B.A. 2ND YR 4TH SEM EXAMINATION 2018 (OLD)

Econometrics

Ref.: EX/UG/ECO/4.3/63/2018

Time : Two hours — Full Marks : 30

Answer any five of the following questions. $6 \times 5 = 30$

1. An econometrician arranges her sample of n (even) observations in ascending order of the regressor X and divides up the sample into two equal parts containing $n/2$ observations each. She then calculates the average values of X and Y for each part separately. Let $(\bar{X_A}, \bar{Y_A})$ be sample means of first $n/2$ values of X and Y and $(\bar{X_B}, \bar{Y_B})$ be sample means of last $n/2$ values of X and Y. The model she considers is

$$Y_i = \alpha + \beta X_i + \varepsilon_i,$$

where X is non-stochastic, $\varepsilon_i \overset{iid}{\sim} (0, \sigma^2)$ and the estimated regression line is the straight line passing through the points $(\bar{X_A}, \bar{Y_A})$ and $(\bar{X_B}, \bar{Y_B})$. If the estimates of α and β in this method are a and b, show that

   (a) a and b are unbiased.

   (b) $var(b) \geq var(\hat{\beta}_{OLS})$.

2. Let $y_t = \beta_1 + \beta_2 y_{t-1} + u_t$, where $u_t \overset{iid}{\sim} \mathrm{N}(0, \sigma^2)$. Is $\hat{\beta}_{2,OLS}$ unbiased? Is it consistent?

3. If $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$, where $E(\mathbf{u}) = \mathbf{0}$ and $E(\mathbf{uu'}) = \Sigma$, a positive definite (non-diagonal) matrix, prove that $\mathbf{var}(\hat{\boldsymbol{\beta}}_{OLS})$ is greater than $\mathbf{var}(\hat{\boldsymbol{\beta}}_{GLS})$ in a matrix sense.

4. Consider the following regression model:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u},$$

where $\mathbf{y}$ and $\mathbf{u}$ are n $\times$ 1, $\mathbf{X}$ is n $\times$ K (where $X_{1i} = 1\ \forall i = 1, \ldots, n$) and $\boldsymbol{\beta}$ is K $\times$ 1. $\mathbf{u} \sim N(\mathbf{0}, \sigma^2\mathbf{I})$. Show that

$$var(\hat{\beta}_k) = \frac{\sigma^2}{(1 - R_k^2)\sum_{i=1}^{n}(X_{ki} - \overline{X}_k)^2}, \quad \forall\, k = 2, \ldots, K,$$

where R_k^2 is the R-squared from regressing X_k on all other regressors, including an intercept.

5. Let the model be

$$y_t = \mathbf{x}_t'\boldsymbol{\beta} + u_t,$$

where $\mathbf{x}$ is $1 \times K$ and $u_t = \rho u_{t-1} + \varepsilon_t$, $|\rho| < 1$ and $\varepsilon_t \overset{iid}{} (0, \sigma_\varepsilon^2)$. What is the autocorrelation of u in this model? Now consider the model

$$y_t - y_{t-1} = (\mathbf{x}_t - \mathbf{x}_{t-1})'\boldsymbol{\beta} + v_t,$$

where $v_t = u_t - u_{t-1}$. Compare autocorrelation of v with that of u.

[Turn over

6. Suppose $\hat{\boldsymbol{\beta}}_{OLS}$ is the $K \times 1$ least square coefficient vector in the regression of $\mathbf{y}$ on $\mathbf{X}$ and that $\mathbf{c}$ is any other $K \times 1$ non-zero vector. Prove that the difference in the two sums of squared residuals is

$$(\mathbf{y} - \mathbf{Xc})'(\mathbf{y} - \mathbf{Xc}) - (\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}_{OLS})'(\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}_{OLS}) = (\mathbf{c} - \hat{\boldsymbol{\beta}}_{OLS})'\mathbf{X}'\mathbf{X}(\mathbf{c} - \hat{\boldsymbol{\beta}}_{OLS}).$$

   Prove that this difference is positive.

7. Let the model be

$$y_i = \mathbf{x}_i'\boldsymbol{\beta} + u_i,$$

   where $\mathbf{x}$ is $1 \times K$ and pdf of y, $f(y) = (1/\mathbf{x}'\boldsymbol{\beta})e^{(-y/\mathbf{x}'\boldsymbol{\beta})}$, $y > 0$. Derive the most efficient estimator of $\boldsymbol{\beta}$.

8. Let $y_i = \mu + \varepsilon_i$, where $\mathrm{E}(\varepsilon_i) = 0$ and $\mathrm{V}(\varepsilon_i) = \sigma^2 \ \forall\, i = 1, \ldots, n$. $\mathrm{cov}(\varepsilon_i, \varepsilon_j) = \sigma^2\rho \ \forall\, i \neq j$. Show that the OLS estimator of μ is inconsistent.