(Ref.: EX/PG/ECO/14/6/2018)

MA 1ST YEAR 1ST SEM 2018

Econometrics I

Time: 2 Hours — Full Marks: 30

Answer any five of the following questions. $6 \times 5 = 30$

1. Consider the linear regression model $y_i = \mathbf{x}_i\boldsymbol{\beta} + u_i$ where $\mathbf{x}_i$ is non-stochastic and $E(u_iu_j) = \sigma^2$ if $i = j$, $E(u_iu_j) = \rho\sigma^2$ if $|i-j| = 1$ and $E(u_iu_j) = 0$ if $|i-j| > 1$.
   (a) What is a consistent estimator of $var(\hat{\boldsymbol{\beta}})$?
   (b) Is White's heteroskedasticity consistent robust estimate of $var(\hat{\boldsymbol{\beta}})$ consistent here?

2. Consider the three equation model: $y = \beta x + u$, $x = \lambda u + \varepsilon$ and $z = \gamma\varepsilon + v$, where the mutually independent errors are normally distributed with zero mean and variances σ_u^2, σ_ε^2 and σ_v^2, respectively.
   (a) Is $\hat{\beta}_{OLS}$ consistent? What is the asymptotic bias in $\hat{\beta}_{OLS}$ if any?
   (b) Is $\hat{\beta}_{IV}$ consistent if z is used as an instrument for x?

3. Suppose y has the pdf $f(y|\mathbf{x},\boldsymbol{\beta}) = \left(\dfrac{1}{\mathbf{x}\boldsymbol{\beta}}\right) e^{-\dfrac{y}{\mathbf{x}\boldsymbol{\beta}}}$, $y > 0$.
   (a) What are $E(y|\mathbf{x})$ and $var(y|\mathbf{x})$?
   (b) Prove that for this model the GLS and ML estimates of $\boldsymbol{\beta}$ are same.

4. For the simple regression model $y_i = \mu + \varepsilon_i$, $\mu \overset{\text{i.i.d.}}{\sim} (0, \sigma^2)$, consider the estimator $\hat{\mu} = \sum_{i=1}^{N} w_i y_i$, $w_i = \dfrac{i}{N(N+1)/2}$. Prove that this is a consistent but inefficient estimator of μ.

5. (a) Consider the model $y = \alpha \exp(\mathbf{x}'\boldsymbol{\beta}) + u$, where $\text{E}(u|\mathbf{x}) = 0$. Under what conditions are the parameters of the model identified?
   (b) Suppose conditions in (a) are satisfied. Suggest two sets of $K \times 1$ moment conditions for the model to enable GMM estimation.

6. Let the N-vector $\mathbf{y}$ be a vector of mutually independent realizations from the uniform distribution on the interval $[\beta_1, \beta_2]$. Let $\hat{\beta}_1$ be the maximum likelihood estimator of β_1 given by $\hat{\beta}_1 = \min(y_t)$, $t = 1, \ldots, N$ and the true values of β_1 and β_2 are 0 and 1, respectively. Find the cdf of $\hat{\beta}_1$.

7. Consider the contribution made by observation i in the loglikelihood function of the binary choice model: $y_i\log F(\mathbf{x}\boldsymbol{\beta}) + (1 - y_i)\log(1 - F(\mathbf{x}\boldsymbol{\beta}))$. Show that this contribution is globally concave with respect to $\boldsymbol{\beta}$ if the function $F(.)$, its derivative $f(.)$ and its second derivative $f'(.)$ satisfy the following conditions: $f'(w).F(w) - f^2(w) < 0$ and $F(-w) = 1 - F(w)$.

8. Consider the model $y_1 = \mathbf{z}_1\boldsymbol{\delta}_1 + \alpha_1 y_2 + u$, where y_2 is endogenous. Consider the reduced form for y_2: $y_2 = \mathbf{z}_2\boldsymbol{\pi}_2 + v_2$, where $\mathbf{z}_2$ has at least one exogenous element more than $\mathbf{z}_1$. Estimate the reduced form by OLS and save the residuals in $\hat{v}_2$. Estimate the following by OLS: $y_1 = \mathbf{z}_1\boldsymbol{\delta}_1 + \alpha_1 y_2 + \rho_1\hat{v}_2 + \text{error}$. Show that OLS estimates of $\boldsymbol{\delta}_1$ and α_1 from this equation are identical to the 2SLS estimates of the same parameters.