MASTER OF ARTS EXAMINATION, 2018

(1st Year, 1st Semester)

ECONOMICS

MATHEMATICAL ECONOMICS

Time : Two hours

Full Marks : 30

Use separate Answer Script for each group

Group A

1. Consider the case of a firm in which the stock of capital, x_t, changes continuously over time according to the following equation of motion:

$$\dot{x}_t = ax_t - bx_t^2 - z_t$$

where a>0 and b>0 are positive constant and z_t is the rate of production. Firm's profit depends on the production at the rate $\ln(z_t)$, and the goal of the firm is to maximize the discounted present value of profit over an infinite horizon, the rate of discount being r.

A formal statement of the firm's problem, therefore is:

$$\max \int_0^\infty e^{-rt} \ln(z_t)\, dt$$

subject to $\dot{x}_t = ax_t - bx_t^2 - z_t$ and $x_t > 0$

a) Set up the current value Hamiltonian.

b) What is the transversality condition? Interpret it.

c) Write down the costate equation and interpret it.

d) Explore the dynamics using phase diagram.   1+2+2+3

2. Solve the following dynamic programming problem using Benveniste Scheinkman method.

$$\text{Max } \sum_0^\infty \beta^t \ln(c_t)$$

Subject to $a_{t+1} = (1 + r)a_t - c_t$ and $a_0 = \overline{a_0}$

[Turn over

Group B

Answer any 3 questions. Each question carries 5 marks.

1) Show that there always exists a rational number between 2 rational numbers. Use the proved proposition to explain that there always exists an infinite number of rational numbers between 2 rational numbers.

$$3 + 2 = 5$$

2) Define a 'sequence' and explain the concept of a 'convergent sequence'. Give an example of a convergent sequence and demonstrate that the sequence provided by you actually converges.

$$1+1+1+2=5$$

3) Prove rigorously that the set of cubes of natural numbers is unbounded above.

$$5$$

4) Prove rigorously that the sum of absolute values of 3 real numbers is never less than the absolute value of their sum.

$$5$$