THERMAL ANALYSIS FOR FOURIER HEAT CONDUCTION IN A SLAB FOR ISOTROPIC AND ORTHOTROPIC MATERIALS

Thesis submitted by

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Under the Supervision of

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JADAVPUR UNIVERSITY
FACULTY OF ENGINEERING AND TECHNOLOGY
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SELF ATTESTATION

This is to certify that, myself, Ms. Tanusri Sen, have personally worked on the research project entitled "Thermal Analysis for Fourier Heat Conduction in a Slab for Isotropic and Orthotropic Materials". The parameters mentioned in this report were obtained during genuine work done & collected by me. Data obtained from other agencies have been duly acknowledged. None of the findings pertaining to the work has been concealed. The results embodied in this project have not been submitted to any other university or institute for the award of any degree or diploma.

Place: Jadavpur University

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CERTIFICATE OF APPROVAL*

The foregoing thesis entitled "Thermal Analysis for Fourier Heat Conduction in a Slab for Isotropic and Orthotropic Materials" is hereby accepted as a creditable study in the area of 'Heat Power & Thermal Engineering' presented by **Ms. Tanusri Sen** in a satisfactory manner to guarantee its acceptance as a prerequisite to the degree for which it has been submitted. It is understood that by this approval, the undersigned do not necessarily endorse or approve any statement made, opinion expressed and conclusion drawn therein but approve the thesis only for the purpose for which it has been submitted.

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RECOMMENDATION CERTIFICATE

I hereby recommend that the thesis entitled "Thermal Analysis for Fourier Heat Conduction through a Slab for Isotropic and Orthotropic Materials" carried out under my supervision and guidance by Ms.Tanusri Sen may be accepted for the partial fulfillment of the requirements for the degree of "Master of Engineering in Mechanical Engineering".

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CONTENTS

ABSTRACT

Thermal analyses for Fourier Heat Conduction in a slab of composite material have been considered. The application of composite materials has been spread in a wide range of industries. Composite wall, Roof slab are the example of composite materials.

The analytical solutions of transient heat conduction in a Slab for isotropic and orthotropic materials are discussed in this paper by using separation of variable method. Transient thermal responses for both the cases are investigated step by step. How the temperature profiles are changed with respect to time and position while internal heat generations are varied is analyzed in this paper. The changes of temperature profiles are observed subjected to various design conditions.

We have considered a slab which is initially at a uniform temperature and is exposed to ambient fluid. Slab temperature is higher than that of ambient fluid. The slab is subjected to convection heat transfer from both sides. Length and height are very high compared to its thickness. Heat resistance is so high along these two directions that heat transfer is neglected in these directions. Thus, heat transfer analysis can be approximated to be one dimensional which is discussed in chapter-4. For example, heat gain through window of a house is of one dimensional heat conduction.

Further, we have considered a slab of higher thickness which can be compared with length but not with height. Height is so high that heat resistance is very high in this direction compared to other two directions i.e. along length and thickness. Heat transfer analysis can be performed along these two directions which are known as two dimensional heat transfer analysis discussed in chapter-5.

NOMENCLATURE

Bi	Biot Number
h	Heat transfer co efficient
k	Thermal conductivity ($W.m^{-1}.\,^{0}C^{-1}$)
t	Time (s)
T	Temperature (^{0}C)
q	Heat transfer rate (W)
Q	Dimensionless heat transfer rate
F	Fourier Number ($\alpha t/a^2$)
Ve	Vernotte number ($\sqrt{\tau\alpha}/a$)
X	Dimensionless spatial variable
x	Spatial variable(m)
q”	Internal Heat generation
φ,γ	Variables, see Eq.(4.4)
ψ,ρ	Variables, see Eq.(4.12)
A, B	Variables, see Eq.(4.20),
C1, C2…C5	Integration constant

Greek Symbol

α	Thermal diffusivity
θ	Dimensionless temperature
λ	Eigen value
μ	Eigen Value

CHAPTER -1

INTRODUCTION

INTRODUCTION

1.1 GENERAL BACKGROUND

Transfer of heat through a medium due to temperature gradient is called conduction heat transfer which is a microscopic level mechanism. During the conduction heat transfer, translational, rotational, and vibrational energy among the molecules comprising the medium changes whereas convection is a macroscopic form of transfer of energy through a fluid which is the result of the both processes i.e. conduction in the fluid and the bulk motion of the fluid.

Heat conduction has a vital role in our daily life, mainly in earth science, physical science, biological science, social science. 1n 1822, Joseph Fourier, the French mathematical physicist, studied the results obtained through experiments on heat conduction and thus derived a powerful tool named as Fourier's Law which dictates a linear relationship between a heat flux and temperature gradient, at infinite propagation velocity of the thermal wave. Simultaneously, in most of engineering applications, validity of the Fourier's Law has been proved. Fourier's law leads to infinite propagation velocities for thermal disturbances because of its parabolic nature of the relevant governing equations.

1.2 FOURIER'S LAW AND THERMAL CONDUCTIVITY

The transfer of heat by conduction can generally be described by Fourier Law i.e.

$$q'' = -k\nabla T \tag{1.1}$$

where k is the thermal conductivity, ∇T is the temperature gradient. Negative sign appears in the equation as heat flows in the direction of decreasing temperature. In general, thermal conductivity is a tensor which relates one vector to another. In engineering application, materials are isotropic i.e. these materials have the same properties in all directions. For these materials, we need not to consider the properties along X, Y and Z directions separately whereas for anisotropic materials, we need to calculate these properties separately in X, Y, Z directions.

Fourier's law for tensor k in Cartesian co ordinate for anisotropic material is as follows:

$$\begin{bmatrix} q_x^{"} \\ q_y^{"} \\ q_z^{"} \end{bmatrix} = \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{xy} & k_{yy} & k_{yz} \\ k_{xz} & k_{yz} & k_{zz} \end{bmatrix} \times \begin{bmatrix} \frac{\partial T}{\partial x} \\ \frac{\partial T}{\partial y} \\ \frac{\partial T}{\partial z} \end{bmatrix} \quad (1.2)$$

1.3 COMPOSITE MATERIAL

When two or more materials of which physical and chemical properties are different are combined among themselves to produce a new material which gives higher strength-weight ratio compared to its constituent materials, it is called composite material.

Concrete slab is an example of composite material. Foils or sheets or foils combined with fibers are the combinations of different particulate composites which are called hybrid composites which enhances the impact resistance. Adding fibers help in resisting damage. In the construction field, there are lots of uses of structural composites, for example, sandwich panels which include roofs, floors, and walls of buildings.

The physical and chemical properties of these composite materials are dependent on the direction of the applied force. All these composite materials' properties are different along X-axis and Y-axis. In our analysis, we have considered a slab of such composites so that properties differ along co ordinates.

1.4 ORTHOTROPIC MATERIAL

Materials have properties that differ along three mutually orthogonal axes of rotational symmetry are called orthotropic materials. These are subset of anisotropic materials. A familiar example of an orthotropic material is Wood. In this case properties are different in three mutually perpendicular directions i.e. axial direction, radial direction and circumferential direction. Polar orthotropy is the type of orthotropy where preferred co-ordinate system is cylindrical polar. Mechanical properties i.e. strength, stiffness are typically better in axial direction than that of measured in other two directions. Hankinson's equation explains these directional differences in strength.

Sheet metal is another example of orthotropic material. It is formed by squeezing thick sections of metal between heavy rollers and for this, grain structure changed. As a result, material becomes anisotropic.

Hence, all the composite materials are anisotropic in nature. All orthotropic materials are anisotropic but all anisotropic materials are not orthotropic in nature. Isotropic materials posses equal strength in all directions and for this reason, efficient structure can't be made by this type of material whereas it is possible by anisotropic material. In most of the application, there is a necessity of these variations in strength along directions. As for example, in a beam, it is the transverse direction on which load is imposed whereas beam bent along lengthwise direction and strength along lateral direction is utilized. There is no load imposed in lateral direction.

Thermal conductivity has nine components. For anisotropic material, if all the components except the diagonal components become zero, then it is called orthotropic material which described by the following matrix:

$$K = \begin{bmatrix} K_{11} & 0 & 0 \\ 0 & K_{22} & 0 \\ 0 & 0 & K_{33} \end{bmatrix} \tag{1.3}$$

1.5 OBJECTIVE

The attention of this paper is to focus on the thermal analysis of the orthotropic composite material. The aim of the analysis is to find out the variation of the temperature as a function of time and position within the body.

Here analysis is performed across a square shaped slab. Face dimensions are considered very large compared to the thickness. One dimensional analysis is done in chapter-4 of this paper and two dimensional analysis is done in chapter 5 of this paper. Boundary conditions applied to each surface is uniform.

The objective of the analysis of the heat conduction is to observe the temperature profiles and how the temperature changes inside the body with time as well as with spatial co-ordinates. To calculate the heat flux at any point of the body, we have to know the temperature profile first. Further, if we know the temperature field we can also come to know the following parameters: thermal stresses, heat treatment method, expansion, design insulation thickness, deflection etc.

CHAPTER -2

LITERATURE SURVEY

LITERATURE SURVEY

2.1 LITERATURE SURVEY

The heat conduction equation of Fourier provides the solution displayed propagation speed of thermal signals infinitesimally. Cattaneo and Vernotte [1] individually evaluated a relaxation model of time dependent for heat flux in solid in order to avoid the failure of parabolic heat conduction equation in high heat flux and highly unsteady (transient) situations applications e.g. laser pulse annealing of semi conductors. A finite speed of heat flux has been considered in this model, which leads to the following equation:

$$q = -k\nabla T - \tau_0 \frac{\partial q}{\partial t} \tag{2.1}$$

Where q is the heat flux, τ_0 is the relaxation time, k is the thermal conductivity and T is the temperature. The classical Fourier's diffusion model is invalid [2-4] for large temperature gradient, low temperature or a very high heating speed. Non Fourier heat conduction can arise in practical engineering problems e.g. in cryogenic engineering and surface melting of metals [5, 6]. There is no formula of physically realistic principal except Fourier's heat conduction for the transient diffusion problem. There is of fundamental importance as the equation can't be confidently integrated over an arbitrary domain unless exact form of the integration is known.

Eq. (2.1) and the energy conservation law, together, provide a profile of hyperbolic equation for an unsteady temperature. Fushinobu et al. [7] analyzed the classical Fourier heat conduction models which predict the temperature distribution in general engineering problems. Cimmelli [8] discussed that zero time lag between imposing temperature gradient and conduction heat transfer is not true because two linked phenomenon can't happen simultaneously.

How the temperature profile of thermal wave varies with sudden change in temperature at the wall in a semi infinite solid is analyzed by Baumeister and Hamill [9]. He has used Laplace Transformation Method here. Taitel [10] established a solution analytically for a thin layer on both sides with respect to a step change of temperature in 1972. A numerical solution for a thin layer on one side with respect to a step change of temperature was provided by Garey [11] in 1982. An analytical solution for a finite slab with insulated boundaries using a volumetric energy source has been given by Ozisik [12] in 1984.For a finite slab under boundary condition of rectangular heat pulse, an analytical solution of hyperbolic heat conduction equation using flux formulation was given by Frankel [13] in 1985. Glass [14, 15] solved the HHC equation with surface radiation in 1985 and temperature dependent conductivity in a finite medium in 1986. In a semi infinite solid subjected to a periodic on-off heat flux, an explicit analytical solution was presented by Glass [16] for a hyperbolic heat conduction equation. In the above mentioned presentation, non linear profiles with surface radiation were solved by using a finite difference scheme. Gembarovic [17] analytically solved HHC equation under instantaneous and extended heat pulse in a finite slab. . Tzou [18, 19] analyzed the damping and resonance characteristics of thermal waves of a heat source subjected to periodic heating. Under pulse surface heating, in a finite medium, Fourier heat conduction equation was analyzed by Tang and Araki [20]. Mikhailov and Cotta [21] gave the solution of a hyperbolic heat conduction equation for periodic nature and steady condition in a finite slab using the computer algebra system Mathematica. Tang and Araki [22] provided the solution of non Fourier heat conduction under periodic surface disturbances in a finite medium. The characteristics of thermal waves were studied analytically by Juhng [23]. Solution of non Fourier heat conduction equation under periodic surface disturbances using the finite integral transform was given by Abdel and Hamid [24]. Wang [25] analyzed the structure of the solution of hyperbolic heat conduction equation. Lor and Chu [26] gave solution of the problem considering thermal resistance in the interface. Using hermite approximation for integration and considering circumferential symmetry heat flux in radial direction in a nuclear fuel rod, transient heat conduction was explained by Clarissa [27]. An Orthogonality relationship has been built by F.de.Monte [28] and obtained a final series of solution for a composite slab. The unsteady heat transfer analysis in a nuclear fuel rod has been performed by Su and Cotta [29].An improved lumped parameter approach had been used here. By using hermite approximation method, average fuel and cladding temperature is calculated. Transient cooling of a long slab has been analyzed by Alhama and Campo [30] by using asymmetric heat convection. With surface radiation and periodic thermal disturbances, in a finite medium, a semi-analytical numerical solution was provided by Yen and Wu [31]. Using the transfer function method, Fourier heat conduction across a one dimensional slab was analyzed by Cossali [32] subject to periodic boundary condition. Transient, one dimensional heat conduction problem for a slab has been analyzed by H.Sadat [33] by using perturbation method. A second order model for unsteady heat conduction in a slab has also been derived by H.Sadat [34] with perturbation method. It has been observed that for high values of Biot number in surrounding the center of the slab, the simple model is more accurate. Symmetrically heated on both sides for a finite medium, solution of a hyperbolic heat conduction equation was analyzed by Lewandowska and Malinowski [35]. Under the circumstances of arbitrary periodic and non-periodic surface disturbances, Moosai [36, 37] gave solution of hyperbolic heat conduction equation (HHC). The unsteady, one dimensional heat conduction of a slab has been analyzed by Keshavarz and Taheri [38] by using Polynomial Approximation method. Considering temperature dependent thermal conductivity; lumped parameter model's development was suggested by Gesu [39]. At uniform temperature and finite Biot numbers, on Fourier heat conduction on a sphere exposed to surrounding was analyzed by Astrogorsky [40]. Laplace transformation method is used here. Thermal wave propagation analysis in a finite slab was performed by Monterio [41]. Integral transformation method is used here. Hyperbolic heat conduction subject to laser heating varying with time was analyzed by Lam and Fong [42].Considering 1-D cylindrical and spherical geometries; non-Fourier heat conduction was analyzed by Mishra and Sahai [43]. Lattice Boltzman method is used here. Temperature variation in thin die-electric material was analyzed by Sarkar and Hazi Sheikh [44]. Laplace transformation method is used here. One dimensional transient heat conduction of a slab was solved by Keshavarz and Taheri [45] in 2007 by using Polynomial Approximation Method.

The Literature survey clearly shows that last few years, works are exhibited related to one dimensional HHC equation for various types of initial and boundary conditions or different source term in a finite and semi infinite mediums. Also, some research works are done for two dimensional HHC which are solved numerically.

Quasilinear diffusion equations with non linear source were analytically solved by Separation of variable method by Changzheng Qu [46] and generalized porous medium equation with non linear source was solved analytically by P.G.Estevez [47]. Laplace's Equation for a particular set of boundary condition i.e. $\nabla^2 V = 0$ is solved by separation of variable method by W. Miller [48]. Wei Tao Zhao [49] analyzed non Fourier heat conduction in a solid sphere considering arbitrary surface thermal disturbances using separation of variable method and the comparison of temperature response between the hyperbolic and parabolic equations are established. Non linear wave equation was solved analytically by R.Z. Zhdanov [50]. Qu [51] provided exact solutions to nonlinear diffusion equations obtained by generalized conditional symmetry in 1999. Qu [52] provided exact solutions to quasilinear diffusion equations with the nonlinear source in 2000. Lu et al. [53] analytically solved transient heat conduction in a composite circular cylinder in 2006.

2.2 SCOPE OF PRESENT WORK

Lot of research works has been done on transient heat conduction which are solved analytically. There are many procedures in solving analytically e.g. - The orthogonal expansion technique [54-57] which technique was first developed by Vodicka [54]; the quasi-orthogonal expansion technique [58, 59] which was developed by Tittle [58]; the Laplace transform method [60]; the Green's function approach [60-63]; the Galerkin procedure [64-66]; the finite integral transform technique [67]; Separation of variable method [68].

The above mentioned literature review showed that lot of works has been performed based on other methods compared to separation of variable method. 2-D Fourier heat conduction equation has not yet solved by this method considering different heat transfer co efficient in both sides of the slab of orthotropic material.

Very little work has been exhibited on analytical solution of Fourier Heat conduction for orthotropic materials by Separation of Variable Method. Present work is humbly focused in that field.

In this paper, under the guidance of arbitrary initial conditions, 2 dimensional heat conduction equations are solved analytically by using Fourier series method. The main purpose of this work is to provide an analytical benchmark for one dimensional and two dimensional Fourier heat conduction for Isotropic and Orthotropic materials.

Main aim of the present work is to compare the temperature profiles obtained from Isotropic and Orthotropic material subjected to different design conditions.

CHAPTER -3

METHODOLOGY

METHODOLOGY

3.1 METHODOLOGY

In this paper, heat transfer of a slab for isotropic and orthotropic material has been solved analytically. There are some cases where it is very difficult to solve the system numerically. However, solution is much secured by analytical methods. Analytical methods provide an exact solution of how the model behaves under any circumstances. Accuracy is also much higher than Numerical Methods.

Separation of variable method is one of the simplest analytical methods for solving partial differential equations. This method is also known as "Fourier Method". Separation of Variable Method gives a closed form solution. This method provides an exact and unique solution. It is assumed that the dependent variable is the product of a number of functions which are single independent variables. Thus, a partial differential equation is reduced to a system of ordinary differential equations, each being a function of single independent variable.

For example, for a case of a non-steady heat conduction for a plane wall, the dependent variable 'u' is the solution function $u(x,t)$. In this case, partial differential equation, 'u' is a function of two independent variables 'x' and 't'. This method depends upon the assumption that a function of the form:

$$u(x,t) = \eta(x)\varepsilon(t) \tag{3.1}$$

will be a solution to the partial differential equation which is called as product separable solutions. Application of this method results two ordinary differential equations, one equation is function of 'x' and the other one is function of 't'. Many partial differential equations have this type of separable solutions. By using separation of variables, we were able to convert our linear homogeneous partial differential equation with linear homogeneous boundary conditions into an ordinary differential equation. The boundary value problem is an Eigen value problem. We have to identify the Eigen values λ during solving the boundary value problem. This Eigen value will generate non-trivial solutions to the corresponding Eigen functions.

When the equation is non homogeneous, we have to convert this equation to homogeneous equation first, then we can apply separation of variable method (SOV). SOV Method for Quasilinear equation is first studied by Changzheng Qu et al. [69]. Generalized conditional symmetry approach is used here.

3.2 APPLICATION OF ANALYTICAL METHOD

For calculating the heat transfer rates and analyzing the temperature history, analytical method can be applied for finite shaped bodies. Analytical method is applicable where boundary conditions are uniform or where temperature is also uniform throughout the body initially or where heat transfer co efficient does not change with time.

3.3 APPLICATION OF SOV METHOD

This method is applicable to simple and finite geometry (for example, a cylinder, a sphere, a rectangular slab etc). Boundary surfaces have to be explained by simple mathematical functions. The partial differential equation and associated boundary conditions and initial conditions must be linear. The main governing equation may contain one homogeneous term.

CHAPTER -4

1-D TRANSIENT HEAT CONDUCTION

1-D TRANSIENT HEAT CONDUCTION

4.1 1-D TRANSIENT HEAT CONDUCTION

One dimensional transient heat conduction occurs while quenching a steel rod and then soaking the same by a bath. Another example is perishable vegetables immersed in a chilled bath in a cold storage. Here geometry is different. Walls of a building are heated gradually as time passed. Heat flows from outer wall to inner wall which is an example of this phenomenon.

The one dimensional transient heat conduction equation can be solved numerically as well as analytically. Here, we solve the partial differential equation analytically by using "separation of variable method" (SOV). Our first aim is to convert the partial differential equation to an ordinary differential equation.

Heat conduction in a square shaped slab of thickness '*2a*' is considered here. Initially, the slab is at temperature T_0 and ambient temperature is T_a assumed to be constant. Origin of the axis has been chosen at the center of the slab so that there persists symmetry of temperature distribution about $X=0$. Thermal conductivity has been considered as k_x. The governing differential equations considering no heat generation and with heat generation to be solved are as follows:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{4.1a}$$

And

$$\frac{\partial^2 T}{\partial x^2} + \frac{q"}{k_x} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{4.1b}$$

The results obtained by solving Eq. (4.1a) by SOV method can be checked by Heisler Charts. Heisler [70] presented the transient temperature charts for a large plane wall, long cylinder and sphere in 1947. These charts are called Heisler chart. H. Grober [71] modified these charts in 1961. There are three charts for each Geometry.1st chart determines the temperature at the center of the geometry for a given time‘t’. Temperatures at other locations are determined by second chart. The total amount of heat transfer is determined by 3rd chart. Results obtained by solving Eq. (4.1b) by SOV method can’t be checked through this chart as there is heat generation inside the slab. This is the limitation of Heisler Chart.

4.2 PROBLEM STATEMENT

The both sides of the slab are subjected to convective heat transfer where convective heat transfer co efficient of both sides of the slab is ‘h_1’ and ‘h_2’ for left and right sides respectively.

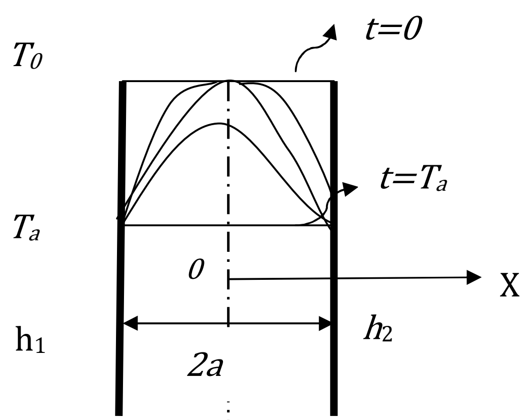


Fig: 4.1 Heat transfer across a 1-D Slab

The non dimensional form of the governing differential equation without heat generation is:

$$\frac{\partial^2\theta}{\partial X^2} = \frac{\partial\theta}{\partial F} \tag{4.2}$$

Where θ is the dimensionless temperature, F is the Fourier number and X is the dimensionless distances are defined as follows:

$$X = \frac{x}{a} \text{ and } Y = \frac{y}{a} \tag{4.2a}$$

$$\theta = \frac{(T - T_a)}{(T_0 - T_a)} \tag{4.2b}$$

$$F = k_x \left(\frac{t}{\rho C_p a^2}\right) \tag{4.2c}$$

The non dimensional form of the governing differential equation with heat generation is:

$$\frac{\partial^2 \theta}{\partial X^2} + Q_0(1 + \beta\theta) = \frac{\partial \theta}{\partial F} \tag{4.3}$$

where,

$$Q_0 = \frac{(q_0 a^2)}{k_x(T_0 - T_a)} \tag{4.3a}$$

$$q'' = q_0[1 + \alpha'(T - T_a)] \tag{4.3b}$$

$$q'' = q_0(1 + \beta\theta) \tag{4.3c}$$

$$\beta = \alpha'(T_0 - T_a) \tag{4.3d}$$

The analytical solution provides exact solution whereas infinite series and transcendental equations arriving during other type of solutions.

4.3 BOUNDARY AND INITIAL CONDITION

Dimensionless boundary conditions are:

$$at \;\; X = 0, \; \frac{\partial \theta}{\partial X} = 0 \tag{4.4a}$$

$$at \;\; X = 1, \; \frac{\partial \theta}{\partial X} = -Bi_2\theta \tag{4.4b}$$

In non dimensional form, the initial condition is:

$$at\ F = 0,\ \theta = 1 \tag{4.4c}$$

These conditions are same for both types of heat conduction-(i) without heat generation (ii) with heat generation.

4.4 Mathematical Formulation

4.4.1 Without Heat Generation:

The non dimensional temperature '*θ*' can be expressed as a product function of '*X*' and '*F*' as follows:

$$\theta = \emptyset(X)\gamma(F) \tag{4.5}$$

Substituting the value of θ in the governing equation:

$$\frac{d^2\emptyset}{dX^2} + \lambda^2\emptyset = 0 \tag{4.5a}$$

and

$$\frac{d\gamma}{dF} + \lambda^2\gamma = 0 \tag{4.5b}$$

The general solution of the above two equations are:

$$\theta = e^{-\lambda^2 F}\{A\,cos(\lambda X) + B\,sin(\lambda X)\} \tag{4.6}$$

where *A* & *B* are arbitrary constants. Now we have to find out the value of *A* and *B* to get the solution of the problem.

Applying the first boundary conditions, we get as follows:

$$\theta = Ae^{-\lambda^2 F}\,cos(\lambda X) \tag{4.7}$$

Applying the second boundary condition, we get the following characteristic equation:

$$\lambda_n\,tan\,\lambda_n = Bi \tag{4.8}$$

The final solution is

$$\theta = A_n e^{-\lambda_n{}^2 F} \cos(\lambda_n X) \tag{4.9}$$

The constant A_n can be found out by applying the initial condition:

$$A_n = \frac{4\sin(\lambda_n)}{2\lambda_n + \sin(2\lambda_n)} \tag{4.10}$$

Substituting the value of A_n, we get the value of θ as:

$$\theta = \sum_{n=1}^{\infty} \frac{4\sin(\lambda_n)}{2\lambda_n + \sin(2\lambda_n)} e^{-\lambda_n{}^2 F} \cos(\lambda_n X) \tag{4.11}$$

4.4.2 With Heat Generation:

Governing differential Eq. (4.3) is a non homogeneous equation. The non dimensional temperature 'θ' can be expressed as a summation function of 'X' and 'X' &' F' as follows:

$$\theta = \psi(X) + \rho(X, F) \tag{4.12}$$

Substituting the value of 'θ' in terms of 'ψ' and 'ρ' in Eq. (4.3), we have the following two equations:

$$\frac{d^2\psi}{dX^2} + Q_0 + Q_0\beta\psi = 0 \tag{4.13}$$

and

$$\frac{\partial^2\rho}{\partial X^2} + Q_0\beta\rho = \frac{\partial\rho}{\partial F} \tag{4.14}$$

Eq. (4.13) is non homogeneous which can be converted to homogeneous equation as follows:

$$\frac{d^2\emptyset}{dX^2} + Q_0\beta\emptyset = 0 \tag{4.15}$$

where,

$$\emptyset = Q_0 + Q_0\beta\psi \tag{4.16}$$

General solution of Eq. (4.15) is:

$$\emptyset = C_1 \cos(\sqrt{Q_0\beta}X) + C_2 \sin(\sqrt{Q_0\beta}X) \tag{4.17}$$

Solving Eq. (4.17), we get the value of ‘ψ’ as:

$$\psi = [\frac{Bi_2\, Q_0\, cos(\sqrt{Q_0\beta}\, X)}{\beta\sqrt{(Bi^2+Q_0\beta)}\, sin(\gamma-\sqrt{Q_0\beta})} - \frac{1}{\beta}] \tag{4.18}$$

where,

$$\gamma = tan^{-1}(\frac{Bi_2}{\sqrt{Q_0\beta}}) \tag{4.19}$$

For solving Eq. (4.14), ‘ρ’ can be expressed as a product function of ‘*X*’ and ’*F*’ as follows:

$$\rho(X, F) = A(X)B(F) \tag{4.20}$$

Substituting the value of ‘*ρ*’ in terms of ‘*A*’ and ‘*B*’ in Eq. (4.14), we have the following two equations:

$$\frac{d^2A}{dX^2} + (Q_0\beta + \lambda^2)A = 0 \tag{4.21}$$

and

$$\frac{dB}{dF} + \lambda^2\, B = 0 \tag{4.22}$$

Solving the above two equations, we get the values of ‘*A*’ and ‘*B*’ as follows:

$$A = C_3\, cos(\sqrt{Q_0\beta + \lambda^2}\, X) \tag{4.23}$$

and

$$B = C_5 e^{-\lambda_n^2 F} \tag{4.24}$$

Putting these values of ‘*A*’ and ‘*B*’ in Eq. (4.20):

$$\rho = \sum_1^\infty C_n\ cos(\sqrt{Q_0\beta + {\lambda_n}^2}\, X)e^{-\lambda_n^2 F} \tag{4.25}$$

By integration, value of C_n arrived as:

$$C_n = \frac{(P_n+Q_n)}{\{(P_n+Q_n)+sin(P_n+Q_n)\}}[4\left(1+\frac{1}{\beta}\right)\frac{sin\left(\frac{P_n+Q_n}{2}\right)}{P_n+Q_n}$$

$$\frac{Bi_2}{\beta\sqrt{\{Bi_2{}^2+(\frac{P_n-Q_n}{2})^2\}}\,sin(\gamma-\frac{P_n-Q_n}{2})}\{\frac{Q_n\,sin\,P_n+P_n\,sin\,Q_n}{P_nQ_n}\}] \tag{4.26}$$

where,

$$P_n = \sqrt{(Q_0\beta+\lambda_n{}^2)} + \sqrt{Q_0\beta} \tag{4.27}$$

and

$$Q_n = \sqrt{(Q_0\beta+\lambda_n{}^2)} - \sqrt{Q_0\beta} \tag{4.28}$$

$$\gamma = tan^{-1}\frac{2Bi_2}{(P_n-Q_n)} \tag{4.29}$$

$$Z = tan^{-1}(\frac{2Bi_2}{P_n+Q_n}) \tag{4.30}$$

$$\frac{P_n+Q_n}{2} = (Z-n\pi) \tag{4.31}$$

where, $(Z-n\pi)^2 > (Q_0\,\beta)$ to avoid the imaginary values of λ

Final expression of 'θ' is as follows:

$$\theta = \left[\frac{Bi_2\ cos(\sqrt{Q_0\beta}\,X)}{\beta\sqrt{(Bi_2{}^2+Q_0\,\beta)}\,sin(\gamma-\sqrt{Q_0\beta})}-\frac{1}{\beta}\right]+$$

$$\sum_1^\infty C_n\ cos(\sqrt{Q_0\beta+\lambda_n{}^2}\,X)e^{-\lambda_n^2F} \tag{4.32}$$

CHAPTER -5

2-D TRANSIENT HEAT CONDUCTION

2-D TRANSIENT HEAT CONDUCTION

5.1 2-D TRANSIENT HEAT CONDUCTION

When the temperature is a function of two spatial co-ordinates i.e. along X-axis and Y-axis and time, the heat transfer is known as two dimensional transient heat conduction. Here, Thickness of the slab is large compared to previous case.

Two dimensional transient heat transfer phenomenon plays an important role in industry as well as in environmental problems. As for example, inside the steam boiler, transfers of heat through the furnace walls occur along two directions i.e. along width and thickness. In Air conditioning duct which carries cold air is of 15-18 Deg C range whereas ambient temperature is of 35-40 Deg C. The duct is of square shaped and length is too high that heat transfer along that direction is neglible. But heat transfer occurs in other two directions i.e. along thickness and width of the duct. This is also an example of two dimensional transient heat conduction.

In this analysis, we have considered the slab material as orthotropic. Thermal conductivity along X-axis and Y-axis are different. However, we have considered thermal conductivity as a constant parameter. The two dimensional transient heat conduction equations can also be solved by various ways. Here, we solve the same by using Separation of Variables Method.

The Governing differential equation is:

$$\frac{\partial^2 T}{\partial x^2} + K_r \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{5.1}$$

5.2 PROBLEM STATEMENT

We consider the heat conduction across a square shaped slab of dimension, '$2a$' as shown in figure 5.1. Surrounding temperature across the slab in all direction is uniform i.e. T_a. The slab is initially at temperature T_0, having heat flux at one side and exchanging heat by convection in another side. Constant heat transfer co-efficient 'h_1' and 'h_2' are assumed along the horizontal surface of the slab and the vertical surface of the slab respectively.

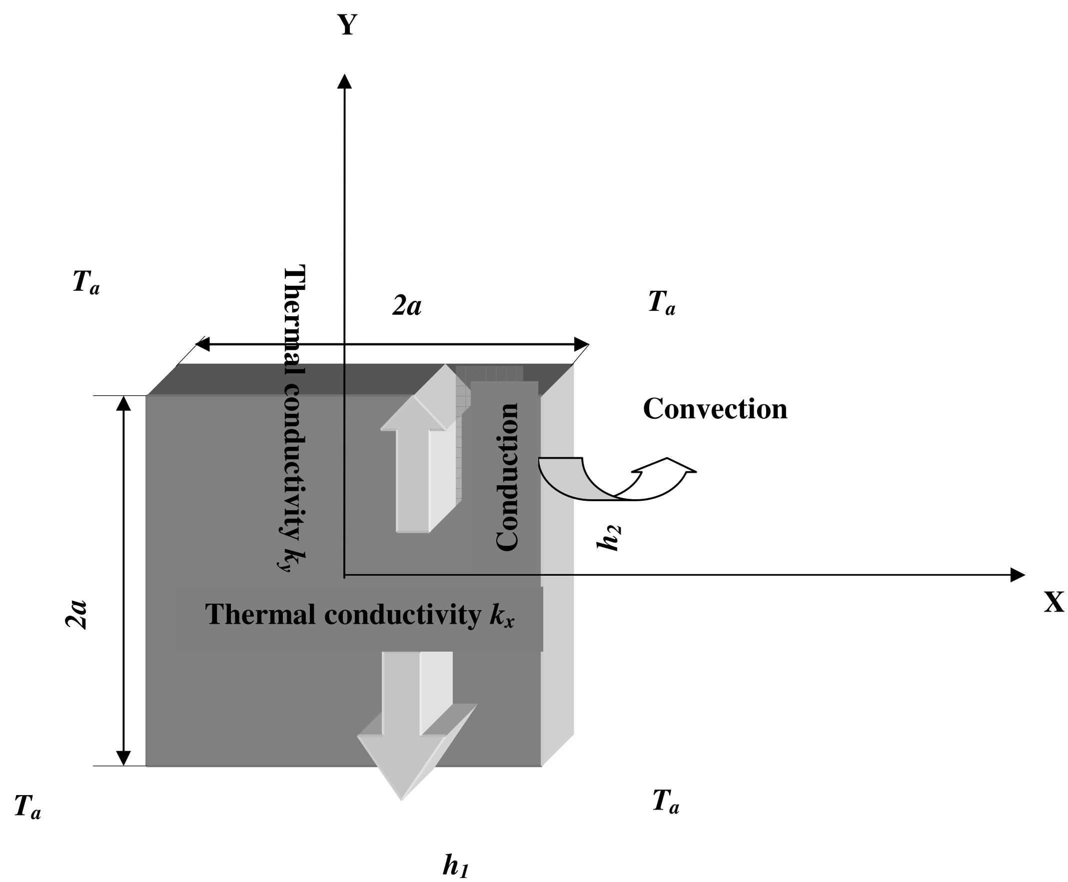


Fig 5.1: Heat Conduction across square shaped slab

The origin of the axis has been selected at the center of the slab. We divide this slab into four equal sections around the origin. Heat transfer analysis has been performed across a cross sectional module of length 'a' and thickness is also 'a'.

5.3 BOUNDARY AND INITIAL CONDITIONS

5.3.1 BOUNDARY CONDITIONS

$$\text{at } \; x = 0\,; -k_x \frac{\partial T}{\partial x} = 0 \tag{5.2a}$$

$$\text{at } \; x = 1\,; -k_x \frac{\partial T}{\partial x} = h_1(T - T_a) \tag{5.2b}$$

$$\text{at } \; y = 0\,; -k_y \frac{\partial T}{\partial y} = 0 \tag{5.2c}$$

$$\text{at } \; y = 1\,; -k_y \frac{\partial T}{\partial y} = h_2(T - T_a) \tag{5.2d}$$

5.3.2 INITIAL CONDITIONS

$$\text{at } \; t = 0\,; T = T_0 \tag{5.3}$$

5.4 GOVERNING DIFFERENTIAL EQUATION

The governing differential equation (GDE) can be expressed as:

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right) = \rho c_p \frac{\partial T}{\partial t} \tag{5.4}$$

With considering constant thermal conductivity, GDE becomes:

$$\frac{\partial^2 T}{\partial x^2} + \frac{k_y}{k_x}\frac{\partial^2 T}{\partial y^2} = \frac{\rho c_p}{k_x}\frac{\partial T}{\partial t} \tag{5.5}$$

The dimensionless parameters are defined as follows:

$$X = \frac{x}{a}\ , Y = \frac{y}{a}\ , \theta = \frac{T-T_a}{T_0-T_a}\ , F = \frac{K_x\, t}{\rho c_p a^2} \tag{5.6}$$

GDE in dimensionless form is:

$$\frac{\partial^2\theta}{\partial X^2} + \frac{k_y}{k_x}\frac{\partial^2\theta}{\partial Y^2} = \frac{\rho c_p a^2}{k_x}\frac{\partial\theta}{\partial t} \tag{5.7}$$

$$\alpha = \frac{k_x}{\rho c_p}\ ;\ \beta = \alpha'(T_0 - T_a); k_r = \frac{k_y}{k_x}\ ; F = \frac{\alpha t}{a^2} \tag{5.8}$$

Replacing 'k_x' and 'k_y' by 'k_r'; and 't' by 'F', Governing Equation in dimensionless form is as follows:

$$\frac{\partial^2\theta}{\partial X^2} + k_r\,\frac{\partial^2\theta}{\partial Y^2} = \frac{\partial\theta}{\partial F} \tag{5.9}$$

Discussion about the Initial and boundary conditions in dimensionless form:

at $X = 0\ ;\ \frac{\partial\theta}{\partial X} = 0$ (5.10a)

at $X = 1\ ;\ \frac{\partial\theta}{\partial X} = -Bi_1\theta$ (5.10b)

at $Y = 0\ ;\ \frac{\partial\theta}{\partial Y} = 0$ (5.10c)

at $Y = 1\ ;\ \frac{\partial\theta}{\partial Y} = -Bi_2\theta$ (5.10d)

5.4 MATHEMATICAL FORMULATION

The governing equation is:

$$\frac{\partial^2\theta}{\partial X^2} + k_r\,\frac{\partial^2\theta}{\partial Y^2} = \frac{\partial\theta}{\partial F} \tag{5.11}$$

Temperature 'θ' is a product function of two new variables. By separating of variables, Eq. (5.11) can be solved with considering the above mentioned boundary conditions. We use the trial solution of the form:

$$\theta(X,Y,F) = \psi(X,F)\emptyset(Y,F) \qquad (5.12)$$

Replacing 'θ' by 'ψ' and 'ϕ' into Eq. (5.11), the governing equations are as follows:

$$\frac{\partial^2\psi}{\partial X^2} = \frac{\partial\psi}{\partial F} \qquad (5.13)$$

and

$$k_r\frac{\partial^2\emptyset}{\partial Y^2} = \frac{\partial\emptyset}{\partial F} \qquad (5.14)$$

Solutions to the Eqs. (5.13) & (5.14) are sought in the following forms:

$$\Psi(X,F) = A(X)B(F) \qquad (5.15)$$

and

$$\emptyset(Y,F) = M(Y)N(F) \qquad (5.16)$$

Replacing the value of 'ψ' with the terms of '*A*' and '*B*' in Eq. (5.13), we have the following two equations:

$$\frac{d^2A}{dX^2} + \lambda^2 A = 0 \qquad (5.17)$$

and

$$\frac{dB}{dF} + \lambda^2 B = 0 \qquad (5.18)$$

The solutions for the above space dependent functions '*A*' are obtained by solving Helmholtz Eq. (5.17) and then applying boundary conditions from Eqs. (5.10a) and (5.10b) as following:

$$A(X) = C_1\, cos(\lambda_n X) \qquad (5.19)$$

The solution for the time dependent function '*B*' are immediately obtained from Eq. (5.18) as:

$$B(F) = C_3 e^{-\lambda_n^2 F} \tag{5.20}$$

Where C_1, C_3, C_n are integration constants and λ_n is the separation constant.

Putting the above mentioned values of '*A*' and '*B*' in Eq. (5.15), we have the following equation:

$$\Psi(X,F) = C_n e^{-\lambda_n^2 F} cos(\lambda_n X) \tag{5.21}$$

Substituting the value of ϕ in terms of '*M*' & '*N*' in Eq. (5.14), we have the following two equations:

$$\frac{d^2M}{dY^2} + \frac{\mu^2}{K_r} M = 0 \tag{5.22}$$

and

$$\frac{dN}{dF} + \mu^2 N = 0 \tag{5.23}$$

Solution of the above function '*M*' is obtained by solving Eq. (5.22) and applying boundary conditions from Eqs. (5.10c) and (5.10d) in the above equations, we have the solutions as:

$$M(Y) = C_4 \, cos(\frac{\mu}{\sqrt{K_r}} Y) + C_5 \, sin(\frac{\mu}{\sqrt{K_r}} Y) \tag{5.24}$$

The solution for the time dependent function '*N*' is immediately obtained from Eq. (5.23) as:

$$N(F) = C_6 \, e^{-\mu^2 F} \tag{5.25}$$

Replacing '*M*' and '*N*' with the above mentioned values in Eq. (5.16):

$$\emptyset(Y,F) = C_m \cos(\frac{\mu_m}{\sqrt{K_r}}Y)\, e^{-\mu_m{}^2 F} \quad (5.26)$$

Determination of C_n and C_m from Eqs. (5.21) and (5.26):

Applying initial condition at $F=0$, $\psi(X, F) = 1$, in Eq. (5.21):

$$C_n \cos(\lambda_n X) = 1 \quad (5.27)$$

Multiplying both sides by $\{\cos(\lambda_n X)\}$ and then integrating both sides within the range X=0 and X=1, value of C_n arrived as follows:

$$C_n = \left[\frac{4\sin\lambda_n}{2\lambda_n + \sin(2\lambda_n)}\right] \quad (5.28)$$

Similarly,

$$C_m = \left[\frac{4\sqrt{K_r}\,\sin(\frac{\mu_m}{\sqrt{K_r}})}{\{2\mu_m + \sqrt{K_r}\,\sin(\frac{2\mu_m}{\sqrt{K_r}})\}}\right] \quad (5.29)$$

Replacing the value of C_n and C_m in Eqs. (5.21) & (5.26) and then substituting these values in Eq. (5.12), we get the final expression of dimensionless temperature 'θ' as follows:

$$\theta = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\Big[\frac{16\sqrt{K_r}\,\sin\lambda_n\,\sin(\frac{\mu_m}{\sqrt{K_r}})}{\{2\lambda_n + \sin(2\lambda_n)\}\{2\mu_m + \sqrt{K_r}\,\sin(\frac{2\mu_m}{\sqrt{K_r}})\}}$$
$$e^{-\lambda_n^2 F}\, e^{-\mu_m{}^2 F} \cos(\lambda_n X)\cos(\frac{\mu_m}{\sqrt{K_r}}Y)\Big] \quad (5.30)$$

CHAPTER -6

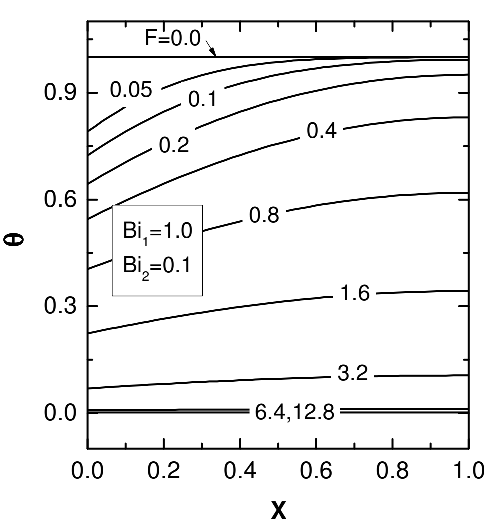
RESULTS AND DISCUSSIONS

RESULTS AND DISCUSSIONS

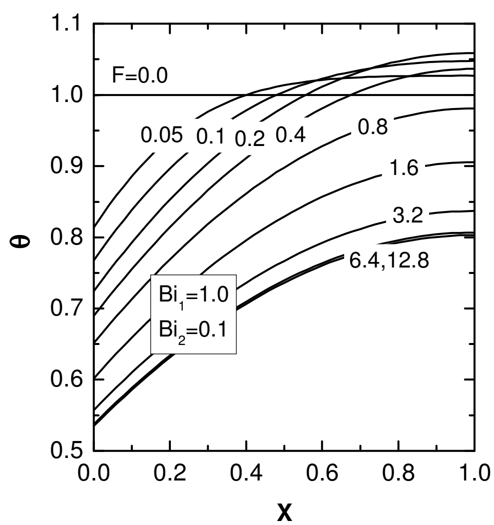
Initially at $t = 0$, temperature of the slab, $T = T_0$. When $T = T_0$, $\theta = 1$.At final stage when the system becomes steady, $T=T_a$. At this stage, $\theta= 0$. All the graphs have been plotted within this range of θ i.e. θ varies from '1' to '0'.

6.1 TEMPERATURE PROFILES IN 1-D SLAB

We have tried to analyze one dimensional heat conduction considering constant heat generation. Temperature profiles are observed as a function of Fourier number as shown in previous section. Fig 6.1(a) and 6.1(b) shows the variation of temperature with respect to spatial co-ordinates at a particular time subject to constant heat generation.



(a) $Q_0 = \beta = 0.001$



(b) $Q_0 = 0.5$, $\beta = 0.1$

Fig 6.1. Temperature response for 1-D heat conduction in a Slab

In Fig.6.1, Bi_1 value at $X = 0$ side has been considered large compared to that of the other end. Lower value of heat generation has been considered in Fig.6.1 (a) i.e. β value is less compared to Fig.6.1 (b).

We have plotted the Fig.6.1 against some values of '*F*' (i.e. 0.0, 0.05, 0.1, 0.2, 0.4…..6.4, 12.8).From the Fig. 6.1 (a), it is clear that at $F = 6.4$, the system converges towards steady state and at $F = 12.8$, the system reaches at steady state.

In Fig. 6.1(b), as heat generation considered is high, system does not reach at steady state. At $X=0$ side, Biot number considered as 1.0 and at the other end of the slab, Biot number considered less as 0.1. As Biot number is less at $X =1$ side, convective heat transfer is also less compared to other side of the slab. Hence, variation of 'θ' is less in right hand side i.e. at $X=1$ end compared to that of the other end. For internal heat generation as well as lower value of Biot number at $X = 1$ side of the slab, there must exist some points where $\theta > 1$.

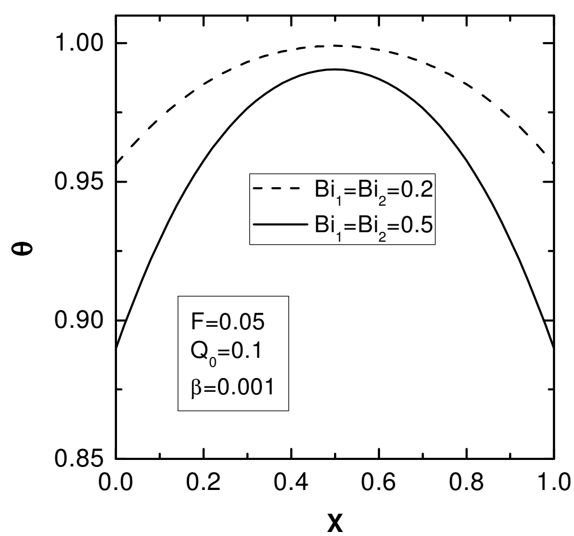


Fig 6.2. Temperature distribution in 1-D Slab for different convected boundary condition at their boundaries

We have also analyzed the case where Biot numbers at both ends of the slab are equal. Fig 6.2 shows the variation of temperature along with the spatial co-ordinates of the slab for a particular value of dimensionless time *F* and also subject to constant heat generation. As Biot numbers are equal at both ends, convective heat transfer becomes equal at both ends of the slab. Heat transfer rate increases along with the length of the slab from both ends of the slab and *'θ'* becomes maximum at the midpoint of the slab which has been clearly reflected in the Fig.6.2. Thus symmetric temperature variation obtained at both ends of the slab.

The Fig 6.2 also shows two curves for two different values of Biot numbers. From this figure, it is clear that with decreasing value of Biot number, the curve becomes flatten. As amount of convective heat transfer decreasing, variation of 'θ' with respect to position 'X' also decreases.

At very lower value of Biot number, convective heat transfer becomes so less that variation of 'θ' with respect to 'X' is negligible and the curve becomes almost flat. If Biot numbers being very less i.e. 0.0001, then rate of heat transfer may be neglected. It can be treated as insulated ends. In this case, variation of 'θ' along with length of the slab is very negligible.

In the above mentioned phenomenon, if we consider higher value of internal heat generation inside the slab, then there may be chance of the fact that value of 'θ' may exceed 1 and the curve tends to slope in upward direction. On the contrary, if we consider the higher *Bi* value and negligible amount of heat generation, then variation of 'θ' increases and time required for achieving steady state reduced drastically.

In Fig. 6.3, we have analyzed the case, where, one end of the slab is insulated. Fig. 6.3 shows that at *X=1* end of the slab, which is insulated, the dimensionless temperature 'θ' is equal to one which indicates that *T=Ti*, temperature of the slab is same as initial temperature. This is because of no heat transfer occurs at insulated end of the slab. With increasing the value of Biot number at the other end of the slab i.e. at *X =0*, variation of 'θ' is also increasing.

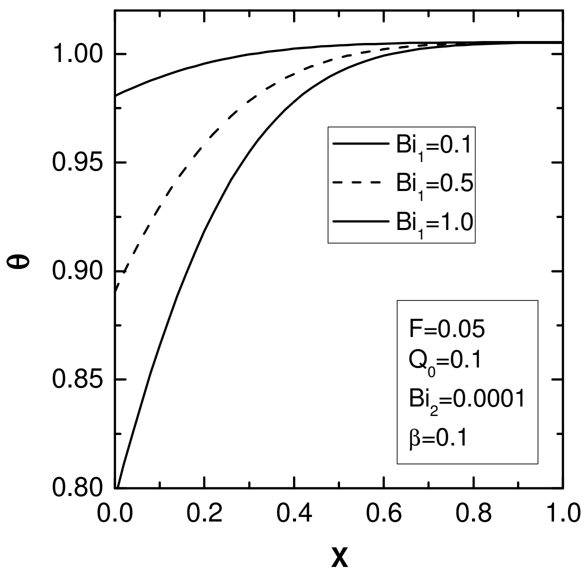


Fig 6.3. Temperature distribution in a one-dimensional slab for different convected boundary conditions at $X = 0$ and insulated condition at $X = 1$

For very lower value of Biot number '0.1', at *X=0* end of the slab, convective Heat transfer is very less and at other end, there is no heat transfer. Hence, the curve will be flat type at *X=1* end and then it starts sloping down slowly. If we further decrease the value of Biot number at *X=0* end, the curve becomes almost flatten. Middle curve in the figure is for Bi_1 value of 0.5 and the lower one is for Bi_1 value of 1.0. Variation of 'θ' is higher in case of higher Bi_1 value. Variation of 'θ' for the Bi_1 value of 0.5 lies in between the curves of Bi_1 values 0.1 and 1.0.

Further, we have analyzed the case where internal heat generation has been varied, keeping other parameters as constant. We consider the constant parameters as following:
$Bi_1=0.5$; $Bi_2=0.1$; $F=0.05$; $Q_0=0.1$ and value of 'β' has been changed from '0.1' to '0.5'.

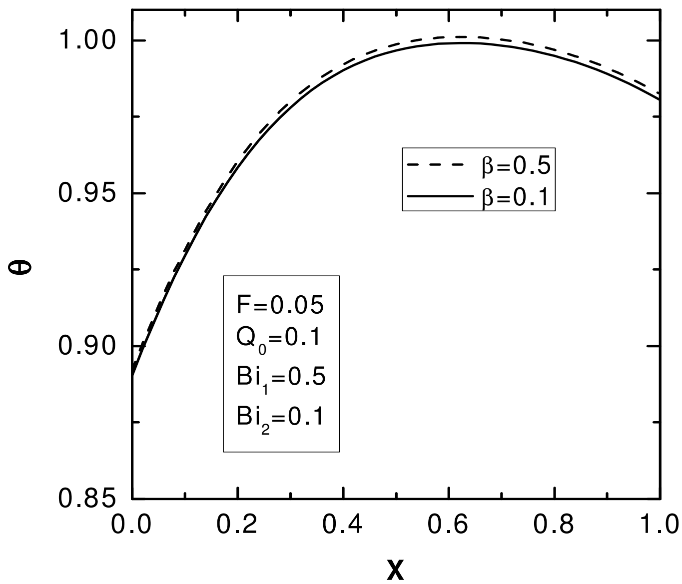


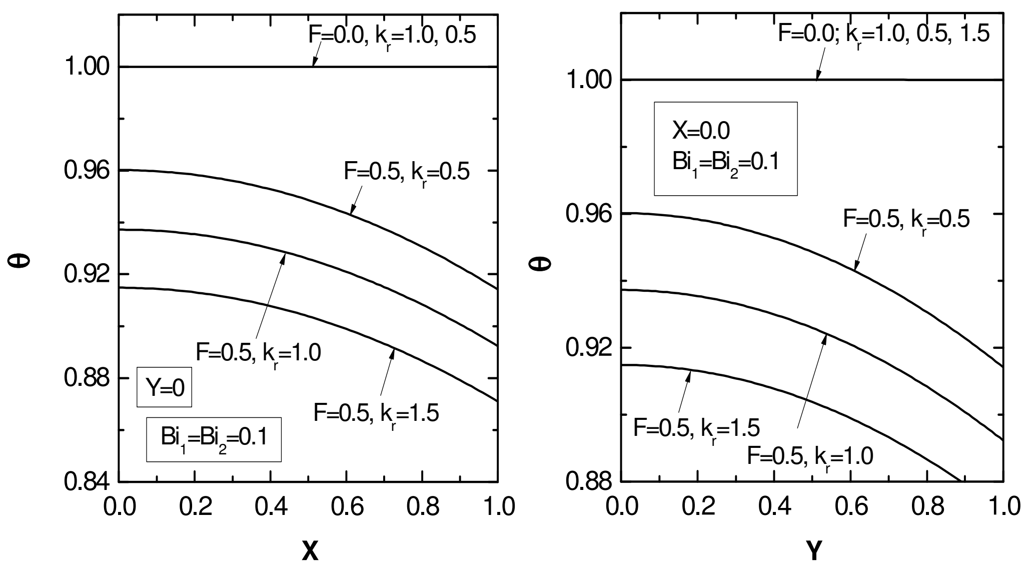
Fig 6.4. Effects of variable internal heat generation parameter on temperature distribution in a slab

Fig 6.4 shows the variation of dimensionless temperature along with spatial co ordinates with respect to changes of internal heat generation at a particular time. Variation of dimensionless temperature decreases along with decrease in value of 'β'. We have plotted two curves for two different values of 'β' i.e. 0.1 and 0.5. But for this amount of difference in 'β', variation of 'θ' is nominal.

As Biot number is higher at $X=0$ end than that of the other end, heat transfer is higher at $X=0$ end. Hence, value of 'θ' decreases rapidly at this end compared to other end of the slab. If we further increase the Bi_1 value and further decrease the value of 'β' then the slab approaches towards equilibrium point.

6.2 TEMPERATURE PROFILES IN 2-D SLAB

For a two dimensional slab, we have analyzed the variation of 'θ' along X-axis of the slab and also along Y-axis of the slab. Also we have compared the variation of 'θ' along with length of the slab for isotropic and orthotropic materials.

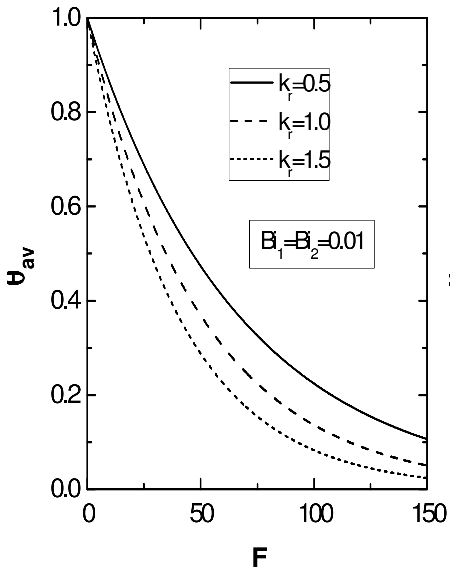


(a) $Y = 0$ (b) $X = 0$

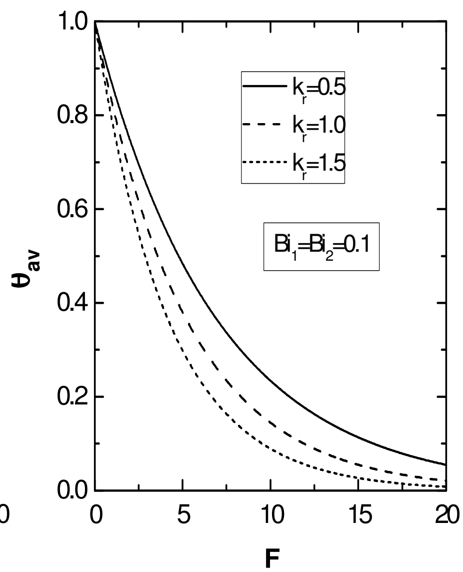
Fig 6.5. Variation of temperature distribution in a slab: A comparison of Isotropic and orthotropic materials

Fig 6.5 shows the temperature profiles for isotropic and orthotropic materials. For Isotropic material k_r=*1.0*. For orthotropic material as discussed previous, k_x and k_y values are different. The variation of 'θ' with respect to X-axis, at *Y=0* has been shown in Fig.6.5 (a) for 2-D Slab. Whereas in Fig.6.5 (b), variation of 'θ' along with Y-axis has been shown. These variations along X and Y axis are of similar patterns. When $k_r>1$,with increase of k_r, temperature decreases and variation of 'θ' increases.

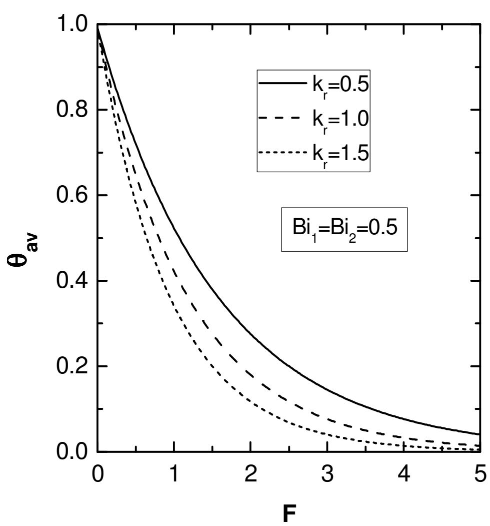
In Fig. 6.6, we have plotted the curves showing the variation of 'θ_{av}' and F subject to the condition that Biot number is constant. We have varied the thermal conductivity ratio and observed the three different curves for three different values of k_r. Also we have varied the Biot number for a constant value of k_r.



(a) $Bi_1=Bi_2=0.01$



(b) $Bi_1 = Bi_2 = 0.1$



(c) $Bi_1 = Bi_2 = 0.5$

Fig 6.6. A comparison of average temperature response in a slab as a function of F for different design conditions

When k_r is decreasing but less than one which implies that value k_x is increasing and Biot number along the horizontal surface of the slab i.e Bi_1 value is increased but Bi_1 value is constant. As a result, value of convective heat transfer co efficient h_1 or length of the body to be increased which implies that conductive resistance increased, rate of heat transfer decreases.

In Fig.6.6 (a), Biot numbers are of lower value i.e. 0.01 and in Fig.6.6 (b), Biot numbers are of higher value i.e. 0.1. Biot number is high which means convective heat transfer rate is high compared to that of shown in Fig.6.6 (a). As convective heat transfer is higher in Fig.6.6 (b), time required for to reach a steady state is very less than that of required in other design conditions as shown in Fig6.6 (a). This is clearly reflected in the above figures. Time required to reach a steady state is reduced when $k_r > 1$. The k_r value of isotropic material is 1 lies in between 0.5 and 1.5. Thereby, in this design conditions, the curve indicates the variation of 'θ_{av}' with respect to F for isotropic material lies in between the curves indicate the variation of orthotropic materials of k_r value of 0.5 and 1.5.

We have further increased the value of Biot numbers, which is shown in Fig. 6.6 (c), where Biot numbers is of 0.5, time required to reach a steady state, got drastically reduced. However the variations of 'θ_{av}' with F for isotropic material lies in between that of the variations of k_r value of 0.5 and 1.5 for orthotropic material. If we increase the thermal conductivity ratio when $k_r > 1$, heat transfer rate increases and time required for reaching the steady condition got reduced which is also cleared from the above figures.

CHAPTER -7

CONCLUSIONS

CONCLUSIONS

7.1 CONCLUSIONS

Use of Composite materials in the field of composite technology progressed a lot over the last two decades. Structural integrity is the main features which governs the choice of materials and form of construction for any component. In composite materials, high specific strength and low weight are achieved. As for example, plywood is a composite material, widely used in the field of construction. Constituents of concrete are cement and aggregates. A robust strong material, concrete is widely used in infrastructure. FRP fills used in cooling tower in power plants are also of composite materials. An analytical model considering a slab of composite material was developed for predicting the variation of dimensionless temperature 'θ' along with dimensionless time 'F' and dimensionless position 'X' by separation of variable method.

From one dimensional slab analysis, it has been observed that if Biot numbers are same at both ends of the slab, variation of 'θ' is symmetric at both ends of the slab. With increase of heat generation, variation of 'θ' also increases with respect to position 'X' subject to other parameters constant e.g. F,Q_0, Bi_1 and Bi_2.

From the above mentioned graphs of two dimensional slab analyses, it is clear that with increase in thermal conductivity ratio, time taken to reach at steady condition being reduced subject to other design conditions e.g. heat generation and Biot number is being constant. With increase of Biot number (e.g.-from 0.01 to 0.1), time required for reaching steady condition got reduced drastically and variation of 'θ' with respect to position 'X' increases. From these analyses, it is observed that pattern for variation of 'θ' for an isotropic and orthotropic material are same and thermal conductivity ratio for an orthotropic material plays an important role in heat transfer phenomenon.

Temperature profiles for transient heat conduction in a slab are observed for both isotropic and orthotropic materials and a comparison between the curves are also focused for composite materials. The necessity of composite materials has increased tremendously in power plants, infrastructures and aerospace and in many other fields. Gypsum board used as false ceiling material in the field of infrastructure is also a good example of orthotropic slab. Hence, for thermal analysis of composite material, this present analysis can be considered.

CHAPTER -8

SCOPE OF FUTURE WORK

SCOPE OF FUTURE WORK

8.1 SCOPE OF FUTURE WORK

A little attempt of analyzing the 2-D Fourier heat conduction for Isotropic and Orthotropic materials is executed in this paper. However the following works can also be done analytically:

2-D Fourier Heat Conduction considering heat generation can be done in the same way. 2-D Non-Fourier heat conduction considering with and without heat generation can also be done by SOV Method. Present work has been performed on a square shaped slab. However, these works can also be performed on other type of Geometries i.e. Sphere, Cylinder etc.

Heat transfer analysis can be done for the case if temperature of one of the surface of the slab being less than its condensing temperature i.e. vapors formed at the slab surface. Three dimensional unsteady problems along with variable thermal conductivity can be solved analytically.

All these works can be performed considering different boundary conditions. All these problems can also be solved by considering the radiation. Even, all the above mentioned works can also be solved analytically by different methods e.g. – Polynomial Approximation Method, Finite Integral Transform Technique, Laplace Transform Method, Green's Function Approach, etc.

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