MA 1ST YEAR 1ST SEM 2018
Econometrics I

Time: 2 Hours   Full Marks: 30

Answer any five of the following questions. $6 \times 5 = 30$

1. Consider the linear regression model $y_i = \mathbf{x}_i\boldsymbol{\beta} + u_i$ where $\mathbf{x}_i$ is non-stochastic and $E(u_i u_j) = \sigma^2$ if $i = j$, $E(u_i u_j) = \rho\sigma^2$ if $|i - j| = 1$ and $E(u_i u_j) = 0$ if $|i - j| > 1$.

   (a) What is a consistent estimator of $var(\hat{\boldsymbol{\beta}})$?

   (b) Is White's heteroskedasticity consistent robust estimate of $var(\hat{\boldsymbol{\beta}})$ consistent here?

2. Consider the three equation model: $y = \beta x + u$, $x = \lambda u + \varepsilon$ and $z = \gamma\varepsilon + v$, where the mutually independent errors are normally distributed with zero mean and variances σ_u^2, σ_ε^2 and σ_v^2, respectively.

   (a) Is $\hat{\beta}_{OLS}$ consistent? What is the asymptotic bias in $\hat{\beta}_{OLS}$ if any?

   (b) Is $\hat{\beta}_{IV}$ consistent if z is used as an instrument for x?

3. Suppose y has the pdf $f(y|\mathbf{x}, \boldsymbol{\beta}) = \left(\dfrac{1}{\mathbf{x}\boldsymbol{\beta}}\right) e^{-\frac{y}{\mathbf{x}\boldsymbol{\beta}}}$, $y > 0$.

   (a) What are $E(y|\mathbf{x})$ and $var(y|\mathbf{x})$?

   (b) What is the GLS estimator of $\boldsymbol{\beta}$?

4. For the simple regression model $y_i = \mu + \varepsilon_i$, $\mu \overset{\text{i.i.d.}}{\sim} (0, \sigma^2)$, consider the estimator $\hat{\mu} = \sum_{i=1}^{N} w_i y_i$, $w_i = \dfrac{i}{N(N+1)/2}$. Prove that this is a consistent but inefficient estimator of μ.

5. Consider the population model $E(y|\mathbf{x}, z) = \mathbf{x}\boldsymbol{\beta} + \gamma z$ with $var(y|\mathbf{x}, z) = \sigma^2$; where y and z are scalars, $\mathbf{x}$ is $1 \times K$ and $\boldsymbol{\beta}$ is $K \times 1$. Let $E(\mathbf{x}'z) = \mathbf{0}$. Consider two estimators of $\boldsymbol{\beta}$ based on N independently and identically distributed sample observations: (i) $\hat{\boldsymbol{\beta}}$ from regression of y on $\mathbf{x}$ and z, and (ii) $\tilde{\boldsymbol{\beta}}$ from regression of y on x alone. Show that $Avar\sqrt{N}(\tilde{\boldsymbol{\beta}} - \boldsymbol{\beta}) - Avar\sqrt{N}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})$ is always positive semidefinite.

6. Discuss Hausman Specification Test for random effects in a model with panel data.

7. Consider the contribution made by observation i in the loglikelihood function of the binary choice model: $y_i \log F(\mathbf{x}\boldsymbol{\beta}) + (1 - y_i)\log(1 - F(\mathbf{x}\boldsymbol{\beta}))$. Show that this contribution is globally concave with respect to $\boldsymbol{\beta}$ if the function $F(.)$, its derivative $f(.)$ and its second derivative $f'(.)$ satisfy the following conditions: $f'(w).F(w) - f^2(w) < 0$ and $F(-w) = 1 - F(w)$.

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8. Consider the model $y_1 = \mathbf{z}_1\boldsymbol{\delta}_1 + \alpha_1 y_2 + u$, where y_2 is endogenous. Consider the reduced form for y_2: $y_2 = \mathbf{z}_2\boldsymbol{\pi}_2 + v_2$, where $\mathbf{z}_2$ has at least one exogenous element more than $\mathbf{z}_1$. Estimate the reduced form by OLS and save the residuals in $\hat{v}_2$. Estimate the following by OLS: $y_1 = \mathbf{z}_1\boldsymbol{\delta}_1 + \alpha_1 y_2 + \rho_1\hat{v}_2 + \text{error}$. Show that OLS estimates of $\boldsymbol{\delta}_1$ and α_1 from this equation are identical to the 2SLS estimates of the same parameters.