Ref. No.: EX/PHY/TG/112/20/2019

M. Sc. Physics (Day) Examination 2019

(2nd Year, 1st Semester)

PHYSICS

SOLID STATE PHYSICS AND X-RAY

PHY/TG/112

Time: Two hours **Full marks: 40**

Answer any FOUR questions.

1. (a) Considering the periodicity requirement, show that the crystals can posses only certain rotational symmetry. [4]

(b) Explain using a clean diagram as to why base centered cubic lattice does not exist. [3]

(c) Prove Braggs law using Ewalds construction. [3]

2. (a) Mention the fundamental symmetry elements which lead to the point group symmetry of the crystals? [2]

(b) What are the proper and improper axes? [1.5]

(c) Draw the stereographic projections of the following point groups (i) $6/m$, (ii) $\acute{6}$, (iii) $\acute{3}$. [4.5]

(d) If in a crystal, a mirror plane is combined perpendicularly to a rotation axis, which combinations will you obtain? Also prove any ONE equivalence that exists. [2]

3. (a) If $\vec{a}_1$, $\vec{a}_2$ and $\vec{a}_3$, be the primitive vectors of a Bravais lattice, define the primitive vectors, $\vec{b}_1$, $\vec{b}_2$ and $\vec{b}_3$, of the corresponding reciprocal lattice.

(b) If $\vec{R}$ and $\vec{G}$ be the Bravais vectors in a direct and the corresponding reciprocal lattices, respectively, show that $e^{i\vec{R}\cdot\vec{G}} = 1$.

(c) Let $\vec{a}_1 = \frac{a}{2}(\hat{j}+\hat{k})$, $\vec{a}_2 = \frac{a}{2}(\hat{k}+\hat{i})$ and $\vec{a}_3 = \frac{a}{2}(\hat{i}+\hat{j})$ be the primitive vectors of a FCC lattice. find the primitive vectors. $\vec{b}_1$, $\vec{b}_2$ and $\vec{b}_3$, of the corresponding reciprocal lattice.

(d) Show that the inter-planar separation of the lattice planes (hkl), satisfies the relation

$$d_{hkl} = \frac{2\pi}{G_{hkl}},$$

where G_{hkl} is the magnitude of shortest reciprocal Bravais vector normal to the lattice plane (hkl), *i. e.*, $\vec{G}_{hkl} = h\,\vec{b}_1 + k\,\vec{b}_2 + l\,\vec{b}_3$.

(e) If $\vec{b}_1 = \frac{2\pi}{a}\hat{i}$, $\vec{b}_2 = \frac{2\pi}{a}\hat{j}$ and $\vec{b}_3 = \frac{2\pi}{a}\hat{k}$, for the simple cubic crystal system, show that $G_{hkl} = \frac{2\pi}{a}\sqrt{h^2+k^2+l^2}$.

[1+1+2+4+2=10]

4. (a) Considering the spin-$\frac{1}{2}$ degrees of freedom, the single-electron Hamiltonian can be written as

$$H = -\frac{\hbar^2}{2m}\left(\vec{\sigma}\cdot\vec{\nabla}\right)^2, \text{ where } \sigma^\alpha, (\alpha = x, y, z) \text{ are the Pauli matrices.}$$

Show that in presence of a magnetic field $\vec{B}\,(=\vec{\nabla}\times\vec{A})$ this Hamiltonian can be expressed as

$$H = \frac{1}{2m}\left(-i\,\hbar\vec{\nabla} + e\,\vec{A}\right)^2 - \vec{\mu}\cdot\vec{B}, \quad \text{where } \vec{\mu} = -\frac{e\,\hbar\,\vec{\sigma}}{2m}.$$

(b) Consider a two dimensional ($L_x \times L_y$) non-interacting electron system in the presence of a magnetic field along the z-direction ($\vec{B} = B\,\hat{k}$). Write down the Hamiltonian of the moving electron. Obtain the eigenvalues and the degeneracy (D) of the eigenstates (Landau levels) by solving the Schrödinger's equation. [3+(2+4+1)=10]

5. (a) Let the energy eigen states of a three-dimensional non-interacting electron system confined within a rectangular parallelopiped having volume, $V = L_xL_yL_z$, be $\psi_{\vec{k}}(\vec{r}) = \frac{1}{\sqrt{V}}\,e^{i\vec{k}\cdot\vec{r}}$, where $\vec{k} = \frac{2\pi n_x}{L_x}\hat{i} + \frac{2\pi n_y}{L_y}\hat{j} + \frac{2\pi n_z}{L_z}\hat{k}$ and n_x, n_y, n_z are integers. By considering the electrons obey the Fermi-Dirac distribution function, derive the expression of density of states, $g(E)$, for this electron system.

(b) By using the Sommerfeld expansion,

$$\int_0^\infty f(E)\,\frac{\partial F}{\partial E}\,dE = F(E_F) + \frac{\pi^2}{6}\,(k_BT)^2\left(\frac{\partial^2 F}{\partial E^2}\right)_{E_F},$$

show that the energy of Fermi level at room temperature T can be expressed as

$$E_F(T) \approx E_F(0)\left[1 - \frac{\pi^2}{12}\left(\frac{T}{T_F}\right)^2\right].$$

Symbols have their usual meaning.

(c) Now derive the expression of specific heat at low temperatures, $C_V(T)$ for the three-dimensional free electron system. And show that $C_V(T) \propto T$ at low temperatures.

[2+4+4=10]

6. (a) Derive the expression of energy dispersion relation for the three-dimensional crystal in tight-binding approximation:

$$E(\vec{k}) = E_A - \alpha - 4\gamma\sum_m e^{i\vec{k}\cdot(\vec{R}_j - \vec{R}_m)},$$

where $\vec{R}_m$ are the nearest neighbours of $\vec{R}_j$. Other symbols have their usual meaning.

(b) Derive the following expressions of dispersion relations in tight-binding approximation for the FCC crystal, *i. e.*,

$$E_k = E_A - \alpha - 4\gamma\left[\cos\frac{k_xa}{2}\cos\frac{k_ya}{2} + \cos\frac{k_ya}{2}\cos\frac{k_za}{2} + \cos\frac{k_za}{2}\cos\frac{k_xa}{2}\right].$$

(c) In the phenomenological description of superconductivity, derive the first and the second London's equations. [5+3+2=10]