MASTER OF SCIENCE EXAMINATION, 2019.
(2nd Year, 1st Semester)

Subject: PHYSICS
Paper: QUANTUM FIELD THEORY
(Ref. No. *Ex/PHY/TE/203/20/2019*)
PHY/TE/203
Use separate answer script for each group.
Time: Two Hours
Full Marks: 40

GROUP - A

Answer any TWO questions.

1. (a) Consider the field operator given by the expansion $\hat{\Psi}(\vec{r},t) = \sum_n \hat{f}_n(t)\psi_n((\vec{r})$. Find the equation of motion obeyed by the coefficients $\hat{f}_n(t)$ when they are fermionic.

   (b) Show that the fermionic character of these operators leads to the exclusion principle.
   Marks: 5 + 5 = 10

2. Evaluate the commutator $[\hat{N}, \hat{H}]$ where $\hat{N}$ and $\hat{H}$ represent the field number operator and the field Hamiltionian in position space.
   Marks: 10

3. Starting from the second quantized form of a two-body interaction potential $V(x - x')$ in position space, find the corresponding form in momentum space by effecting appropriate Fourier transforms.
   Marks: 10

GROUP - B

Answer any one question from serial no. 1 to 2 and any two questions from serial no. 3 to 5.

$10 + 2 \times 5 = 20$

1. a) Write down the expression for Green's function of Klein-Gordon operator in space-time coordinates. b) Transform it in 4-momentum space using Fourier transformation. c) Then describe it following the Feynmann prescription. d) Explain using time ordering operator T that the Green's function represents the amplitude of propagation of virtual particles. Distinguish beteen virtual and physical particles. e) Prove that $E_{\vec{p}}\delta^{(3)}(\vec{p} - \vec{q})$ is Lorentz **invariant.**

2. a) Write down the free Hamiltonian H_0 for Klein-Gordon Field $\phi(t, \vec{x})$. b) Show that in presence of interaction Hamiltonian H_{int}, the field $\phi(t, \vec{x})$ in Heisenberg picture can be expressed as $U^\dagger(t, t_0)\phi(t, \vec{x})_I U(t, t_0)$, where $\phi(t, \vec{x})_I = \phi(t, \vec{x})$ for $H_{\text{int}} = 0$. c) Find out the time evolution equation for $U(t, t_0)$. d) Then solve it to find a general expression of $U(t, t_0)$ for any interaction Hamiltonian H_{int}.

3. State and prove Wick's theorem. Find the Feynman diagram for $\langle 0| T\{\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\} |0\rangle$.

4. State and prove Noether's theorem. Find out the energy-momentum tensor of the scalar field $\phi(x)$.

5. i) Express Klein-Gordon Field as independent harmonic oscillators with its own creation and annihilation operators.
   ii) Find out all the commutation relations among the creation and annihilation operators $a^\dagger$ and a from the second quantization relations of the field.