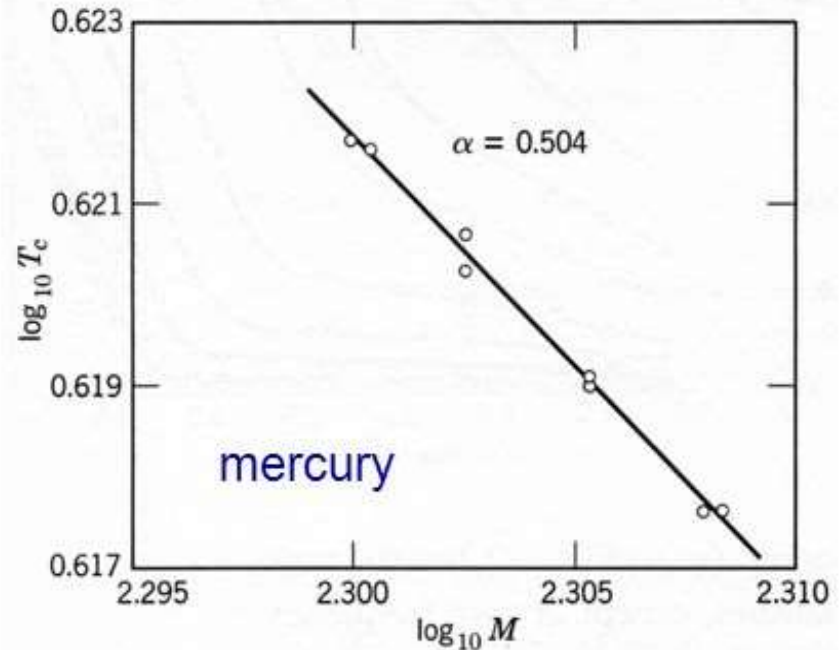
For any particular superconductor T_c depends upon the *isotopic composition* of the sample

Experimentally it is found that

$$M^\alpha T_c = \text{constant}$$

with α usually taking a value close to 0.5

This, together with the observation that good superconductors are poor normal state conductors, suggests that a **strong electron-phonon interaction** may be a necessary condition for superconductivity



Substance	α	Substance	α
Zn	0.45 ± 0.05	Ru	$0.00 \pm$
Cd	0.32 ± 0.07	Os	$0.15 \pm$
Sn	0.47 ± 0.02	Mo	0.33
Hg	0.50 ± 0.03	Nb ₃ Sn	$0.08 \pm$
Pb	0.49 ± 0.02	Mo ₃ Ir	$0.33 \pm$
Tl	0.61 ± 0.10	Zr	$0.00 \pm$

Rather than inhibiting superconductivity, phonons actually seem to help it!

isotope effect, was early evidence that lattice motion played an important role in superconductivity

The full microscopic theory of
superconductivity presented in
1957

by

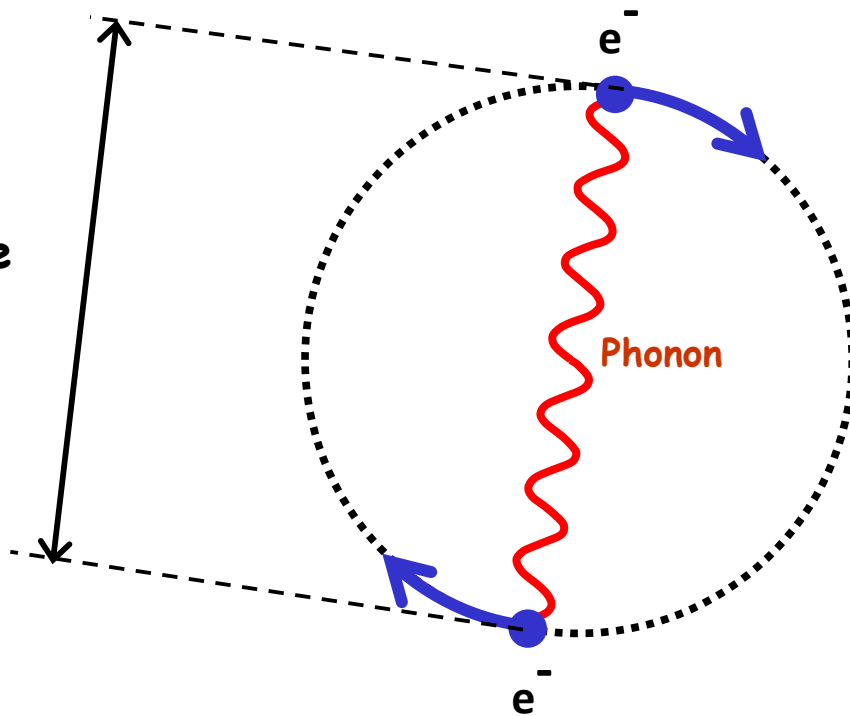
Bardeen, Cooper, and Schrieffer



Nobel Prize in Physics 1972

"for their jointly developed theory of superconductivity, called the BCS-theory"

John Bardeen, Leon Neil Cooper, John Robert Schrieffer



Cooper pair model

The central feature of the BCS theory is that two electrons in the superconductor are able to form a bound pair called a Cooper pair

BCS THEORY

- BCS theory is the first microscopic theory of superconductivity since its discovery in 1911.

It explains,

- The interaction of phonons and electrons
- Cooper pairs : A key conceptual element in this theory is the pairing of electrons close to the Fermi level into Cooper pairs through interaction with the crystal lattice.
- Superconducting Energy Gap

What are Cooper pairs ?

How Cooper pairs are formed ?

Cooper Pairs

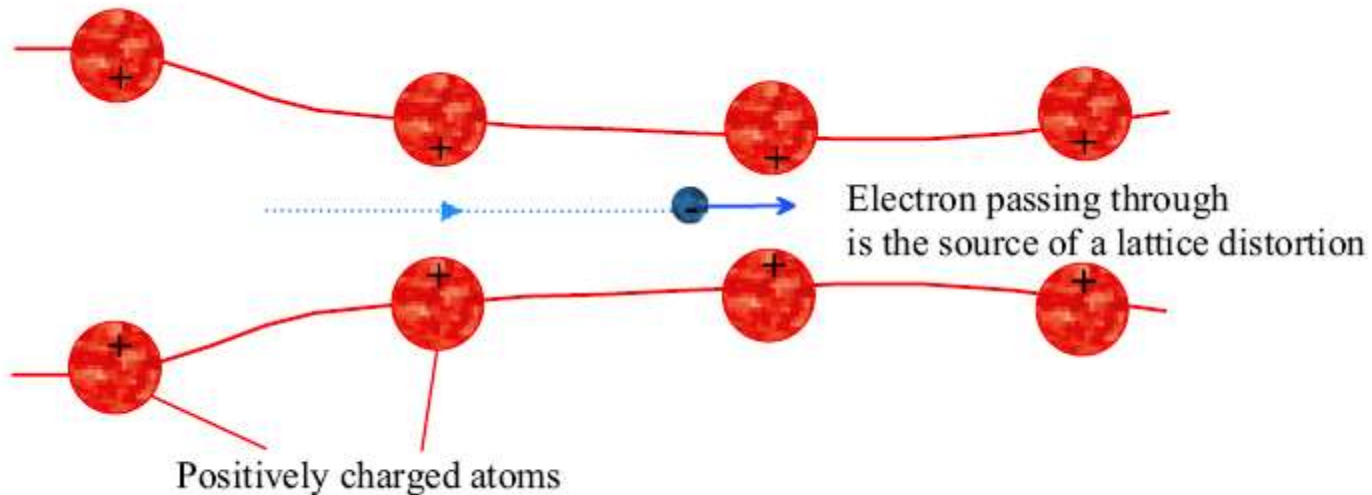
- **Cooper pair** is a pair of electrons (or fermions) bound together at low temperatures in a certain manner first described in 1956 by American physicist Leon Cooper
- An electron in a metal normally behaves as a free particle. The electron is repelled from other electrons due to their negative charge, but it also attracts the positive ions that make up the rigid lattice of the metal.
- This attraction distorts the ion lattice, moving the ions slightly toward the electron, increasing the positive charge density of the lattice in the vicinity.

Cooper Pair:

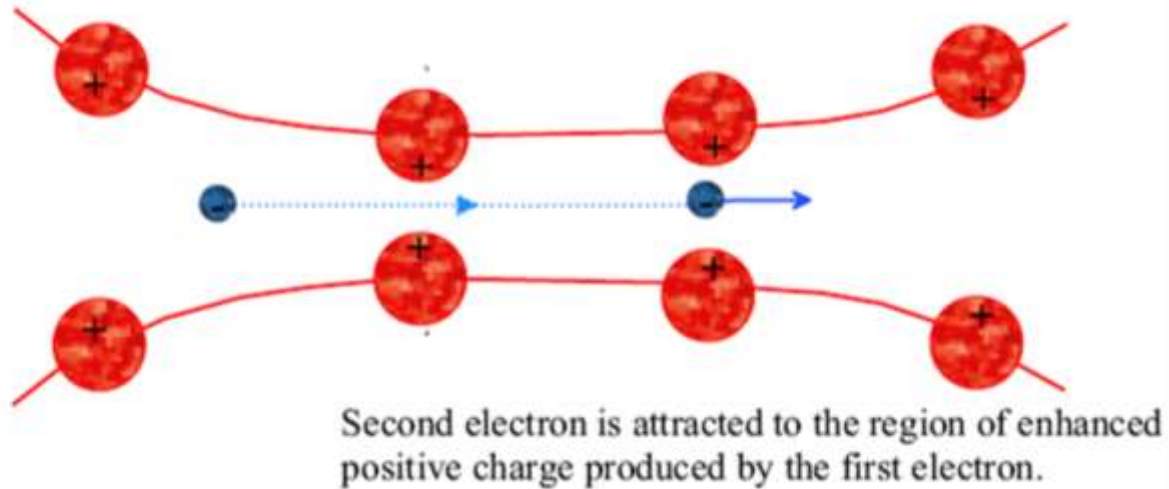
Two electrons that appear to "team up" – despite the fact that they both have a negative charge and normally repel each other.

Below the superconducting transition temperature, paired electrons form a condensate – a macroscopically occupied single quantum state – which flows without resistance

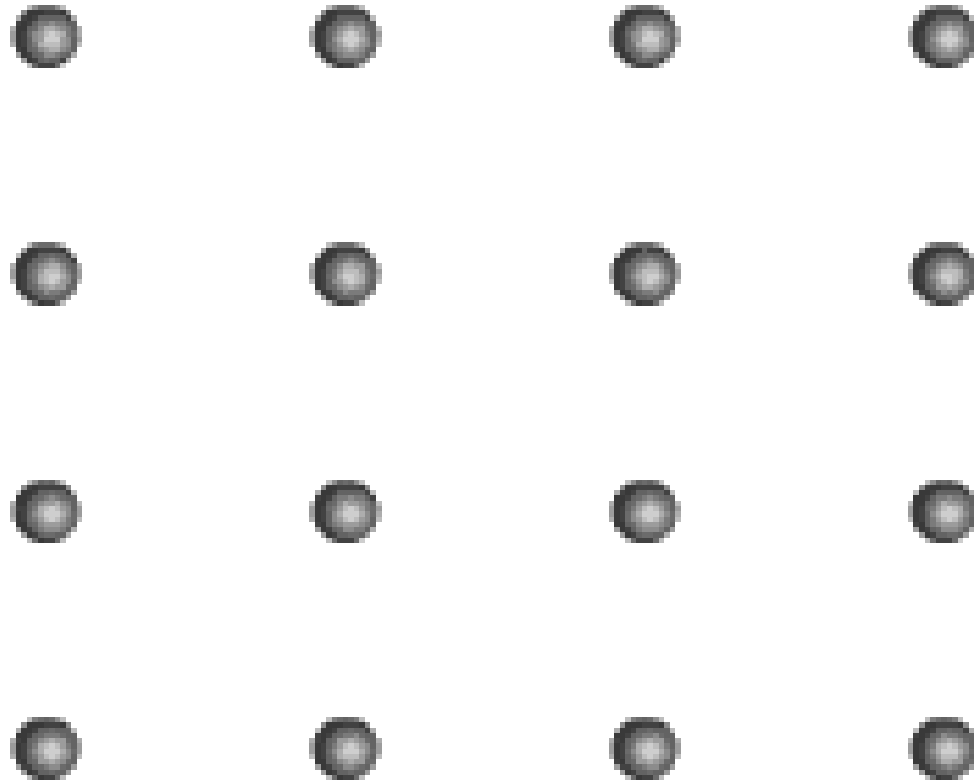
The passage of electron 1 causes nearby ions to move inward toward the electron, resulting in a slight increase in the concentration of positive charge in this region.

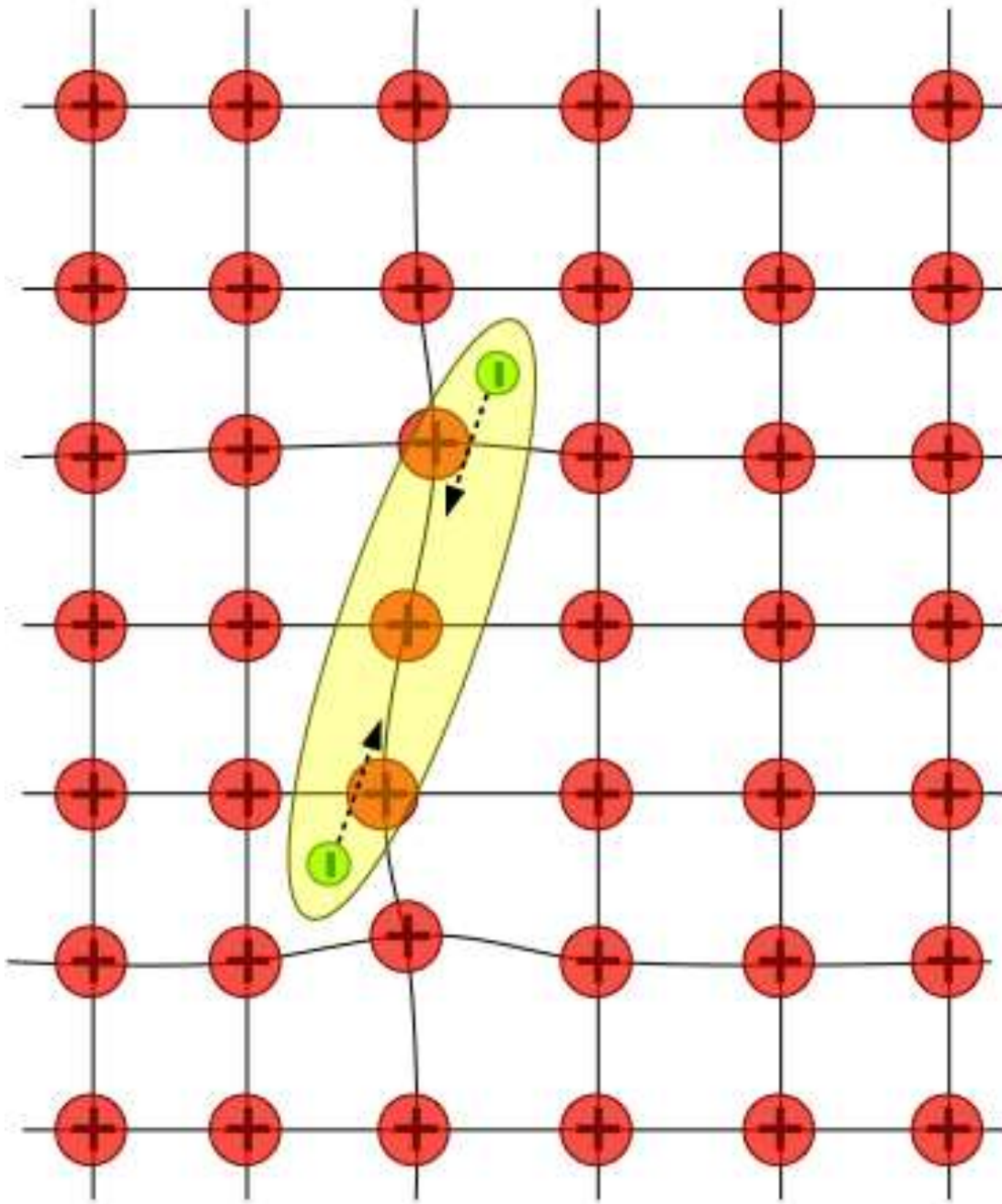


Electron 2 approaching before the ions have had a chance to return to their equilibrium positions, is attracted to the distorted (positively charged) region.



Animation of Cooper pairs:



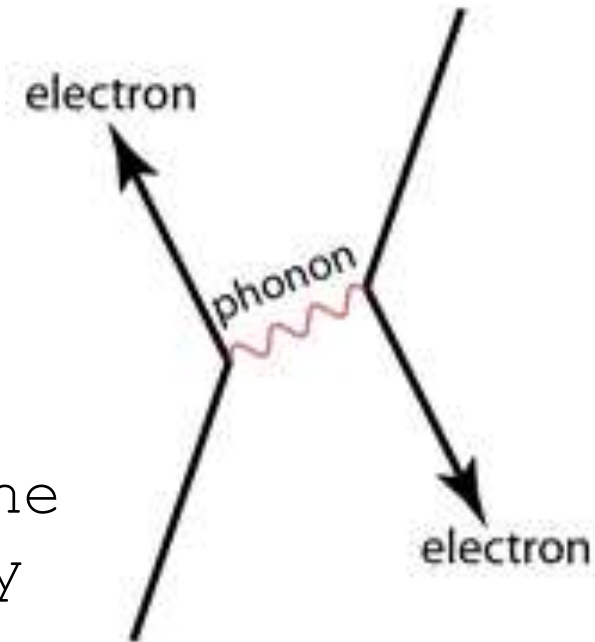


The net effect is a weak delayed attractive force between the two electrons, resulting from the motion of the positive ions.

one can say that the attractive force between two Cooper electrons is an *electron-lattice-electron interaction*, where the crystal lattice serves as the mediator of the attractive force.

Some scientists refer to this as a *phonon-mediated mechanism*, because quantized lattice vibrations are called *phonons*.

The energy of the pairing interaction is quite weak, of the order of meV, and thermal energy can easily break the pairs.

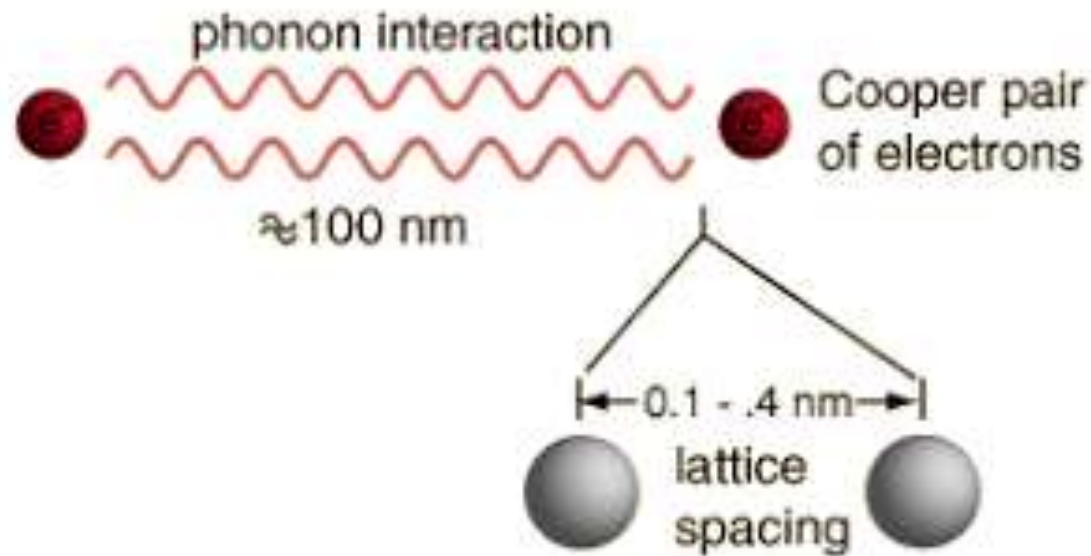


The electrons in a pair are not necessarily close together.

Because the interaction is long range, paired electrons may

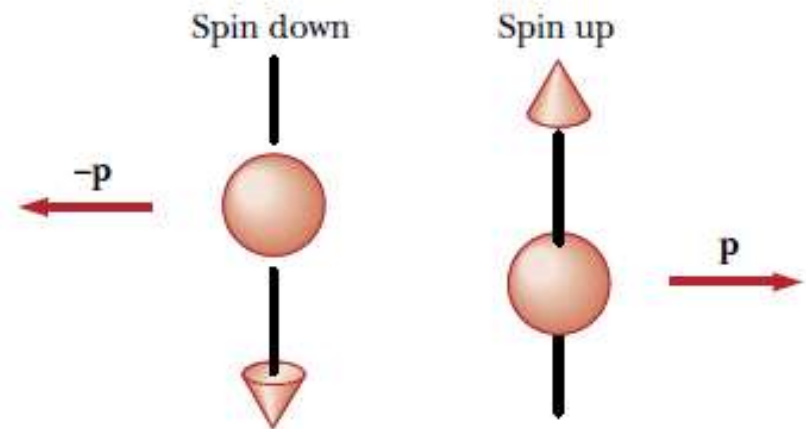
still be many hundreds of nanometers apart.

This distance is usually greater than the average inter-electron distance; and three orders of magnitude larger than lattice spacing.



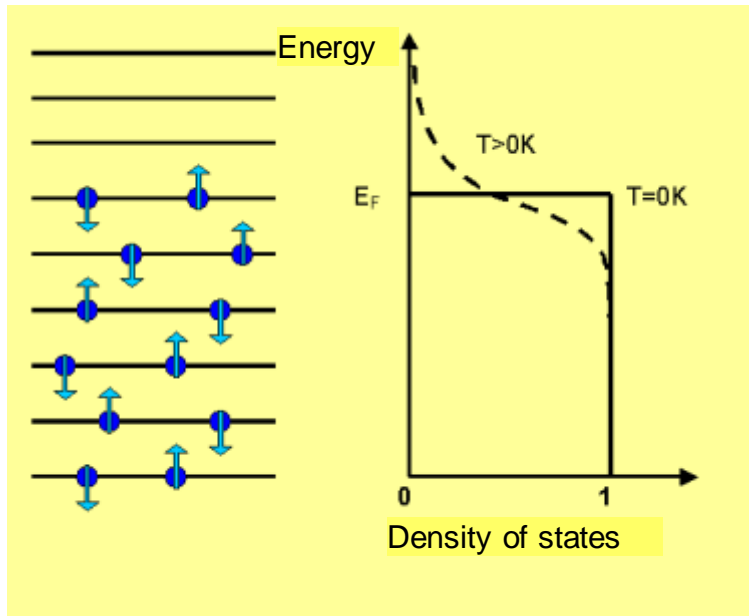
A Cooper pair in a superconductor consists of two electrons having opposite momenta and spin.

In the superconducting state, the linear momenta can be equal and opposite, corresponding to no net current, or slightly different and opposite, corresponding to a net superconducting current.

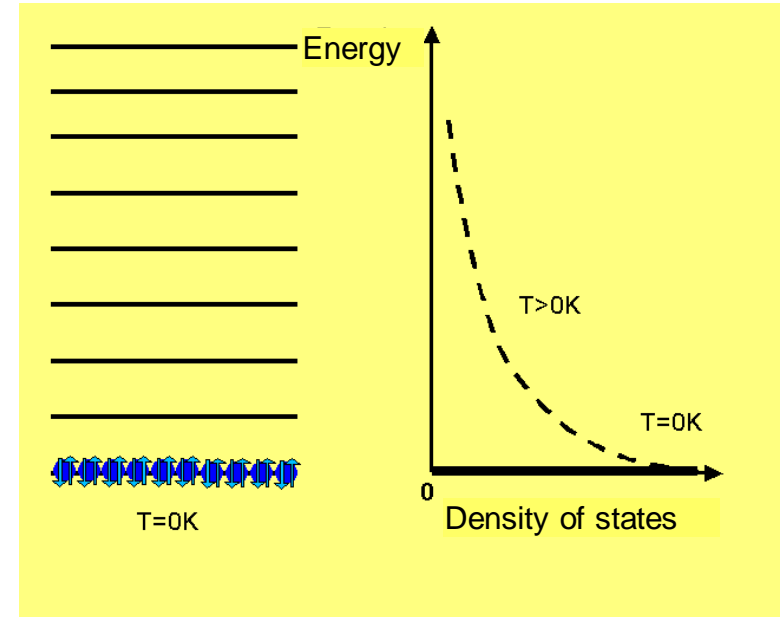


Because Cooper pairs have zero spin, they can all be in the same state.

Fermi und Bose-Statistic



- **Fermions**- elemental particles with $1/2$ spin (e.g. electrons, protons, neutrons..)
- Pauli-Principle -every energy level can be occupied with maximum two electrons with opposite spins.



- Cooper-Pairs are created with electrons with opposite spins.
- Total spin of C-P is zero. C-P are **bosons**. Pauli-Principle doesn't obey.
- All C-P can have the same quantum state with the same energy.

In a Cooper pair, the electron spins ($+1/2$ or $-1/2$) cancel and the total spin is 0;

Cooper pairs are bosons;

As all bosons, Cooper pairs can all be in the same energetic state.

In effect, all Cooper pairs are “locked” into the *same quantum state*.

One can view this state of affairs as a condensation of all electrons into the same state (Bose-Einstein condensation);

- The Cooper pairs are described by one wave function;
- Collisions can only alter the *whole* systems of pairs, such a change requires a large amount of energy.

