STUDIES ON FLUID FLOW AND HEAT TRANSFER CHARACTERISTICS FOR FLOW THROUGH A RECTANGULAR MICROCHANNEL – AN ANALYTICAL APPROACH

A THESIS SUBMITTED FOR PARTIAL FULFILMENT OF THE REQUIREMENT FOR

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*This is to certify that, I Mr. Arnab Ash, have personally worked on the research project entitled "**Studies on Fluid Flow and Heat Transfer Characteristics for Flow through a Rectangular Microchannel—An Analytical Approach**". The data mentioned in this report were obtained during genuine work done and collected by me. Data obtained from other agencies have been duly acknowledged. None of the findings pertaining to the work has been concealed. The results embodied in this project have not been submitted to any other university or institute for the award of any degree or diploma.*

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ABSTRACT

Flow through microchannels are of substantial importance as on date due to its multi dimensional applications in modern electronic industries, Micro Electro Mechanical Systems (MEM) as well as micro biological systems and various other applications. Due to the typical sizes of the microchannels as well as flow parameters involved in the microchannel flow, it closely resembles fluid flow through some typical biological systems like blood flow through veins etc. Also with the gradual implementation of prosthetic limbs and artificial organs in medical sciences, study of microchannel flow is receiving more attention. A very fundamental way of studying flow through microchannels are finding out basic flow parameters like velocity, temperature etc for a given flow situations.

From the existing literatures it is found that, these areas of fluid flow & heat transfer are quiet less developed compared to its conventional counterpart. Likewise conventional flow through macro channels, Navier-Stokes equation is also valid for microchannel flows. However, depending upon the situation, boundary conditions get changed in case of flow through microchannels. For example, no slip boundary condition for velocity and temperature is sufficiently accurate approximation in case of conventional macrochannel flows. But the same approximation no longer holds good in case of microchannel flows as for later case the mean free path of the fluid molecules under consideration becomes comparable with the channel characteristics dimensions. Therefore, possibility of momentum exchange with the channel wall gets increased, hence velocity as well as temperature slip phenomenon is observed. As the governing Navier-Stokes equations are still valid for microchannel flows as well, therefore the same have been simplified under the initial & boundary conditions of the considered microchannel and discussed in detail in chapter-3 of this thesis. In the subsequent chapters simplified Navier-Stokes equations along with the boundary conditions have been solved numerically to trace the exact velocity and temperature distributions.

Though numerical method can be applied for any kind of flow situation and for any geometry but it is also to be noted that, it is quite laborious and time consuming to obtain numerical solution for any flow situations. Also the numerical analysis needs to be carried out freshly even with a minor change in flow geometry. An alternative approach could be obtaining the analytical solution, so that flow parameters like velocity, temperature etc. could be found out easily at any point in the flow field. However, like conventional flow, it is not an easy to find out analytical solutions for microchannel flow; rather thing is more complicated due to the presence of slip. Therefore, a simpler alternative approach to find the solution is the utilization of approximate analytical solution. From the existing literatures, it is observed that approximate analytical techniques are less developed for microchannel flows compared to its conventional counterparts. Therefore, in this work, effort has been given to apply some approximate analytical techniques to determine the velocity and temperature distribution for laminar fluid flow through a straight rectangular microchannel.

The results obtained by various approximate analytical techniques have been compared with the results obtained by numerical methods. For the purpose of better understanding of accuracy obtained by various approximate methods (Integral Ritz, Integral Kantorovich, Variational Ritz, Variational Kantorovich etc.), results have been plotted graphically. Also for the purpose of showing influence of channel geometry on results three different aspect ratio of the channels (A=1, 0.5 and 0.25) have been considered.

As sufficient accuracy may be obtained by adopting different approximate analytical techniques, therefore it can be considered that, rather than going for complicated numerical analysis or exact analytical solutions, approximate analytical methods may be utilized for almost all practical purposes.

List of figures and plots

NOMENCLATURE

K	Thermal conductivity ($W.m^{-1}.\,^{0}C^{-1}$)
T	Temperature (^{0}C)
T_W	Wall temperature (^{0}C)
T_s	Slip temperature (^{0}C)
q	Heat transfer rate (W)
x	Spatial variable in the longitudinal direction (m)
y	Spatial variable in the transverse direction (m)
X	Dimensionless spatial variable (in the longitudinal direction)
Y	Dimensionless spatial variable (in the transverse direction)
L	Dimension of the channel in the x direction
l	Dimension of the channel in the transverse direction
U	Non dimensional velocity in the flow direction
u_s	Slip velocity (m/s)
u	Velocity in the flow direction (m/s)
A	Aspect ratio of the channel ($=l/L$)
B	Coefficient to accommodate the effect of slip phenomenon
F	Tangential momentum accommodation coefficient
F_t	Thermal momentum accommodation coefficient
Kn	Knudsen number
Pr	Prandtl Number of the considered fluid
D_H	Hydraulic diameter of the channel
p	Wetted perimeter of the channel
A_C	Cross sectional area of the channel
L_C	Characteristic length of the channel

Greek symbols

α	Thermal diffusivity
θ	Dimensionless temperature
λ	Mean free path of the fluid molecules
μ	Dynamic viscosity of the fluid considered

CHAPTER-1

INTRODUCTION

INTRODUCTION

1.1 Background

With the advancement of micro electro mechanical system, its application areas like micro-heat exchangers, micro-valves and many other micro-fluidic systems have been started developing. Therefore fabrication of MEMS and understanding of flow through microchannels have gained considerable importance in modern days.

Basic dynamic laws of Physics such as Newton's law of motion, Newton's law of viscosity, Pascal's law etc together culminate the Navier-Stokes equation of fluid mechanics, which is basically the governing equation of fluid mechanics. Theoretically Navier-Stokes equations are believed to be complete and any kind of fluid flow problems can be solved with the help of these laws. However the classical Navier-Stokes equations fail to predict the behavior of ideal gas flows through microchannels at high Knudsen numbers [1], known as slip flow. Due to the confined area of the channel the mean free path of the fluid is comparable to the characteristic length of the channel (hydraulic diameter can be considered as he characteristic length of the channel) leading to slip-like flow behavior and strong diffusion-enhanced transport of mass and momentum, To solve the slip flow problems theoretically as well as numerically, a tuning parameter is required to be incorporated, which is known as tangential momentum accommodation coefficient (TMAC). The introduction of TMAC considers the effect of slip at the wall. And this is worth saying that this is the only possible way developed as of now to solve the slip flow problems numerically as well as theoretically. From the time of Maxwell it is known that the no-slip boundary condition can be violated in case of rarified gas flows [2]. Other deviations from classical macro scale flows include observations of higher mass flow rates through the channel [3] and nonlinear pressure drop along the channel [4].

1.2 Utilization of microchannels

To mitigate the criteria of very high rate of heat transfer, microchannels are often utilized. Its application area is increasing day by day in areas of electronic equipments. Microchannel heat transfer has the potential of cooling high power density microchips [5]. In their paper Gawali *et al.* [5] suggested that, to enhance the heat transfer it is necessary to study simultaneous effects of various parameters like size of channel, shape of channel, fluid properties, Reynolds number, friction factor, pressure drop, pumping power etc. In order to fabricate such micro devices effectively, it is necessary to understand the fundamental mechanisms involved in fluid flow and heat transfer characteristics in microchannels since these parameters affects the transport phenomena for the bulk of MEMS and micro-fluidic applications.

1.3 General characteristics of flow through microchannels

Published studies based on an extensive literature reviews include a variety of fluid types, microchannel cross-section configurations, flow rates, analytical techniques, and channel materials. The issues and related areas associated with the microchannels are summarized in the following table.

Table-1: Channel classification schemes

Definition	The range of channel dimensions
Conventional channels	$D_c > 3mm$
Minichannels	$3mm \geq D_c > 100\ \mu m$
Microchannels	$100\ \mu m \geq D_c > 10\ \mu m$
Transitional microchannels	$10\ \mu m \geq D_c > 1\ \mu m$
Transitional nanochannels	$1\ \mu m \geq D_c > 0.1\ \mu m$
Nanochannels	$D_c \leq 0.1\ \mu m$

When the molecular mean free path of the of the fluid is comparable with the system's characteristic lengths then the fluid's molecular structure become more important and the traditional continuum assumption does not hold good any longer. In such situations fluid exhibits non continuum behavior such as slip flow (that is non zero velocity at the boundary) and thermodynamic temperature jump (that is fluid temperature at the boundary is different from that of the boundary itself) at the solid-fluid interface.

Some common example of fluid flow in slip-flow regime includes low pressure fluid flow, such as air flow occurring at high atmospheres and in vacuum. Micro fluidic systems typically have characteristic lengths of the order of 1µm to 100µm. Analysis of micro fluidic systems must consider rarefaction and other continuum effects even at very standard conditions.

The flow in the following systems like micro-gyroscope, accelerometer, flow sensors, microchannels, micro valves etc. in slip flow regime is of practical interest.

In this context a dimensionless parameter called Knudsen Number plays an important role to determine whether a flow should be considered as a microchannel flow or not. In another word, Knudsen No. basically determines whether one should to carry out flow analysis by assuming the as continuum or non continuum. The Knudsen number is defined by

$$Kn = \frac{\text{Mean free path of the fluid particles}}{\text{Characteristic Length of the duct}}$$

This number is very small for continuum flows. For conventional flows the mean free path of the fluid particles are much lower compared to other dimensions associated with the flow channel. However for microchannel flows mean free path of the particles are comparable with dimension of the flow passage. When the Knudsen number for a flow system lies in the range $0.001 \leq Kn \leq 0.1$ we consider the flow as microchannel flow and the analysis should be carried out by considering non-continuum model of fluid flow.

Several effects which are normally neglected in macroscopic flow become significant in microchannel flow situations. The first of these micro scale phenomena is the two or three dimensional transport effects. As characteristic lengths are reduced to the same order of magnitude as the hydrodynamic boundary layer thickness, momentum transfer in directions other than stream wise direction can increase significantly [10].

Another significant micro scale effect is the temperature variation in the transport fluid that can cause a significant variation in fluid properties (e.g. apparent viscosity of fluid etc) throughout the micro system, which readily invalidates the assumptions of constant properties of transport fluid [10]. Along with the above mentioned characteristics significant viscosity variation near the solid wall, slip flow at the wall and micro polar fluid effects etc. are observed in microchannel flow. Because of the size of the flow passage a huge pressure drop generally occurs and therefore the flow through a microchannel in general lies predominantly within the laminar flow regime.

1.4 Importance of studying flow through microchannels

Study of flow through microchannels is of great importance because of its applications in the micro devices in engineering, medical and other scientific areas. Microchannel flow play a key role in biological systems like blood flow inside veins etc beside its numerous other applications like the cooling of electronic equipments like Integrated chips, Computer motherboard etc.

Microchannels offer certain advantages due to their high surface-to-volume ratio and their small volumes over conventional fluid flow passages. The large surface-to-volume ratio leads to high rate of heat transfer, making micro devices excellent tools for compact heat exchangers. That is why microchannels are used so extensively in thermal management of high-power-density microprocessors. Study of micro-channel flow has recently been recognized as a prime contender for cooling of next generation cutting edge electronics.

Microchannels are also used in the area of Micro Electro Mechanical system (MEMS) devices for biological and chemical analysis. The primary advantage of micro scale devices in these applications are the very good match with the scale of biological structures and the potential for placing multiple functions for chemical analysis on a small area. Microchannels are used to transport biological materials such as proteins, DNA, cells, and embryos or to transport chemical samples. Typical of such devices is the i-STAT blood sample analysis cartridge. Flows in biological devices and chemical analysis micro devices are usually much slower than those in heat transfer and chemical reactor micro devices.

1.5 Aim of the present work

Though numerical method can be applied for any kind of flow situation and for any geometry but it is also to be noted that, it is quite laborious and time consuming to obtain numerical solution for any flow situations. Also the numerical analysis needs to be carried out freshly even with a minor change in flow geometry. An alternative approach could be obtaining the analytical solution, so that flow parameters like velocity, temperature etc. could be found out easily with less effort. However, like conventional flow, it is not an easy task to find out exact analytical solutions for flow within microchannel; rather thing is more complicated due to the presence of slip at boundary walls. Therefore, a simpler alternative approach to find the solution is the utilization of approximate analytical solution. From the existing literatures, it is observed that analytical techniques are less developed for flow through microchannel. Therefore, in this work, effort has been given to apply some approximate analytical techniques to determine the velocity and temperature distribution for laminar fluid flow through a straight microchannel of rectangular cross section.

CHAPTER-2

LITERATURE REVIEW

LITERATURE REVIEW:

It is already stated that flow within microchannels are of substantial importance due to its multi dimensional applications in modern electronic industries, MEMs as well as micro-biological systems and various other applications. Few related literatures on flow through microchannels are reviewed and reported as follows:

1. Kundu *et al.* [6] described a procedure for approximate analytical solution of the no slip flow through a straight rectangular microchannel as a function of aspect ratio of the channel. The variation of different flow and heat transfer parameters (like velocity distribution, temperature distribution, Nusselt Number of the flow etc) for different aspect ratio of the rectangular channel has been described. The result determined by different techniques had been presented in a comparative way for easy understanding of the accuracy level of predictions. Results so obtained had been compared with the results obtained by solving the governing differential equation that is with the exact result.

2. Hooman [7] presented a superposition approach to investigate forced convection in microducts of arbitrary cross sections subjected to H1 and H2 boundary conditions in the slip flow regime with farther assumption of a temperature jump condition. Utilizing the TMAC and thermal accommodation coefficients, results obtained for no slip calculation had been extended to find out values for slip flow situations. It was also shown in the paper that this superposition method is strictly valid for the steady laminar fully developed flow (both thermally and hydrodynamically) through microchannels.

3. Sobhan *et al.* [8] have shown in their research paper on comparative study of flow through microchannels that, the continuum hypothesis does not get violated as long as channels have hydraulic diameter 50μm or more. As a result analysis based on Navier-Stokes and energy equations are expected to model the phenomena observed sufficiently as long as the experimental conditions and measurements are identified and simulated correctly. The discrepancies in predictions are due to entrance and exit

conditions etc. They have also suggested that there is a need of further study to closely monitor each parameter influencing the transport fluid in microchannels.

4. Sharp *et al.* [11] has described that microchannel flow differs from their conventional macroscopic counterpart from two aspects. The small scale of the flow passage makes molecular effects such as wall slip more important and it amplifies the magnitudes of certain ordinary continuum effects to extreme levels for example strain rate, shear rate etc. Fluids that are Newtonian at ordinary rates of shear and extension can become non-Newtonian at very high rates. The pressure gradient generally also becomes very large at microchannel flows. For liquid fluid flowing through a solid microchannel, electro kinetic effects become significant at the solid liquid interface due to small characteristic length of the flow passage. For example an electrically charged double layer forms which includes an electrical charge distribution in a very thin fluid layer adjacent to the wall. Application of an electric field to this layer creates a body force capable of moving the fluid as if it were slipping over the wall. It was also shown that, As the Knudsen number continues to increase; continuum assumptions and fluid theory are no longer applicable. Analysis of such flow requires consideration of different physical phenomena. As per Sharp *et al.*, despite considerable simplicity of the laminar flow equations through a microchannel, experimental results often deviates from the theoretical results in case of friction factor and Reynolds number relationship.

5. Meolans *et al.* [12] has developed an analytical approach to take account of the influence of the lateral wall on stationary isothermal gas flow through a rectangular microchannel. The study concerns pressure gradient driven flows in channels where the length is large compared to the critical smallest dimension namely the channel height. It was also shown that calculation of velocity is based on stokes equation treatment and uses the property of Laplace operator. The mass flow rates also measured along such systems were fitted to first or second order polynomial form s following the mean Knudsen number of the flow. The slip and accommodation

coefficients were extracted from mass flow rates through the rectangular microchannels.

6. Dongari *et al.* [13] showed that the flow of liquid in microchannel is quite different from the flow of gas. Standard results may be applied for liquid flow through a microchannels but this is not the case for the gases. It was shown that the non dimensional Knudsen number plays a major role in the heat transfer analysis of microchannel flows. The Knudsen number is large if the mean free path of the gas is large or the characteristic dimension of the channel is small. Former can be accomplished by reducing the pressure in the channel.

   It was also told that the difficulties come from the fact that flow is generally compressible and the slip at the wall has to be appropriately modeled. In this paper an integral form of Navier-Stokes equation was employed which was assumed valid in the slip flow region also. A second order model was used to find out the slip velocity. A change in curvature of the pressure versus the stream wise coordinates at high Knudsen number was observed by the analysis. From their analysis it is observed that Navier – Stokes equation along with second order slip model and appropriate co-efficient can be used for simulating a large range of Knudsen number and secondly any simulation which wants to capture behavior of pressure and volume flux correctly, should employ at least a second order accurate boundary condition.

7. From the work of Teng *et al.* [14] it is understood that, rectangular microchannels with LVG (Longitudinal vortex generations) have better heat transfer enhancement than smooth rectangular microchannels. It has also been established that, at Reynolds number in the range of 600- 750 transition from laminar to turbulent occurs for a flow through a microchannel. Like conventional macro channels flows here also Nusselt number of a function of Reynolds numbers. It has also been demonstrated that for flow with low Reynolds number through a microchannel (essentially laminar flow) generates a parabolic velocity profile in the fully developed region.

8. Chakraborty [15] analyzed the flow problems within a straight microchannel of arbitrary cross-section either exactly or approximately using three general solution methods namely, complex function analysis, membrane vibration analogy and variational method. With these three methods, velocity profile within a channel of arbitrary cross-section is calculated. In his paper, the flow considered viscous, incompressible and pressure driven and governed by Navier-Stokes equation.

CHAPTER-3

MODELING & ANALYSIS

3. MODELING AND ANALYSIS

3.1 Consideration of the Physical problem

For the purpose of analysis a rectangular microchannel of arbitrary dimensions (having dimension $(2l \times 2L)$) has been chosen satisfying the criteria of microchannels. The flow of the fluid through the duct has been considered axi-symmetric. Following assumptions have also been made for the purpose of analysis.

a) Incompressible flow.
b) Flow through the duct is laminar and fully developed and steady flow.
c) Fluid properties remain unchanged in the flow field.
d) No axial conduction is present, viscous dissipation effect is neglected.
e) The boundary walls are maintained at a constant temperature.
f) Flow occurs only in one direction (here z direction)
g) No body forces are present in the flow field.

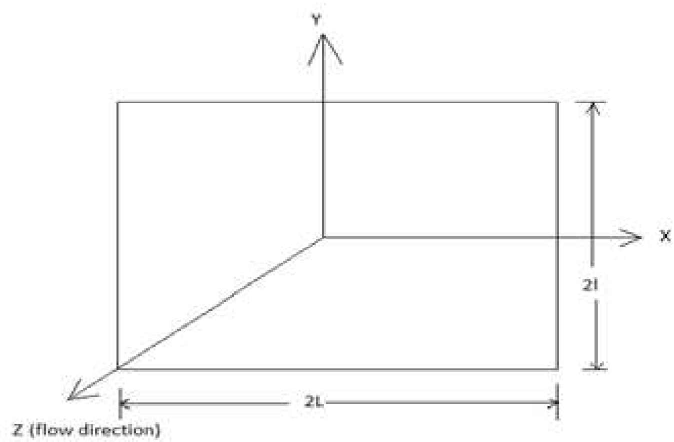

Fig. 1: Cross section of the rectangular microchannel

3.2 Mathematical modeling

Two separate analyses namely, *Fluid flow analysis* and *Heat transfer analysis* have been carried out for determining velocity and temperature distribution respectively.

3.2.1 Analytical approach

3.2.1.1 Fluid flow analysis

The Navier-Stokes equation which is valid for all types of fluid flow problems is also applicable for the present case. Therefore writing the generalized Navier-Stokes equation in Cartesian coordinate we get,

$$\rho\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right]=-\frac{\partial p}{\partial x}+\mu\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right]+b_x \tag{1}$$

$$\rho\left[\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right]=-\frac{\partial p}{\partial y}+\mu\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right]+b_y \tag{2}$$

$$\rho\left[\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right]=-\frac{\partial p}{\partial z}+\mu\left[\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right]+b_z \tag{3}$$

Due to the assumptions referred in points 2(a) to 2(f) above,

a) All the $\frac{\partial}{\partial t}$ terms are zero (Steady state)

b) $u=v=0$, due to the assumptions that flow is in only one direction.

c) Flow is steady, therefore $\frac{\partial w}{\partial z}$ term becomes zero.

d) $b_x=b_y=b_z=0$ (As from the assumptions, body force equal to 0).

Therefore the equations 1, 2 and 3 will be simplified as,

$$\frac{\partial p}{\partial z}=\mu\left[\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right] \tag{4}$$

Corresponding boundary conditions are as follows:

$w=u_s$, at $x=\pm l$ and $y=\pm L$ (5)

Where, u_s is the slip velocity at the wall. Now to simplify the analysis and making the results unit free governing momentum equation of the flow has been non-dimensionalized by introducing following non dimensional variables:

$$U = \frac{-\mu(w-u_S)}{L^2{}_C\left(\frac{dp}{dx}\right)} \;;\quad X = \frac{x}{L} \;;\; Y = \frac{y}{l} \quad \text{and} \quad A = \frac{l}{L} \tag{6}$$

Defining average velocity by w_m is the average velocity defined as,

$$w_m = \frac{\iint w dx dy}{\iint dx dy} \tag{7}$$

Substituting these variables in the above equations, the governing momentum equation becomes:

$$\frac{\partial^2 U}{\partial X^2} + \frac{1}{A^2}\frac{\partial^2 U}{\partial Y^2} = -1 \tag{8}$$

Where $\frac{dp}{dz}$ is the pressure gradient in the flow direction and assumed to be constant for the present situation (that is why partial derivatives have been converted to ordinary differentials).

Despite simpler look of Eq. 8 it is to be noted that, the slip velocity at the wall that is u_S is known. Therefore some provision must exist to find out the value of u_S.

From the work of Hooman [7] the slip velocity is given by:

$$u_S = \frac{F-2}{F} Kn D_H \frac{\partial u}{\partial n}_{wall} \tag{9}$$

The original boundary condition $w = u_S$ at the boundary also gets non-dimensionalized after substituting non dimensional variables as mentioned in Eq. 6 above.

Therefore the non dimensionalized boundary conditions are

$$U = 0 \text{ at } X = \pm 1 \text{ and } Y = \pm 1 \tag{10}$$

Eq. 8 resembles the velocity field of the no slip condition. Therefore solution obtained for U by solving Eq. 8 has been denoted by U_{NS}. However due to the presence of slip phenomenon the actual solution takes the following form:

$$U = U_{NS} + \frac{2-F}{F} \cdot \left(\frac{D_H}{2L_C}\right)^2 \tag{11}$$

Here D_H is the hydraulic diameter of the channel and L_C is the characteristic length of the flow passage. As per Hooman [7] the simplified relationship for obtaining the velocity field is given by:

$$U = BU_{NS} + 1 - B \tag{12}$$

where B is defined by the relation:

$$B = \frac{1}{1 + \dfrac{2-F}{F.U_{NS},M} Kn\left(\dfrac{D_H}{2L_C}\right)^2} \tag{13}$$

Putting, $D_H = \dfrac{4A_C}{P}$ and $L_C = L$ (14)

Eq. 13 gets modified as, $B = \dfrac{1}{1 + \dfrac{2-F}{F.U_{NS},M} Kn\left(\dfrac{2A}{1+A}\right)^2}$ (15)

Here $U_{NS,M} = \dfrac{\iint_{Ac} U_{NS}\, dXdY}{\iint_{A_C} dXdY}$ (16)

The factor VF defined by the following Eq. 17 plays a vital role in accommodating slip velocity effects through the duct.

$$VF = \frac{2-F}{F} Kn \cdot D_H \tag{17}$$

Approximate analytical solutions of Eq. 8 have been obtained subjected to the boundary conditions (Eq. 10) as mentioned in the following sections.

From Eq. 12 it may be noted that (1-B) closely resembles the slip flow contribution on velocity as the same gets added along with the non dimensional velocity term to get the total velocity for slip flow situations. Therefore the slip factor may be defined by Eq. 18,

$$\beta = 1 - B \tag{18}$$

3.2.1.1.1 *Analytical approach for velocity distribution*

For the purpose of finding approximate analytical solutions Integral (Integral Ritz, Integral Kantorovich etc) and Variational (Variational Ritz, Variational Kantorovich etc) methods have been considered. Results obtained by applying those methods are listed below:

Integral Ritz methods

Here the governing differential equation has been solved by integral method using second order approximation of Ritz profile. The generalized equation is given by

$$w(x,y) = \left(L^2 - x^2\right)\left(l^2 - y^2\right)\left(a_0 + a_1 x^2 + b_1 y^2 + c_1 x^2 y^2 + a_2 x^4 + b_2 y^4 + ...\right) \tag{19}$$

For the purpose of calculation, a second order truncated series is considered. For different types of second order approximation we obtain different series.

In this method the second order series is approximated according to Eq. 20 given below.

$$w(x,y) = \left(L^2 - x^2\right)\left(l^2 - y^2\right)\left(a_0 + a_1 x^2\right) \tag{20}$$

Eq. 20 is subjected to the following two conditions

$$4\int_0^L\int_0^l \left(\partial^2 w/\partial x^2 + \partial^2 w/\partial y^2\right) dxdy = 4\int_0^L\int_0^l \left(dp/\mu dz\right) dxdy \tag{21}$$

And $$\frac{\partial^2 w}{\partial x^2}\Big|_{x=0,y=0} + \frac{\partial^2 w}{\partial y^2}\Big|_{x=0,y=0} = \frac{1}{\mu}\frac{dp}{dz} \tag{22}$$

From Eq. 21 and Eq. 22 two unknowns namely a_0 and a_1 may be obtained. Now substituting values as per Eq. 6 and Eq. 7 the non dimensional velocity distribution through the flow may be obtained. Thus non dimensional velocity distribution by Integral Ritz method-1 is therefore given by:

$$U(X,Y) = \frac{9(1-X^2)}{4(3+26A^2)}\left[\frac{2+25A^2}{(1+A^2)} + 5X^2\right](1 - Y^2 - 2Y \cdot VF) \tag{23}$$

Integral Kantorovich method

Here also the solution is obtained by integral method considering the Kantorovich profile given by:

$$w(x,y) = \left(l^2 - y^2\right)\left[X_1(x) + y^2 X_2(x)\right] \tag{24}$$

Where X_1 and X_2 are functions of x and to be determined. By introducing the operators

$D \equiv \frac{d}{dx}$, two differential equations obtained can be re arranged as follows:

$$(l^2D^2/3-1)X_1+l^2(l^2D^2/15-1)X_2=dp/2\mu dz \tag{25}$$

$$\left(l^2D^2-2\right)X_1+2l^2X_2=\left(dp/\mu dz\right) \tag{26}$$

The above two differential equation may be solved simultaneously to obtain X_1 and X_2 for the boundary condition $X_1(L)=X_2(l)=u_s$ (27)

And the velocity profile for Integral Kantorovich method is given by following Eq. 28.

$$\begin{aligned}U(X,Y)&=\frac{3}{2M_3}(1-Y^2)\left[1-(1-k_1Y^2)X_1+(1-k_2Y^2)X_2\right]\\&-\frac{3Y\cdot VF}{M_3}\left[1-(1-k_1Y^2)X_1+(1-k_2Y^2)X_2\right]+\frac{3Y\cdot VF}{M_3}(1-Y^2)(k_1X_1-k_2X_2)\end{aligned} \tag{28}$$

Where $k_1=\frac{1}{2}(\alpha^2-2)$ and $k_2=\frac{1}{2}(\beta^2-2)$ (29)

$$X_1=\frac{(\beta^2-2)\exp(\alpha X/A)}{(\beta^2-\alpha^2)\exp(\alpha/A)}\quad\text{and}\quad X_2=\frac{(\alpha^2-2)\exp(\beta X/A)}{(\beta^2-\alpha^2)\exp(\beta/A)} \tag{30}$$

$$M_3=1-\frac{A(12-\alpha^2)(\beta^2-2)}{10\alpha(\beta^2-\alpha^2)}+\frac{A(12-\beta^2)(\alpha^2-2)}{10\beta(\beta^2-\alpha^2)} \tag{31}$$

$$\alpha=(27/2-\sqrt{489}/2)\quad\text{and}\quad\beta=(27/2+\sqrt{489}/2) \tag{32}$$

Variational Methods

In this method of solution a variation of the function is considered at a particular point, and the summation of these variations over the entire zone is considered to be zero. Therefore the necessary condition for variational formulation is is given by:

$$\delta I=\int_{-L}^{L}\int_{-l}^{l}\left(\partial^2u/\partial x^2+\partial^2u/\partial y^2-dp/\mu dz\right)\delta u\,dx\,dy=0 \tag{33}$$

Where I is a functional form and δI is the variation of the function.

Variational Ritz method:

The profile used for first order approximation first order approximation is:

$$w(x,y)=\left(L^2-x^2\right)\left(l^2-y^2\right)a_0 \tag{34}$$

Eq. 34 may be substituted in Eq. 33 and following result may be obtained upon non dimensionalisation.

$$U(X,Y) = \frac{9}{4}(1 - X^2)(1 - 2Y \cdot VF - Y^2) \tag{35}$$

Variational Kantorovich method:

$$U(X,Y) = \frac{3}{2M_4}\left[\frac{\cosh\{\sqrt{5/2}(X/A)\}}{\cosh(\sqrt{5/2}/A)} - 1\right](1 - Y^2 - 2 \cdot VF \cdot Y) \tag{36}$$

where

$$M_4 = \frac{A}{\sqrt{5/2}}\tanh(\sqrt{5/2}/A) - 1 \quad \text{and} \quad \alpha = (14 - \sqrt{133})^{1/2} \;\; ; \; \beta = (14 + \sqrt{133})^{1/2} \tag{37}$$

3.2.1.2 Heat transfer analysis

To find out the temperature distribution through the duct cross section Navier-Stokes energy equation is to be considered. The most generalized Navier-Stokes energy equation is as follows:

$$\rho c_p\left[\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}\right] = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + q''' + \mu\Phi + \beta T\frac{D\rho}{Dt} \tag{38}$$

Where the operator $\frac{D}{Dt}$ is defined as $\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$ (39)

and $\Phi =$ Dissipation function (40)

Which up on simplification subjected to the assumptions 2(a) to 2(f) becomes:

Eq. 38 is subjected to the boundary condition that at the wall $T = T_S$ (41)

Similar to the momentum equations, energy equation also needs to be non dimensionalized. For this purpose following non dimensional variables have been introduced:

$$\theta = \frac{T - T_S}{(q/k)} \quad \text{and} \quad q = mC_p\frac{\partial T}{\partial Z} = \rho A_C w_m C_p\frac{\partial T}{\partial Z} = \rho(4lL)w_m C_p\frac{\partial T}{\partial Z} \tag{42}$$

Where T_s is the slip temperature at the wall.

Eq. 38 upon substituting non dimensional variables in accordance with Eq. 6 and Eq. 42 becomes

$$A^2 \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} = \frac{A}{4} U(X,Y) \tag{43}$$

From Eq. 43 it is clear that once velocity field is known, temperature profile can be obtained by solving the differential equation (Eq. 43) subjected to non dimensional boundary conditions:

$$\theta = 0 \text{ at } X = \pm 1 \text{ and } Y = \pm 1 \tag{44}$$

Similar to the VF factor defined by Eq. 17 for slip velocity fields, TF factor defined by Eq. 45 considers the effects temperature slip in the microchannels.

$$TF = \frac{2 - F_t}{F_t} D_H \cdot \frac{Kn}{\Pr} \cdot \frac{2\gamma}{1+\gamma} \tag{45}$$

Non dimensional temperature fields have been analytically as well as numerically solved by many researchers for the case of non slip flow. The solution procedure for temperature is similar to the solution procedure of velocity. Therefore different approximate analytical techniques, similar to those which were utilized in non dimensional velocity field have been considered for temperature as well in the next section.

3.2.1.2.1 Analytical approach for temperature distribution

For the purpose of getting approximate analytical solution both Integral (Integral Ritz, Integral Kantorovich) and Variational (Variational Ritz and Variational Kantorovich) methods have been considered. The expressions θ as function of X and Y have been mentioned against each method.

Integral Ritz method

Similar to the Integral Ritz method described for velocity, a second order temperature profile given by Eq. 46 below has been considered and the same has been solved using the conditions mentioned in Eq. 53 and 54 below.

$$\theta(X,Y) = (1 - X^2)(1 - Y^2)(a_0 + a_1 X^2) \tag{46}$$

$$4\int_0^1\int_0^1 \left(A^2\, \partial^2\theta/\partial X^2 + \partial^2\theta/\partial Y^2\right) dX dY = 4\int_0^1\int_0^1 (A/4) U(X,Y)\, dY dX \tag{47}$$

$$A^2 \left.\frac{\partial^2\theta}{\partial X^2}\right|_{X=0,Y=0} + \left.\frac{\partial^2\theta}{\partial Y^2}\right|_{X=0,Y=0} = \frac{A}{4}U(X,Y)\Big|_{X=0,Y=0} \quad (48)$$

Eq. 46 upon solving subjected to the condition of Eq. 47 and 48 gives the following result:

$$\theta(X,Y) = (1-X^2)(a_0 + a_1 X^2)(1-2Y\cdot TF - Y^2) \quad (49)$$

Where

$$a_0 = -\frac{A(18+405A^2+1905A^4)}{32(1+A^2)(1+10A^2)(3+26A^2)} \quad (50)$$

$$a_1 = \frac{345A^3}{32(1+A^2)(1+10A^2)(3+26A^2)} \quad (51)$$

Integral Kantorovich method

Similar to the approximation mentioned in case of velocity profile, temperature profile for the Integral Kantorovich method may be approximated by Eq. 582 mentioned below.

$$\theta = (1-Y^2)[X_1(X) + Y^2 X_2(X)] \quad (52)$$

Eq. 58 is solved utilizing the condition mentioned in Eq. 53 and 54 and following result is obtained.

$$\theta(X,Y) = (1-Y^2)[X_1(X) + Y^2 X_2(X)] - 2Y\cdot TF\cdot X_1(X) + X_2(X)\cdot TF\cdot(2Y - 4Y^3) \quad (53)$$

Where

$$X_1(X) = N_1 + C_1 e^{\alpha X/A} + C_2 e^{\beta X/A} \quad (54)$$

$$X_2(X) = N_2 + C_3 e^{\alpha X/A} + C_4 e^{\beta X/A} \quad (55)$$

$$N_1 = -(B + A/4) \quad \text{and} \quad N_2 = -(B - A/4) \quad (56)$$

$$B = \frac{3A}{8M_3}\left[1 - \frac{(\beta^2-2)}{(\beta^2-\alpha^2)e^{\alpha/A}} + \frac{(\alpha^2-2)}{(\beta^2-\alpha^2)e^{\beta/A}}\right] \quad (57)$$

$$C_1 = \frac{E_2}{(\beta^2-\alpha^2)e^{\alpha/A}} \quad \text{and} \quad C_2 = \frac{E_1}{(\beta^2-\alpha^2)e^{\beta/A}} \quad (58)$$

$$C_3 = -C_1(\alpha^2-2)/2 \quad \text{and} \quad C_4 = -C_2(\alpha^2-2)/2 \quad (59)$$

$$E_1 = (B - A/4)/2 - (\alpha^2-2)(B + A/4)/4 \quad (60)$$

$$E_2 = (B + A/4)/2 - (\beta^2-\alpha^2)/4 - E_1 \quad (61)$$

$$\alpha = (27/2 - \sqrt{489}/2)^{1/2} \quad \text{and} \quad \beta = (27/2 + \sqrt{489}/2)^{1/2} \quad (62)$$

Variational Ritz method

The necessary condition for the formulation of Variational Ritz is given by Eq. 69 below.

$$\delta I = \int_{-1}^{1}\int_{-1}^{1}\left[A^2\,\partial^2\theta/\partial X^2 + \partial^2\theta/\partial Y^2 - (A/4)U\right]\delta\theta dXdY = 0 \tag{63}$$

And the temperature profile considered is

$$\theta = (1 - X^2)(1 - Y^2)a_0 \tag{64}$$

Which upon substitution into Eq. 63 gives the temperature profile as:

$$\theta(X,Y) = -\frac{9A}{40(1+A^2)}(1 - X^2)(1 - 2\cdot TF\cdot Y - Y^2) \tag{65}$$

Variational Kantorovich method

$$\theta(X,Y) = \frac{3A}{20}\left[\frac{\cosh\sqrt{(5/2)}(X/A)}{\cosh(\sqrt{5/2}/A)} - 1\right]\left[(1 - 2Y\cdot TF - Y^2)\right] \tag{66}$$

3.2.2 Numerical Approach

The governing differential equation for slip flow problems through a rectangular microchannel has been solved numerically particularly implementing finite difference method discritization technique to compare the results obtained by applying different approximate analytical techniques.

For the purpose of solving the governing equations numerically discritization is must. Entire cross section of the channel has been divided by m vertical and n horizontal points and therefore a total of $(m\times n)$ grid points have been obtained as shown in Fig-2.

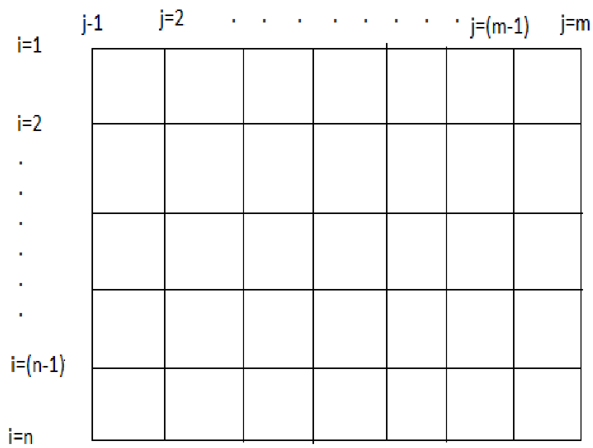

Fig. 2: Grid points obtained on the channel cross section for discritization

Therefore the region $-1 \le X \le 1$ has been subdivided into $(m-1)$ equal divisions and similarly the region $-1 \le Y \le 1$ has been subdivided into $(n-1)$ equal subdivisions. Denoting each division by ΔX and ΔY along the x and y direction respectively, we may write:

$$\Delta X = \frac{2}{(m-1)} \quad \text{and} \quad \Delta Y = \frac{2}{(n-1)} \tag{67}$$

Now discritizing Eq. 8 we get,

for all $i = 2:(n-1)$, $j = 2:(m-1)$

$$\frac{U_{i-1,j} - 2U_{i,j} + U_{i+1,j}}{(\Delta X)^2} + \frac{1}{A^2}\left[\frac{U_{i,j-1} - 2U_{i,j} + U_{i,j+1}}{(\Delta Y)^2}\right] = -1$$

$$\Rightarrow \frac{U_{i-1,j}}{(\Delta X)^2} + \frac{U_{i+1,j}}{(\Delta X)^2} + \frac{U_{i,j-1}}{(A\Delta Y)^2} + \frac{U_{i,j+1}}{(A\Delta Y)^2} - U_{i,j}\left[\frac{2}{(\Delta X)^2} + \frac{2}{(A\Delta Y)^2}\right] = -1$$

$$\Rightarrow U_{i,j} = \frac{(\Delta X)^2 + (\Delta Y)^2}{2\{(\Delta X)^2 + (\Delta Y)^2\}} \cdot \left[1 + \frac{U_{i-1,j}}{(\Delta X)^2} + \frac{U_{i+1,j}}{(\Delta X)^2} + \frac{U_{i,j-1}}{(A\Delta Y)^2} + \frac{U_{i,j+1}}{(A\Delta Y)^2}\right] \tag{68}$$

Subjected to the following discritized boundary conditions:

for $i = 1:n$

$$U_{i,1} = 0 \quad \text{and} \quad U_{i,m} = 0 \tag{69}$$

for $j = 1 : m$

$$U_{1,j} = 0 \quad \text{and} \quad U_{n,j} = 0 \tag{70}$$

Here for (i,j) is the considered point.

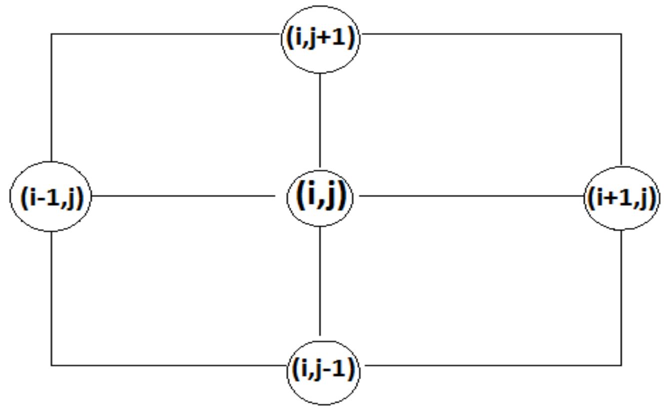

Fig. 3: Discritization scheme

A tri diagonal matrix can easily be formed from Eq. 74, 75 and 76. Therefore the same can easily be solved by any commercial mathworking software by Gaussian elimination or any other suitable techniques. The present discritized equations have been solved using MATLAB software.

Solving Eq. 68 to Eq. 70 U_{NS} values has been obtained for the entire cross section. $U_{NS,M}$ have been calculated applying Eq. 71

$$U_{NS,M} = \sum_i \sum_j U_{i,j} \Delta X \Delta Y \tag{71}$$

By putting the values of $U_{NS,M}$ obtained from Eq. 71 into Eq. 15 value of B can be found out. Once B value is obtained, Eq. 12 can be easily solved and velocity field would be obtained, also slip coefficient given by Eq. 18 can be found out.

Poiseuille number of the flow, which is of particular interest, may also be found out by numerical methods by Eq. 72 mentioned below.

$$Po = 2\left[\left(\sum_j \sum_i U_{i,j} \Delta X \Delta Y\right) + \left(\frac{2-F}{F} Kn \left(\frac{2A}{1+A}\right)^2\right)\right]^{-1} \left(\frac{2A}{1+A}\right)^2 \tag{72}$$

To obtain the temperature field, similar technique adopted for velocity is to be followed. However while solving for temperature profile; one thing should be kept in mind that, prior solving discritized equation for temperature velocity fields should be known. Discritizing Eq. 39 and Eq. 40:

$$A^2\frac{\theta_{i-1,j}-2\theta_{i,j}+\theta_{i+1,j}}{(\Delta X)^2}+\frac{\theta_{i,j-1}-2\theta_{i,j}+\theta_{i,j+1}}{(\Delta Y)^2}=\frac{A}{4}U_{i,j}$$

$$\theta_{i,j}=\frac{(\Delta X\Delta Y)^2}{2[(A\Delta Y)^2+(\Delta X)^2]}\left[\frac{A^2}{(\Delta X)^2}(\theta_{i-1,j}+\theta_{i+1,j})+\frac{1}{(\Delta Y)^2}(\theta_{i,j-1}+\theta_{i,j+1})-\frac{A}{4}U_{i,j}\right] \quad (73)$$

for $i=1:n$

$$\theta_{i,1}=0 \quad \text{and} \quad \theta_{i,m}=0 \quad (74)$$

for $j=1:m$

$$\theta_{1,j}=0 \quad \text{and} \quad \theta_{n,j}=0 \quad (75)$$

Discritized equation Eq. 79 to Eq. 81 forms a tri diagonal matrix structure which can easily be solved by matrix method to find out the solution of the temperature field similar to the velocity field.

CHAPTER-4

RESULTS & DISCUSSIONS

4. RESULTS AND DISCUSSIONS

Result obtained for velocity and temperature distribution using different approximate analytical methods have been compared with the results obtained by numerical methods for fluid having $\gamma = 1.4 \;\&\; \Pr = 0.7$.

For the purpose of better understanding results have been compared by plotting graphs. Such plots for velocity distribution are depicted in the following figures (Fig. 4 to Fig. 6). For the purpose of plotting the graphs three different aspect ratios of the channels have been considered to get a clear picture about the dependence of velocity on channel geometry. From the plots it is evident that, Integral Kantorovich method gives the best result among different approximate methods considered. However the accuracy level varies with the channel aspect ratios. It is observed that for A=0.25 IK gives the best estimate whereas for A=1 it gives the least accurate estimation. Also from the plots it is observed, that velocity at the wall is non-zero which confirms the presence of slip effect at the boundary wall.

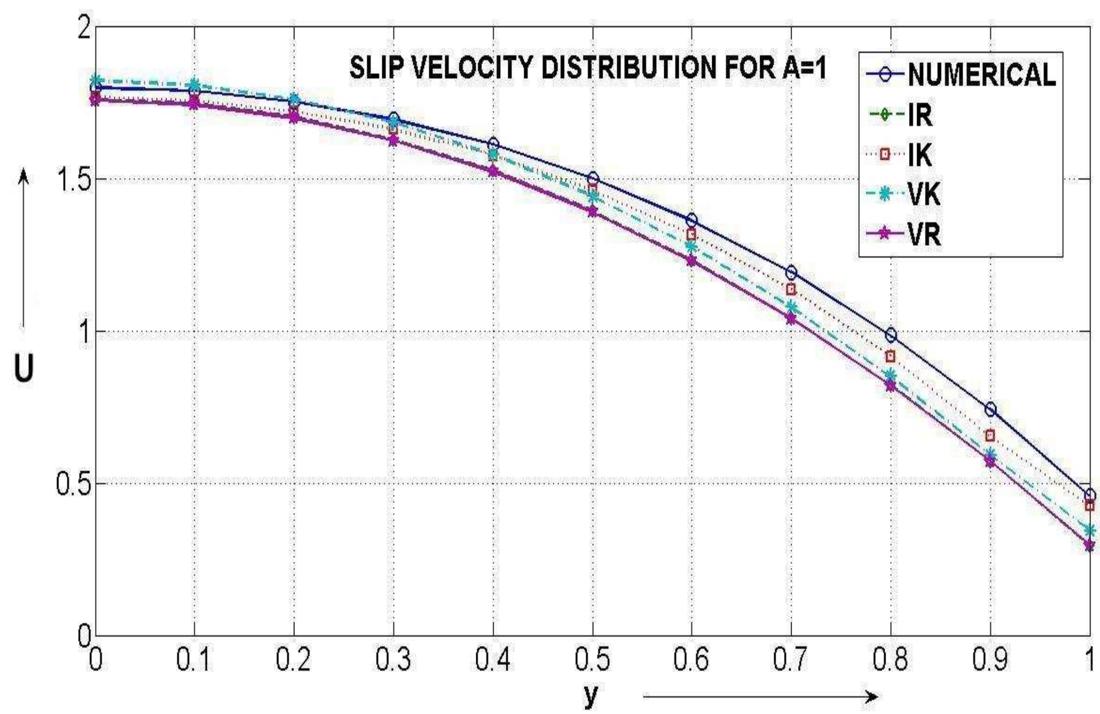

Fig. 4: Slip velocity variation for A=1 along the height of the channel at x=0

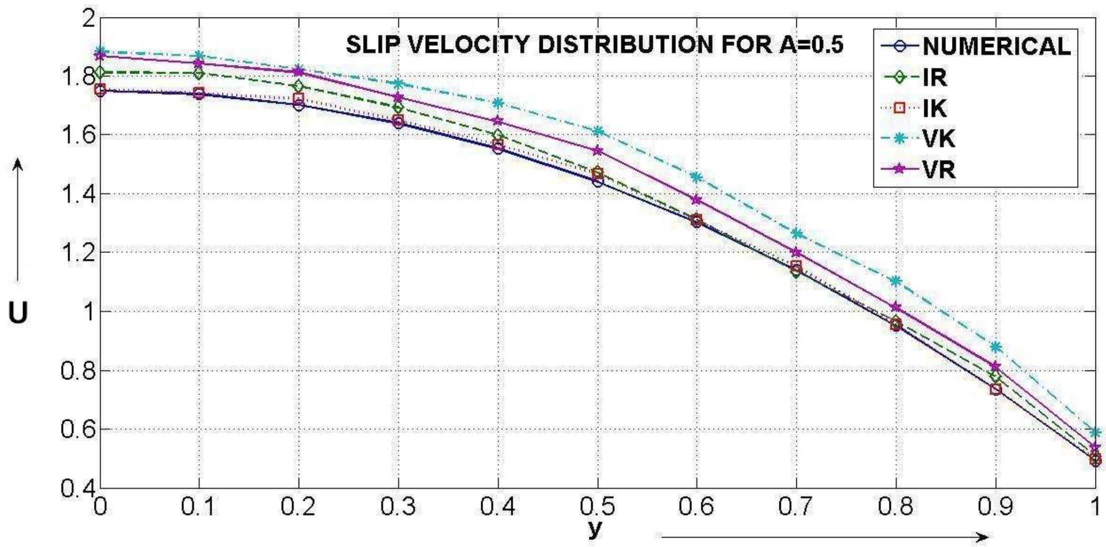

Fig. 5: Slip velocity variation for A=0.5 along the height of the channel at x=0

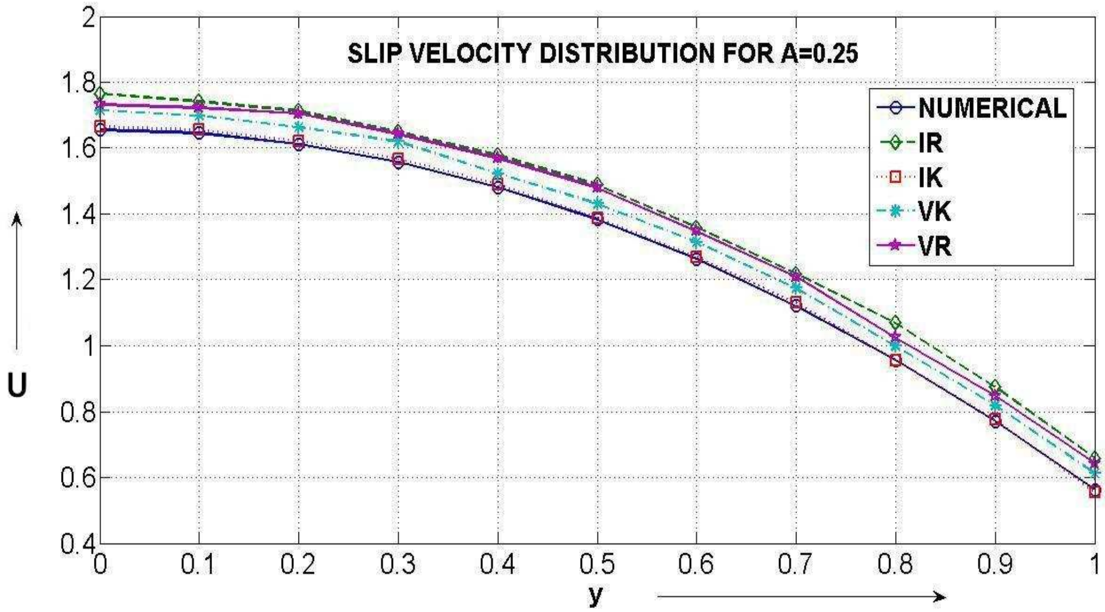

Fig. 6: Slip velocity variation for A=0.25 along the height of the channel at x=0

For the purpose of understanding the difference between slip velocity field and non slip velocity field, for otherwise identical flow situations, a slip coefficient have been introduced in chapter-3, Eq. 18. It is worth noting that, like all other parameters, the slip velocity also depends upon the channel aspect ratio. The comparative values of slip coefficients for different aspect ratios of the channel are depicted in the following table:

Table-2: Comparison of slip coefficients for different aspect ratio of the channels for different Knudsen number and aspect ratio of the channel

Name of the method	**Slip Coefficient (β)**								
	Kn=0.1			**Kn=0.05**			**Kn=0.001**		
	A=1	**A=0.5**	**A=0.25**	**A=1**	**A=0.5**	**A=0.25**	**A=1**	**A=0.5**	**A=0.25**
Numerical	0.152	0.164	0.187	0.143	0.157	0.179	0.129	0.147	0.158
Integral Ritz	0.137	0.139	0.155	0.139	0.151	0.171	0.117	0.139	0.144
Integral Kantorovich	0.159	0.171	0.193	0.154	0.164	0.176	0.131	0.145	0.156
Variational Ritz	0.181	0.187	0.201	0.174	0.181	0.187	0.137	0.159	0.167
Variational Kantorovich	0.173	0.192	0.199	0.167	0.175	0.183	0.135	0.157	0.165

From the above Table-2 it is evident that as the channel aspect ratio is decreasing, the slip coefficient is increasing, which shows that with decreasing channel aspect ratio tendency of slip is increasing. Also from the data depicted on the above table it is seen that, Integral Kantorovich method gives the best estimate for slip coefficient also. From the above table it is also evident that as the Knudsen number is decreasing the value of slip co efficient is also decreasing, that is flow is approaching towards the conventional flow. From the definition of Knudsen number lower value of Knudsen number means higher channel characteristic length compared to mean free path of the fluid particles which indicated that flow tend to become conventional macrochannel flow. Therefore for lower Knudsen number, lower slip coefficient obtained as expected.

Similarly in line with the velocity fields, temperature fields have also been plotted on the same co ordinate. Likewise velocities, temperatures have also been plotted for three different aspect ratios (that is for A=1, 0.5 and 0.25) in Fig. 7 to Fig. 9. In case of temperature profiles it is observed that among the chosen approximate methods, Integral Kantorovich method gives the best estimate. However, in accuracy level is more or less same with different aspect ratio. From the plots it is also observed that, non dimensional temperature values are becoming negative, which indicates that, the temperature of the fluid is lower than the slip temperature that is temperature of the fluid layer adjacent to the wall.. In case of non slip flow situations non dimensional temperature obtained at the wall would be 0, however for slip condition a non zero value is observed, which indicates better cooling that is more heat removal from the bulk fluid to the wall in the direction of flow due to the presence of finite temperature difference.

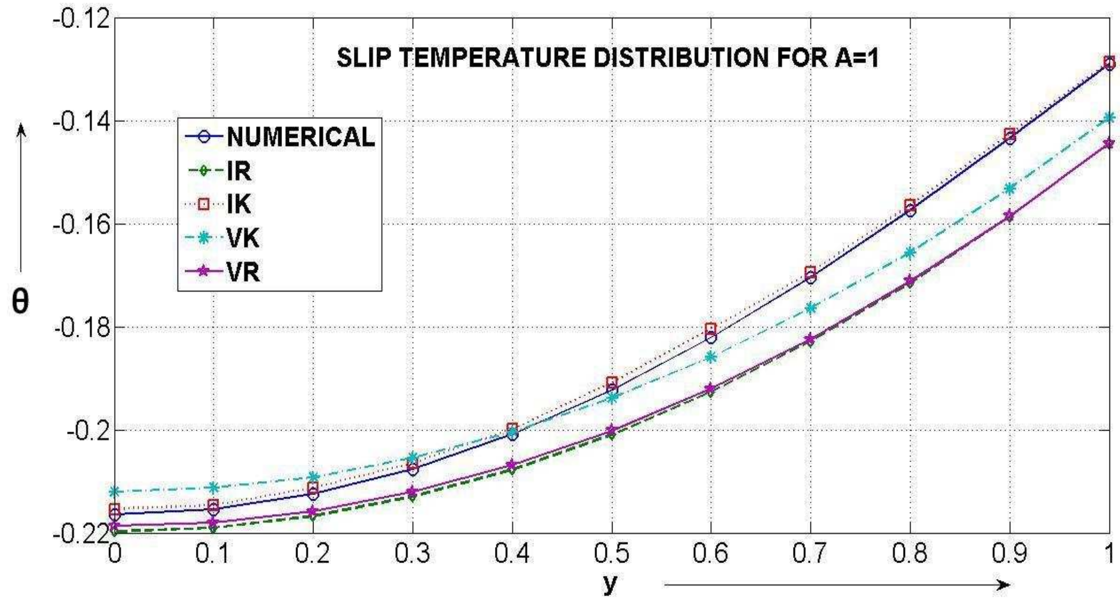

Fig. 7: Slip temperature variation for A=1 along the height of the channel at x=0

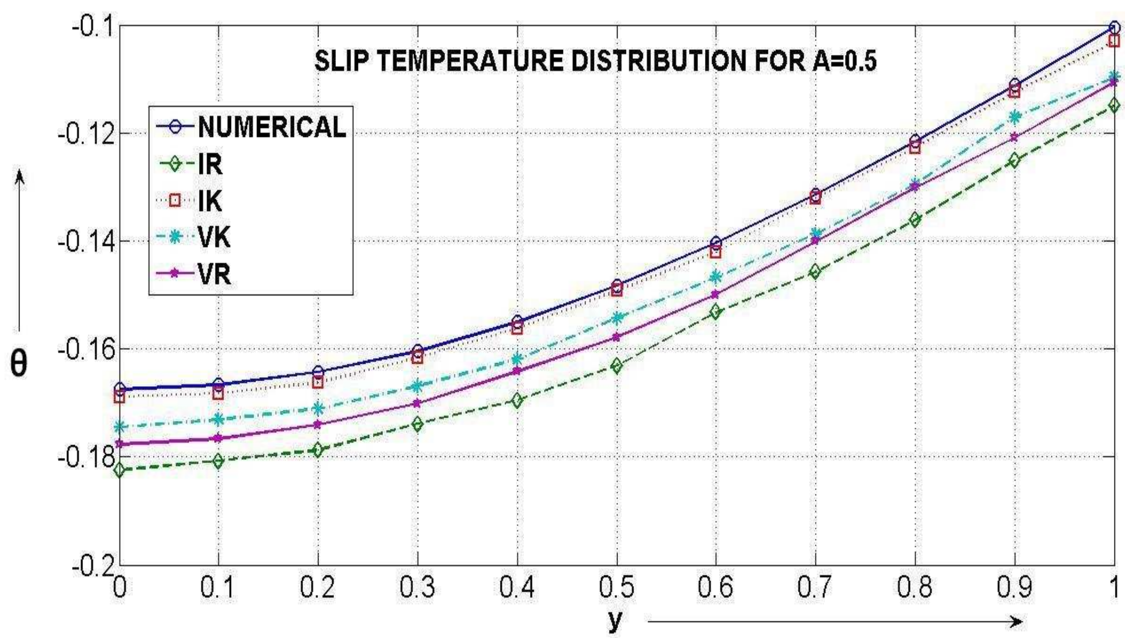

Fig. 8: Slip temperature variation for A=0.5 along the height of the channel at x=0

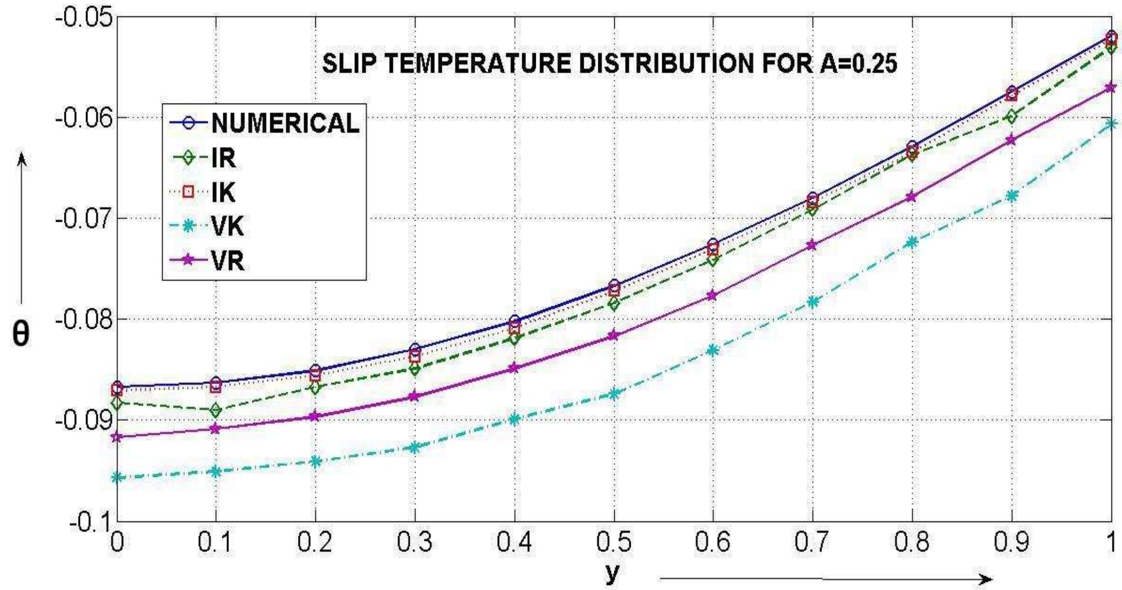

Fig. 9: Slip temperature variation for A=0.25 along the height of the channel at x=0

Due to the symmetrical nature of the flow through the channel, all the values have been considered at X=0. Though the graphs have been plotted for different Y values at X=0, the same could be plotted for any region of the channel and similar result expected.

From the above plots it is evident that, both velocity and temperature fields are dependent upon the aspect ratio of the channel. Therefore the accuracy level of different approximate analytical techniques also depends on the channel aspect ratio hence channel geometry. It is also observed that, the results obtained by various approximate analytical methods are very close to the exact value (less than 5% error). Therefore results obtained by various methods can be utilized for all practical purposes. In addition to these from Fig: 10 it is also observed that the Poiseuille Numbers for flow is also dependent on the aspect ratio of the channel as well as on the Knudsen number of the flow considered.

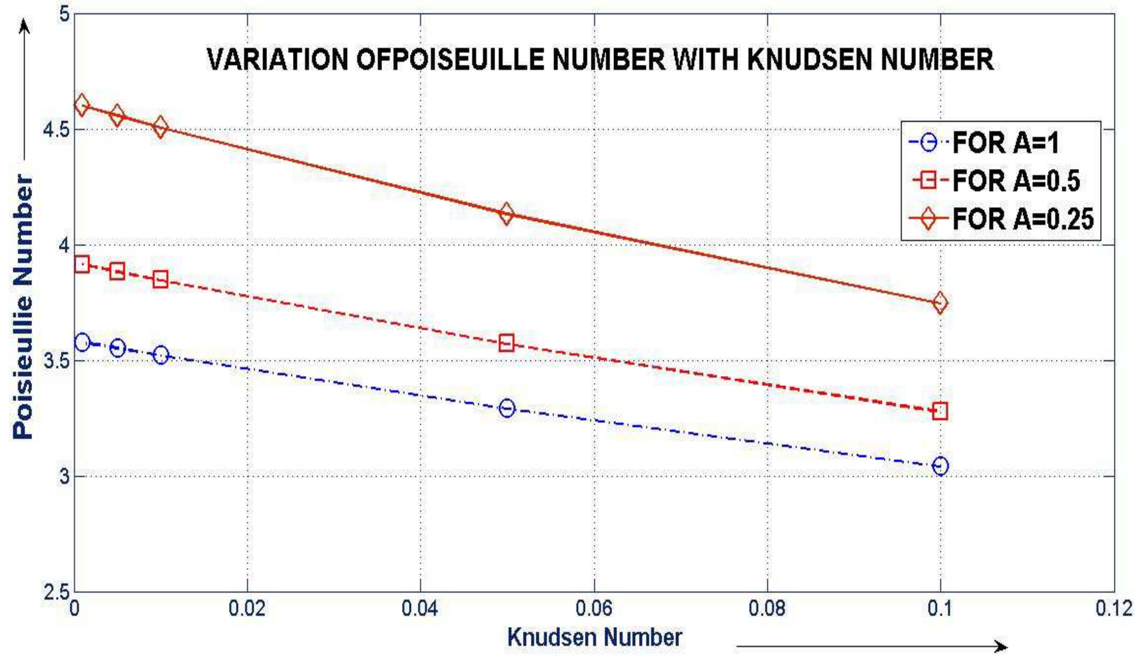

Fig. 10: Variation of Poiseuille Number with Knudsen number

From Fig: 10, it is seen that Poiseuille number for the flow is in the range of 5 to 3.5. Such a low value of the Poiseuille number suggests that flow regime considered is of laminar nature, therefore writing the Navier-Stokes equations for laminar flow regime is a correct approximation. Also for lower values of Knudsen number higher Poiseuille number is observed, which suggests that at lower Knudsen number higher friction coefficient is observed (as *Po*=*f*. *Re*). As the Knudsen number goes on increasing, the value of *Po* goes on decreasing suggesting a decline of friction factor which is expected. Also the same pattern is observed as aspect ratios of the channels are increasing. With channel aspect ratio tends towards 1, the channel cross section tends to become symmetric; therefore it is very natural to expect that frictional effect gets lowered. Studying the variation of Po number the power required for maintaining the flow can be approximated.

CHAPTER-5

CONCLUSIONS & SCOPE OF FUTURE WORK

CONCLUSION AND SCOPE OF FUTURE WORK

Conclusions

Different approximate analytical solutions for calculating velocity profile and temperature profile has been found out for hydrodynamically as well as thermally fully developed flow for a constant wall temperature rectangular channel under steady state condition but for the slip flow situations. The solution of mathematical modeling has been achieved for the slip flow situations by different approximate and numerical methods both for momentum as well as energy equations. However it should also be noted that the energy equation mentioned in chapter 3 is coupled with velocity fields. Therefore most appropriate method for velocity fields required to be used to get the best result for temperature fields also. The same is also clear from the plots, which show that Integral Kanotorovich method is the most appropriate method for velocity as well as temperature.

As the approximate analytical methods are a function of aspect ratio and the accuracy of different approximate methods depend the aspect ratio of the channel, therefore proper approximate methods must be chosen for analytical velocity and temperature profile, depending upon the aspect ratio of the channel.

Therefore it can be concluded that without going in to the cumbersome numerical solution method or exact solutions approximate analytical solutions may be utilized for slip velocities and temperatures for all practical purposes as the results gives less than 5% deviation.

Scope of future work:

In the present work, analytical approach for flow through a microchannel has been considered for analysis of fluid flow and heat transfer characteristics. The Navier-Stokes equation has been reduced to simpler form to simplify the mathematical analysis. As for example, the viscous dissipation term is neglected. As a result, the predicted results show some deviation from the numerical ones. Therefore, the scope of future work may be analysis of fluid flow and heat transfer characteristics for flow within microchannels considering all the terms in the Navier-Stokes equation.

Also present work has been carried out by considering a rectangular cross section of the microchannels. However the research work can be carried out farther for the channels having any other cross sections rather than rectangular. Though in line with the existing literatures it has been concluded that results obtained for flow through rectangular microchannels are valid for any other channels of arbitrary cross sections, still the results can be verified considering a wide range of cross sectional shapes and tallying the result.

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