M.E. (Water Resources & Hydraulic Engineering) Examination, 2019

(2nd Semester)

COMPUTATIONAL HYDRO DYNAMICS

(Paper- VII)

Time : Three Hours | Full Marks : 100

Answer any ***four*** questions.

1. a) Find a root of the equation $x^3 - 4x - 9 = 0$, using the bisection method in four stages.

   b) Find root of the equation $2x - \log_{10} x = 7$, which lies between 3.5 and 4 by regula-falsi method.

   c) Evaluate $\sqrt{65}$ to four decimal places by Newton's iterative method.

   d) Apply Gauss elimination method to solve the equations $10x + 2y + z = 9$; $2x + 20y - 2z = -44$; $-2x + 3y + 10z = 22$.

   6+6+6+7 = 25

2. a) State different advantages and disadvantages of experimental, theoretical and computational approaches for solving non-linear problems.

   b) Give examples of elliptic and parabolic partial differential equations as found in fluid dynamics. Briefly explain the characteristics of such equations.

   c) Consider the function $f(x,y) = e^{4x} + e^{-2y}$. Consider the point (x,y) = 1,2. Assume $\Delta x = \Delta y = 1$.

      (i) Calculate the exact values of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at this point.

      (ii) Calculate the same using 1st order accurate forward and backward differences. Also calculate the percentage difference compared with the exact values.

      (iii) Calculate the same using 2nd order accurate central differences. Also calculate the percentage difference compared with the exact values.

   9+4+(3+6+3) = 25

3. a) Distinguish between Discretisation Error, Round-off Error.

   b) What do you mean by explicit approach of CHD Technique? Give examples.

   c) Discretise the vorticity – transport equation for two-dimensional fluid flow using forward difference for the temporal variation and central difference for the spatial variation. Hence find out the stability criteria for the finite difference scheme.

   4+3+18 = 25

4. a) Find out the numerical solution for the unsteady two-dimensional inviscid flow by MacCormack's technique. Use Euler equations in non-conservation form. Assume that there is no body force and no volumetric heat addition.

   b) Write short note on Reflection boundary condition.

   c) Obtain 1st order and 2nd order accurate finite difference expressions for the terms $\frac{\partial^2 u}{\partial x \partial y}$ at the backward boundaries of the flow domain.

13+4+8 = 25

5. a) By Taylor series expansion show how can you obtain 1st order and 2nd order accurate finite difference expressions for the terms $\frac{\partial v}{\partial y}$ and $\frac{\partial^2 u}{\partial z^2}$ at the forward and backward boundaries of the flow domain.

   b) Consider the viscous flow of water over a river bed. At a given station in the flow direction, the variation of the flow velocity (u) in the direction perpendicular to the bed (the z direction) is given by the expression $u = 100(2p^3 - p^2 + 0.5p)$ where $p = z/L$ and L = total depth of water = 10 cm, velocity along the flow (u) is in meter per second. The dynamic coefficient of viscosity of water at 20^{0}C (μ) = 0.001002 kg/(m·s). Values of u were measured at discrete grid points with Δz = 5 mm. Calculate the shear stress (τ_o) at the wall using the following ways: (a) Using 1st order accurate forward difference, (b) Using the 2nd order accurate forward/backward difference and using the 3rd order accurate forward difference using Polynomial approach. Finally, compare these calculated finite-difference results with the exact value of τ_o.

12+13 = 25

6. a) Explain briefly the effectiveness of Lax-Wendroff technique.

   b) Define consistency with suitable example.

   c) Write a short note on Dirichlet and Neumann boundary conditions.

18+3+4 = 25