

**B.A. EXAMINATION, 2018**

( 1st Year, 2nd Semester )

**MATHEMATICAL ECONOMICS – I**

Time : Two hours

Full Marks : 30

Answer any two

1. a Let The total cost function of the product be

$$C = 2x \left( \frac{x+7}{x+5} \right) + 7$$

Find the average cost function and marginal cost function. Comment on their slopes. 3

- b) Consider the following production function:. Find the values of 'a' and 'b' for which it is continuous. Then draw the production function and hence find the optimum level of output. 3

$$f(x) = \begin{cases} x & \text{if } x < a \\ x^2 & \text{if } 1 \leq x \leq b \\ 8x^{1/2} & \text{if } x > 4 \end{cases}$$

- c) Which of the following functions are homogenous or homothetic? Give reasons for your answer.

$$\text{i) } f = \frac{x^2 - y^2}{x^2 + y^2} + 3 \quad \text{ii) } f = x_1^2 + x_2^3 \quad \text{iii) } f = \frac{x^2 y^2}{xy + 1} \quad 3$$

- d) Consider the following utility function:
- $Q = x_1^2 + 2x_1x_2 + 3x_1x_3 - 2x_2^2 + 6x_2x_3 + 3x_3^2$

$$\text{Max } U = \alpha x + \sqrt{y}$$

$$\text{Subject to } px + y = 1$$

- a) Find optimal values for x and y.  
 b) Give an interpretation of Lagrange multiplier.  
 c) Also check for the second order condition of maximization. 3+1+2

2. a) Do detailed graphing for the following function:
- $\frac{x+1}{(x-2)^2}$
- 4

- b) Using Kuhn Tucker conditions find the optimum solution for this problem:

$$\text{Max } x^2 + y^2$$

$$\text{Subject to } x \geq 0 \quad y \geq 0 \quad 5$$

$$2x + y \leq 2$$

- c) Consider a quadratic form

Check whether Q has a maxima, minima or saddle point at (0,0,0) subject to

$$\text{the constraint } x_2 = x_3. \quad 3$$

- d) State and prove Roy's identity. 3

3. a) Define a convex set with example. Solve the following LPP problem graphically:

[ Turn over

minimize  $4x+3y$

subject to

$$2x + y \geq 8$$

$$x + y \geq 5$$

$$x \geq 0 \quad y \geq 0$$

Also graphically show that feasible set is convex. 2+4+1

b) A domestic auto producer is facing intense competition in the market from Chinese auto imports. The executive officer decides that one way to counter this competition is by producing at that quantity at which total costs are minimized. The firm's cost structure can be illustrated by the

$$\text{function } TC = \frac{1}{3}Q^3 - 10Q^2 + 80Q + 500.$$

i. Calculate the cost-minimizing quantity.

ii. Assume that the total revenue function is  $TR(Q) = 8Q - Q^2$ . Show that the profit-maximizing quantity is different from the cost-minimizing quantity. 1+2

c) Check whether the statements are true or false:

i) The level curves of the plane  $z = d - (ax + by)$ , where  $a; b; c; d \neq 0$ , are parallel lines in the  $xy$ -plane. 2.5

ii) The domain of the function  $g(x; y) = \ln((x + 1)^2 + (y - 2)^2 - 1)$  consists

of all points  $(x; y)$  lying strictly in the interior of a circle centered at  $(1; 2)$  of radius 1. 2.5